# Polar Coordinates

MATH 1112

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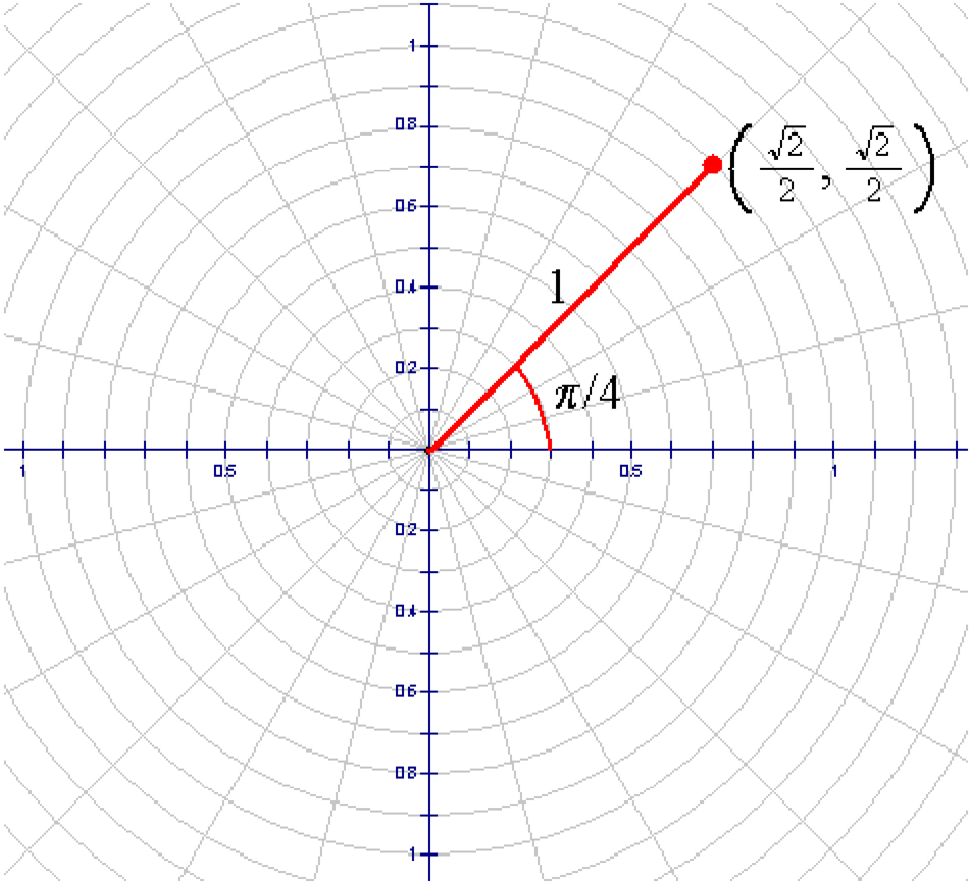
# Rectangular vs. Polar Coordinates

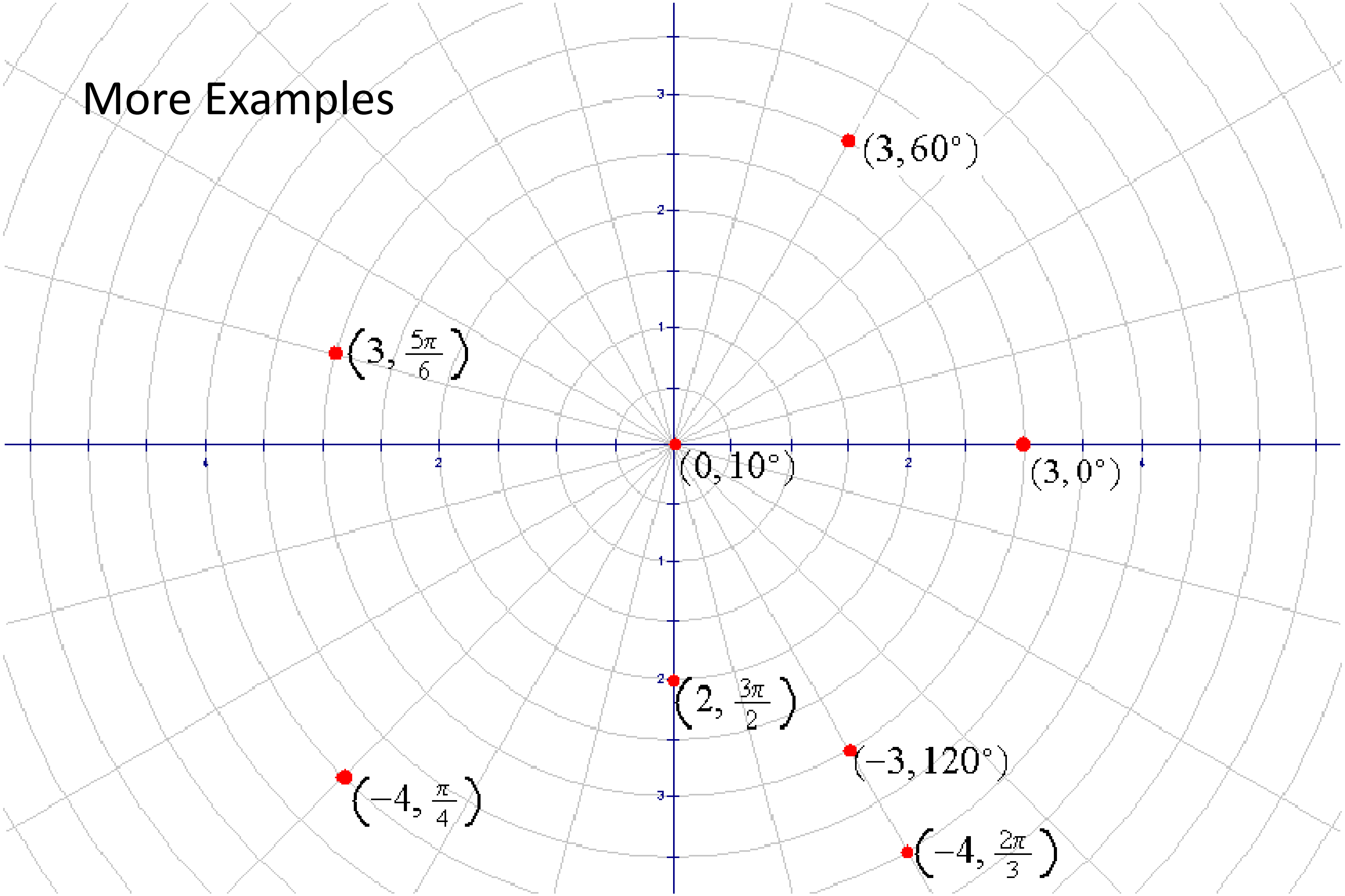
- Rectangular coordinates are the usual (x,y) coordinates.
- Polar coordinates are $(r,\theta)$ coordinates – where $\theta$ is the directed angle measured in the usual way and r is the directed distance from the origin to the point in question. "Directed distance" means that we travel in the direction of the terminal side of $\theta$ if $r>0$ and in the opposite direction if $r<0$.

The rectangular coordinates of a point are unique, but the polar coordinates are not unique. Every point has infinitely many polar coordinate representations.

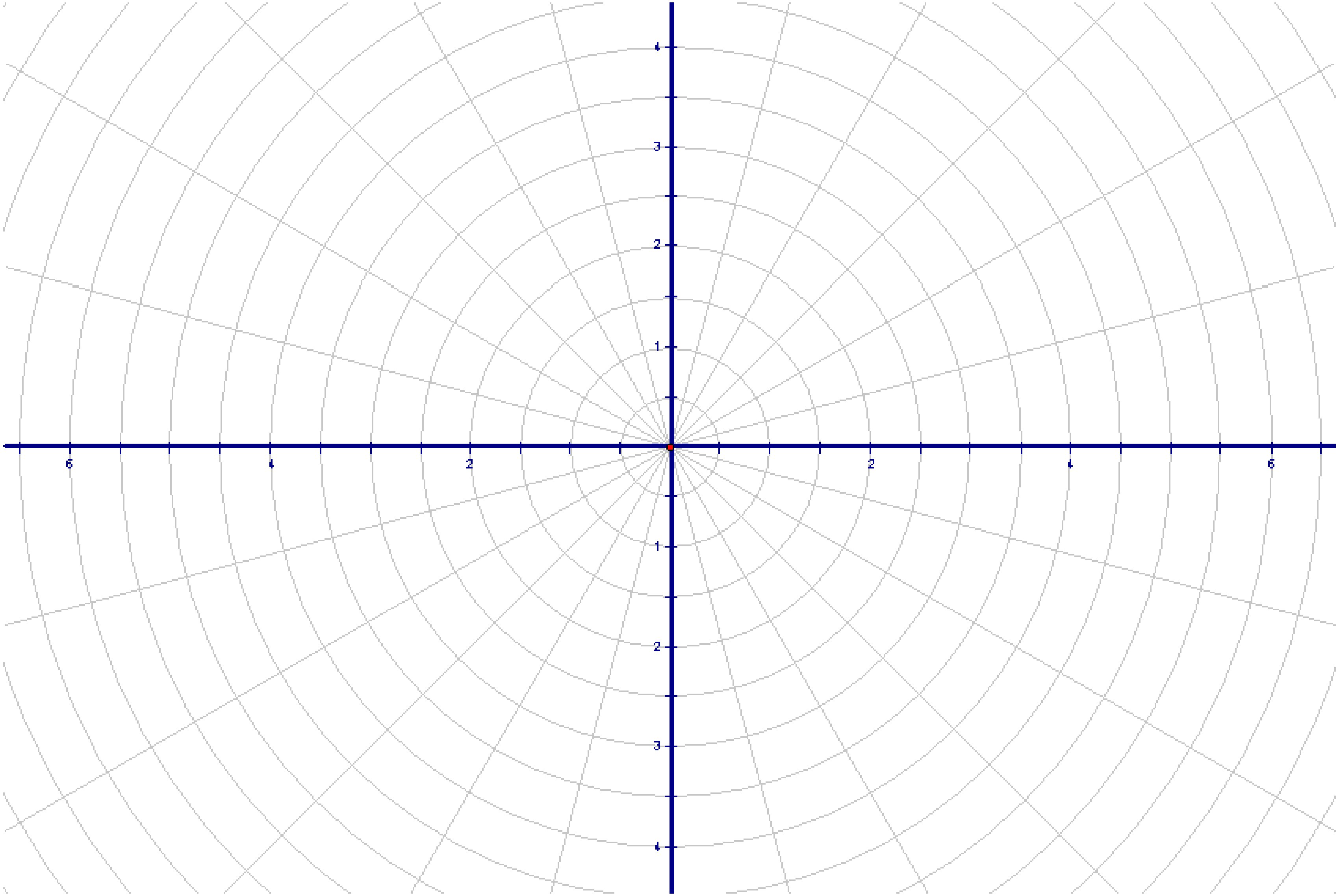
For example, the point $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ (in rectangular coordinates) has polar coordinate represenation $\left(1, \frac{\pi}{4}\right)$ or $\left(1, \frac{9\pi}{4}\right)$ or $\left(1, -\frac{7\pi}{4}\right)$ or $\left(-1, \frac{5\pi}{4}\right)$ and there are infinitely many other ways to represent this point.



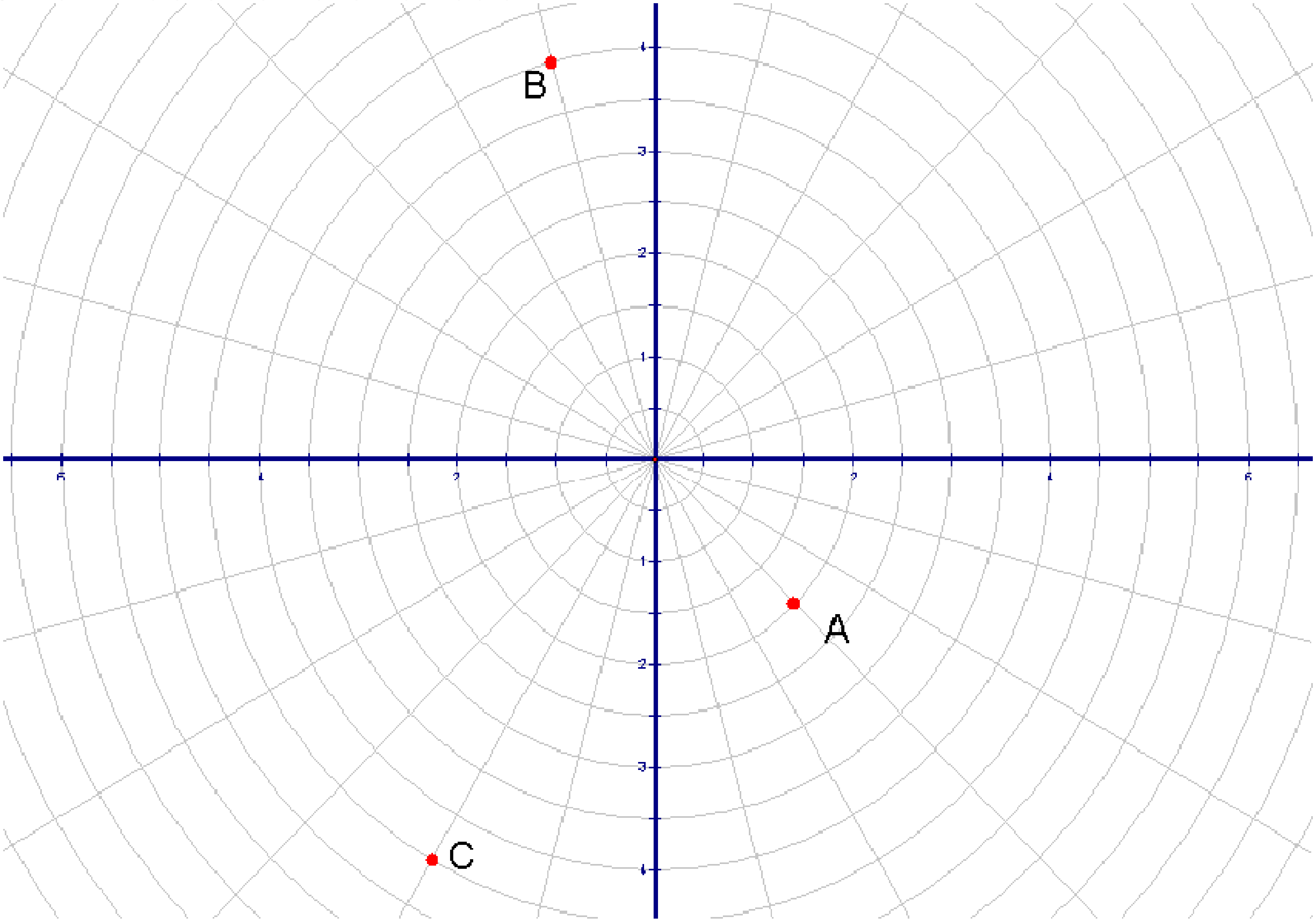
# More Examples



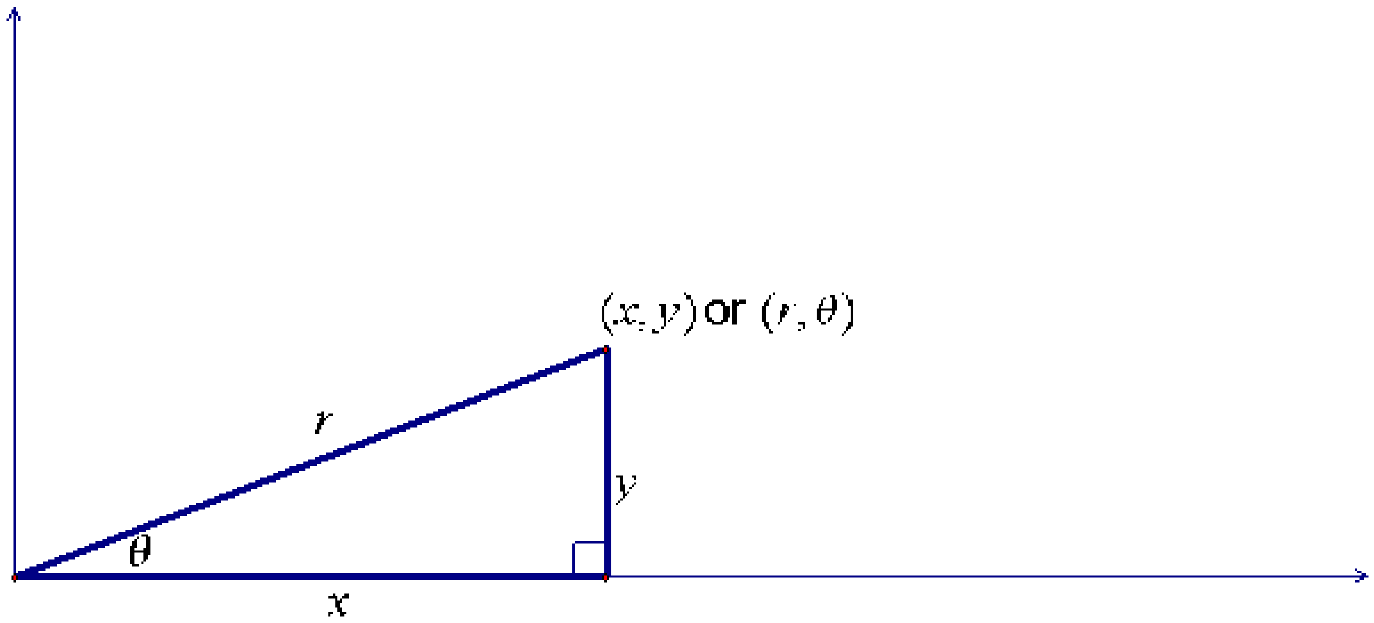
**Problem:** Graph the point that has polar coordinates $(4, \pi)$.
Also graph the points $(-3, 135^\circ)$, $(2.75, 150^\circ)$, and $(1.2, -2\pi/3)$.



**Problem:** Find polar coordinates of the labelled points $A$, $B$, and $C$. Give three answers for each.

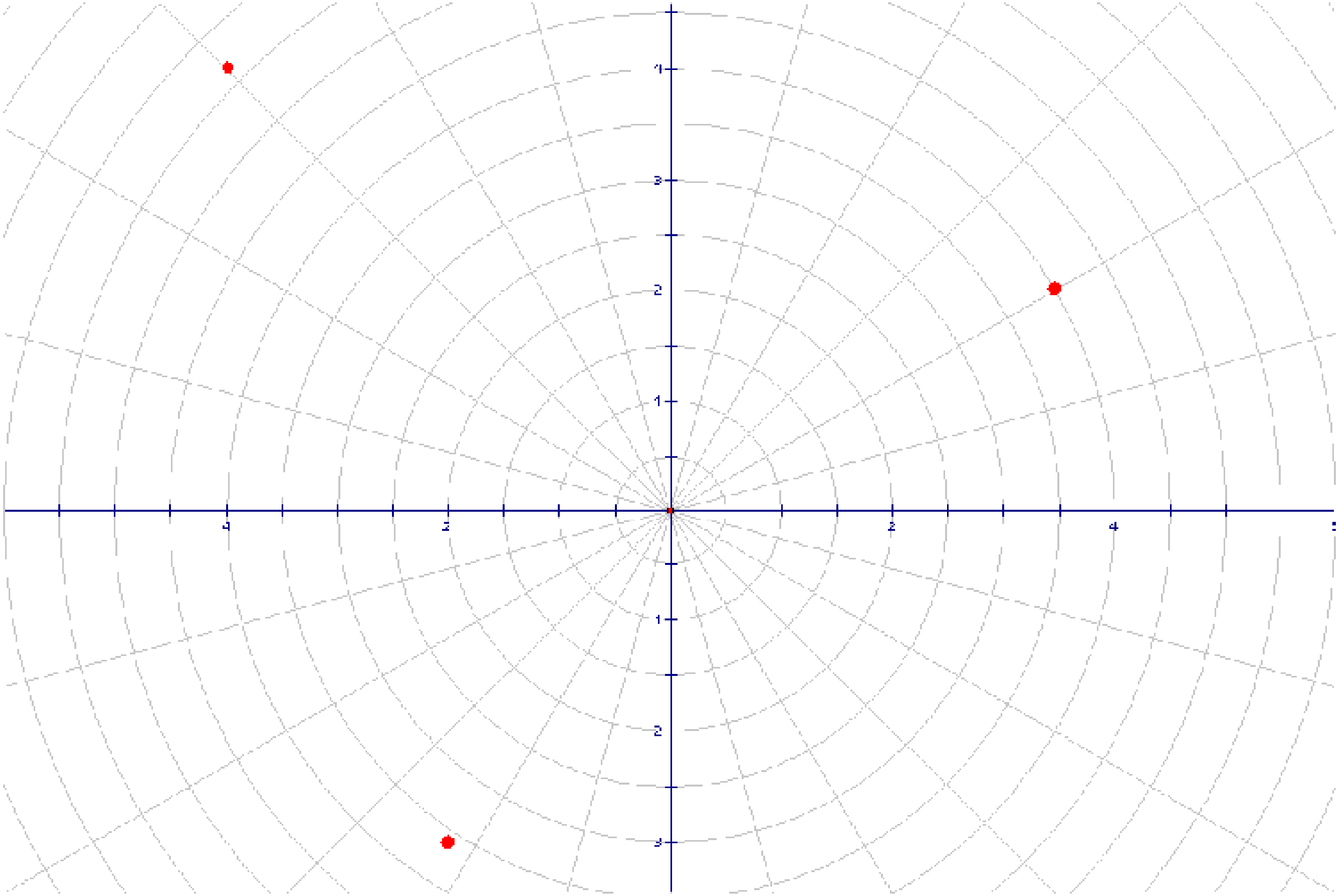


# The Relationship between Rectangular and Polar Coordinates




$$r^2 = x^2 + y^2$$
$$\tan(\theta) = y/x$$
$$x = r\cos(\theta)$$
$$y = r\sin(\theta)$$

**Problem:** Find polar coordinates for the point whose rectangular coordinates are $(-4, 4)$. Express the angle in degrees, using the smallest possible positive angle. Do the same for the points whose rectangular coordinates are $\left(2\sqrt{3}, 2\right)$ and $(-2, -3)$. (A calculator will be needed for the last one.)



**Problem:** Find the rectangular coordinates of the point that has polar coordinates $(7, \pi/6)$. Do the same for the points $(0, -23^\circ)$ and $(-3, 2)$. (The last one will require a calculator.)

# Polar vs. Rectangular Equations of Curves

Some curves are easier to describe using equations in rectangular coordinates and some are easier to describe using polar coordinate equations. Curves that have a “circular” nature are the ones that are easier to describe using polar coordinates.

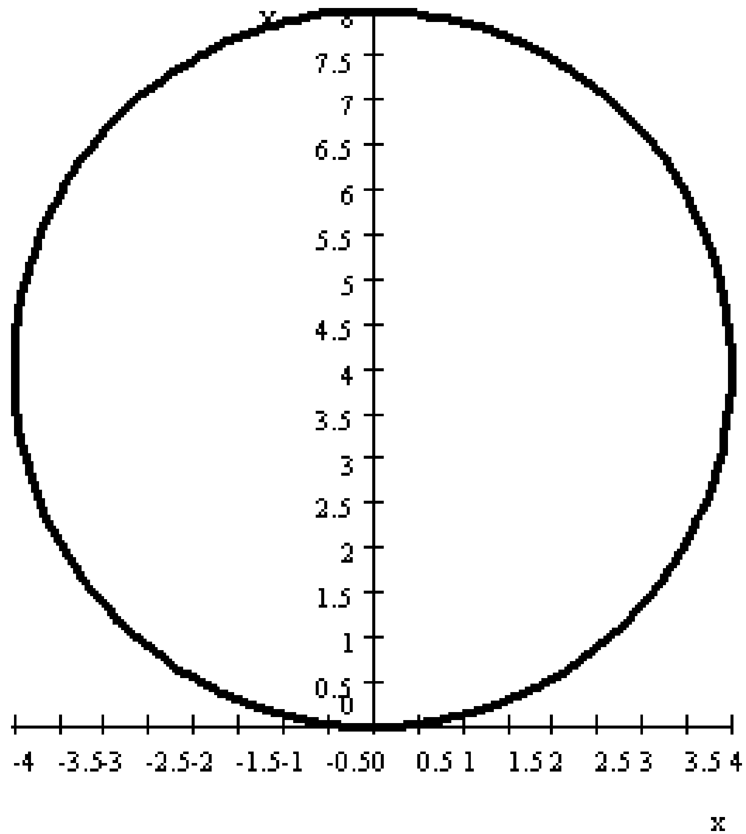
**Example:** The curve $x^2 + y^2 = 16$ is a circle of radius $4$ centered at the origin. Write a polar coordinate equation for this curve. (Is it simpler? Yes!)

The curve $2x - y = 3$ is a line that does not pass through the origin. Write a polar coordinate equation for this curve. (Is it simpler? No!)

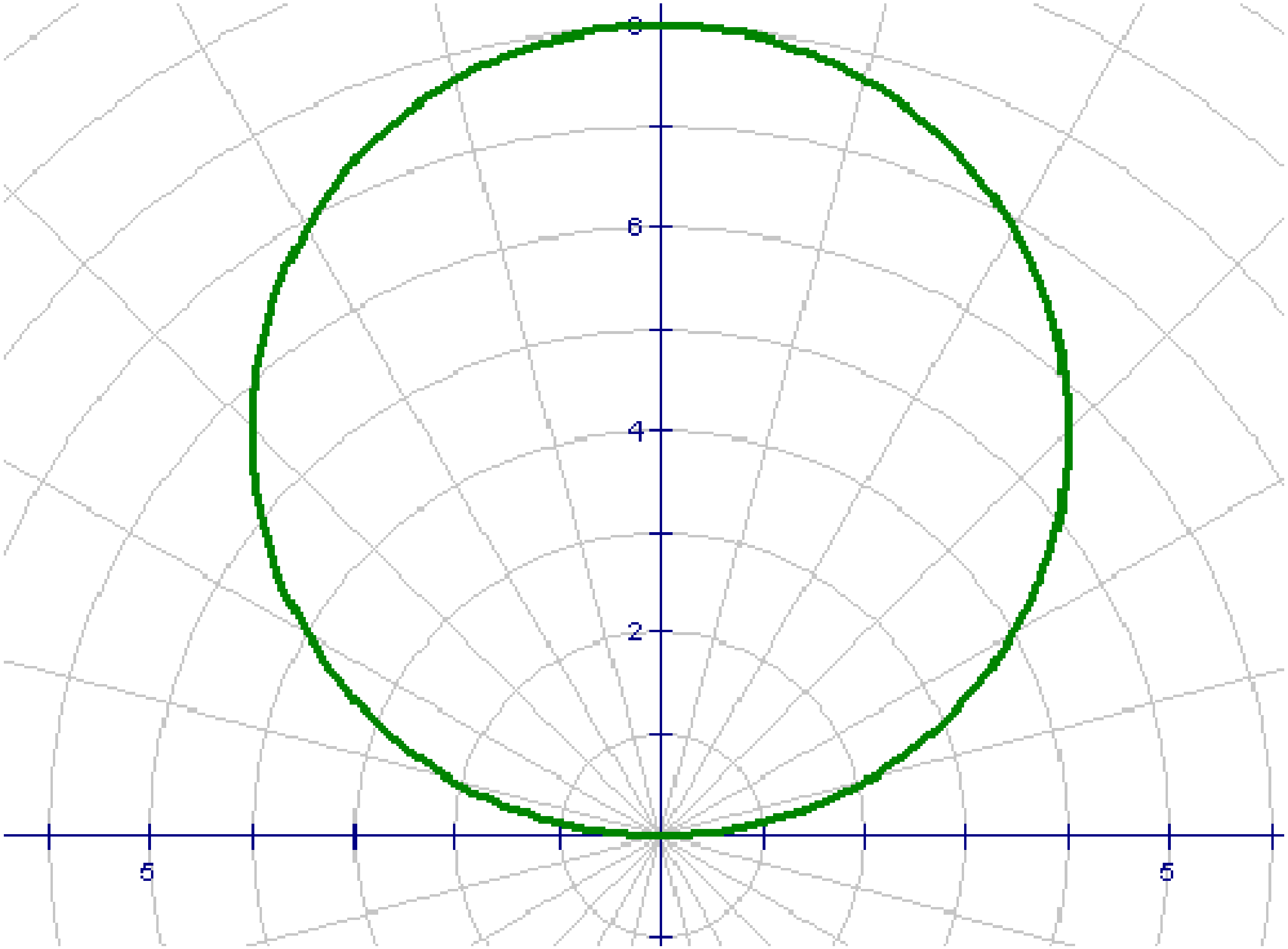
The curve $y = x$ is a line passing through the origin. Write a polar coordinate equation for this curve.

(Is it simpler? Perhaps a tie!)

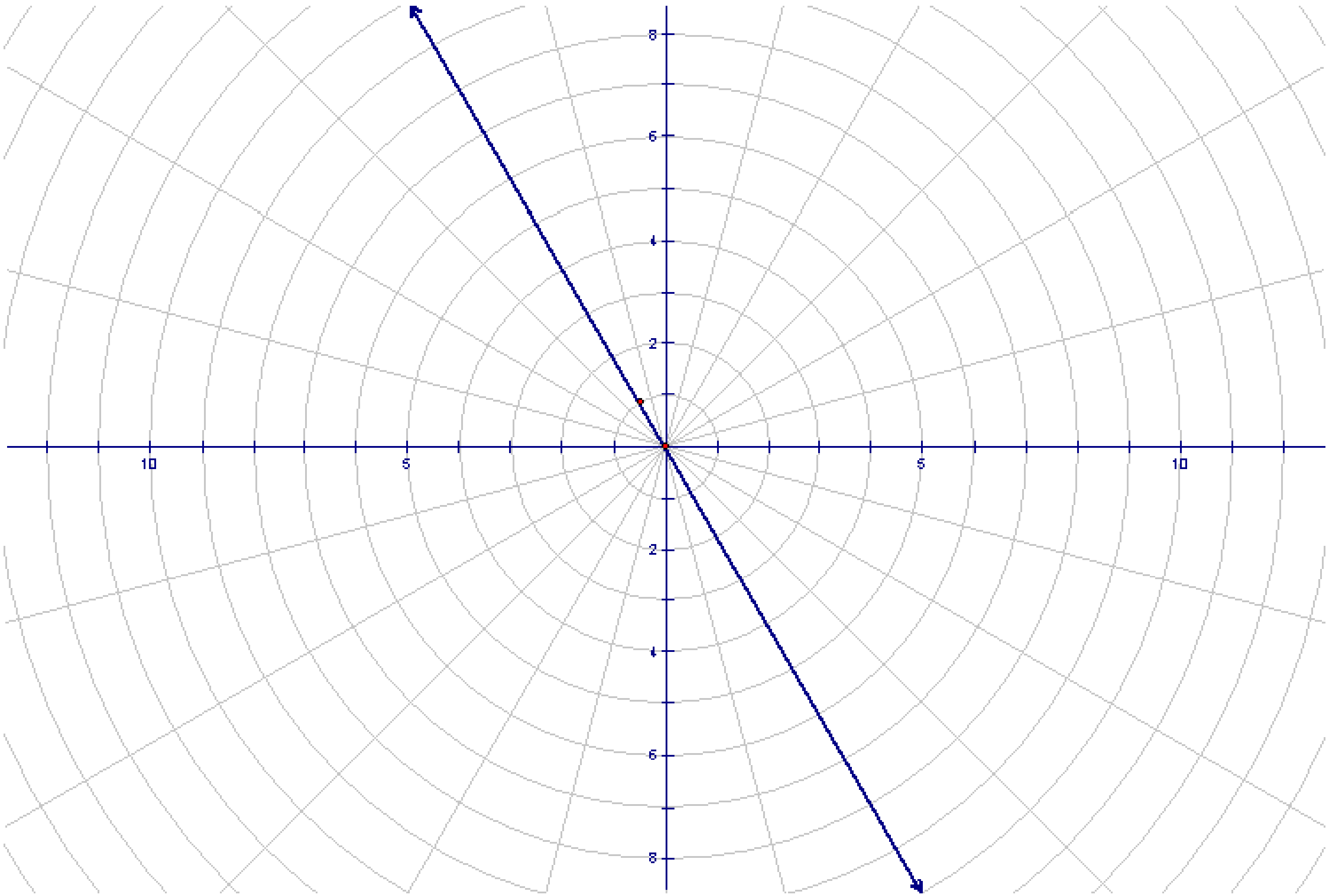
**Example:** The graph of $x^2 + y^2 = 8y$ is shown below.
This curve is a circle of radius 4 centered at $(0, 4)$.
Write an equation for this curve in polar coordinates.



# r=8sin(θ)



**Example:** The graph of $\theta = 2\pi/3$ is shown below.
This curve is a line passing through the origin.
Write an equation for this curve in rectangular coordinates.



# More Examples

Graph $r=1-\sin(\theta)$.

Graph $r=4\sin(3\theta)$.

Graph $r=4\sin(5\theta)$.

Graph $r=1-2\sin(\theta)$.