Graph Theory Homework 8

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due Monday, December 4, 2023
(Note the unusual due date: Monday rather than Friday!)

0 Course evaluations

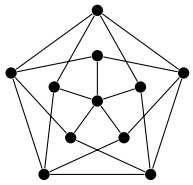
Course evaluations for this class can now be filled out!! There should be a link, I believe, from the D2L home page. (Let me know if you have trouble finding them.)

The *last* day that you can fill out your course evaluations is Monday, December 4^{th}.

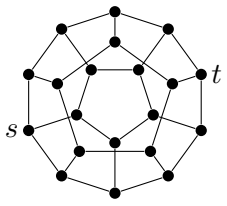
If more than 50% of the class fills out the course evaluation, then I will wear a funny hat for the final exam.

1 Short answer

1. Give an example of:
   (a) A graph with 8 vertices, 6 of which are cut vertices.
   (b) A graph with 9 vertices, exactly one cut vertex, and minimum degree 4.
2. Find an ear decomposition of the Grötzsch graph, shown below:



3. Let G be the dodecahedron graph, shown below; let s and t be two opposite vertices of the dodecahedron.



   Determine $\kappa(s, t)$; explain why the value cannot be larger or smaller.

4. Answer briefly (one sentence should be enough for each part). Feel free to cite theorems from class if they are relevant.

   (a) In a tree, which vertices are cut vertices?

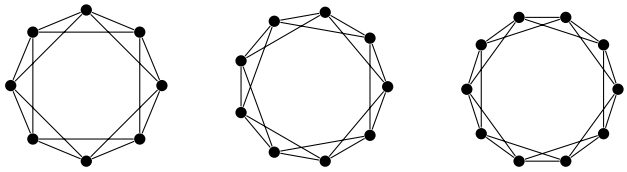
   (b) Why are all Hamiltonian graphs 2-connected?

   (c) What is the clique number $\omega(K_{m,n})$ of the complete bipartite graph $K_{m,n}$?

   (d) If H is a subgraph of G, what can we say about their independence numbers: is $\alpha(G) \leq \alpha(H)$, or is $\alpha(H) \leq \alpha(G)$, or is neither one necessarily true?

2 Proof

5. The Harary graph $H_{n,4}$, as defined in Lecture 7, has n vertices that can be drawn in a circle so that vertices are adjacent exactly when they are one or two steps apart around the circle. The diagram below shows $H_{8,4}$, $H_{9,4}$, and $H_{10,4}$, as illustration.



   For all $n \geq 6$, the chromatic number of $H_{n,4}$ is either 3 or 4. Prove this, and determine which values of n give which chromatic number.

   You have already written a rough draft of the solution; now, write a final draft.