

Graph Theory Homework 6

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due Friday, November 3, 2023

1 Short answer

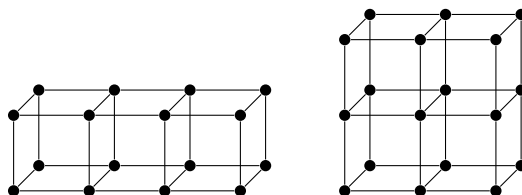
1. The webpage <https://umop.com/rps7.htm> gives an example of a 7-vertex tournament.

Is this tournament strongly connected? If so, find a cycle of each length between 3 and 7 in it. If not, explain why not.

2. A connected planar graph G has an embedding in which there are 2 triangular faces, 4 square faces, and 2 faces of length k .

Is it possible to determine the number of vertices, edges, and faces of G ? If it is, do so. If not, explain why not.

3. For each graph below, determine whether it is planar:



2 Proof

4. Let G be a bipartite graph, with bipartition (A, B) , that has the following properties:

- Every vertex on side A has degree 3 or 5;
- Every vertex on side B has degree 2 or 4;
- There are no edges between vertices of degree 3 and vertices of degree 4.

Prove that G has a matching that covers all vertices in A .

You have already written a rough draft of the solution; now, write a final draft.

5. Prove that if two vertices in a tournament have the same degree, then there is a cycle that contains both of them.

Write a rough draft of the solution. I will give you feedback, and you will write a final draft of your proof as part of Homework 7.