

MATH 1112 – Quiz 3 (Version 1) Solutions

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Name _____

Instructions. Your work on this quiz will be graded according to two criteria: **mathematical correctness** and **clarity of presentation**. In other words, you must know what you are doing (mathematically) and you must also express yourself clearly. In particular, write answers to questions using correct notation and using **complete sentences** where appropriate. Also, you must supply sufficient detail in your solutions (relevant calculations, written explanations of why you are doing these calculations, etc.). It is not sufficient to just write down an “answer” with no explanation of how you arrived at that answer. As a rule of thumb, the harder that I have to work to interpret what you are trying to say, the less credit you will get. **No calculators allowed on this quiz!**

1. Use an appropriate sum or difference identity to find the **exact** value of $\sin\left(\frac{7\pi}{12}\right)$. A calculator-generated answer will not suffice. Show all of your work.

Solution: Note that

$$\frac{7\pi}{12} = \frac{4\pi}{12} + \frac{3\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}.$$

Thus

$$\begin{aligned}\sin\left(\frac{7\pi}{12}\right) &= \sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right) \\ &= \sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right) \\ &= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \\ &= \frac{\sqrt{6} + \sqrt{2}}{4}.\end{aligned}$$

2. Prove the identity

$$\frac{1 + \cos^2(x)}{\sin^2(x)} = 2 \csc^2(x) - 1.$$

Your proof must be written correctly. If you begin the proof by writing

$$\frac{1 + \cos^2(x)}{\sin^2(x)} = 2 \csc^2(x) - 1,$$

then you will get at most 4 points.

Proof:

$$\begin{aligned}\frac{1 + \cos^2(x)}{\sin^2(x)} &= \frac{1}{\sin^2(x)} + \frac{\cos^2(x)}{\sin^2(x)} \\ &= \csc^2(x) + \cot^2(x) \\ &= \csc^2(x) + (\csc^2(x) - 1) \\ &= 2 \csc^2(x) - 1.\end{aligned}$$

Therefore

$$\frac{1 + \cos^2(x)}{\sin^2(x)} = 2 \csc^2(x) - 1.$$