Graph Theory Homework 5

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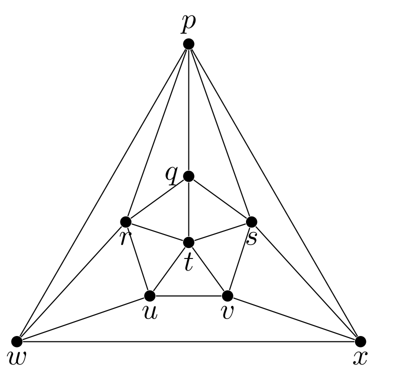
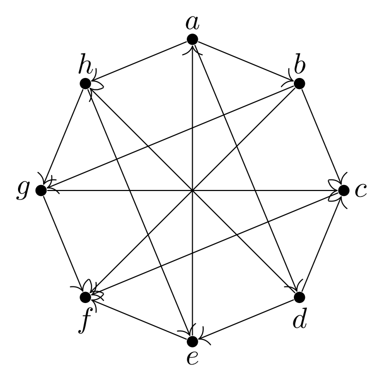
due Friday, October 20, 2023

1 Short answer

Important note: *even after we cover multigraphs and directed graphs in this class, it is still the case that when I say "graph", I mean an undirected graph with no loops or parallel edges. If I want to ask about directed graphs or multigraphs, I will specify.*

1. Let J_{10} be the graph below on the left. *(Irrelevant trivia: this is the skeleton graph of the* 10^{th} *Johnson solid, the gyroelongated square pyramid.)* Find a vertex v in J_{10} such that deleting it produces an 8-vertex *Eulerian* graph.



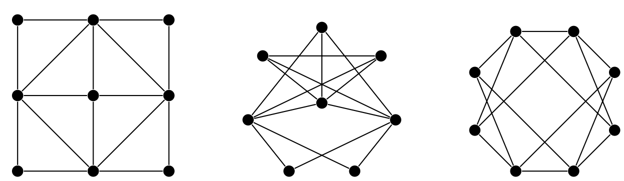


2. In the directed graph shown above on the right, find an arc (u, v) such that if it is reversed (deleted and replaced by (v, u)), the result is acyclic. Give a topological ordering of the resulting directed graph after (u, v) is reversed.

   If the picture is unclear, the set of arcs is $\{(a, b), (a, d), (a, h), (b, c), (b, f), (b, g), (c, f), (d, c), (d, e), (d, h), (e, a), (e, f), (g, c), (g, f), (h, e), (h, g)\}$.

   (Hint: if you're not sure how to get started, try looking for cycles, or try finding the strongly connected components.)

3. In each of the graphs below, either find a Hamiltonian cycle, or say why it is not Hamiltonian.



2 Proof

4. Using Prüfer codes or in some other way, determine the number of trees with vertex set $\{v_1, v_2, \dots, v_n\}$ which have exactly $n - 2$ leaves.

   You have already written a rough draft of the solution; now, write a final draft.

5. Let G be a bipartite graph, with bipartition (A, B), that has the following properties:

   - Every vertex on side A has degree 3 or 5;
   - Every vertex on side B has degree 2 or 4;
   - There are no edges between vertices of degree 3 and vertices of degree 4.

   Prove that G has a matching that covers all vertices in A.

   Write a rough draft of the solution. I will give you feedback, and you will write a final draft of your proof as part of Homework 6.