From Equal Groups to Proportional Reasoning
Multiplicative Comparison as Key Structure

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Ratios & Proportional Relationships

Grade 6 Overview

(1) Connecting ratio and rate to whole number multiplication and division…

By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions.

Multiplication/Division

- Multiplication and division in equal group situations (3.OA.A.1 & 3.OA.A.2)
- Multiplication in array/area situations (3.MD.C)

- Multiplication and division in comparison situation (4.OA.A.1 & 4.OA.A.2)

- Multiplication as scaling (5.NF.B.5)
- Division as?
  - 5.NF.B.7.a: Interpret division of a unit fraction by a non-zero whole number, and compute such quotients.
  - 5.NF.B.7.b: Interpret division of a whole number by a unit fraction, and compute such quotients.

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$, and $18 \div 3 = ?$	$? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[4] Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

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4.OA.A.1 Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

4.OA.A.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

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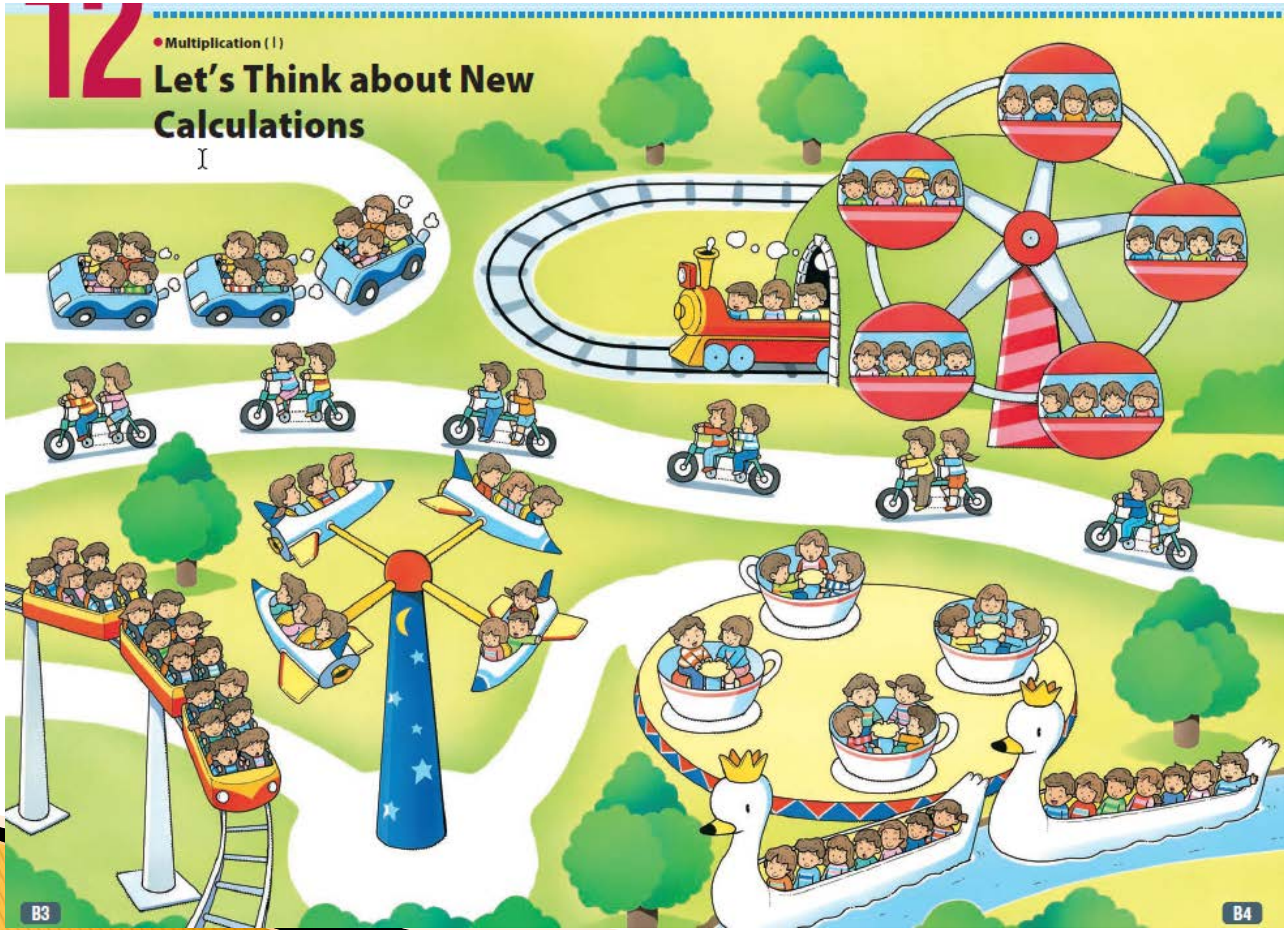
3.OA.A.1 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.

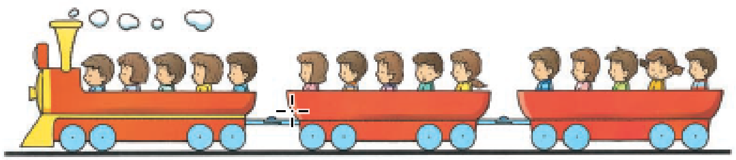
4.OA.A.* Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the base number to which 56 is compared, or as a number of times as much 56 is as 8.

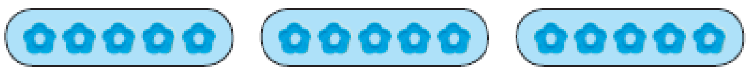
Multiplicative Comparison in Japanese curriculum

- Grade 2: Introduction of multiplication



2 How many children are riding on the train altogether?

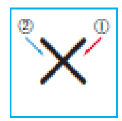




There are 5 children in each train car and there are 3 train cars. There are 15 children altogether.
You can write this using the following math sentence.

$$5 \times 3 = 15$$

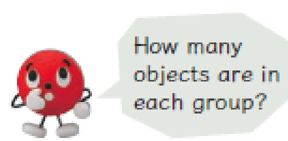
Five multiplied by three equals fifteen.



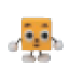
5	×	3	=	15
Number of objects in each group		Number of groups		Total number of objects

★1 Write a math sentence for the number of people riding [tandem bicycle] on page 5. Write the math sentence just like we did above.

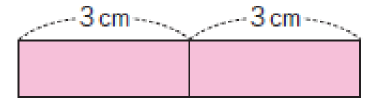
Math Sentence: ☐ (Number of objects in each group) × ☐ (Number of groups) = ☐ (Total number of objects)



Calculations like 5×3 and 2×6 are called **multiplication**.


This is a new calculation.

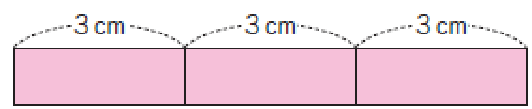
4 What is the length of two 3cm strips of paper put together?



Math Sentence ______________ Answer ☐ cm

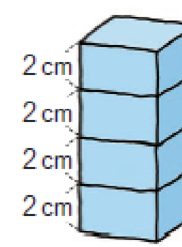
If a piece of tape is as long as two 3cm strips of paper put together, we can say the tape is **2 times** as long as the 3cm tape.
You can use the multiplication math sentence 3×2 to find the length that is two times as long as 3cm.

★1 If a piece of tape is as long as three 3cm strips of paper put together, how many times as long is it as a 3cm strips of paper? How long is the tape?



3 times or 4 times as much as an amount is the same as 3 or 4 sets of the amount put together. 1 time as much is the same as the given amount.

6 What is the height of the stack of boxes if it is 4 times as high as 2cm? Write a multiplication math sentence, and then find the answer.

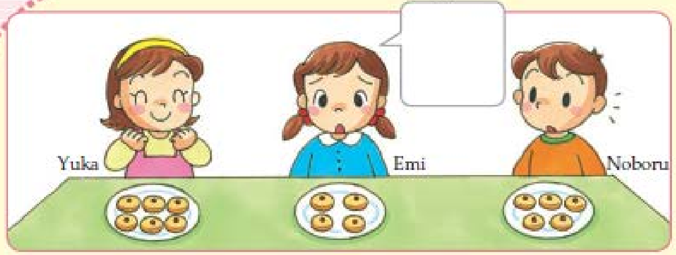


Grade 3: Introduction of division

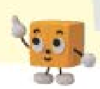
● Division

3 Let's Think about New Kinds of Calculations

Think about what Emi is saying.



? Let's think about what kind of calculation we need when we are dividing evenly.



They are helping each other to make sure everyone gets the same amount.

1 Calculations for Finding How Many for 1 Person

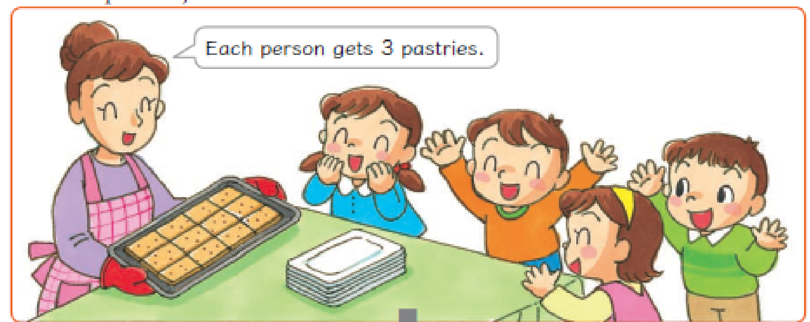
1 There are 12 cookies. If they are divided evenly among 3 people, how many cookies will 1 person get?

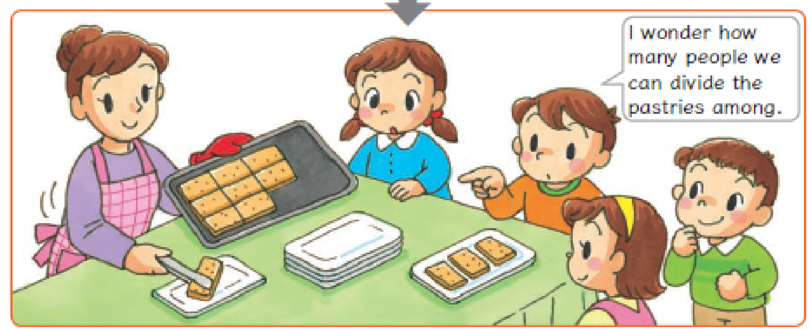
? Let's think about a calculation that finds how many for 1 person.

Investigate the amount for 1 person using counters.

2 Calculations for Finding the Number of People We Can Divide Something into

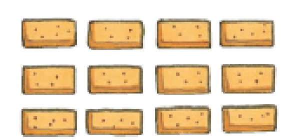
12 pastries just came out of the oven.





? Let's think about what kind of calculation we need to use to find how many people we can divide something into.

1 There are 12 pastries. If we give 3 pastries to each person, how many people can share the pastries?

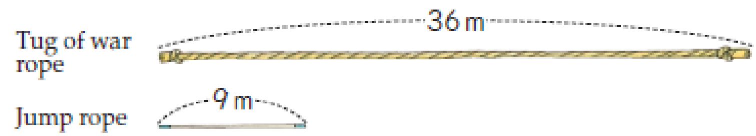


Use counters to investigate how many people can get pastries.

Division as the operation to find how many times as much

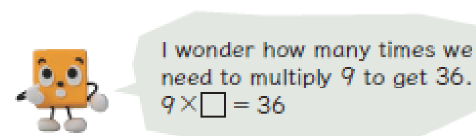
3 Calculations for Finding Times as Much

1 The length of a tug of war rope is 36m. The length of a jump rope is 9m.
How many times as long is the tug of war rope as the jump rope?



Let's think about what kind of calculation we need to use to find how many times as much.

1. Look at the diagram below and think about it.



$36 \div 9 = \square$ Answer □ times

Summary: To find how many times as much, you can use division.

1 During the team jump rope competition, Chiemi's team jumped 21 times and Takashi's team jumped 7 times.
How many times as much did Chiemi's team jump as Takashi's team?

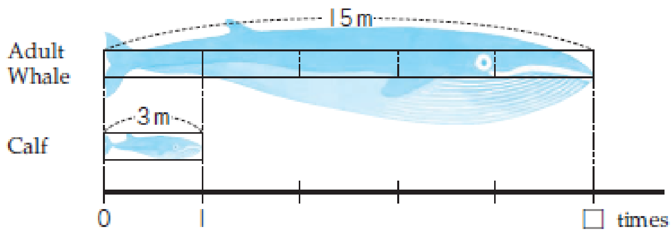
Additional Problems → Page 110 E

Grade 4: Division

4 Calculation with Times as Much

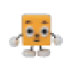
1 An adult whale is 15m long, and its calf is 3m long. How many times as long is the adult whale as the calf?

1 Think about it using the diagram below.



2 Write a math sentence, and then find the answer.

$15 \div 3 = \square$ Answer ☐ times



$3 \times \square = 15$

5 times means that if we consider 3m as 1, 15m corresponds to 5.

If we say 1 piece is 3 m long, 15 m is the same as 5 pieces together, isn't it?

1 Ayumi's class has pet hamsters. At the beginning of the school year, there were 4 hamsters, but now there are 24 of them. Answer the following questions:

① How many times as many hamsters are there now as there were at the beginning of the school year?

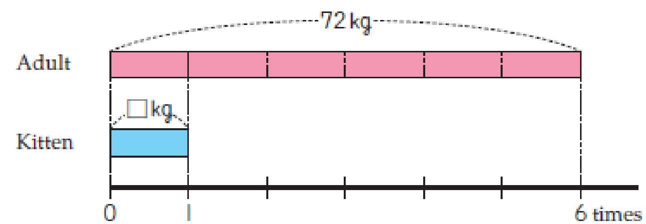
② If we consider 4 hamsters as 1, what number will 24 hamsters correspond to?

3 An adult jaguar weighs 6 times as much as a jaguar kitten. If the adult jaguar weighs 72kg, how much does the kitten weigh in kg?

? **Let's think about how to calculate the amount for 1.**

1 Write a multiplication sentence by considering the weight of the jaguar kitten as ☐ kg.

2 What calculation is necessary to find the number in the ☐? Think about it using the diagram below.



$\square \times 6 = 72$

$\square = 72 \div 6$

$= 12$ Answer 12kg

3 A story-book costs 920 yen. The price of the story-book is 4 times as much as the price of a comic book. Write a multiplication sentence by considering the price of the comic book as ☐ yen. Then, find the number that goes in to the ☐.

Grade 4 Application of multiplicative comparison

Problems with times as much

We are going to buy 12 cups each of yogurt and pudding.

4 You can buy a pack of 3 cups of yogurt for 240 yen.
How much will it cost to buy 12 cups of yogurt?

1 pack
240 yen

1 Explain the two students' thinking.

Shinji
I thought about the price of each cup of yogurt.
$240 \div 3 = 80$
$80 \times 12 = 960$
Answer 960 yen

Yumi
I thought about how many times 12 is compared to 3.
$12 \div 3 = 4$
$240 \times 4 = 960$
Answer 960 yen

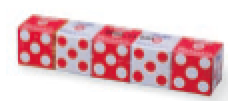
2 Pudding is also sold in packs of 3, and each pack costs 200 yen.
How much will it cost to buy 12 cups of pudding?

1 pack
200 yen

Shinji: If we calculate the price of each cup of pudding....

4 Calculate the cost of the following items.

① The price of 5 pieces of caramel is 120 yen. How much will it cost to buy 15 pieces of caramel?



② The price of 3 sticks of sweet dumpling is 100 yen. How much will it cost to buy 21 sticks of sweet dumpling?

Multiplication as scaling

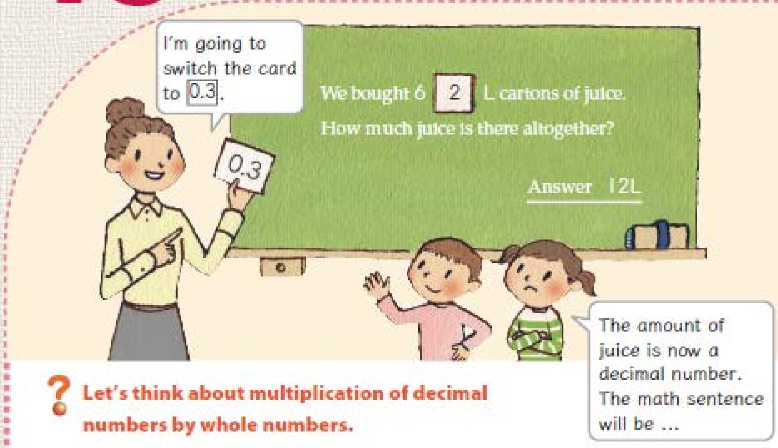
- When the multiplier (# of groups) become something other than whole numbers, we need to extend our interpretation of multiplication – “equal groups” is no longer sufficient.

Multiplying and dividing decimals

15

● Multiplication and Division of Decimal Numbers

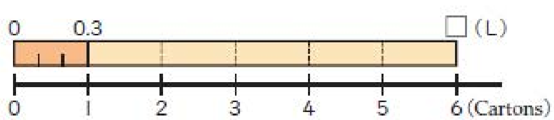
Let's Think about Multiplying and Dividing Decimal Numbers



? **Let's think about multiplication of decimal numbers by whole numbers.**

1 Multiplying Decimal Numbers

1 We bought 6, 0.3L cartons of juice. How much juice is there altogether?



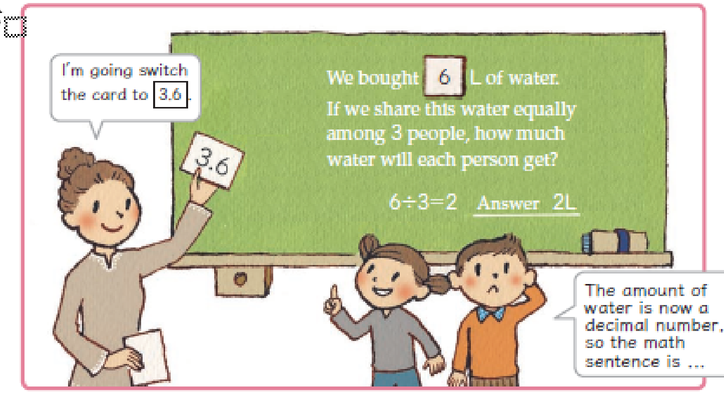
What math sentence should we write?

Math Sentence

Can you explain the reason for the math sentence?

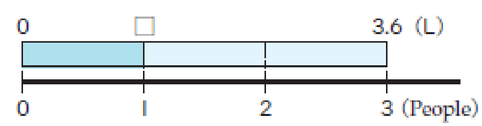
? **Let's think about how to calculate.**

2 Dividing Decimal Numbers



? **Let's think about dividing decimal numbers by whole numbers.**

1 We bought 3.6L of water. If we share this water equally among 3 people, how much water will each person get?



What math sentence should we write?

Math Sentence

Can you explain the idea behind why you wrote this math sentence?

? **Let's think about how to calculate.**

Multiplying and dividing decimals

3 Times as Much and Decimal Numbers

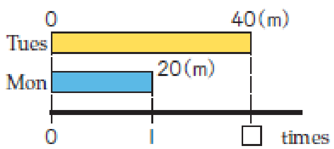
Taichi started learning to ride a unicycle this week.

1 The table on the right shows the longest distance Taichi could ride his unicycle without his feet touching the ground on each day.
Compared to the distance on Monday, how many times as much is the distance for each of the other days of the week?

Taichi's Unicycle Record

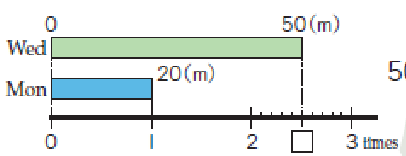
	Distance (m)
Mon	20
Tues	40
Wed	50
Thurs	30

1 The distance on Tuesday is how many times as much as the distance on Monday?

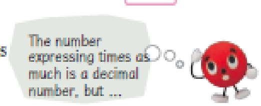


40 ÷ 20 = □ (times)

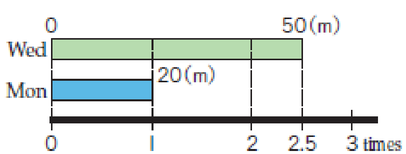
2 The distance on Wednesday is how many times as much as the distance on Monday?



50 ÷ 20 = □ (times)

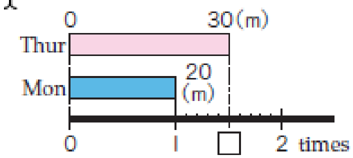


? Let's think about what times as much means with a decimal.



If we consider the distance on Monday as 1, the distance on Wednesday can be considered as 2.5.

3 How many times as much is the distance on Thursday as the distance on Monday?

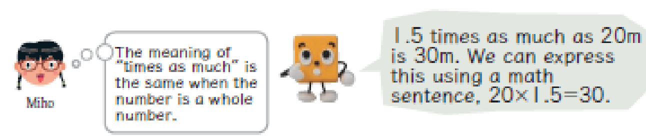


30 ÷ 20 = □ times

Summary

We can use decimal numbers to express "times as much", such as 2.5 times as much or 1.5 times as much.

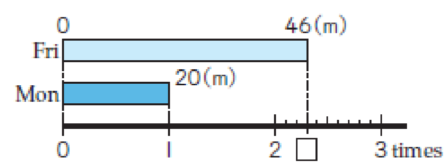
1.5 times means if we consider 20m as 1, 30m will be considered as 1.5.



4 Taichi's record on Friday was 46m. How many times as much is the distance on Friday as the distance on Monday.

Taichi's Unicycle Record

	Distance (m)
Mon	20
Fri	46



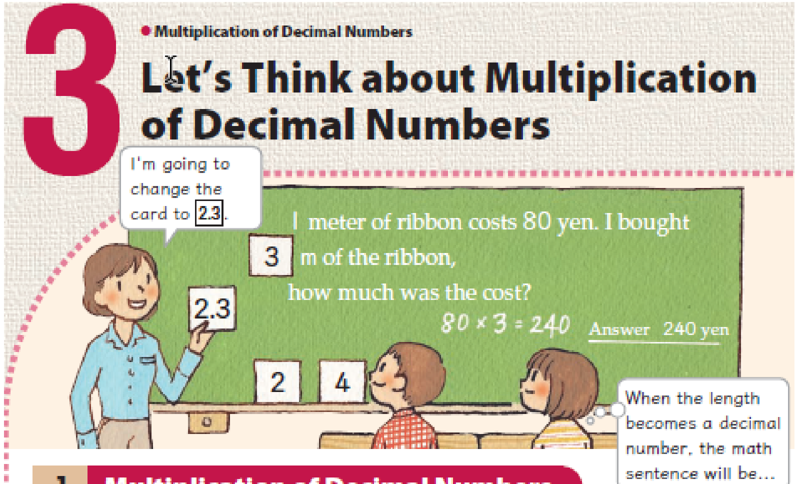
46 ÷ 20 = □ (times)

1 A dictionary costs 2800 yen. A storybook costs 800 yen.
How many times as much does the dictionary cost as the storybook?

Multiplying and dividing by decimals

3 ● Multiplication of Decimal Numbers

Let's Think about Multiplication of Decimal Numbers



1 Multiplication of Decimal Numbers

1 1 meter of ribbon costs 80 yen. I bought 2.3m of the ribbon, how much was the cost?

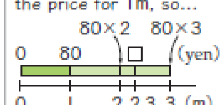
? Let's think about what math sentence we should write.

Math Sentence

Takumi: If it were 3m, we could think of it as three times the cost of 1m of ribbon, but...

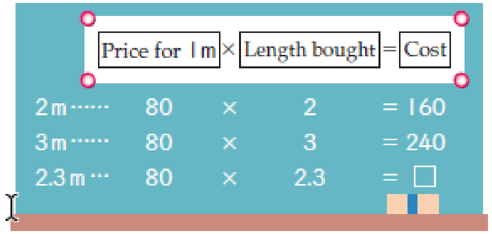
Hiroki: Price for 1m × Length bought = Cost

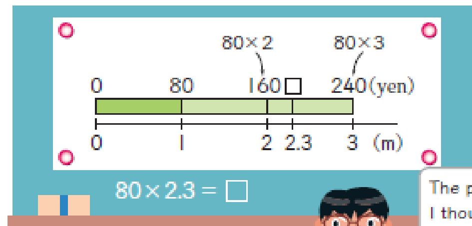
Miho: If we buy 2m or 3m, the cost will be 2 and 3 times the price for 1m, so...



Explain why the math sentence is written in this way.

Hiroki: I thought we could think of the lengths as if they were whole numbers.





Miho: The price for 1m is 80 yen. I thought the cost for 2.3m should be 2.3 times 80 yen, and that's why I thought we could use multiplication.

Summary

Even when the length of ribbon is a decimal number, we can use a multiplication sentence to find the total cost, just like we did when the lengths were whole numbers.

80×2.3 — About how much will it be? It will be greater than 80×2, but 80×3...

? Let's think about how to calculate.

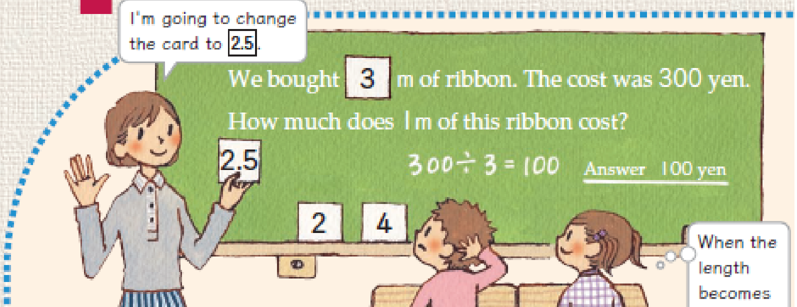
Takumi: We can find the price for 0.1m, then find 23 times as much...

Kaori: We can find the cost of 23m of the ribbon first, then find $\frac{1}{10}$ of that cost...

Multiplying and dividing by decimals

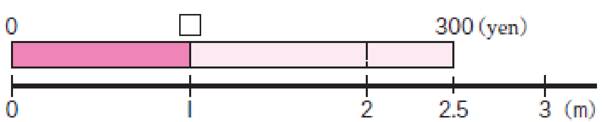
Division of Decimal Numbers

4 Let's Think about Division of Decimal Numbers



1 Division of Decimal Numbers

1 We bought 2.5m of ribbon. The cost was 300 yen. How much does 1 m of this ribbon cost?



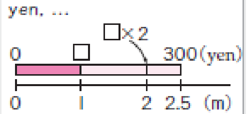
Let's think about what math sentence we should write.

Math Sentence ______________

Kaori: If it were 3m, then, we could think of it as 3 pieces of 1 m and divide 300 yen into 3 equal pieces. But, ...

Shinji: Cost ÷ Length we bought(m) = Price for 1 m

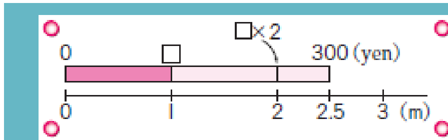
Yumi: Since 2.5 times as much of the price for 1 m is 300 yen, ...



Explain why the math sentence is written in this way.

Shinji: If the length we bought was a whole number, it will be division, ...

Cost ÷ Length we bought(m) = Price for 1 m				
2 m	···300 ÷	2	=	150
3 m	···300 ÷	3	=	100
2.5 m	···300 ÷	2.5	=	□



□×2.5 = 300

□ = 300÷2.5

Yumi: If we say the price for 1 m is □ yen, the math sentence will be □×2.5=300. Since we are trying to find the value of □, I thought it should be 300÷2.5.

Summary

Even when the length of ribbon is a decimal number, we can use division to find the price for 1 m just like we did with whole numbers.

300 ÷ 2.5

About how much will it be? Since 300÷2=150 and 300÷3=100, ...

Let's think about how to calculate.

Miho: What if we find the price for 0.1 m and make that 10 times as much...

Hiroki: What if we find the cost for 25m and divide that by 25...

Possible sequence in the US

Grade 5

- Fractions as quotients of whole numbers divided by whole numbers
- Introducing "Q times as much" idea with Q as a fraction (division to find the scale factor)
- Multiplication by fraction as scaling

A key CCSS standard for ratio/rate reasoning

5.NF.B.4.a Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times (q \div b)$.

Examples with 5.NF.B.4.a

For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)

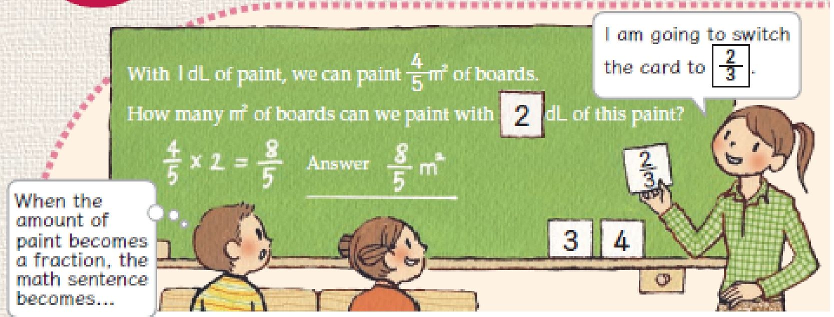
Examples with 5.NF.B.4.a

For example, use a visual fraction model to show $\frac{2}{3} \times 4 = 2 \times (4 \div 3) = \frac{8}{3}$, *and create a story context for this equation. Do the same with* $\frac{2}{3} \times \frac{4}{5} = 2 \times \left(\frac{4}{5} \div 3\right) = \frac{8}{15}$. *(In general, (a/b) × (c/d) = ac/bd.)*

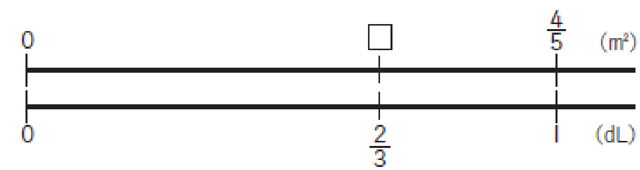
G6: Multiplication by fractions

● Multiplication of Fractions

3 Let's Think about How to Multiply by Fractions



1 With 1 dL of paint, we can paint $\frac{4}{5}$ m² of boards. How many m² of boards can we paint with $\frac{2}{3}$ dL of this paint?



? Let's think about what math sentence we should write.

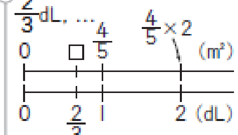
Math Sentence ______________

Takumi: With 2dL, we can think of it as 2 of the amount that can be painted with 1dL, but with $\frac{2}{3}$dL…

Area we can paint with 1 dL	×	Amount of paint (dL)	=	Area we can paint

Shinji

Kaori: The amount of boards we can paint with 2dL will be 2 times the area we can paint with 1dL. So, with $\frac{2}{3}$dL, …

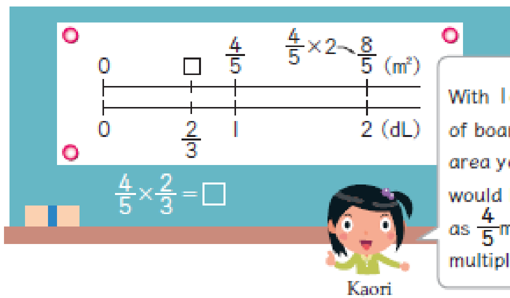


1 Explain the reason for your math sentence.

G6: Multiplication by fractions

I thought we could use the same way of thinking we used when the amount of paint was a whole number.

Area we can paint with 1 dL	×	Amount of paint (dL)	=	Area we can paint
2 dL …… $\frac{4}{5}$	×	2	=	$\frac{8}{5}$
3 dL …… $\frac{4}{5}$	×	3	=	$\frac{12}{5}$
$\frac{2}{3}$ dL …… $\frac{4}{5}$	×	$\frac{2}{3}$	=	□



With 1 dL you can paint $\frac{4}{5}$ m² of boards. I thought that the area you can paint with $\frac{2}{3}$ dL would be $\frac{2}{3}$ times as much as $\frac{4}{5}$ m². I think we can use multiplication.

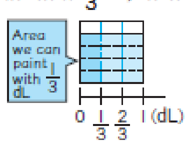
Summary

Even when the amount of paint used is a fraction, we can use multiplication to calculate the total area that can be painted, just like we did with whole numbers.

$$\frac{4}{5}\times\frac{2}{3}$$

? Let's think about how to calculate.

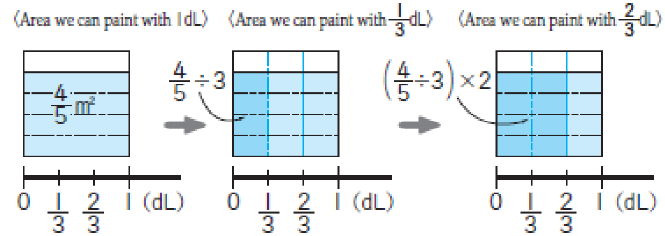
Yumi: First, we can find the area of boards we can paint with $\frac{1}{3}$ dL, then…



Hiroki: I wonder if we can change $\frac{2}{3}$ into a whole number just as we did with decimal multipliers…

Yumi

First, find the area of boards you can paint with $\frac{1}{3}$ dL, and then double that amount.



$$\frac{4}{5}\times\frac{2}{3}=\left(\frac{4}{5}\div 3\right)\times 2$$
$$=\frac{4}{5\times 3}\times 2$$
$$=\frac{\square\times\square}{\square\times\square}$$
$$=\square$$

Hiroki

If we change $\frac{2}{3}$ into a whole number, we can calculate it. We make the multiplier 3 times as much, and then divide the product by 3.

$$\frac{4}{5}\times\frac{2}{3}=\frac{4}{5}\times\left(\frac{2}{3}\times 3\right)\div 3$$
$$=\frac{4}{5}\times 2\div 3$$
$$=\frac{\square\times\square}{\square\times\square}$$
$$=\square$$

$\frac{4}{5}\times\frac{2}{3}=\square$ (↓×3, ↓×3, ÷3)

$\frac{4}{5}\times\left(\frac{2}{3}\times 3\right)=\frac{4}{5}\times 2$

80 × 2.3 = 184 (↓×10, ↓×10, ÷10)

80 × 23 = 1840

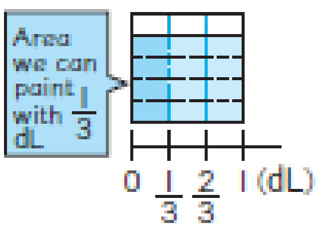
It's the same thinking we used with decimal numbers, isn't it?

2 Compare the last part of the math sentences in these two students' ideas.

G6: Multiplication by fractions

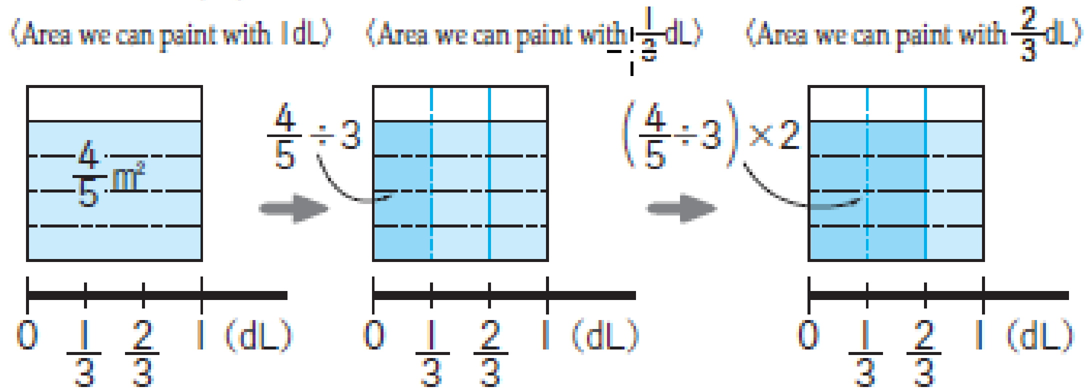
? Let's think about how to calculate.

Yumi: First, we can find the area of boards we can paint with $\frac{1}{3}$dL, then...



Yumi

First, find the area of boards you can paint with $\frac{1}{3}$dL, and then double that amount.



$$\frac{4}{5} \times \frac{2}{3} = \left(\frac{4}{5} \div 3\right) \times 2$$

$$= \frac{4}{5 \times 3} \times 2$$

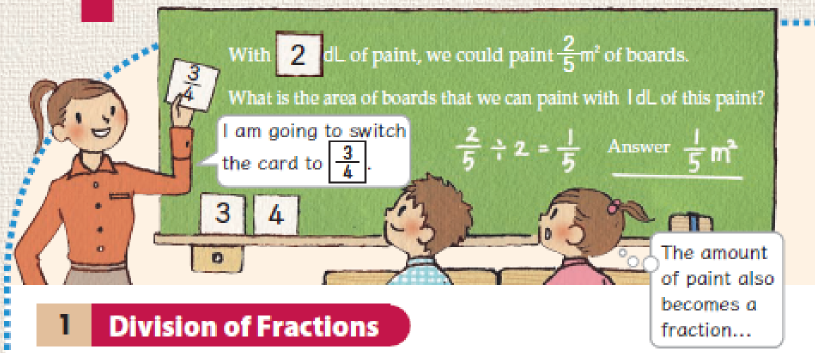
$$= \frac{\square \times \square}{\square \times \square}$$

$$= \square$$

G6: Division by fractions

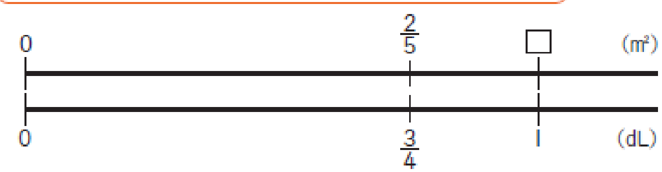
● Division of Fractions

4 Let's Think about How to Divide by Fractions



1 Division of Fractions

1 With $\frac{3}{4}$ dL of paint, we could paint $\frac{2}{5}$ m² of boards. What is the area of boards that we can paint with 1 dL of this paint?



? **Let's think about what math sentence we should write.**

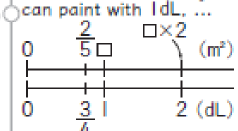
Math Sentence

Miho: If the amount of paint were 2dL, we could think of it as 2 of 1 dL, so we divide $\frac{2}{5}$ m² into 2 equal parts. But...

Area painted	÷	Amount of paint used (dL)	=	Area we can paint with 1 dL

Hiroki

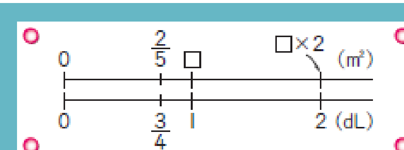
Yumi: $\frac{2}{5}$ m² is the area that you can paint with $\frac{3}{4}$ times as much of the area you can paint with 1 dL, ...



Explain the reason for your math sentence.

Hiroki: If the amount of paint used were a whole number...

Area painted	÷	Amount of paint used (dL)	=	Area we can paint with 1 dL
2 dL ······ $\frac{2}{5}$	÷	2	=	$\frac{1}{5}$
3 dL ······ $\frac{2}{5}$	÷	3	=	$\frac{2}{15}$
$\frac{3}{4}$ dL ······ $\frac{2}{5}$	÷	$\frac{3}{4}$	=	□



$\square \times \frac{3}{4} = \frac{2}{5}$

$\square = \frac{2}{5} \div \frac{3}{4}$

Yumi: If we say we can paint □ m² with 1 dL, we can say that $\square \times \frac{3}{4} = \frac{2}{5}$. Since we are finding the number for □, it will be $\frac{2}{5} \div \frac{3}{4}$.

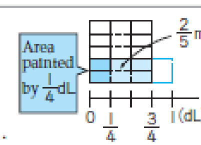
Summary

Even when the amount of paint used is a fraction, we can still use a division sentence to calculate the amount that can be painted with 1 dL, just like we did with whole numbers and decimal numbers.

$$\frac{2}{5} \div \frac{3}{4}$$

? **Let's think about how to calculate.**

Kaori: First, find how much area can be painted with $\frac{1}{4}$ dL, then...

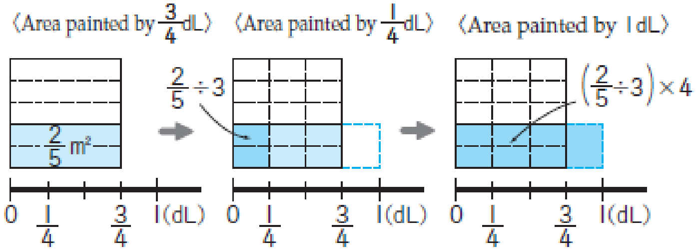


Shinji: I wonder if we can change $\frac{3}{4}$ into a whole number, just like we did when using a decimal divisor.

G6: Division by fractions

Kaori

First, find how much area can be painted with $\frac{1}{4}$ dL, and then find 4 times as much as that number.



$$\frac{2}{5} \div \frac{3}{4} = \left(\frac{2}{5} \div 3\right) \times 4$$

$$= \frac{2}{5 \times 3} \times 4$$

$$= \frac{\square \times \square}{\square \times \square}$$

$$= \square$$

Shinji

We can calculate if we can change $\frac{3}{4}$ into a whole number...

$$\frac{2}{5} \div \frac{3}{4} = \square$$

$\times 4$ $\times 4$ (equal)

$$\left(\frac{2}{5} \times 4\right) \div \left(\frac{3}{4} \times 4\right) = \frac{2}{5} \times 4 \div 3$$

$$\frac{2}{5} \div \frac{3}{4} = \left(\frac{2}{5} \times 4\right) \div \left(\frac{3}{4} \times 4\right)$$

$$= \left(\frac{2}{5} \times 4\right) \div 3$$

$$= \frac{2 \times 4}{5} \div 3$$

$$= \frac{\square \times \square}{\square \times \square}$$

$$= \square$$

$200 \div 2.5 = 80$

$\times 10$ $\times 10$ (equal)

$2000 \div 25 = 80$

It's the same idea we used when we divided by a decimal number, isn't it?

Yumi

We can easily calculate if the divisor is 1. So, I multiplied both $\frac{2}{5}$ and $\frac{3}{4}$ by $\frac{4}{3}$, the reciprocal of $\frac{3}{4}$ ...

$$\frac{2}{5} \div \frac{3}{4} = \square$$

$\times \frac{4}{3}$ $\times \frac{4}{3}$ (equal)

$$\left(\frac{2}{5} \times \frac{4}{3}\right) \div \left(\frac{3}{4} \times \frac{4}{3}\right) = \frac{2}{5} \times \frac{4}{3} \div 1$$

$$\frac{2}{5} \div \frac{3}{4} = \left(\frac{2}{5} \times \frac{4}{3}\right) \div \left(\frac{3}{4} \times \frac{4}{3}\right)$$

$$= \left(\frac{2}{5} \times \frac{4}{3}\right) \div 1$$

$$= \frac{2}{5} \times \frac{4}{3}$$

$$= \frac{\square \times \square}{\square \times \square}$$

$$= \square$$

She is using the same property as the one Shinji used, isn't she?

2 Compare the last part of the math sentences of these three students.

$$\frac{2}{5} \div \frac{3}{4} = \frac{2 \times 4}{5 \times 3}$$

$$= \frac{8}{15}$$

Answer $\frac{8}{15}$ m²

Summary

To divide by a fraction, we can multiply the dividend by the reciprocal of the divisor.

$$\frac{b}{a} \div \frac{d}{c} = \frac{b}{a} \times \frac{c}{d}$$

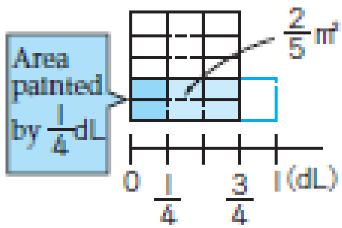
$$= \frac{b \times c}{a \times d}$$

G6: Division by fractions

? Let's think about how to calculate.

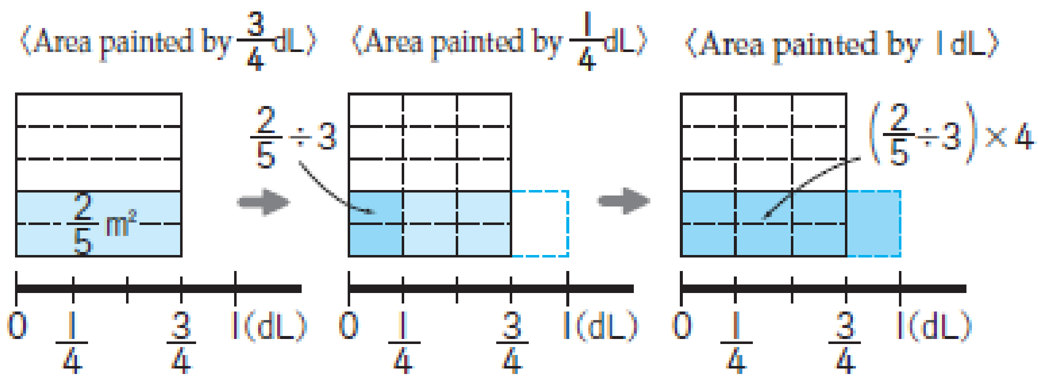
Kaori

First, find how much area can be painted with $\frac{1}{4}$dL, then…



Kaori

First, find how much area can be painted with $\frac{1}{4}$dL, and then find 4 times as much as that number.



$$\frac{2}{5}\div\frac{3}{4}=\left(\frac{2}{5}\div 3\right)\times 4$$

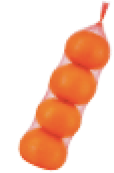
$$=\frac{2}{5\times 3}\times 4$$

$$=\frac{\square\times\square}{\square\times\square}$$

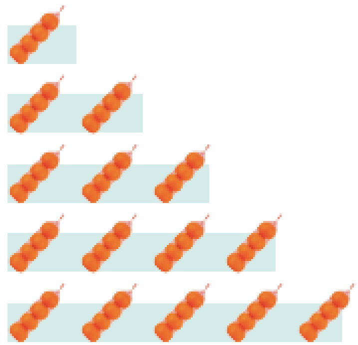
$$=\square$$

Concluding Thoughts

- Teaching of multiplication and division (whole numbers, decimal numbers, and fractions) in Grades 3 – 5 needs to be thought of as the foundation building for proportional reasoning in middle grades.



3 Find multiplication facts for 4.



$4 \times 1 = \square$

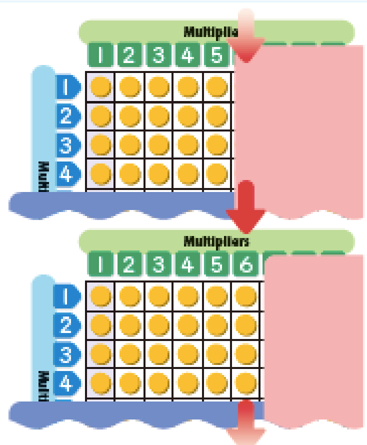
$4 \times 2 = \square$

$4 \times 3 = \square$

$4 \times 4 = \square$

$4 \times 5 = \square$

1 When the multiplier of 4×5 increases by 1, by how many does the answer increase?



$4 \times \boxed{5} = \square$

Increase by 1 | Increase by $\square$

$4 \times \square = \square$

2 Find the answers for 4×6, 4×7, 4×8, and 4×9.

$4 \times 6 = \square$

$4 \times 7 = \square$

$4 \times 8 = \square$

$4 \times 9 = \square$

Concluding Thoughts

- Teaching of multiplication and division (whole numbers, decimal numbers, and fractions) in Grades 3 – 5 needs to be thought of as the foundation building for proportional reasoning in middle grades.
- An important goal of teaching multiplication and division of fractions in Grades 5 and 6 is for students to develop an understanding that multiplication and division are special cases of proportional reasoning.