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1 Introduction

by Dr. Joel Fowler

Welcome to the first issue of the Southern Polytechnic State University Mathematics and Physics Newsletter sponsored by the Student Chapter of the Mathematical Association of America and the Society of Physics Students. In it you’ll find contributions from both students and faculty in the Mathematics and Physics Departments. The contributors and organizers deserve a great deal of credit for taking the idea of a newsletter, first raised by students in Fall 2003, all the way to a finished product in less than one term. They have my thanks and congratulations. The newsletter is a chance to see some of the physics and mathematics that students and faculty are thinking about and working on at Southern Polytechnic. Look for future issues to continue to serve as a forum for interesting articles that range from problems (solved and unsolved) to informative and expository material to new ideas and results. I’m very pleased to introduce what I am sure will become a continuing part of the SPSU community.

2 Contact Information

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3 Gramps’ Dilemma and
the Rotation Matrix
by Jacob Keenum

Before I actually cover the meat of this article, I feel a little background
information is in order. First consider the rotation matrix in $\mathbb{R}^2$.
For $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by

$$T \left( \begin{bmatrix} x \\ y \end{bmatrix} \right) = A \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

we know $A = [T(\hat{e}_1) T(\hat{e}_2)]$, where

$$\hat{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \text{and} \quad \hat{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$ 

We know that

$$T(\hat{e}_1) = \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix} \quad \text{and} \quad T(\hat{e}_2) = \begin{bmatrix} -\sin \theta \\ \cos \theta \end{bmatrix}$$

and so

$$A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix},$$

but I feel compelled to give a “heuristic” derivation of sorts. Consider ro-
tating $\hat{e}_1$ and $\hat{e}_2$ by $\theta$. Pictorially, we can think of this as shown on the
next page (the notation in the figure is slightly different due to “notorious”
difficulties with displaying symbols in MAPLE-generated graphs).
Gran ted, this “derivation” lacks any real formality but if suffices to illustrate the motivation for the rotation matrix $A$ in $\mathbb{R}^2$.

We know that rotating the graph of $y = \kappa / x$ ($\kappa \in \mathbb{R}^*$) will reveal that it is really a hyperbola. Consider taking any point $(x, y)$ that satisfies $x \cdot y = \kappa$ and hitting it with a rotation transformation given by

$$
T \left( \begin{bmatrix} x \\ y \end{bmatrix} \right) = A \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}, \quad A = \begin{bmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{bmatrix}.
$$

This says $x' = x \cos t - y \sin t$ and $y' = x \sin t + y \cos t$. So consider $t = \frac{\pi}{4}$ (radians) so that $\cos t = \sin t = \frac{\sqrt{2}}{2}$. Then

$$
x' = \frac{\sqrt{2}}{2} x - \frac{\sqrt{2}}{2} y
$$

$$
y' = \frac{\sqrt{2}}{2} x + \frac{\sqrt{2}}{2} y.
$$

Now $x' + y' = \sqrt{2} x$ and $y' - x' = \sqrt{2} y$. Thus

$$
(x' + y') (y' - x') = (y')^2 - (x')^2 = 2xy = 2\kappa = m.
$$

Hence

$$
\frac{(y')^2}{m} - \frac{(x')^2}{m} = 1
$$

and clearly $m \neq 0$ since $\kappa \in \mathbb{R}^*$. So $xy = \kappa$ is thought of as a hyperbola.
This, I find, is a very nice application of the rotation matrix in $\mathbb{R}^2$. For the longest time, I was not fully convinced that the shape of $xy = \kappa$ is that of a hyperbola. However, by using the rotation matrix, I have finally created, at least in my mind (and for my intents and purposes), a satisfactory explanation of why the above said is like so.

Be that as it may, my grandfather came visiting recently, and lo and behold, he had a problem for me. Now, gramps, as I like to affectionately call him, isn’t of the mathematical nature any more than your typical high school student. However, he is more keen on his math skills than anyone else in the rest of my immediate family, relative pool, and co. He solved his
problem by implementing some simple geometry methods and then solved it again utilizing some basic algebra. His dilemma went like this:

He has a tower with a rotating “boom” as he called it. This boom is perpendicular to the tower, and on the boom are several elements of varying length, the longest of which is located at the end of the boom. So from the top (i.e., if you were to look down at this setup from above) the setup would look something like this:

![Diagram of a tower with a rotating boom and elements]

He said the boom (and for that matter, the entire setup) experiences continual circular motion. Let \( r_1 \) denote the length of half the boom and \( \ell_1 \) denote the length of half of the longest element. He gave me actual numbers to work with, but they are immaterial here. He wanted to get a new boom of half-length \( r_2 \), where \( r_2 < r_1 \) and the half-length of the longest element (at the end of the boom), denoted \( \ell_2 \), is longer than the old one (i.e., \( \ell_2 > \ell_1 \)). He wanted to know if a new setup would be able to rotate freely without whacking any trees. That was his dilemma.

He had solved it by the time he came up for the visit; he really just wanted to run it by me and see what I had to say. Again, the problem is trivial; however, it triggered something in my mind. A new, more interesting problem surfaced: How to describe the motion of the longest element on the boom as relevant to the boom’s motion?

Well, I considered visualizing the setup like so (but due to MAPLE’s inability to display “\( \ell \)” this figure uses L instead, and recall the use of X for \( x’ \) and Y for \( y’ \)):  

6
Here $\vec{r} = \langle x, y \rangle = \langle r \cos t, r \sin t \rangle$ and $\vec{r} + \vec{\ell} = \langle x', y' \rangle$. But what is $\vec{\ell}$?

Well, $\vec{\ell}$ is clearly orthogonal to $\vec{r}$ and so $\vec{r} \cdot \vec{\ell} = 0$. Since $\vec{r} = \langle x, y \rangle$, consider $\vec{\ell} = c(-y, x)$. If the scalar $c$ were not included, then $\|\vec{r}\| = \|\vec{\ell}\|$, which is not the case in Gramps’ dilemma. His setup (both old and new) would have $\|\vec{\ell}\| > \|\vec{r}\|$. So if $\vec{\ell} = c(-y, x)$ then

$$\|\vec{\ell}\| = c \sqrt{x^2 + y^2} = cr.$$  

But $\|\vec{\ell}\| = \ell$, so $cr = \ell \Rightarrow c = \frac{\ell}{r}$. That is,

$$\vec{\ell} = \langle -\frac{\ell}{r} y, \frac{\ell}{r} x \rangle .$$

So

$$\vec{r} + \vec{\ell} = (x - \frac{\ell}{r} y, \frac{\ell}{r} x + y) = \langle x', y' \rangle .$$

Thinking of this as a system of equations, we have

$$x' = x - \frac{\ell}{r} y$$  
$$y' = \frac{\ell}{r} x + y$$

or

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & -\frac{\ell}{r} \\ \frac{\ell}{r} & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$ (1)
But $x = r \cos t, y = r \sin t$. So

\[
\begin{align*}
x' &= r \cos t - \ell \sin t \\
y' &= r \sin t + \ell \cos t
\end{align*}
\]

or

\[
\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{bmatrix} \begin{bmatrix} r \\ \ell \end{bmatrix}
\]

(2)

Alas, the rotation matrix in $\mathbb{R}^2$ creeps into my life again. I find Eq. 2 to be the nicer system seeing as it expresses $x'$ and $y'$ in terms of three unknowns while Eq. 1 does it in four unknowns.

At this point, from the system in Eq. 1, one might consider $(x')^2 + (y')^2$. Since $x' = x - \frac{\ell}{r} y$,

\[
(x')^2 = x^2 - 2xy\frac{\ell}{r} + \frac{\ell^2}{r^2}y^2,
\]

and since $y' = \frac{\ell}{r}x + y$,

\[
(y')^2 = \frac{\ell^2}{r^2}x^2 + 2xy\frac{\ell}{r} + y^2.
\]

So

\[
(x')^2 + (y')^2 = x^2 + \frac{\ell^2}{r^2}x^2 + \frac{\ell^2}{r^2}y^2 + y^2
\]

\[
= \left(1 + \frac{\ell^2}{r^2}\right)\left(x^2 + y^2\right)
\]

\[
= \left(1 + \frac{\ell^2}{r^2}\right)r^2 = r^2 + \ell^2,
\]

which is easy to intuit from the previous figure.

All in all, I really wasn’t expecting to see the rotation matrix pop up in Gramps’ dilemma, but sure enough it did. And as I said, for those who are curious, this problem (i.e., Gramps’ dilemma) is a triviality but here are his values:

\[
\ell_1 = 16 \quad ; \quad \ell_2 = 18 \\
r_1 = 9 \quad ; \quad r_2 = 6
\]

(the second column are the new values). So I confirmed what he already knew: his new setup might cause some problems.
4 Math Problems

by Dr. James Whitenton

A problem that Steve Edwards showed me, nearly eight years ago, involves the Lambert series

\[ f(r) = \sum_{m=1}^{\infty} \frac{1}{r^m - 1} \]  \hspace{1cm} (3)

for \( r > 1 \). Although Steve’s interest was in the \( r = 2 \) case of this, I became fascinated by the behavior of \( f(r) \) as \( r \to 1 \). Since

\[ r^m - 1 = (r - 1) \left( r^{m-1} + r^{m-2} + \cdots + 1 \right) \]  \hspace{1cm} (4)

then it’s clear that \( f(r) \) diverges as \( r \) approaches 1, but there is more going on in that climb to infinity than one might expect. So the first problem I’ll pose here is the following:

is the limit of \( (r - 1)f(r) \) \hspace{1cm} (5)

finite for \( r \to 1 \)? If so, what is its value?

Something that one might have already seen in Calculus is the indefinite integral of \( \text{csch}^n \) (powers of hyperbolic cosecants):

\[ \int \text{csch}^n(x) \, dx = \int \frac{1}{\sinh^n(x)} \, dx . \]  \hspace{1cm} (6)

It’s quite striking that for \( n \) a positive even integer, the indefinite integrals of \( \text{csch}^n \) give a linear combination of odd powers of hyperbolic cotangents:

\[ \int \frac{1}{\sinh^n(x)} \, dx = \sum_{i} c_i \coth^i(x) \]  \hspace{1cm} (7)

where \( c_i = 0 \) for \( i = \) even. It is not surprising that \( c_i \) can be related to binomial coefficients. The second problem here, then, is to precisely determine the \( c_i \) (and feel free to change the notation if you think that might make the final result [in terms of binomial coefficients] more elegant).

The third problem is to check a couple of series evaluations. This is sort of “true or false” – are the (two) equalities presented below correct or not? I
don’t really expect you to derive them (unless you’re very ambitious); looking
them up or checking them any way you can imagine will provide challenge
enough.
Let \( \chi(k) \) be +1 for \( k \equiv 1 \mod 4 \), -1 for \( k \equiv 3 \) and zero otherwise (in other
words, let \( \chi(k) = \sin(\pi k/2) \) for integer \( k \); \( \chi(k) \) is often written \( \chi_4(n) \) and
described as “the primitive character of modulus 4.” Does
\[
\sum_{k=1}^{\infty} \frac{\chi(k) k^5}{\cosh \left( \frac{\pi k \sqrt{3}}{2} \right)} = 0 \quad ?
\] (8)
And (for \( \beta > 0 \)) does
\[
\sum_{k=1}^{\infty} \frac{k}{e^{\beta k} - 1} = \frac{1}{24} - \frac{1}{2\beta} + \frac{\pi^2}{6\beta^2} \quad ?
\] (9)
Note: for this latter series, it might help to note that
\[
\sum_{k=1}^{\infty} \frac{k}{e^{\beta k} - 1} = \frac{1}{4} \sum_{k=1}^{\infty} \frac{1}{\sinh^2 \left( \frac{k\beta}{2} \right)} .
\]
Now, as the fourth and final problem, I’m going to ask you to calculate the
determinants where the entries are consecutive prime numbers, arranged in
an odd-looking (“anti-diagonal”) way. Note that, nowadays, mathematicians
do not include 1 in the list of prime numbers. Calculate the determinant of
\[
\begin{bmatrix}
2 & 3 & 5 \\
3 & 5 & 7 \\
5 & 7 & 11
\end{bmatrix} .
\] (10)
Calculate the determinant of
\[
\begin{bmatrix}
2 & 3 & 5 \\
3 & 5 & 7 \\
5 & 7 & 11
\end{bmatrix} .
\] (11)
Calculate the determinant of
\[
\begin{bmatrix}
2 & 3 & 5 & 7 \\
3 & 5 & 7 & 11 \\
5 & 7 & 11 & 13 \\
7 & 11 & 13 & 17
\end{bmatrix} .
\] (12)
This last result, particularly, might come as a surprise.
5 Websites

by Aiken Oliver

  MIT’s OpenCourseWare webpage doesn’t give you access to their professors, but it does let you take a look at the sort of things they’d be expecting their students to do. It covers a large variety of topics in varying detail, and is definitely worth taking a look at if you’re not already familiar with it.

- http://science.nasa.gov/RealTime/JTrack/Spacecraft.html
  NASA provides this to track earth orbiting bodies, from spacecraft to satellites, for the amateur observer. J-Track 3D is particularly neat.

- http://www.sciencedaily.com/
  For your daily dose of science news, look here. They’ll present a fairly comprehensive listing of major events in the scientific community.

- http://mathforum.org/
  A massive collection of resources for everyone from Elementary School to Graduate Level. You can take try the problem of the week, or even browse employment resources!

- http://www.curiousmath.com/
  A cute collection of math trivia that might be helpful, might amuse, and might even teach you how to do square roots in your head.

- http://www.aip.org/pt/
  Physics Today for your viewing pleasure. While the entire publication isn’t viewable without a subscription, there are always free articles available.

  Need a quick index of physics laws and experiments? Some notable ones are here, with quick explanations for each. While not usually deep, it’s rather useful for refreshing your memory.
I'd like to acquaint you with a unique program sponsored by the National Science Foundation (NSF). It is called the Research Experience for Undergraduates Program (REU), and it provides opportunities for undergraduates to do research at top universities, national research centers and private industry laboratories. Under the direction of research faculty, students conduct research in an area that interests them. There are many areas to choose from in biology, chemistry, physics, and mathematics. Students selected for this program are granted stipends and, in many cases, housing and travel allowances are provided. Some sites also award semester credits that are transferable to SPSU. These summer programs are usually 9-10 weeks in duration, and in some cases, the start and ending dates are flexible.

Since budget constraints make offering upper level science and math courses in the summer prohibitive, an REU appointment would be a great alternative for SPSU students. It is also an excellent way to meet scientists and mathematicians involved in meaningful research. Some of our undergraduate physics majors have been awarded REU grants to such institutions as the NASA Goddard Space Flight Center, Los Alamos National Laboratory, and Georgia Tech. One recent REU student is senior physics major Sandra Valencia, who worked the previous summer at the Goddard Space Flight Center. Sandra worked in the department of Atmospheric and Hydropheric Sciences and her research dealt with the study of airborne particles. When asked about her experience, she replied “Working at NASA Goddard was a great experience. It was like a dream coming true. The best thing was that you meet very bright people but really down to Earth. Also it was a good challenge to put my physics knowledge in practice by applying it to analyze the structure of aerosols and clouds.”

General information about the REU program can be found at the NSF website:


Posters advertising many REU opportunities in the lab sciences are displayed in the Physics student workroom, E170. Other information and some application forms for some of these programs are in the Physics office located in E183.
7 Remarks and Acknowledgements

Special thanks go to the presidents of the SPSU student groups in Math and Physics: Matt Turner, Opal Ramocan and Aiken Oliver.

The vast majority of Math and Physics research articles are produced using one version or other of Donald’s Knuth’s \text{T\LaTeX}. This newsletter was written using a popular version developed by Leslie Lamport known as \text{L\LaTeX}. All this is shareware; it’s all free (if you know where to find it). Professor Ziegler did a great job generating very nice MAPLE files (which we would be glad to email to anyone who would like them) for the figures in the Gramps’ dilemma article. Unfortunately, the software used to turn the MAPLE figures into postscript figures which in turn were incorporated into the final \text{L\LaTeX}’ed product was fairly primitive and lost some of the detail present in Professor Ziegler’s original figures. Converting the MAPLE graphs to gif figures, on the other hand, did not cause any problems, and – again – if you would like copies of the gif files we would be glad to email them to you (they do look a little better than the figures printed in this newsletter). Professor Dillon has a more sophisticated system to work with \text{L\LaTeX} which can produce pdf files and other formats besides, so – if you are interested in finding out more about \text{L\LaTeX} and how to use it to best effect, she is probably the best person to ask.

We should add that the obvious need for a good way to search for math formulas may soon be met in the form of MathML; see

http://www.ima.umn.edu/complex/spring/searching.html