MATH 1112 Practice Problems for Final Exam
July 15, 2016

1. If we punch in $\tan^{-1}(-1)$ on our calculator we get the answer $-\pi/4 \approx -0.78540$ (if the calculator is set in radians mode) or the answer -45° (if the calculator is set in degrees mode). This is not surprising because $\tan(-45^\circ) = -1$. However there are infinitely many other angles θ for which $\tan(\theta) = -1$. For example , $\tan(315^\circ) = -1$. Explain why $\tan^{-1}(-1) = -45^\circ$ and $\tan^{-1}(-1) \neq 315^\circ$.

2. **a.** Find the value of $\sin(\pi/6)$ and the value of $\sin(-11\pi/6)$. Explain why $\sin^{-1}(1/2) = \pi/6$ and $\sin^{-1}(1/2) \neq -11\pi/6$.

b. Find the value of $\cos(-120^\circ)$. Explain why $\cos^{-1}(-1/2) \neq -120^\circ$. What is the correct value of $\cos^{-1}(-1/2)$? Explain why.

3. Without using a calculator, find the value of $\cos\left(\sin^{-1}\left(-\sqrt{3}/2\right)\right)$ and the value of $\cos^{-1}(\sin(750^\circ))$.

4. **a.** Without using a calculator, explain why

$$\sin\left(\tan^{-1}\left(\frac{1}{3}\right)\right) = \frac{\sqrt{10}}{10}.$$

b. Prove that the identity

$$\sin(\tan^{-1}(a)) = \frac{a}{\sqrt{1+a^2}}.$$

is true for all real numbers a.

5. Without using a calculator, find **all** solutions, θ, of the following equations:

a. $\sin(\theta) = \sqrt{2}/2$. (Give the solutions in radians.)

b. $\cos(\theta) = -1/2$. (Give the solutions in degrees.)

c. $\tan(\theta) = \sqrt{3}$. (Give the solutions in radians.)

6. Using a calculator, find

a. all solutions of $\cos(\theta) = -0.6$ that lie in the interval $[0, 4\pi]$. (Round answers to four decimal places.)

b. all solutions of $\sin(\theta) = 0.6$ that lie in the interval $[-4\pi, 0]$. (Round answers to four decimal places.)

c. all solutions of $\tan(\theta) = -2$ that lie in the interval $[-360^\circ, 360^\circ]$. (Round answers to four decimal places.)

7. Find all solutions of the equation

$$6\cos^2(x) + 5\cos(x) + 1 = 0$$

that lie in the interval $[0, 2\pi]$.

8. Find all solutions of the equation

$$\sin(2x)\sin(x) - \cos(2x)\cos(x) = -\cos(x)$$

that lie in the interval $[0, 2\pi]$.

9. Find all solutions of the equation

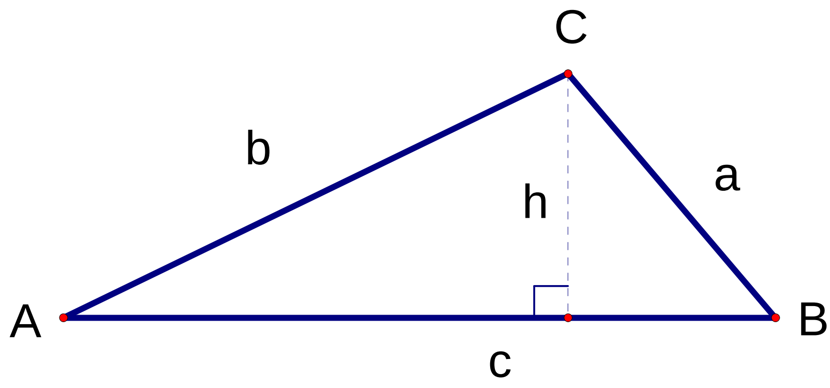
$$\sin(2x) + \sin(x) + 2\cos(x) + 1 = 0$$

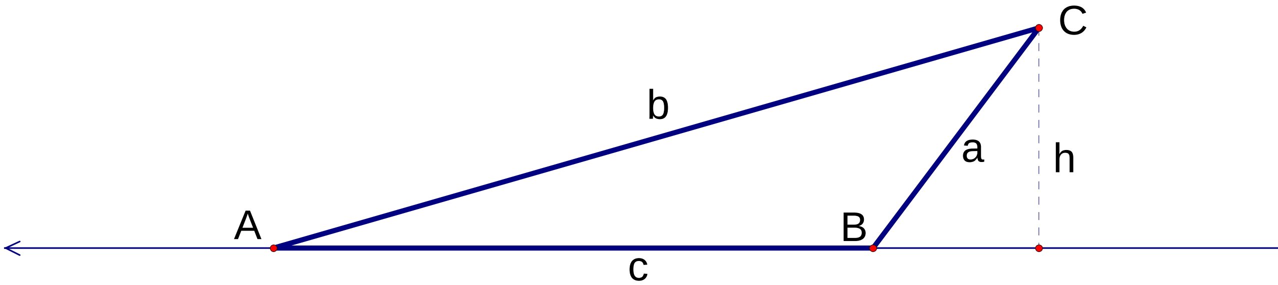
that lie in the interval $[0^\circ, 360^\circ]$.

10. Prove the **Law of Sines**,

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

in the cases illustrated in **each** of the two figures given below.

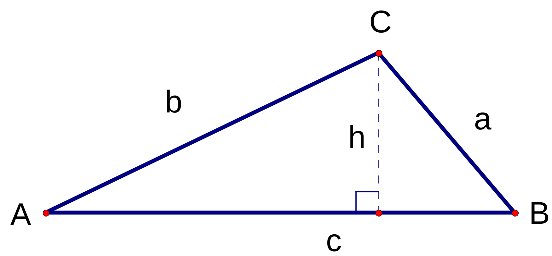


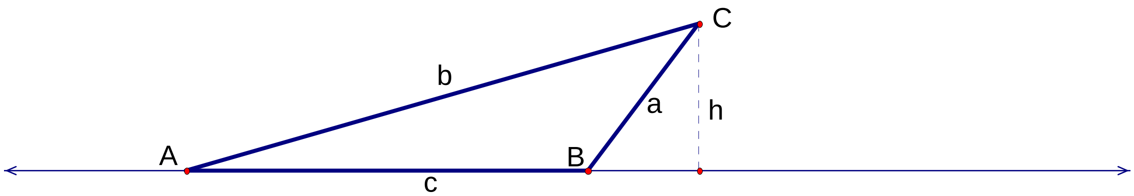


11. Prove the **Law of Cosines**,

$$b^2 = a^2 + c^2 - 2ac\cos(B),$$

in the cases illustrated in **each** of the two figures given below.





12. Find the remaining angles and side lengths for the triangle with the information given below. (There may be no solution, one solution or two solutions.)

$A = 32.76°$ $a = 200$

$B =$ ______ $b =$ ______

$C = 21.97°$ $c =$ ______

13. Find the all of the angles for the triangle with the information given below.

$A =$ ______ $a = 26.1$

$B =$ ______ $b = 21.3$

$C =$ ______ $c = 19.3.$

14. Find the remaining angles and side lengths for the triangle with the information given below. (There may be no solution, one solution or two solutions.)

$A = 89^\circ$ $a = 15.6$

$B =$ ______ $b = 18.4$

$C =$ ______ $c =$ ______.

15. Find all of the remaining angles and side lengths for the triangle with the information given below.

$A = 53.5^\circ$	$a =$ _______
$B =$ _______	$b = 10.2$
$C =$ _______	$c = 17.3.$