Probability Theory Homework 1

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due Friday, January 19, 2024

1. In a children's game, you advance a marker along a track by rolling a fair 6-sided die. You roll the die once, and move forward that many spaces. If the die lands on a 6, you get to roll it again, and move forward that many spaces again. (Even if it lands on a 6 twice, you don't get to roll it a third time.)

   (a) Give the sample space of the random experiment that describes a single turn of this game, as a set of outcomes.

   (b) Are we sampling uniformly from this sample space? Why or why not?

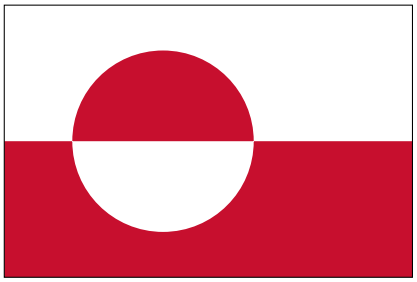
   (c) Express the event "You move forward at least 9 spaces" as a set of outcomes.

   (d) What is the probability that you move forward at least 9 spaces?

2. If a number between 1000 and 9999 is chosen uniformly at random, there is a $\frac{1}{100}$ chance that it is a palindrome (like 2772, for example) and a $\frac{1}{15}$ chance that it is divisible by 15 (like $2190 = 15 \cdot 146$, for example). Only three numbers in this range are both palindromes and divisible by 15: they are 5115, 5445, and 5775.

   What is the probability that a number between 1000 and 9999 chosen uniformly at random is either a palindrome or divisible by 15 (or both)?

3. The flag of Greenland is shown below:



   It is a 12×18 rectangle; if placed on a coordinate plane with bottom left corner at $(0, 0)$ and top right corner at $(18, 12)$, it is divided in half by a line at $y = 6$ and has a circle of radius 4 centered at $(7, 6)$.

   The top half of the flag is white and the bottom half is red; within the circle, the two colors are swapped.

Suppose that a point on this flag is chosen uniformly at random.

(a) Find the probability that the chosen point is red.

(b) Find the probability that the chosen point is red **or** inside the circle. (As usual, "or" in mathematics includes the possibility that both things happen.)

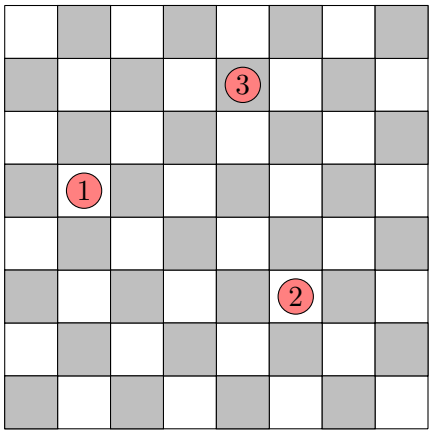
4. You have an unfair coin which lands heads $\frac{2}{3}$ of the time, and tails $\frac{1}{3}$ of the time. You decide to flip the coin over and over until you get the same outcome two times in a row.

(a) Describe the sample space of this experiment as a countably infinite set of outcomes, writing each outcome as a finite sequence like $(\mathsf{H}, \mathsf{T}, \mathsf{H}, \mathsf{T}, \mathsf{T})$. (Of course, you won't be able to write all infinitely many outcomes down, so just convey the idea of what they will be.)

(b) Find the probability that you flip the coin four or more times.

5. Three pieces are randomly placed on an 8×8 chessboard, as shown below. (The circles numbered 1, 2, and 3 are just one possible outcome of the experiment, given as an example.)

The first piece is placed on a uniformly random square; the second piece's location is chosen uniformly from the 63 squares different from the first; the third piece's location is chosen uniformly from the 62 squares different from the first and second.



(a) What is the probability that the second piece does not share a row or column with the first?

(b) What is the probability that all three pieces are in different rows and columns?