Calculus IV Homework 3

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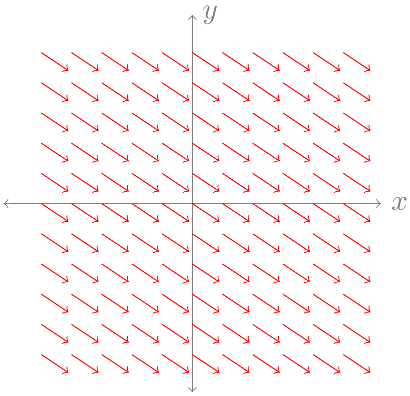
due Friday, September 22, 2023

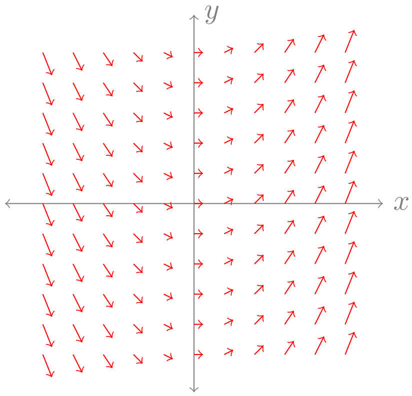
1. Each of the diagrams below is the graph of one of the vector fields

$$\mathbf{F}_1 = \mathbf{i} + x\,\mathbf{j}$$
$$\mathbf{F}_2 = 3\,\mathbf{i} - 2\,\mathbf{j}$$
$$\mathbf{F}_3 = (x-1)^2\,\mathbf{i} + (y-1)^2\,\mathbf{j}$$
$$\mathbf{F}_4 = y\,\mathbf{i}$$
$$\mathbf{F}_5 = y\,\mathbf{i} - 2x\,\mathbf{j}$$

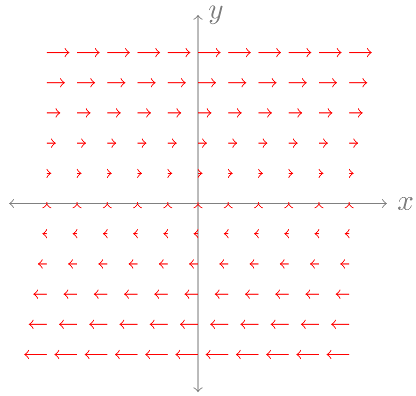
For each diagram, give the vector field (one of $\mathbf{F}_1, \ldots, \mathbf{F}_5$) that it is the graph of.



(a)

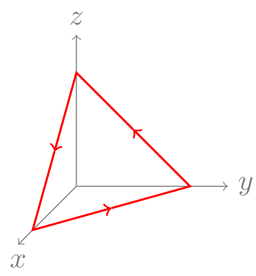


(b)



(c)

2. Let $\mathbf{F} = z\,\mathbf{i} + x\,\mathbf{j} + y\,\mathbf{k}$, and let C be the triangular path from $(1,0,0)$ to $(0,1,0)$ to $(0,0,1)$ back to $(1,0,0)$ shown in the diagram below.



Find the circulation of $\mathbf{F}$ around C.

3. Determine whether these vector fields are gradient fields, and if they are, find a potential function for them.

   (a) $\mathbf{F} = 4x^2y\,\mathbf{i} + \frac{4}{3}(x^3 - y^3)\,\mathbf{j}$.

   (b) $\mathbf{G} = z\cos(y+z)\,\mathbf{i} - xz\sin(y+z)\,\mathbf{j} - xz\sin(y+z)\,\mathbf{k}$.

   (c) $\mathbf{H} = (x+y)\,\mathbf{i} + (x-z)\,\mathbf{j} - (y+z)\,\mathbf{k}$.

4. Let $\mathbf{F} = \mathbf{i} + x^2\,\mathbf{j}$, and let C be the *boundary* of the region $\{(x,y) : x^2 + y^2 \leq 1 \text{ and } x \geq 0\}$: the right half of the unit disk. Find the outward flux of $\mathbf{F}$ across C.

5. (a) Let $f(x,y,z) = xe^{y-z}$. Compute the gradient field $\mathbf{F} = \boldsymbol{\nabla} f$.

   (b) Let C be the coiled spring parameterized by $\mathbf{r}(t) = (\cos t, \sin t, t/\pi)$, where $t \in [0, 4\pi]$. Compute the vector line integral $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$.