Probability Theory Homework 3

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1. How many sequences of 5 letters satisfy **all three** of the following conditions?

   - All 5 letters must be distinct.

   - At least one letter must be a consonant.

   - At least one letter must be a vowel.

   (There are 21 consonants and 5 vowels in the alphabet.)

2. A bag of marbles contains 6 red marbles and 10 blue marbles.

   Suppose I tell you that I took 3 marbles out of the bag (without replacement), and they were all the same color. What is the probability that all 3 marbles were red?

3. You roll a fair six-sided die three times. What is the probability that you roll three different numbers in ascending order?

   (For example, $1, 3, 6$ counts, because $1 < 3 < 6$, but $3, 6, 1$ does not count, because 6 is not less than 1, and $2, 2, 5$ does not count, because 2 is not less than 2.)

4. You draw 4 cards from a standard 52-card deck. What is the probability that you draw one card of each suit if:

   (a) You draw a hand of 4 cards, so you are holding them all at once?

   (b) You draw 4 cards one at a time, putting each card back and shuffling before you draw the next?

   (There are 4 suits in the deck; there are 13 cards of each suit.)

5. A biased coin has a $\frac{2}{3}$ probability of landing heads and a $\frac{1}{3}$ probability of landing tails.

   You toss the coin 9 times. Which of the two options below is more likely to happen?

   - The coin lands heads only 3 times, and lands tails 6 times.

   - The coin lands heads all 9 times.