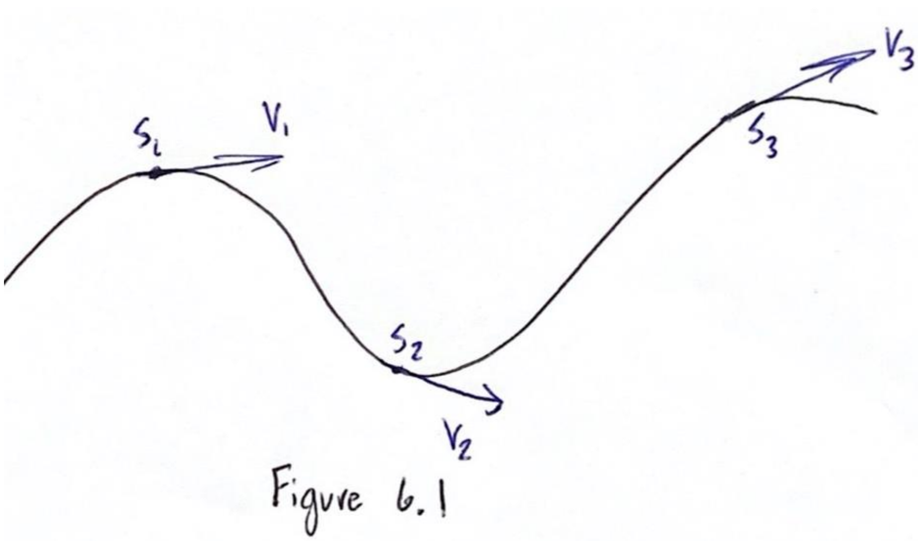
CHAPTER SIX

MOTION IN TWO OR THREE DIMENSIONS

Motion in two or three dimensions is often described as curvilinear motion. Curvilinear motion is the motion of an object as it travels along a curved path. This motion is defined by the objects position, *s*, velocity, *v*, and acceleration, *a*. Velocity and Acceleration each have a magnitude and direction. Within the traditional rectangular (x, y, z) coordinate system, position, velocity, and acceleration can all be described in relation to the *x-axis, y-axis,* and z-*axis.* Position is where an object is, relative to a reference point. Velocity is the change in position within a given time. Acceleration is the change in velocity within a given time

6-1: NORMAL AND TANGENTIAL COMPONENTS

Normal and tangential components can be used when we know the path of an object and its position. This helps us determine its velocity and acceleration. The velocity will always be tangent to the curve, or path. The velocity is shown tangent to the curve below, in *Figure 6.1*.



Acceleration is the change in velocity within a given time. Acceleration can be broken down into two different components: normal component and tangential component. The normal component always points directly into the curve while the tangential component will be parallel to the velocity, either pointed in the same direction, or opposite to the velocity. If the tangential acceleration is point in the same direction as velocity, then the speed of the object is increasing. If it is pointing opposite to the velocity, then the speed is decreasing. This also means the tangential acceleration is the change in speed over a given time. Use the figure below, *Figure 6.2*, to help illustrate the acceleration components.

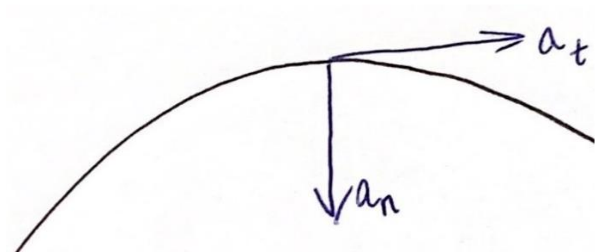


Figure 6.2

The normal acceleration is only responsible for the change in direction of the velocity. Likewise, the tangential velocity is only responsible for the change is magnitude of the velocity, or speed. As mentioned earlier, if it is point with the velocity, the object speeds up; and if it is point opposite to velocity, the object slows down. Tangential acceleration can be described as…

$$a_t = \frac{\Delta speed}{\Delta t} \ or \ \frac{change\ in\ speed}{change\ in\ time}$$

The normal acceleration can also be described as the centripetal acceleration, or circular acceleration. This is because it is responsible for the curve created in a path at that given position and time. Because of this, the following equation describes normal acceleration.

$a_n = \frac{v^2}{r}$; r is the radius of curvature.

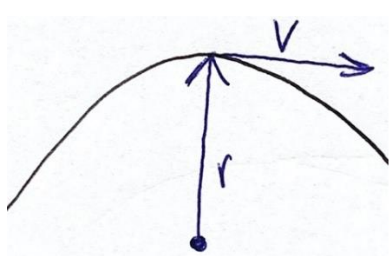


Figure 6.3

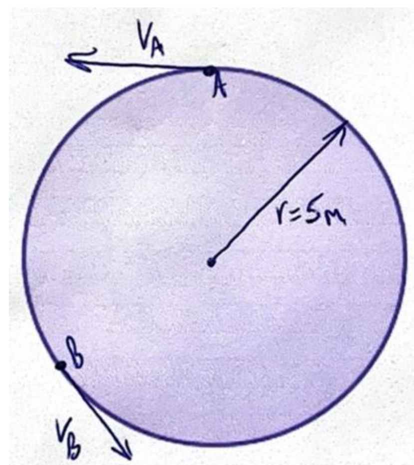
Total acceleration, or resultant acceleration can be found using the following equation…

$$a = \sqrt{{a_n}^2 + {a_t}^2}$$

6:1 HOMEWORK

Problem 1:

A toy is swung in a uniform circle with increasing acceleration. At point A, the velocity is equal to 3 *m/s* at *t = 0s*. At point B, the velocity is now *7m/s* at *t=2s*. First, find both a_n and a_t at *t=1s* Use these to solve for the resultant acceleration.



6-2: POLAR COMPONENTS

Polar coordinates are a way to describe the position, velocity, and acceleration in relation to the origin, rather than axes, such as x, y, z. This is done by measuring the position in terms of *r and θ* instead of x *and y*. *r* is the distance from the origin and θ is the angle between *r and the* x-*axis*. When doing this, we create two new axes at every position to define the components of velocity and acceleration. These are created by first making the first axis, *r-axis,* by drawing a line through the origin and the position. The second axis is perpendicular to the *r-axis and* located at the position. This is called the *θ-axis*. Making these new reference axes allows us to break down the velocity and acceleration into components along each of these axes, respectively. This is shown below in *Figure 6.4*.

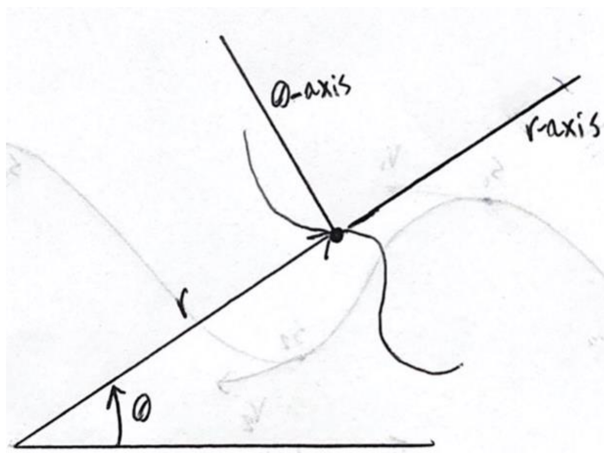
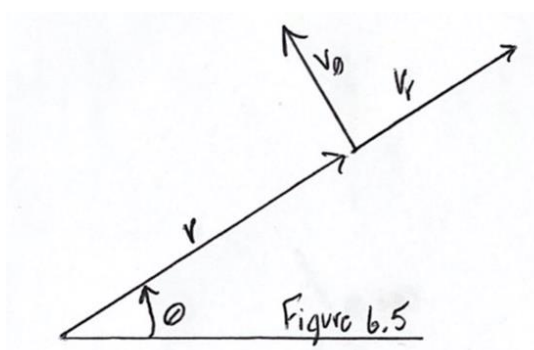


Figure 6.4

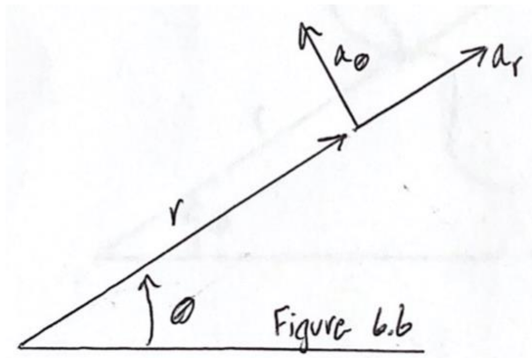
The velocity has a component in the *r* direction and in the θ direction. The v_r is the change in radius over a given time. Similarly, v_θ is the change in θ over a given time. The velocity components are shown in the figure below, *Figure 6.5*.



To calculate the resultant velocity, the following equation can be used…

$$v = \sqrt{{v_r}^2 + {v_\theta}^2}$$

Likewise, acceleration also has a component in the r direction and a component in the θ direction. The a_r is the change in v_r over a given time and a_θ is the change in v_θ over a given time. The acceleration components are shown below in *Figure* 6.6.



To calculate the resultant acceleration, we, again, need to follow the equation…

$$a = \sqrt{{a_r}^2 + {a_\theta}^2}$$