

# Probability Theory Homework 2

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1. Let's consider a simplified card game with a deck of 6 cards:

{red 2, red 3, red 4, black 2, black 3, black 4.}

Alice shuffles the deck and takes the top card, then Bob takes the next card. We define the events

$A$  = Alice's card is black,

$B$  = Bob's card is a 4,

$C$  = Alice and Bob's cards are the same color,

$D$  = Alice's card is a red 2.

Which pairs of these events:

- (a) are disjoint?
  - (b) are independent?
2. You operate a polar research station; every morning, you send a radio message indicating which way the wind is blowing. The wind is equally likely to blow in any of 4 directions, which you indicate with the codewords "000", "110", "101", and "011".

Due to the extreme weather, each bit (0 or 1) of the message independently has a  $\frac{1}{3}$  chance of being flipped in transit: 1 changed to 0 or vice versa. For example, the message "000" has a  $(\frac{1}{3})(\frac{1}{3})(\frac{2}{3}) = \frac{2}{27}$  chance of being changed to "110", flipping the first two bits.

What is the probability that when you send your message, the recipient receives "111" on their end?

3. In the board game Monopoly, if you end up in Jail, then on your turn you roll two fair 6-sided dice; if you roll doubles (the same number on both dice), you get out, and otherwise you stay in Jail and skip your turn. Usually you can also get out by paying a fine, but let's say that we are playing with house rules that prevent that.

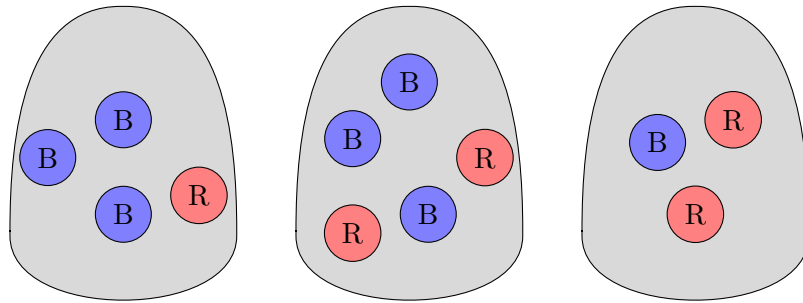
Suppose that you're playing Monopoly and it takes you more than six turns to get out of Jail. Given this information, what is the probability that it takes you exactly eight turns?

4. You're friends with two twins: Ella and Emma. They are identical in every way as far as you can tell, except one. You can tell them apart when playing rock-paper-scissors; Ella beats

you one in three times (as would be expected), but Emma somehow always guesses what you will do and beats you in every game.

One day, you run into a random twin and play rock-paper-scissors five times with her; you lose every game. What is the probability that it's Emma you encountered?

5. You have three bags of marbles, as shown below: the first has 3 blue marbles and 1 red marble, the second has 3 blue marbles and 2 red marbles, and the third has 1 blue marble and 2 red marbles.



- (a) If you pick a bag uniformly at random and draw a marble from it, what is the probability that the marble will be red?
- (b) If you pick a bag uniformly at random and draw a red marble from it, what is the probability that the bag you picked was the third bag?