Calculus IV Homework 1

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due Friday, August 25, 2023

1. Set up each of the following integrals, using cylindrical coordinates. You don't have to evaluate the integrals.

   (a) The integral for the volume of a solid ssphere of radius 1 centered at the origin. (Yes, it would make more sense to use spherical coordinates. I'm being unreasonable.)

   (b) The integral of $x + y + z$ over the interior of a cone whose base is a disk of radius 1 in the xy-plane centered at the origin, and whose tip is the point $(0, 0, 2)$.

   (c) The integral of $x^2+y^2+z^2$ over the half-cylinder bounded by the inequalities $x^2+y^2 \leq 1$, $0 \leq z \leq 3$, and $y \geq 0$.

2. Set up each of the following integrals, using spherical coordinates. You don't have to evaluate the integrals.

   (a) The integral for the volume of the portion of the solid sphere of radius 2 centered at the origin with $x \geq 0$, $y \geq 0$, and $z \geq 0$.

   (b) The integral of $x^2 + y^2$ over a spherical shell of thickness 1 and inner radius 3 centered at the origin.

   (c) The integral for the volume of a sphere of radius $\frac{1}{2}$ centered at $(0, 0, \frac{1}{2})$. (Yes, it would make more sense to center the sphere at the origin. I'm being unreasonable again.)

3. Perform each of the substitutions given below. You don't have to evaluate the resulting integrals. For the sake of variety, use the Jacobian at least once, and use the $\mathrm{d}x \wedge \mathrm{d}y$ method at least once.

   (a) Rewrite the integral

   $$\int_{x=2}^{5} \int_{y=-x}^{2-x} \sqrt{x^2+1}\, \mathrm{d}y\, \mathrm{d}x$$

   using the substitution $u = x^2 + 1$, $v = x + y$.

   (b) Rewrite the sum of two integrals

   $$\int_{x=-1}^{0} \int_{y=-x-1}^{x+1} (x^2 + xy)\, \mathrm{d}y\, \mathrm{d}x + \int_{x=0}^{1} \int_{y=x-1}^{1-x} (x^2 + xy)\, \mathrm{d}y\, \mathrm{d}x$$

   as a single integral using the substitution $u = x + y$, $v = x - y$.

(c) Rewrite the integral

$$\iint_R \mathrm{d}y\,\mathrm{d}x$$

where R is the region bounded by $1 \leq xy \leq 4$ and $\frac{1}{2} \leq \frac{x}{y} \leq 2$, using the substitution $u = xy$, $v = \frac{x}{y}$.

4. Find the centroid of the solid hemisphere which is the portion of the sphere of radius 1 centered at $(0, 0, 0)$ that lies above the plane $z = 0$.

5. Compute the integral

$$\int_{x=1}^{7} \int_{y=x-1}^{x+1} \int_{z=x}^{3x} \frac{y-1}{2x}\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x$$

by first performing the substitution $u = \frac{x-1}{2}$, $v = x - y$, $w = \frac{z}{x}$.