P, T, and ρ Altitudes

- Pressure altitude: Standard altitude that corresponds to a given pressure
- Temperature altitude: Standard altitude that corresponds to a given temperature
- Density altitude: Standard altitude that corresponds to a given density

Example: An airplane is flying at an altitude where the actual pressure and temperature are $4.72 \times 10^4$ N/m$^2$ and 255.7 K, respectively. Determine pressure, temperature, and density altitudes.

1) From Appendix D (5th Edition) *Standard Atmosphere*. Standard altitude value corresponding to $P = 4.72 \times 10^4$ N/m$^2$ is 6000m
2) From Appendix D (5th Edition) *Standard Atmosphere*. Standard altitude value corresponding to $T = 255.7$ K is 5000m
3) For density altitude value corresponding to $\rho = 0.643$ kg/m$^3$ is 6240m

Note: Temperature altitude has limited usefulness – multiple altitudes can have the same temperature.
Basic Aerodynamics

• Airplane flying at 10,000 ft. and 200 knots experiences certain amount of pressure and velocity at a given point near the wing tip
• A Space Shuttle engine experiences certain amount of pressure and temperature during takeoff
• These quantities need to be calculated using the laws of nature
• **Inviscid flow** is flow with no friction (used for simplifying math)
• **Viscous flow** is flow with friction

• **Continuity Equation:**
  • Conservation of mass: Mass can be neither created nor destroyed, except when considering \( E = mc^2 \)

![Diagram of flow through a cylindrical pipe](image)

• Consider streamlines that go through the circumference of the circle
• These streamlines form a tube called a stream tube
• The cross sectional area of the tube may change; e.g. water hose nozzle
• But for steady flow (invariant with time), the mass that flows through the cross section at point 1 is the same as that through point 2

Consider $A_1$ to be the cross sectional area of the stream tube; $V_1$ is the velocity at point 1

In a lapse of time $dt$, all the fluid elements move a distance $= V_1 dt$
Volume swept by the fluid elements $= A_1 V_1 dt$
Mass of gas ‘$dm’ in this volume $= \rho_1 A_1 V_1 dt$

• The mass flow $\dot{m}$ through an area $A$ is the mass crossing $A$ per unit time.
• Mass flow $= \frac{dm}{dt} = \dot{m}_1 = \rho_1 A_1 V_1$ (kg/s or slugs/s)
• Similarly the mass flow through $\dot{m}_2 = \rho_2 A_2 V_2$, because mass can be neither created or destroyed.
• Therefore, $\dot{m}_1 = \dot{m}_2 \rightarrow \rho_1 A_1 V_1 = \rho_2 A_2 V_2$ (1)
• (1) is referred to as the continuity equation for steady fluid flow
• Continuity Equation is applicable to flow through ducts, tubes, wind, tunnels, rocket engines, airfoils, and so on.

\[
\frac{V_1}{A_1, \rho_1} = \frac{V_2}{A_2, \rho_2}
\]

• Compressible Flow:
  • Flow in which the density of the fluid element can change from point to point; e.g. supersonic flow, rocket engines

• Incompressible Flow:
  • Flow in which the density of the fluid element is always constant
  • *In real life, gases get compressed during any flow, but the change in density is negligible.
  • Incompressible flow assumption is a good approximation of liquids in motion
  • Low speed flow of air, where \( V < 100 \text{ m/s (225 mi/hr)} \) can be assumed to be incompressible
For incompressible flow $\rho_1 = \rho_2 = \rho$

$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$
$\rho A_1 V_1 = \rho A_2 V_2$
$V_2 = (A_1/A_2)V_1$
Since $A_2 < A_1 \Rightarrow V_2 > V_1$

Momentum Equation
- Newton’s Second Law of Motion applies to fluids
- $F = ma$
• Pressure on left face = P
• Area of left face = dydz
• Force on left face = Pdydz
• Pressure changes from point to point in the flow
• Change in pressure per unit length is \( \frac{dP}{dx} \)
• At the distance dx from the left face, the pressure is \( \frac{dP}{dx} dx \)
• Therefore, the pressure on the right face = \( P + \frac{dP}{dx} dx \)
• Force on the right face = \( [P + \frac{dP}{dx} dx] dydz \) \{acts in the \( -x \) direction\}
• The net force in the \( x \)-direction is: \( F = P dydz - [P + \frac{dP}{dx} dx] dydz = -\frac{dP}{dx} dx dydz \)
• *mass of the fluid element \( m = \rho dx dydz \)
• *acceleration of the fluid element \( a = \frac{dV}{dt}, V = \frac{dx}{dt} \)
• \( a = \frac{dV}{dt} = \frac{dV}{dx} \frac{dx}{dt} = \frac{dV}{dx} V \)
Now \( F = ma \) yields
\[
\frac{dP}{dx} \, dx \, dy \, dz = \rho \, dx \, dy \, dz \, * \, \frac{dV}{dx} \, V \rightarrow dP = -\rho V \, dv \quad (2) \]
Euler’s Equation

Leonhard Euler (1707 – 1783), Swiss mathematician, physicist, graph theory
Euler Equation relates the rate of change of momentum to force
Euler Equation is valid for inviscid flow
  • Gravity is also neglected
  • Flow is assumed to be steady
Bernoulli’s Equation
  • Daniel Bernoulli (1700 – 1782), Dutch mathematician → fluid mechanics,
  • probability, statistics
  • Consider the flow over the following airfoil
• To relate $P_1$ and $V_1$ at point 1 to $P_2$ and $V_2$ at point 2, the Euler’s Equation is Integrated:

\[
dP = -\rho V \, dv \\
dP + -\rho V \, dv = 0
\]

\[
\int_{P_1}^{P_2} dP + \rho \int_{V_1}^{V_2} V \, dv = 0
\]

\[
P_2 - P_1 + \rho \left(\frac{V_2^2}{2} - \frac{V_1^2}{2}\right) = 0
\]

\[
P_1 + \rho \frac{V_1^2}{2} = P_2 + \rho \frac{V_2^2}{2} \quad (3) \text{ Bernoulli’s Equation}
\]

\[
P + \rho \frac{V^2}{2} = \text{constant along streamline}
\]

Important points about Bernoulli’s Equation:
• Only valid for inviscid (frictionless), incompressible flow
• Relate properties between different points along a streamline
• Not valid for compressible flow, must be treated with $\rho$ as a variable
• Euler’s and Bernoulli’s equations are essentially Newton’s Second Law applied to fluid mechanics