# Graph Theory Homework 7

Mikhail Lavrov

due Friday, November 17, 2023

## 1 Short answer

1. Let $G$ be the interval graph whose vertices are the intervals

$$[1,6],\ [2,4],\ [3,14],\ [5,13],\ [7,8],\ [9,11],\ [10,12]$$

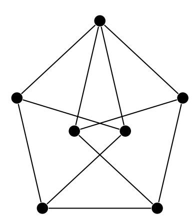
with an edge between two vertices whenever they overlap.

   (a) Draw a diagram of $G$.

   (b) What is the clique number of $G$? Find a clique of that size.

   (c) What is the independence number of $G$? Find an independent set of that size.

2. The graph below is known as "Moser's spindle". It's notable for being a **unit distance graph**: it has a drawing (such as the one below) where two vertices are adjacent whenever the distance between them is 1.



   (a) Find a 4-coloring of Moser's spindle.

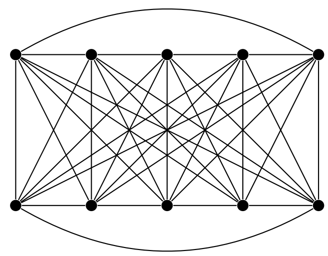
   (b) Convince yourself (no need to write down the convincing) that the clique number of Moser's spindle is 3. What does this tell us about the chromatic number?

   (c) Convince yourself (no need to write down the convincing) that the independence number of Moser's spindle is 2. What does this tell us about the chromatic number?

*Some cultural background: the "chromatic number of the plane" is the least number of colors necessary to color* all *the points in* $\mathbb{R}^2$ *so that no two points at distance* 1 *have the same color.*

*Moser's spindle is interesting because its chromatic number is a lower bound on the chromatic number of the plane. (In order to properly color the plane, you have to properly color every Moser's spindle inside it.) Until 2018, this was the best lower bound known!*

3. Let $G$ be the graph obtained by taking two copies of the 5-cycle and joining them with all possible edges:



   (a) Find the clique number of $G$.

   (b) Find the chromatic number of $G$, and a coloring of $G$ with that number of colors.

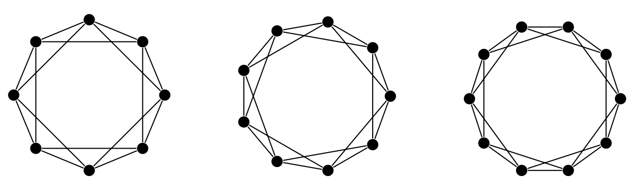
   *(Hint: the chromatic number of the* 5*-cycle is* 3*.)*

# 2 Proof

4. Prove that if two vertices in a tournament have the same degree, then there is a cycle that contains both of them.

   *You have already written a rough draft of the solution; now, write a final draft.*

5. The Harary graph $H_{n,4}$, as defined in Lecture 7, has $n$ vertices that can be drawn in a circle so that vertices are adjacent exactly when they are one or two steps apart around the circle. The diagram below shows $H_{8,4}$, $H_{9,4}$, and $H_{10,4}$, as illustration.



   For all $n \geq 6$, the chromatic number of $H_{n,4}$ is either 3 or 4. Prove this, and determine which values of $n$ give which chromatic number.

   *Write a rough draft of the solution. I will give you feedback, and you will write a final draft of your proof as part of Homework 8.*