MATH 1112 – Quiz 1 (Version 2) Solutions
June 4, 2015

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Instructions. Your work on this quiz will be graded according to two criteria: **mathematical correctness** and **clarity of presentation**. In other words, you must know what you are doing (mathematically) and you must also express yourself clearly. In particular, write answers to questions using correct notation and using **complete sentences** where appropriate. Also, you must supply sufficient detail in your solutions (relevant calculations, written explanations of why you are doing these calculations, etc.). It is not sufficient to just write down an "answer" with no explanation of how you arrived at that answer. As a rule of thumb, the harder that I have to work to interpret what you are trying to say, the less credit you will get. You may use your calculator but you may not use any books or notes.

1. Explain why $\cos(45°) = \frac{\sqrt{2}}{2}$. Your explanation should be written in narrative form (sentences) and should include an appropriate picture which is referred to in your narrative.

   Solution: We draw a right triangle with two $45°$ angles as pictured. Then each of the sides opposite the $45°$ angles must have the same length. We can let this length be 1. If h is the length of the hypotenuse, then by the Pythagorean Theorem, we have
 $$h^2 = 1^2 + 1^2$$
   which yields $h = \sqrt{2}$. Thus
   $$\cos(45°) = \frac{\text{length of side adjacent to } \theta}{\text{length of hypotenuse}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}.$$
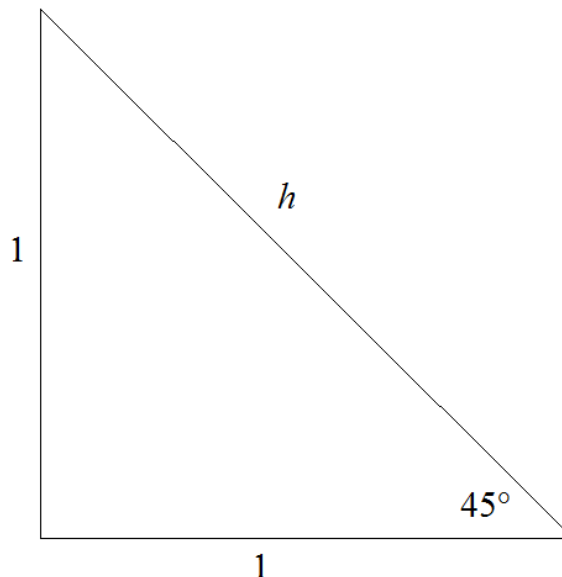
2. While visiting the Mount Rushmore Memorial in Rapid City, South Dakota, Landon approximated the angle of elevation to the top of George Washington's head to be $35°$. After walking 250 feet closer, he approximated that the angle of elevation had increased by $15°$. Approximate the height of the Mount Rushmore Memorial (to the top of George

Washington's head). (The answer you should get is about 424 feet.)
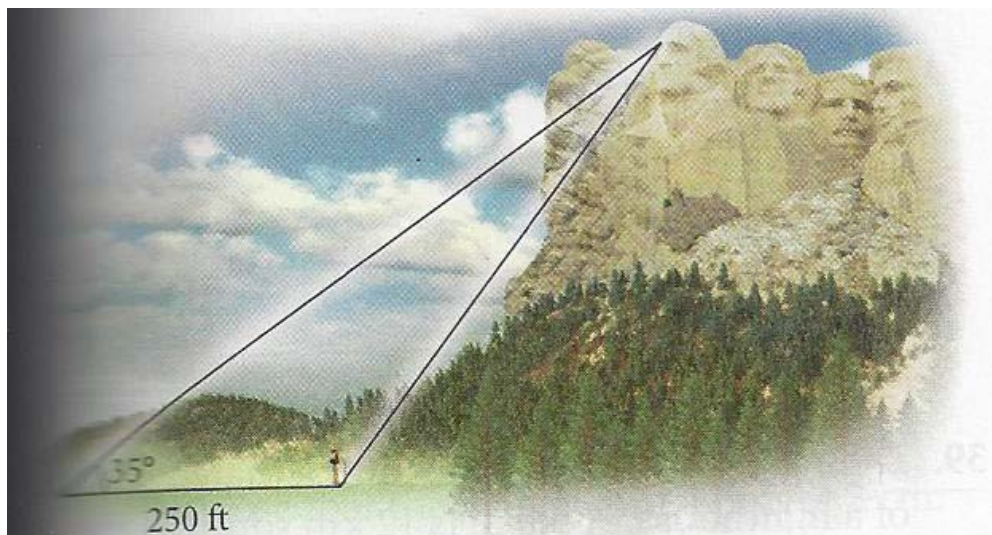
Solution: We form a right triangle as shown in the picture below and label the unknown side lengths as x and y (as shown). The value that we want to find is y. By looking at the figure we see that

$$\tan(35°) = \frac{y}{x + 250}$$

and

$$\tan(50°) = \frac{y}{x}.$$

The first of these equations yields

$$y = \tan(35°)(x + 250)$$

and the second equation yields

$$y = \tan(50°)x.$$

This gives

$$\tan(35°)x + 250\tan(35°) = \tan(50°)x$$
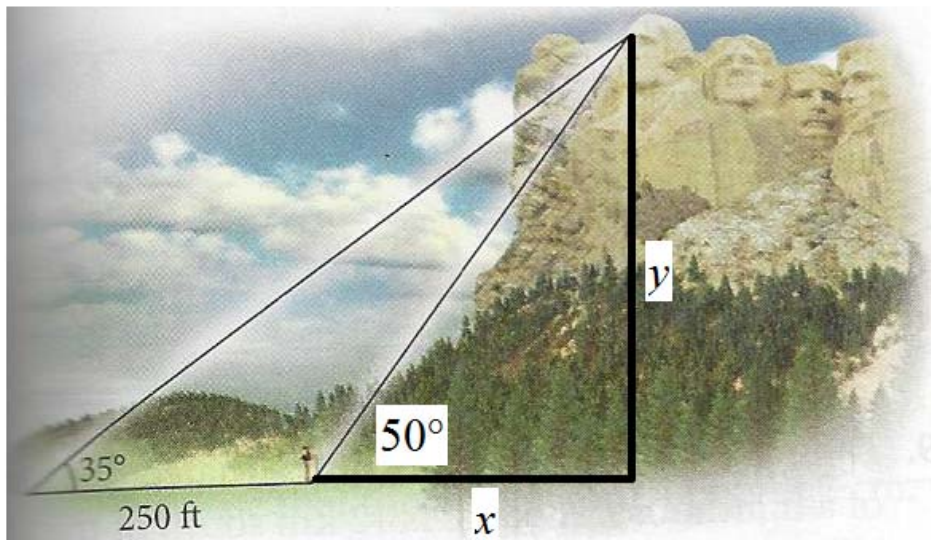
which gives

$$(\tan(50°) - \tan(35°))x = 250\tan(35°).$$

Solving for x gives

$$x = \frac{250\tan(35°)}{\tan(50°) - \tan(35°)}.$$
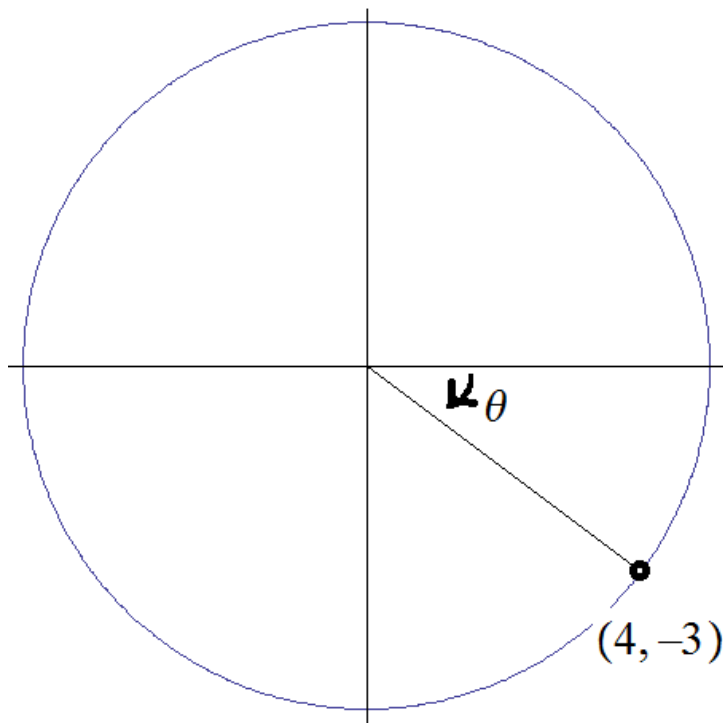
Since $y = \tan(50°)x$, we see that

$$y = \frac{250\tan(50°)\tan(35°)}{\tan(50°) - \tan(35°)} \approx 424 \text{ feet.}$$

Thus the hight of the Mount Rushmore Memorial is about 424 feet.

3. For the angle θ in the picture shown here, find $\sin(\theta)$, $\cos(\theta)$, $\tan(\theta)$, $\cot(\theta)$, $\sec(\theta)$ and $\csc(\theta)$.

Solution: Let r be the radius of the picture circle. Then by the Pythagorean Theorem we obtain

$$r^2 = 3^2 + 4^2$$

which gives $r = 5$. Therefore

$$\sin(\theta) = \frac{y}{r} = -\frac{3}{5}$$

$$\cos(\theta) = \frac{x}{r} = \frac{4}{5}$$

$$\tan(\theta) = \frac{y}{x} = -\frac{3}{4}$$

$$\cot(\theta) = \frac{x}{y} = -\frac{4}{3}$$

$$\sec(\theta) = \frac{r}{x} = \frac{5}{4}$$

$$\csc(\theta) = \frac{r}{y} = -\frac{5}{3}.$$