# Probability Theory Homework 7

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due Friday, April 19, 2024

1. A random variable $\mathbf{A}$ has the probability density function

$$f_{\mathbf{A}}(t) = \begin{cases} c \cdot (2-t) & 0 \le t \le 2 \\ c \cdot (t-3) & 3 \le t \le 4 \\ 0 & \text{otherwise} \end{cases}$$

   for some constant $c$.

   (a) Find the value of $c$ that makes $f_{\mathbf{A}}(t)$ a valid PDF.

   (b) Find the cumulative distribution function of $\mathbf{A}$.

   (c) Find $\Pr[1 \le \mathbf{A} \le 3]$.

2. Let $\mathbf{X} \sim \text{Exponential}(\lambda = \frac{1}{2})$, let $\mathbf{Y} \sim \text{Uniform}(a = 1, b = 3)$, and let $\mathbf{Z}$ be the random variable with PDF

$$f_{\mathbf{Z}}(t) = \begin{cases} 2/t^3 & t \ge 1, \\ 0 & t < 1. \end{cases}$$

   These have something in common: $\mathbb{E}[\mathbf{X}] = \mathbb{E}[\mathbf{Y}] = \mathbb{E}[\mathbf{Z}] = 2$. For each random variable, find the probability that it's less than this expected value.

3. Find the variance of each random variable in question 2.

4. If $\mathbf{\Theta} \sim \text{Uniform}(a = 0, b = 3)$, find the probability density function of $\sin(\mathbf{\Theta})$.

5. Let $\mathbf{W} \sim \text{Normal}(\mu = 0, \sigma^2 = 1)$ and let $Q(1) = \Pr[\mathbf{W} > 1]$. *(There is no closed-form expression for* $Q(1)$*, but it's approximately* 0.159*.)*

   Find the conditional expectation $\mathbb{E}[\mathbf{W} \mid \mathbf{W} > 1]$ in terms of $Q(1)$.