MATH 1112 – Quiz 1 (Version 1) Solutions

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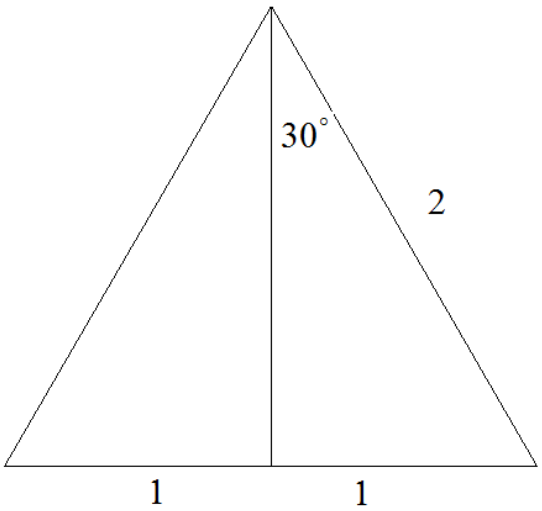
S. F. Ellermeyer Name_____

Instructions. Your work on this quiz will be graded according to two criteria: **mathematical correctness** and **clarity of presentation**. In other words, you must know what you are doing (mathematically) and you must also express yourself clearly. In particular, write answers to questions using correct notation and using **complete sentences** where appropriate. Also, you must supply sufficient detail in your solutions (relevant calculations, written explanations of why you are doing these calculations, etc.). It is not sufficient to just write down an "answer" with no explanation of how you arrived at that answer. As a rule of thumb, the harder that I have to work to interpret what you are trying to say, the less credit you will get. You may use your calculator but you may not use any books or notes.

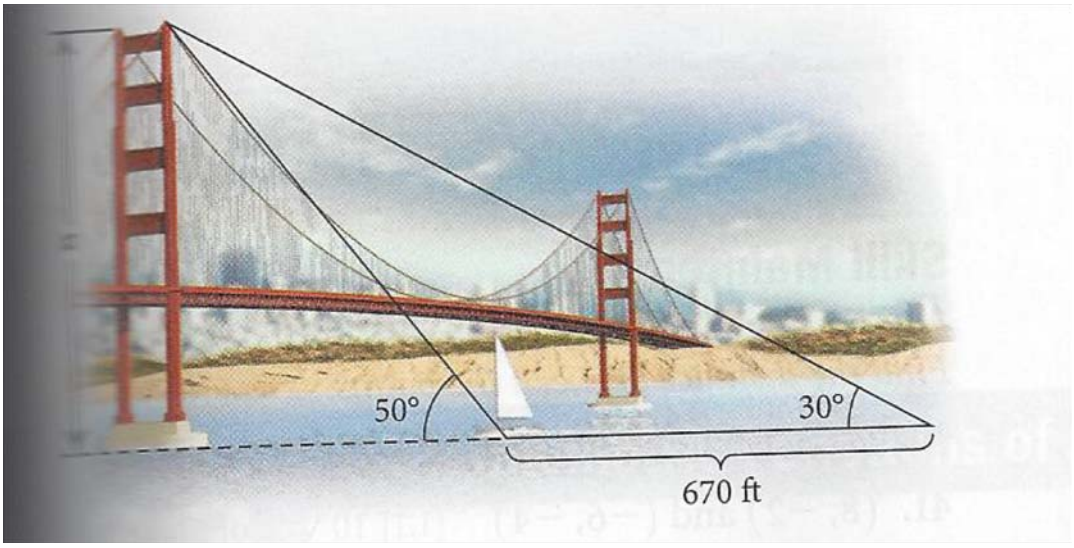
1. Explain why $\sin(30°) = \frac{1}{2}$. Your explanation should be written in narrative form (sentences) and should include an appropriate picture which is referred to in your narrative.

Solution: We draw an equilateral triangle with side lengths all equal to 2. All angles of the triangle are equal to 60°. If we bisect the top angle into two 30° angles, then the sides of the two right triangles opposite the 30° angles each have length 1. From this we see that

$$\sin(30^\circ) = \frac{\text{length of side opposite } \theta}{\text{length of hypotenuse}} = \frac{1}{2}.$$



2. The Golden Gate Bridge has two main towers of equal height that support the two main cables. A visitor on a tour boat passing through San Fransisco bay views the top of one of the towers and estimates the angle of elevation to be 30°. After sailing 670 feet closer, he estimates the angle of elevation of the same tower to be 50°. Approximate the height of the tower. (The answer you should get should be about 750 feet.)



Solution: We construct a right triangle with unknown side lengths x and y as shown in the picture below: We want to find y. From the picture we see that

$$\tan(30°) = \frac{y}{x+670}$$

and also

$$\tan(50°) = \frac{y}{x}.$$

The first of these equations yields

$$y = \tan(30°)(x+670)$$

and the second equation yields

$$y = \tan(50°)x.$$

This gives

$$\tan(30°)x + 670\tan(30°) = \tan(50°)x$$

which gives

$$(\tan(50°) - \tan(30°))x = 670\tan(30°).$$

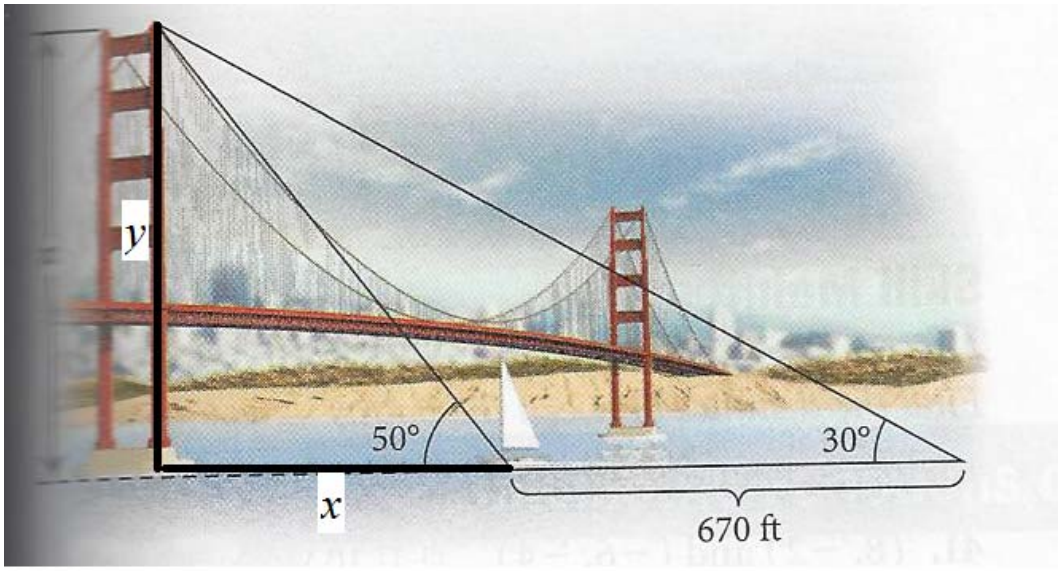
Solving for x gives

$$x = \frac{670\tan(30°)}{\tan(50°) - \tan(30°)}.$$

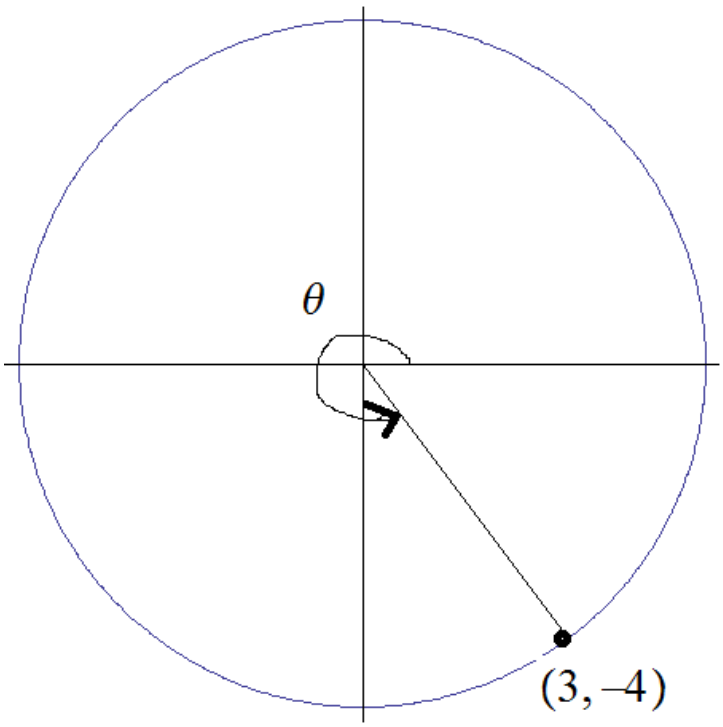
Since $y = \tan(50°)x$, we see that

$$y = \frac{670\tan(50°)\tan(30°)}{\tan(50°) - \tan(30°)} \approx 750 \text{ feet.}$$

Thus the height of the tower is about 750 feet.



3. For the angle θ in the picture shown here, find $\sin(\theta)$, $\cos(\theta)$, $\tan(\theta)$, $\cot(\theta)$, $\sec(\theta)$ and $\csc(\theta)$.



Solution: Let r be the radius of the picture circle. Then by the Pythagorean Theorem we obtain

$$r^2 = 3^2 + 4^2$$

which gives $r = 5$. Therefore

$$\begin{aligned}
\sin(\theta) &= \frac{y}{r} = -\frac{4}{5} \\
\cos(\theta) &= \frac{x}{r} = \frac{3}{5} \\
\tan(\theta) &= \frac{y}{x} = -\frac{4}{3} \\
\cot(\theta) &= \frac{x}{y} = -\frac{3}{4} \\
\sec(\theta) &= \frac{r}{x} = \frac{5}{3} \\
\csc(\theta) &= \frac{r}{y} = -\frac{5}{4}.
\end{aligned}$$