Discrete Math Homework 2

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due Friday, February 3, 2023

1   Counting problems

1. You order a burger from a fast-food restaurant. If there are 10 toppings available, how many ways are there for you to choose *at least* 2 toppings to put on your burger?

2. You got a box of chocolates for your birthday: 3 pieces of dark chocolate, 3 pieces of milk chocolate, and 3 pieces of white chocolate. The pieces of chocolate of each kind are identical. In how many distinguishable orders can you eat your chocolate?

3. You have to choose 4 out of the 7 days of the week to work on building a robot for the robotics club. You don't want to give up your entire weekend to do this, so you can choose at most one of Saturday or Sunday (not both). How many ways are there to choose 4 days?

4. How many subsets of the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$ have between 3 and 5 elements?

5. My fridge has fridge magnets on it; each magnet has a word on it, and they can be put together to form a sentence. Unfortunately, I've lost most of the magnets, and now I only have six:

   | my | | cat | | meowed | | at | | the | | dog |

   How many sentences can I make using exactly 4 of the magnets? (The sentence doesn't have to be coherent: something like | dog | | at | | meowed | | the | counts.)

6. A homework assignment has 8 problems on it and is graded out of 20 points. How many ways are there to assign point values to the problems so that each problem is worth a positive number of points (and the total number of points is 20)?

   (In reality, problems 1–6 are worth 2 points each, and problems 7–8 are worth 4 points each.)

2   Logical statements

7. Consider the statement "If either Pauli or Quinn goes to see a movie, then both of them go."

   (a) Write the statement above in terms of p, q, and logical connectives, where p is "Pauli goes to see a movie" and q is "Quinn goes to see a movie".

   (b) Write a truth table for the statement (with rows for all possible truth values of p and q).

8. Asssuming the statement "If you haven't learned the meaning of friendship, you haven't learned anything" is true, is "You have learned something" a necessary condition for "You have learned the meaning of friendship", a sufficient condition, or both? Briefly explain why.