Graph Theory Homework 4

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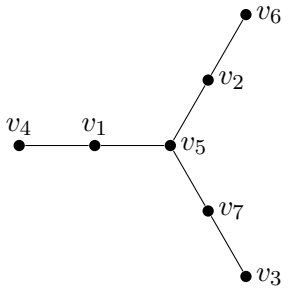
due Friday, October 6, 2023

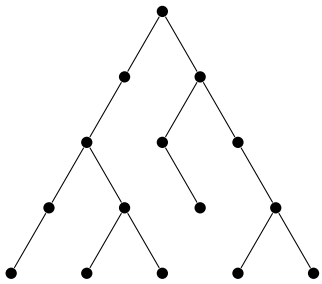
1 Short answer

1. (a) Find the tree with Prüfer code 11611.

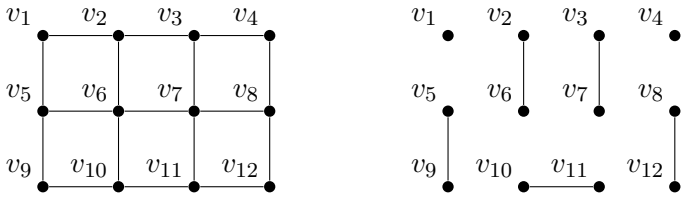
   (b) Find the Prüfer code of the tree below:



2. In the graph below, find a matching M and a vertex cover U with $|M| = |U|$.



3. In the graph shown on the left below, find an augmenting path for the matching $M = \{v_2v_6, v_3v_7, v_5v_9, v_8v_{12}, v_{10}v_{11}\}$ (shown on the right). Then, augment M by that path to get a bigger matching.



2 Proof

4. Let G be a connected graph with n vertices and exactly one cycle. (That is, exactly one cycle if we don't count starting at a different vertex or going in a different direction.) Prove that G has n edges.

   You have already written a rough draft of the solution; now, write a final draft.

5. Using Prüfer codes or in some other way, determine the number of trees with vertex set $\{v_1, v_2, \ldots, v_n\}$ which have exactly $n - 2$ leaves.

   Write a rough draft of the solution. I will give you feedback, and you will write a final draft of your proof as part of Homework 5.