

# Solving Trigonometric Equations

MATH 1112

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# Three Basic Equations to Know How to Solve

$$\cos(\theta) = k \quad (\text{where } -1 \leq k \leq 1)$$

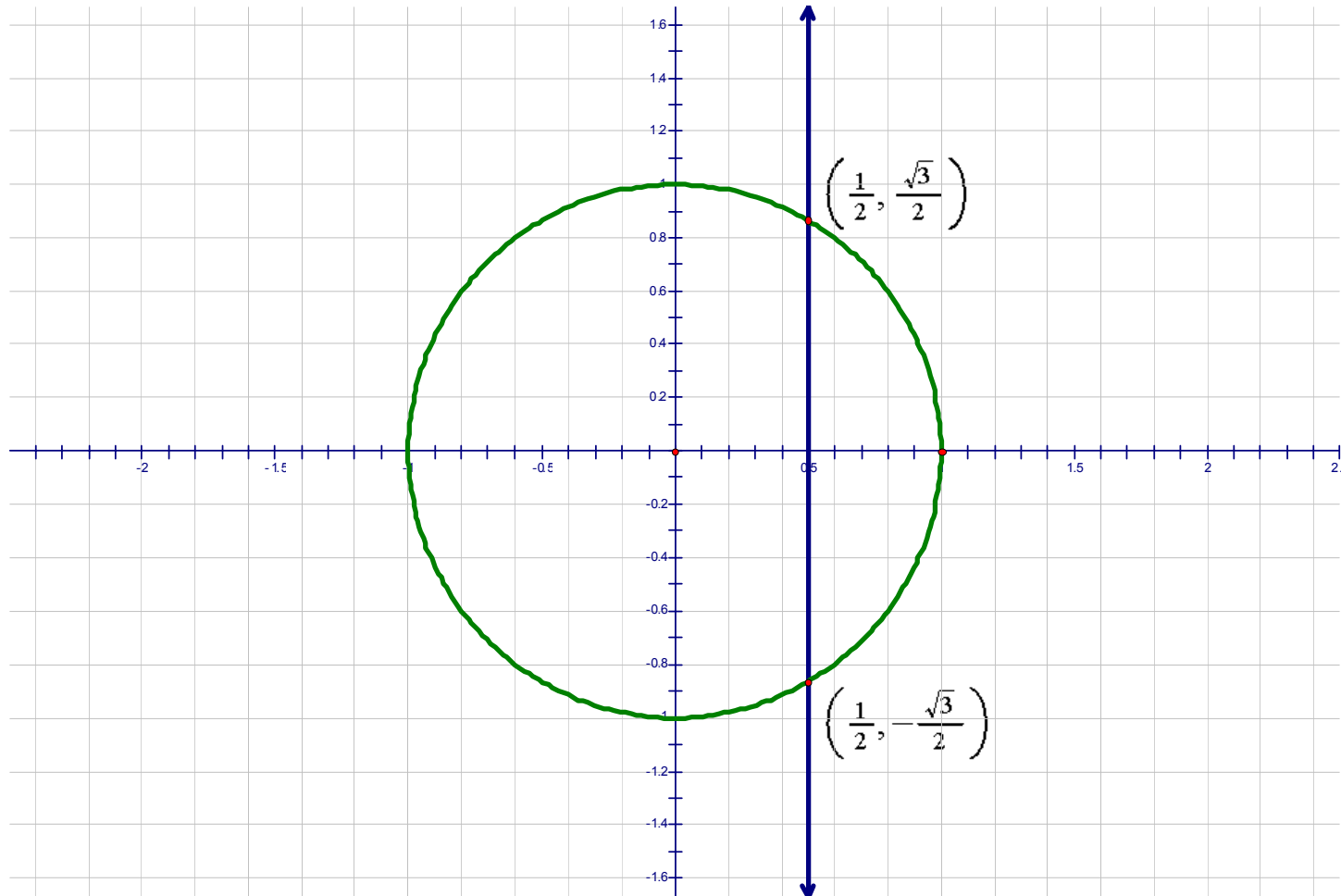
$$\sin(\theta) = k \quad (\text{where } -1 \leq k \leq 1)$$

$$\tan(\theta) = k \quad (\text{where } -\infty < k < \infty)$$

# Solving $\cos(\theta) = k$

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Example: Solve  $\cos(\theta) = \frac{1}{2}$



The solutions of  $\cos(\theta) = \frac{1}{2}$  (in degrees) are

$$\theta = 60^\circ + n \cdot 360^\circ$$

and

$$\theta = -60^\circ + n \cdot 360^\circ.$$

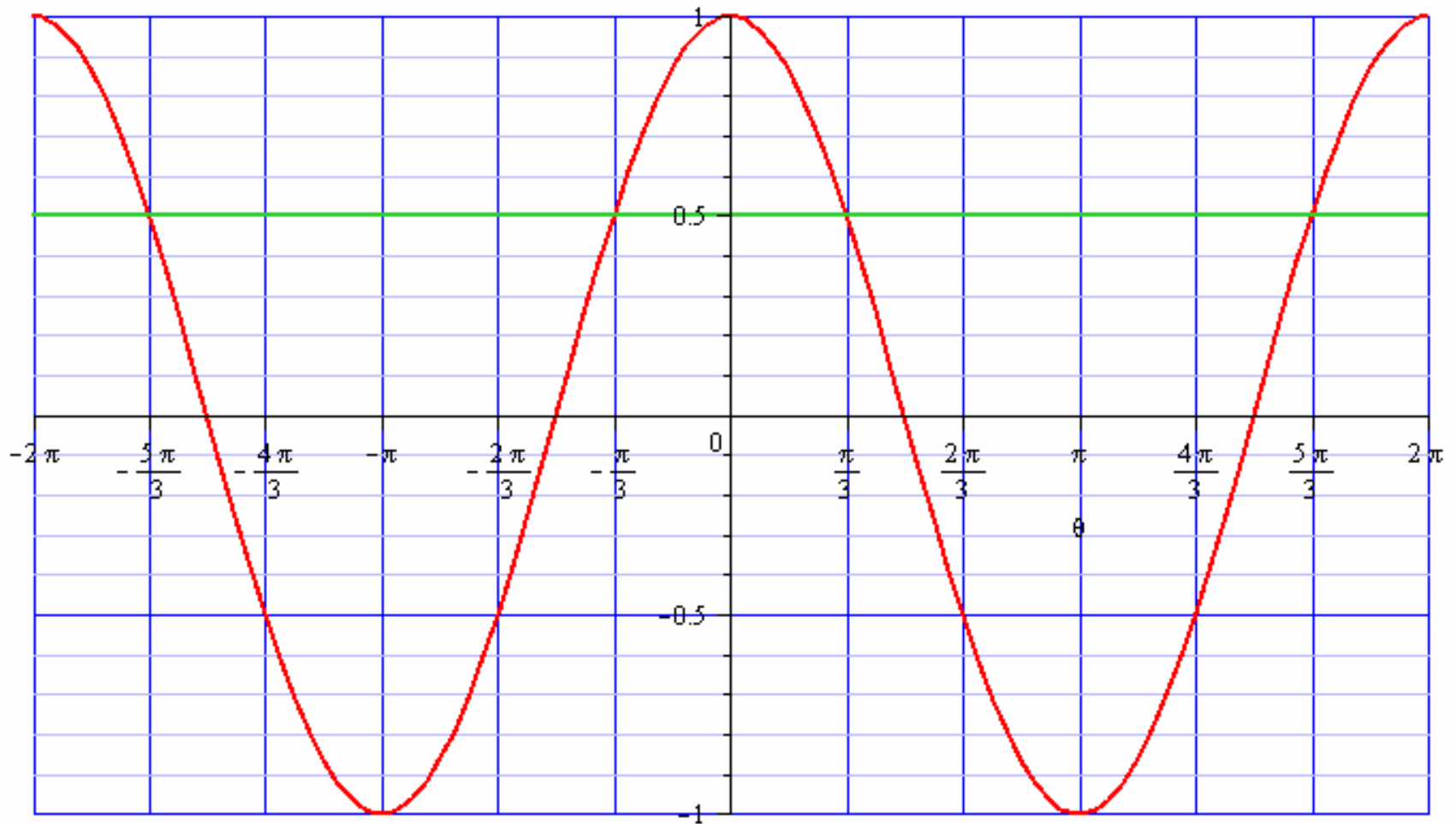
In radians, the solutions are

$$\theta = \frac{\pi}{3} + n \cdot 2\pi$$

and

$$\theta = -\frac{\pi}{3} + n \cdot 2\pi.$$

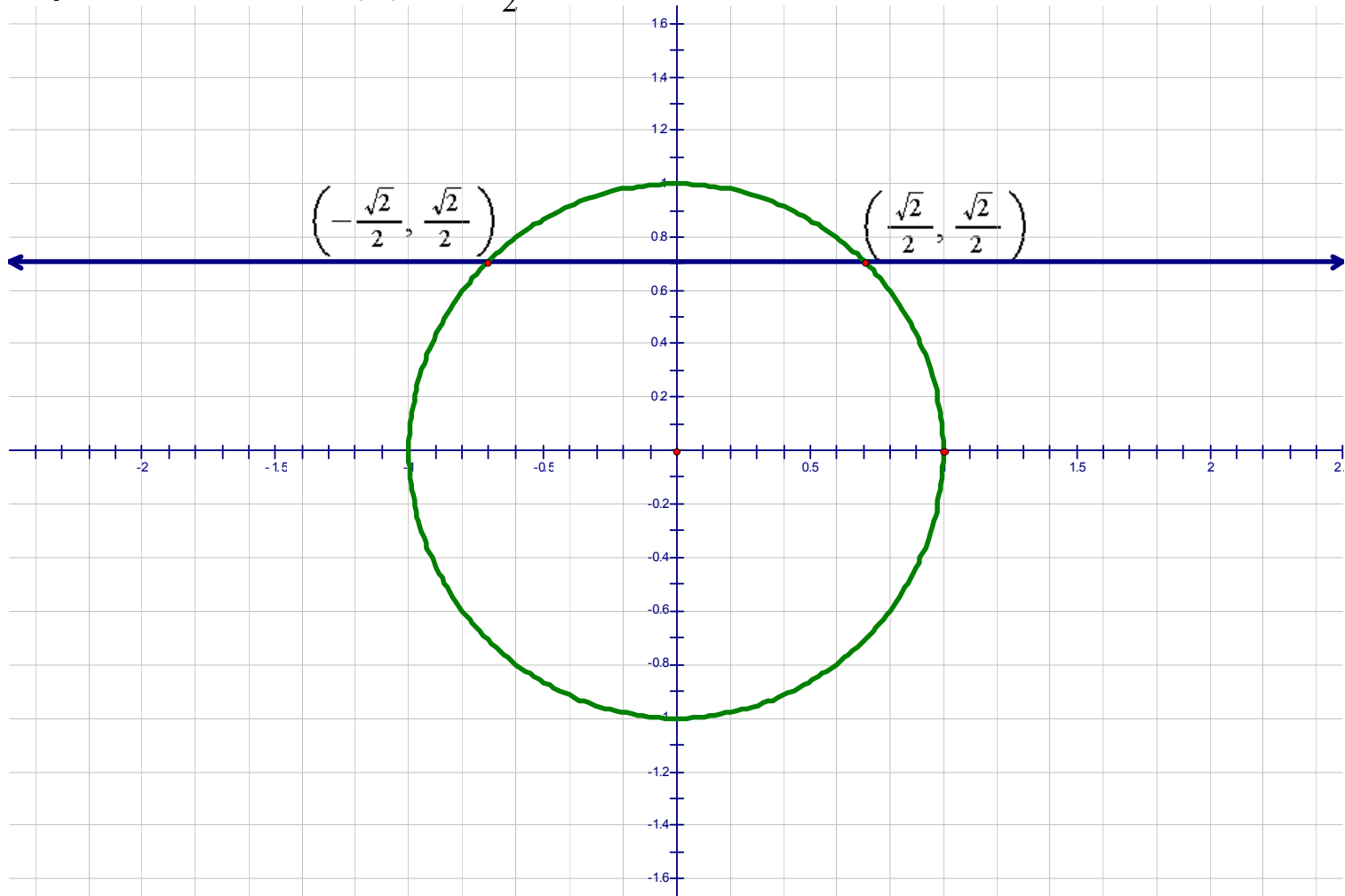
# Graphical Check



# Solving $\sin(\theta) = k$

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Example: Solve  $\sin(\theta) = \frac{\sqrt{2}}{2}$ .



The solutions of  $\sin(\theta) = \frac{\sqrt{2}}{2}$  (in degrees) are

$$\theta = 45^\circ + n \cdot 360^\circ$$

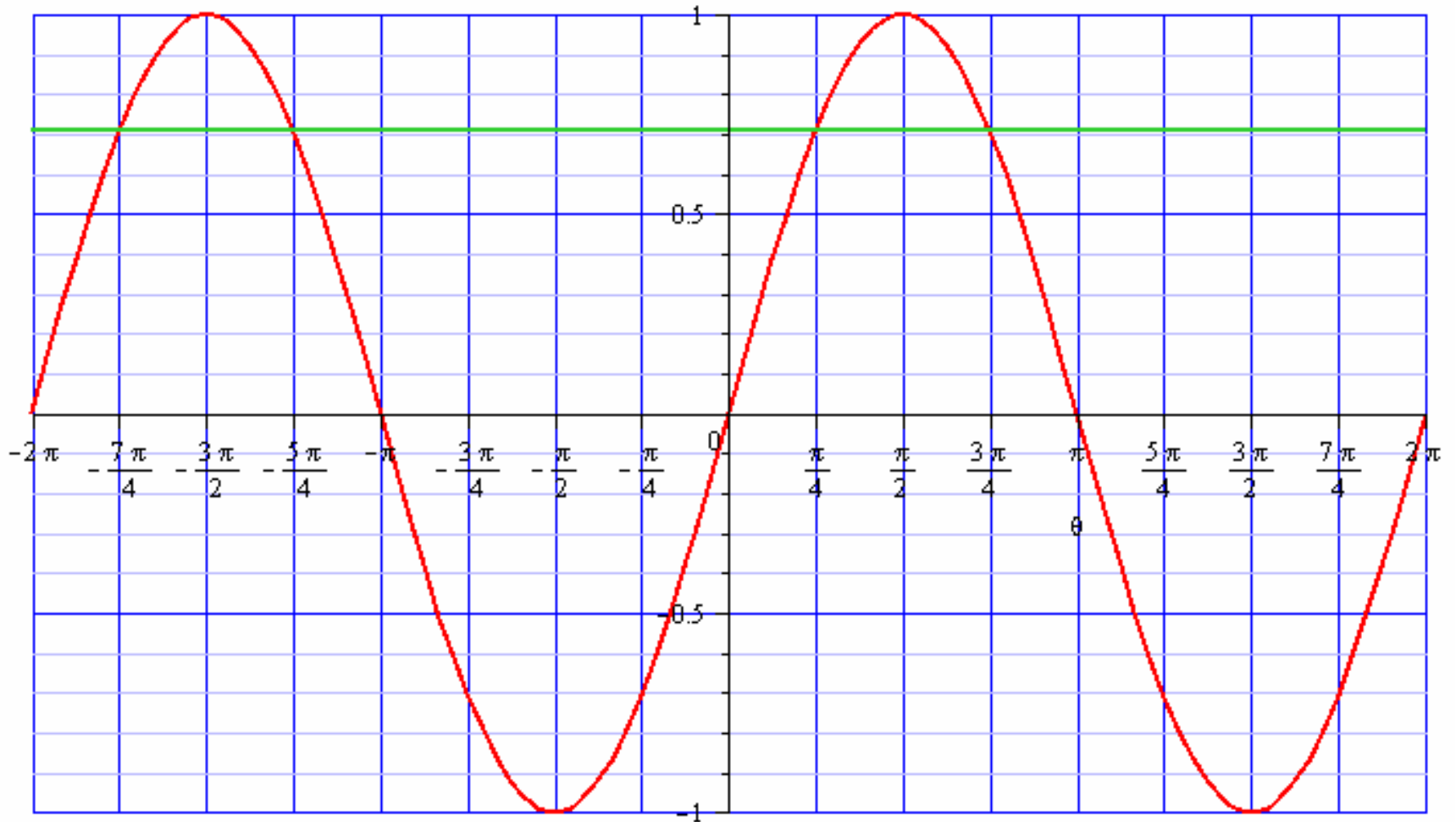
and  $\theta = (180^\circ - 45^\circ) + n \cdot 360^\circ = 135^\circ + n \cdot 360^\circ$

In radians, the solutions are

$$\theta = \frac{\pi}{4} + n \cdot 2\pi$$

and  $\theta = \frac{3\pi}{4} + n \cdot 2\pi.$

# Graphical Check

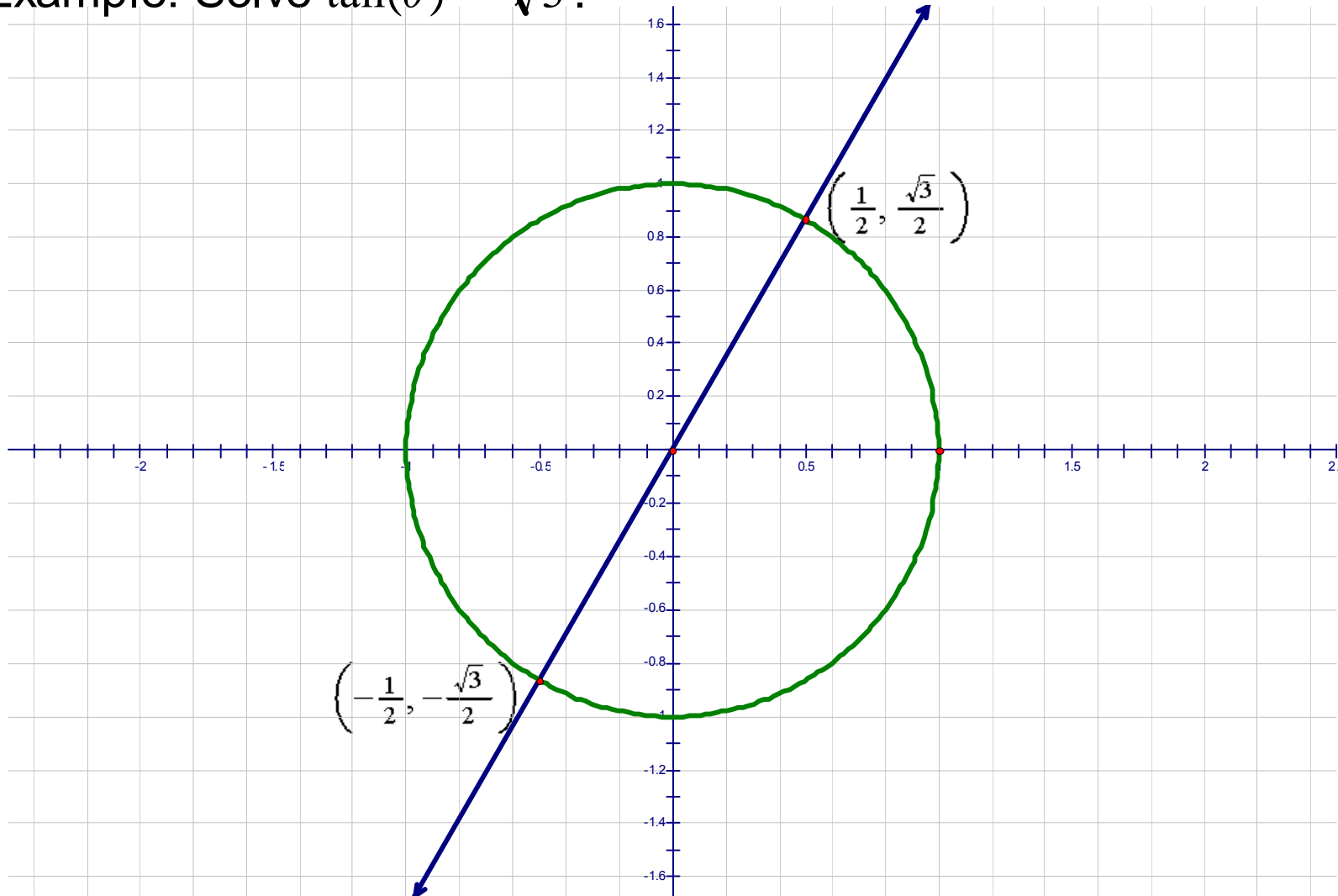




# Solving $\tan(\theta) = k$

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Example: Solve  $\tan(\theta) = \sqrt{3}$ .



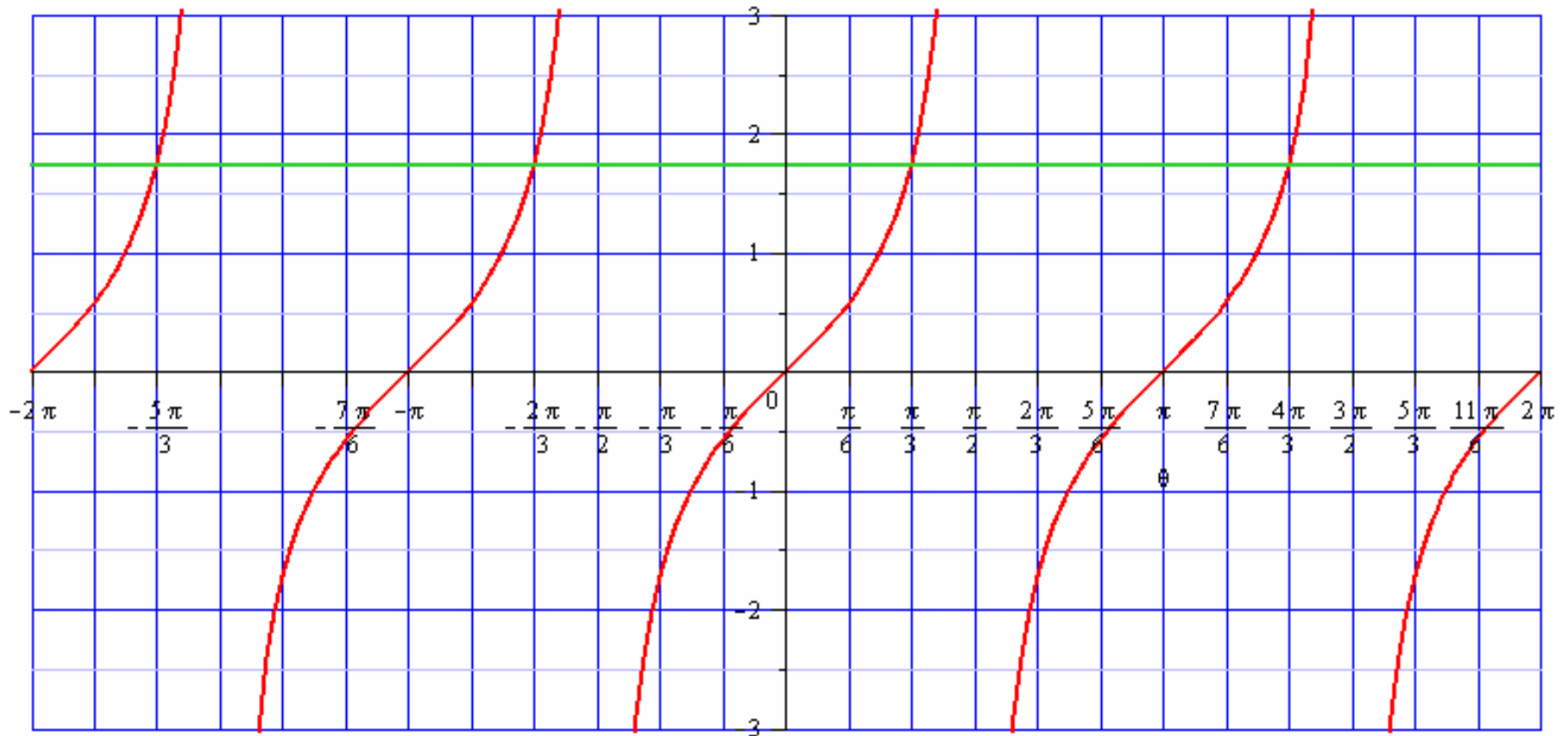
The solutions of  $\tan(\theta) = \sqrt{3}$  (in degrees) are

$$\theta = 60^\circ + n \cdot 180^\circ.$$

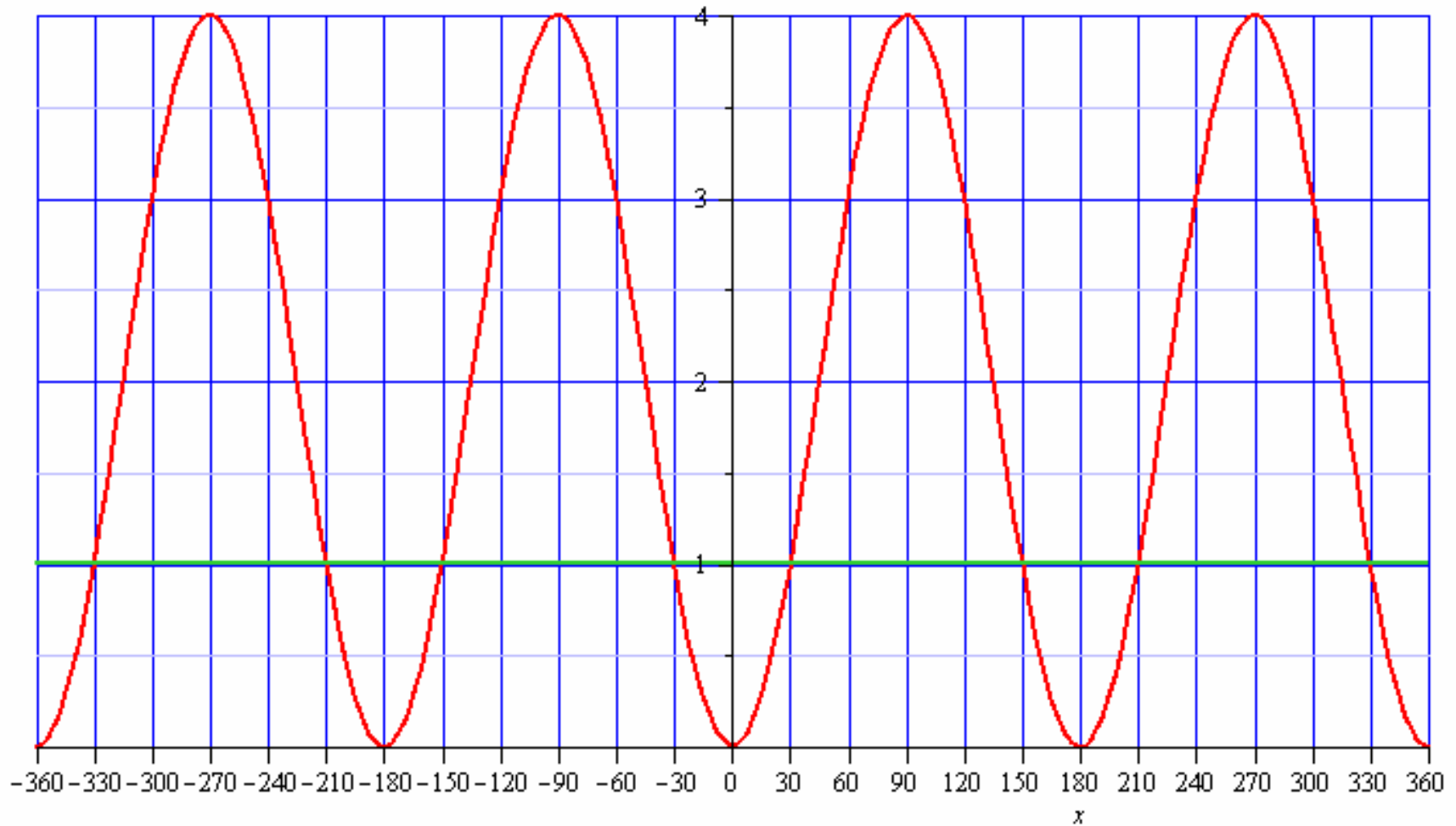
In radians, the solutions are

$$\theta = \frac{\pi}{3} + n \cdot \pi.$$

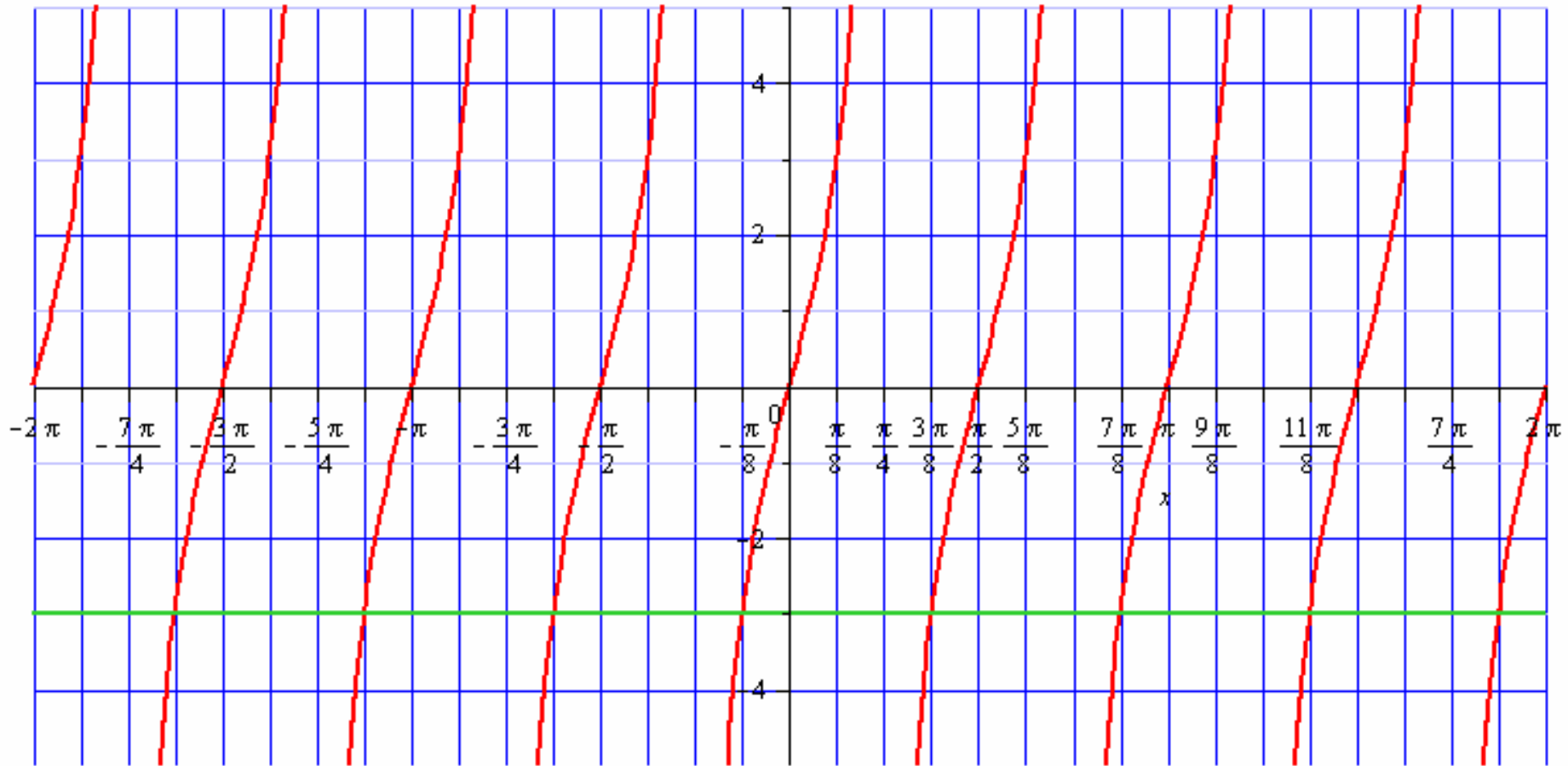
# Graphical Check



Problem: Solve  $4 \sin^2(x) = 1$  in degrees.



Problem: Solve  $3 \tan(2x) = -3$  in radians.



Problem: Find all solutions of  $\cos(2x) + \sin(x) = 1$  that lie in the interval  $[0, 2\pi)$ .

Problem: Find all solutions of  $\sqrt{3} \cos(x) - \sin(x) = 1$  that lie in the interval  $[0^\circ, 360^\circ)$ .