1. Explain why $\sin(30°) = \frac{1}{2}$. Your explanation should be written in narrative form (sentences) and should include an appropriate picture which is referred to in your narrative.

Solution: We draw an equilateral triangle with side lengths all equal to 2. All angles of the triangle are equal to 60°. If we bisect the top angle into two 30° angles, then the sides of the two right triangles opposite the 30° angles each have length 1. From this we see that

$$\sin(30°) = \frac{\text{length of side opposite } \theta}{\text{length of hypotenuse}} = \frac{1}{2}.$$
Solution: We construct a right triangle with unknown side lengths $x$ and $y$ as shown in the picture below: We want to find $y$. From the picture we see that
\[
\tan(30°) = \frac{y}{x + 670}
\]
and also
\[
\tan(50°) = \frac{y}{x}.
\]
The first of these equations yields
\[
y = \tan(30°)(x + 670)
\]
and the second equation yields
\[
y = \tan(50°)x.
\]
This gives
\[
\tan(30°)x + 670\tan(30°) = \tan(50°)x
\]
which gives
\[
(tan(50°) - \tan(30°))x = 670\tan(30°).
\]
Solving for $x$ gives
\[
x = \frac{670\tan(30°)}{\tan(50°) - \tan(30°)}.
\]
Since $y = \tan(50°)x$, we see that
\[
y = \frac{670\tan(50°)\tan(30°)}{\tan(50°) - \tan(30°)} \approx 750 \text{ feet}.
\]
Thus the height of the tower is about 750 feet.
3. For the angle $\theta$ in the picture shown here, find $\sin(\theta)$, $\cos(\theta)$, $\tan(\theta)$, $\cot(\theta)$, $\sec(\theta)$ and $\csc(\theta)$.

Solution: Let $r$ be the radius of the picture circle. Then by the Pythagorean Theorem we obtain

$$r^2 = 3^2 + 4^2$$

which gives $r = 5$. Therefore
\[
\begin{align*}
\sin(\theta) &= \frac{y}{r} = -\frac{4}{5} \\
\cos(\theta) &= \frac{x}{r} = \frac{3}{5} \\
\tan(\theta) &= \frac{y}{x} = -\frac{4}{3} \\
\cot(\theta) &= \frac{x}{y} = -\frac{3}{4} \\
\sec(\theta) &= \frac{r}{x} = \frac{5}{3} \\
\csc(\theta) &= \frac{r}{y} = -\frac{5}{4}.
\end{align*}
\]