Calculus IV Homework 4

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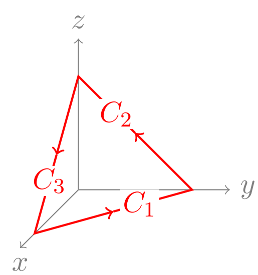
due Friday, October 6, 2023

1. Find the circulation density and the flux density of each of the following vector fields:
   (a) $\mathbf{F} = e^{x^2}\,\mathbf{i} - e^{y^2}\,\mathbf{j}$.
   (b) $\mathbf{F} = \frac{\mathbf{i}-\mathbf{j}}{x+y}$.
   (c) $\mathbf{F} = (x^3+y^3)\,\mathbf{i} - (x^2y+xy^2)\,\mathbf{j}$.

2. Let C be the triangular path from $(1,0,0)$ to $(0,1,0)$ to $(0,0,1)$ back to $(1,0,0)$ shown in the diagram below. Let C_1, C_2, and C_3 be the three individual segments making up C.



Suppose that $\mathbf{F}$ is a conservative vector field, and that you've already computed

$$\int_{C_1} \mathbf{F}\cdot d\mathbf{r} = \frac{1}{2}, \qquad \int_{C_2} \mathbf{F}\cdot d\mathbf{r} = \frac{1}{2}.$$

What must the value of $\displaystyle\int_{C_3} \mathbf{F}\cdot d\mathbf{r}$ be?

3. For each part of this problem, come up with an example of a 2-dimensional vector field with the required properties.
   (a) A nonzero vector field for which both the circulation density and the flux density are 0.
   (b) A vector field for which the circulation density and flux density are both a positive constant at every point.
   (c) A conservative vector field for which the flux density at (x,y) is proportional to x^2+y^2.

4. Use Green's theorem to compute the counterclockwise circulation of $\mathbf{F} = e^{x^2}\,\mathbf{i} + \frac{4}{3}x^3y^2\,\mathbf{j}$ around the boundary of the square with vertices at $(\pm1,\pm1)$.

5. Let C be the curve made up of two segments with the parameterizations

$$\mathbf{r}_1(t) = (t, -2) \qquad t \in [-2, 2]$$
$$\mathbf{r}_2(t) = (-t, 2 - t^2) \qquad t \in [-2, 2]$$

and let $\mathbf{F} = x\,\mathbf{i} + xy\,\mathbf{j}$. Use Green's theorem to calculate the outward flux of $\mathbf{F}$ across C.