The Zariski Topology

A topology of an infinite set whose open sets have finite complements. The Zariski topology is a topology which is well-suited for the study of polynomial equations in algebraic geometry, since in Zariski topology, there are many fewer open sets than in the usual metric topology. In fact, the only closed sets are the algebraic sets, which are the zeros of polynomials.

For example, in **C**, the only nontrivial closed sets are finite collections of points. In C^2, there are also the zeros of polynomials such as lines ax + by and cusps $x^2 + y^3$

The Zariski topology is not Hausdorff. In fact, any two open sets must intersect, and cannot be disjoint. Also, the open sets are dense, in the Zariski topology as well as in the usual metric topology.

Because there are fewer open sets than in the usual topology, it is more difficult for a function to be continuous in Zariski topology. For example, a continuous function $f\colon (C^n, \mathit{Zariski}) \to (C, \mathit{usual})$ must be a constant function. Conversely, when the range has the Zariski topology, it is easier for a function to be continuous. In particular, for functions of the form $f\colon (C^n, \mathit{Zariski}) \to (C, \mathit{Zariski})$, the polynomials are continuous functions .

If X has the Zariski topology, a basis for the open sets in X is given by the sets

$$U_f = \{p \in X \mid f(p) \neq 0\}$$

where f ranges over polynomials. In other words, the closed sets can be obtained as mutual zeroes of a set of polynomial equations.

Borrowed from MathWorld, http://mathworld.wolfram.com/ZariskiTopology.html, and http://mathcircle.berkeley.edu/BMC3/alg-geom/node3.html.