

# Summary of Trigonometric Identities

## The Basic Identities

$$\begin{aligned}\csc(\theta) &= 1/\sin(\theta) & \sin(\theta) &= 1/\csc(\theta) \\ \sec(\theta) &= 1/\cos(\theta) & \cos(\theta) &= 1/\sec(\theta) \\ \cot(\theta) &= 1/\tan(\theta) & \tan(\theta) &= 1/\cot(\theta) \\ \cot(\theta) &= \cos(\theta)/\sin(\theta) & \tan(\theta) &= \sin(\theta)/\cos(\theta)\end{aligned}$$

## The Even–Odd Identities

$$\begin{aligned}\cos(-\theta) &= \cos(\theta) \\ \sin(-\theta) &= -\sin(\theta) \\ \tan(-\theta) &= -\tan(\theta) \\ \cot(-\theta) &= -\cot(\theta) \\ \sec(-\theta) &= \sec(\theta) \\ \csc(-\theta) &= -\csc(\theta)\end{aligned}$$

## The Pythagorean Identities

$$\begin{aligned}\cos^2(\theta) + \sin^2(\theta) &= 1 \\ 1 + \tan^2(\theta) &= \sec^2(\theta) \\ \cot^2(\theta) + 1 &= \csc^2(\theta)\end{aligned}$$

## Sum and Difference Identities for Sine and Cosine

$$\begin{aligned}\cos(\alpha - \beta) &= \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta) \\ \cos(\alpha + \beta) &= \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta) \\ \sin(\alpha - \beta) &= \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta) \\ \sin(\alpha + \beta) &= \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)\end{aligned}$$

### The Cofunction Identities

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin(\theta)$$

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos(\theta)$$

### Sum and Difference Identities for Tangent

$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)}$$

$$\tan(\alpha - \beta) = \frac{\tan(\alpha) - \tan(\beta)}{1 + \tan(\alpha)\tan(\beta)}$$

### The Double Angle Identities

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

$$\cos(2\theta) = 1 - 2\sin^2(\theta)$$

$$\cos(2\theta) = 2\cos^2(\theta) - 1$$

$$\tan(2\theta) = \frac{2\tan(\theta)}{1 - \tan^2(\theta)}$$

### Product to Sum Identities

$$\sin(\alpha)\sin(\beta) = \frac{1}{2}(\cos(\alpha - \beta) - \cos(\alpha + \beta))$$

$$\cos(\alpha)\cos(\beta) = \frac{1}{2}(\cos(\alpha - \beta) + \cos(\alpha + \beta))$$

$$\sin(\alpha)\cos(\beta) = \frac{1}{2}(\sin(\alpha + \beta) + \sin(\alpha - \beta))$$

$$\cos(\alpha)\sin(\beta) = \frac{1}{2}(\sin(\alpha + \beta) - \sin(\alpha - \beta))$$

### Sum to Product Identities

$$\sin(x) + \sin(y) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin(x) - \sin(y) = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos(y) + \cos(x) = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\cos(y) - \cos(x) = 2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$