**MATH 1112 – Exam 1 (Version 1) Solutions**

June 11, 2015

S. F. Ellermeyer Name__________________________________

**Instructions**. Your work on this exam will be graded according to two criteria: **mathematical correctness** and **clarity of presentation**. In other words, you must know what you are doing (mathematically) and you must also express yourself clearly. In particular, write answers to questions using correct notation and using **complete sentences** where appropriate. Also, you must supply sufficient detail in your solutions (relevant calculations, written explanations of why you are doing these calculations, etc.). It is not sufficient to just write down an "answer" with no explanation of how you arrived at that answer. As a rule of thumb, the harder that I have to work to interpret what you are trying to say, the less credit you will get. You may use your calculator on this exam but you may not use any books or notes.

**1**. The Pythagorean Theorem states that if a right triangle has sides of length $a$ and $b$ and hypotenuse of length $c$, then

$$a^2 + b^2 = c^2.$$

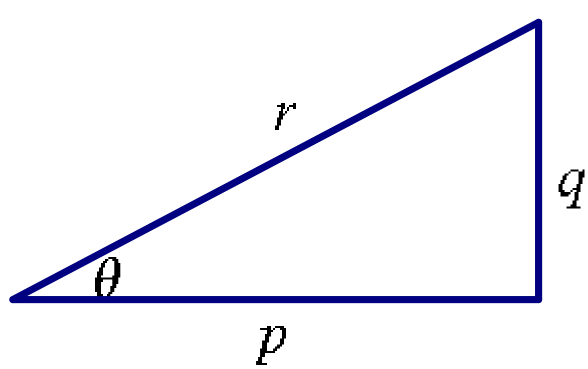
Give James Garfield's proof of this theorem. The proof must be written using complete sentences and correct mathematical notation. (It is essential to include a picture with your proof.)

In giving this proof, you make take it as already known that the sum of the angles of any triangle is $180^\circ$ and that the area of a trapezoid with parallel sides of length $b_1$ and $b_2$ and height $h$ is

$$A = \frac{1}{2}(b_1 + b_2)h.$$

(You don't have to prove these two facts.)

**2**. For the right triangle pictured here, find the six trigonometric ratios of the angle $\theta$ in terms of $p$, $q$, and $r$.




$\sin(\theta) =$ __________ $\csc(\theta) =$ __________

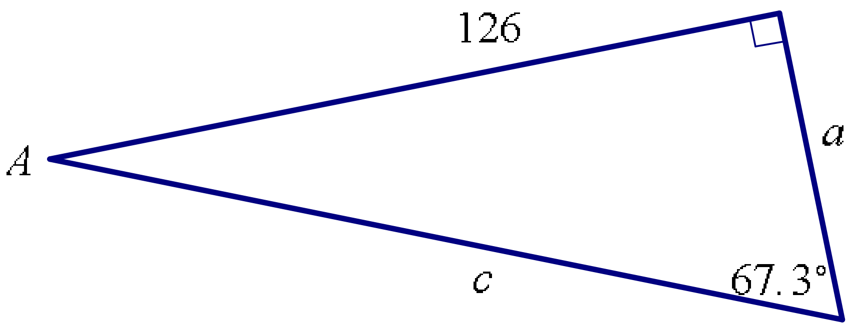
$\cos(\theta) =$ __________ $\sec(\theta) =$ __________ .

$\tan(\theta) =$ __________ $\cot(\theta) =$ __________

**Answers**:

$$\sin(\theta) = \tfrac{q}{r} \qquad \csc(\theta) = \tfrac{r}{q}$$
$$\cos(\theta) = \tfrac{p}{r} \qquad \sec(\theta) = \tfrac{r}{p} \quad .$$
$$\tan(\theta) = \tfrac{q}{p} \qquad \cot(\theta) = \tfrac{p}{q}$$

**3**. For the right triangle pictured here, find angle $A$ and side lengths $a$ and $c$. You must explain your reasoning. (Round answers to two decimal places.)




**Solution**:

$$A = 90° - 67.3° = 22.7°.$$

Also

$$\tan(A) = \frac{a}{126}$$

and hence

$$a = 126\tan(22.7°) \approx 52.71.$$

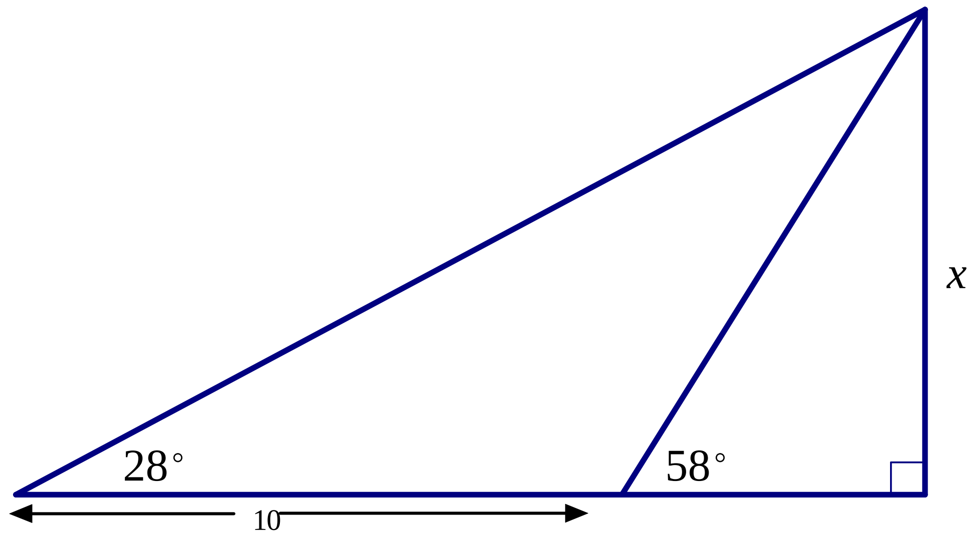
To find $c$ we can use

$$\sin(67.3°) = \frac{126}{c}$$

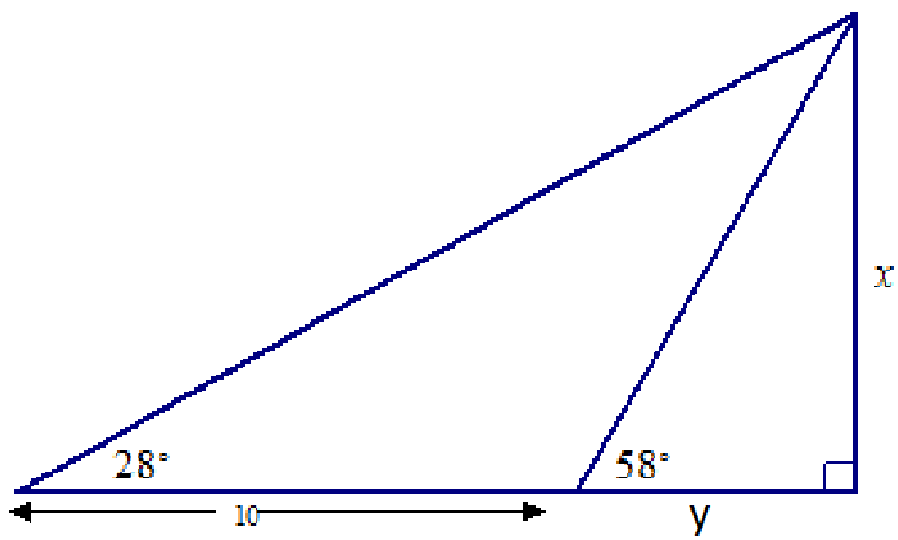
which gives

$$c = \frac{126}{\sin(67.3°)} \approx 136.58.$$

**4**. In the figure shown below, find the length $x$. You must include all details. (You can round your answer to two decimal places.)




**Solution**: Let *y* be the length pictured:




Then

$$\tan(28°) = \frac{x}{y+10}$$

and

$$\tan(58°) = \frac{x}{y}.$$

This gives

$$y + 10 = \frac{x}{\tan(28°)}$$

and

$$y = \frac{x}{\tan(58°)}.$$

From the above two equations we obtain

$$\frac{x}{\tan(28°)} - \frac{x}{\tan(58°)} = 10.$$

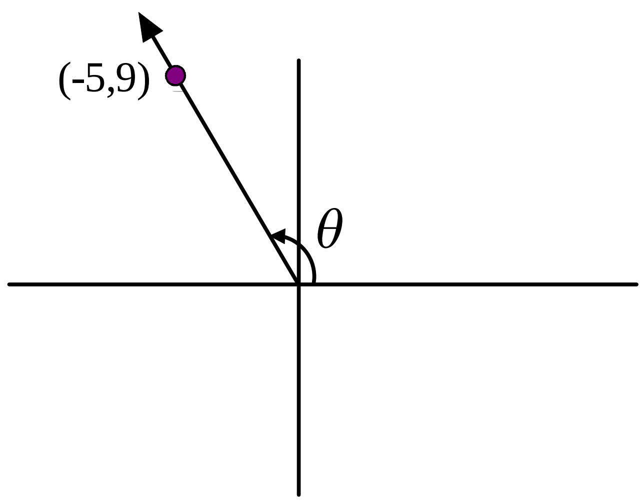
Multiplying both sides of the above equation by $\tan(28°)\tan(58°)$ gives

$$x(\tan(58°) - \tan(28°)) = 10\tan(28°)\tan(58°)$$

which gives

$$x = \frac{10\tan(28°)\tan(58°)}{\tan(58°) - \tan(28°)} \approx 7.96.$$

**5**. Find the six trigonometric function values for the angle, $\theta$, pictured here. Show all of your work.




$\sin(\theta) =$ ____________ $\csc(\theta) =$ ____________

$\cos(\theta) =$ ____________ $\sec(\theta) =$ ____________

$\tan(\theta) =$ ____________ $\cot(\theta) =$ ____________

**Show work here**:

**Solution**: By the Pythagorean Theorem, the radius of the circle on which the point $(-5, 9)$ lies is

$$r = \sqrt{5^2 + 9^2} = \sqrt{106}.$$

Thus

$$\sin(\theta) = \frac{9}{\sqrt{106}} = \frac{9\sqrt{106}}{106} \qquad \csc(\theta) = \frac{\sqrt{106}}{9}$$

$$\cos(\theta) = \frac{-5}{\sqrt{106}} = \frac{-5\sqrt{106}}{106} \qquad \sec(\theta) = -\frac{\sqrt{106}}{5}$$

$$\tan(\theta) = -\frac{9}{5} \qquad \cot(\theta) = -\frac{5}{9}.$$

**6**.

**a**. Find a positive angle and a negative angle that are coterminal with $5\pi/3$.

**b**. Convert $5\pi/3$ to degrees. (You must show the calculation that you use to do this.)

**c**. Use your calculator to compute $\sin(739)$ and $\cos(739)$. (Note that the angle being referred to is 739 radians – not degrees.) Based on your calculations, which quadrant (I, II, III or IV) does the angle 739 radians lie in? Explain.

**d**. Suppose that $\theta$ is an angle such that $\tan(\theta) < 0$ and $\sin(\theta) > 0$. Which quadrant (I, II, III or IV) must $\theta$ lie in? Explain.

**Answers**: A positive angle that is coterminal with $5\pi/3$ is

$$\frac{5\pi}{3} + 2\pi = \frac{11\pi}{3}.$$

A negative angle that is coterminal with $5\pi/3$ is

$$\frac{5\pi}{3} - 2\pi = -\frac{\pi}{3}.$$

We convert $5\pi/3$ to degrees as follows:

$$\frac{5\pi}{3} = \frac{5\pi}{3}\left(\frac{180°}{\pi}\right) = 300°.$$

On my calculator (set in radians) I get

$$\sin(739) \approx -0.66 < 0$$

and

$$\tan(739) \approx 0.89 > 0.$$

Since $\sin(739) < 0$ and $\tan(739) > 0$, then the angle 739 radians lies in Quadrant III.

If $\theta$ is an angle such that $\tan(\theta) < 0$ and $\sin(\theta) > 0$, then $\theta$ must lie in Quadrant II.

**7**. A wheel with a 30 cm radius is rotating at a rate of 3 radians per second. Find the linear speed of a point on the rim of the wheel. Show all calculations. Give your answer in units of meters per minute. (You will use the fact that 1 meter equals 100 cm.)

**Solution**: The radius of the wheel is

$$r = 30\,\text{cm} = (30\,\text{cm})\left(\frac{1\,\text{m}}{100\,\text{cm}}\right) = 0.3\,\text{m}.$$

The angular speed of the wheel is

$$\omega = \frac{3}{1\,\text{s}} = \left(\frac{3}{1\,\text{s}}\right)\left(\frac{60\,\text{s}}{1\,\text{min}}\right).$$

Thus the linear speed is

$$v = r\omega = (30\,\text{cm})\left(\frac{1\,\text{m}}{100\,\text{cm}}\right)\left(\frac{3}{1\,\text{s}}\right)\left(\frac{60\,\text{s}}{1\,\text{min}}\right) = 54\frac{\text{m}}{\text{min}}.$$