

In Pursuit of Multiple Scalars @ Colliders



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@ Manchester [May 31st 2024]



KENNESAW STATE
UNIVERSITY



Disambiguation: “Multiple Scalars”

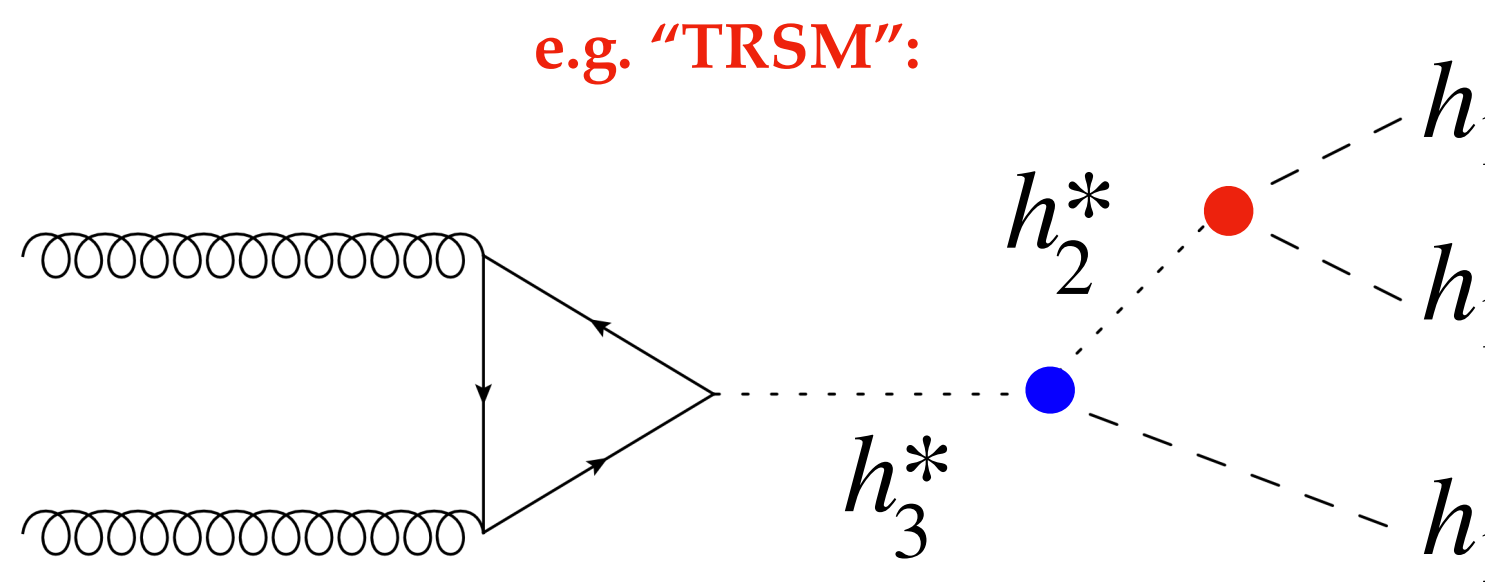
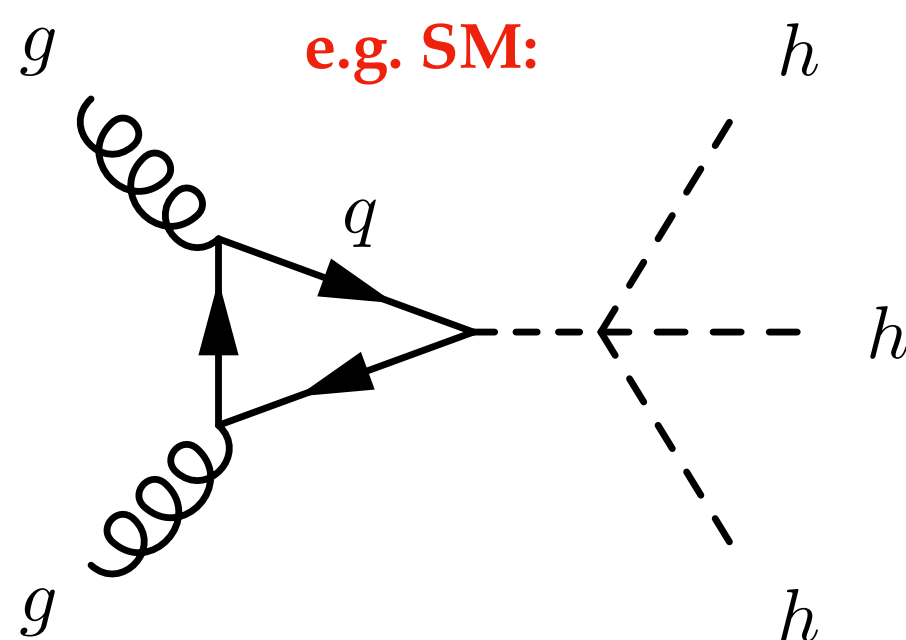


= New scalar fields added to the Standard Model,

e.g. SM + one singlet scalar field: $\mathcal{V}(\phi, S) = \color{green}\bullet |\phi|^2 + \color{blue}\blacksquare |\phi|^4 + \color{magenta}\bullet S^2 + \color{cyan}\blacktriangle S^3 + \color{red}\blacksquare S^4 + \color{red}\blacktriangle |\phi|^2 S + \color{purple}\blacksquare |\phi|^2 S^2$

and/or

= The production of multiple physical scalar states at colliders.



The Plan:



The Plan:

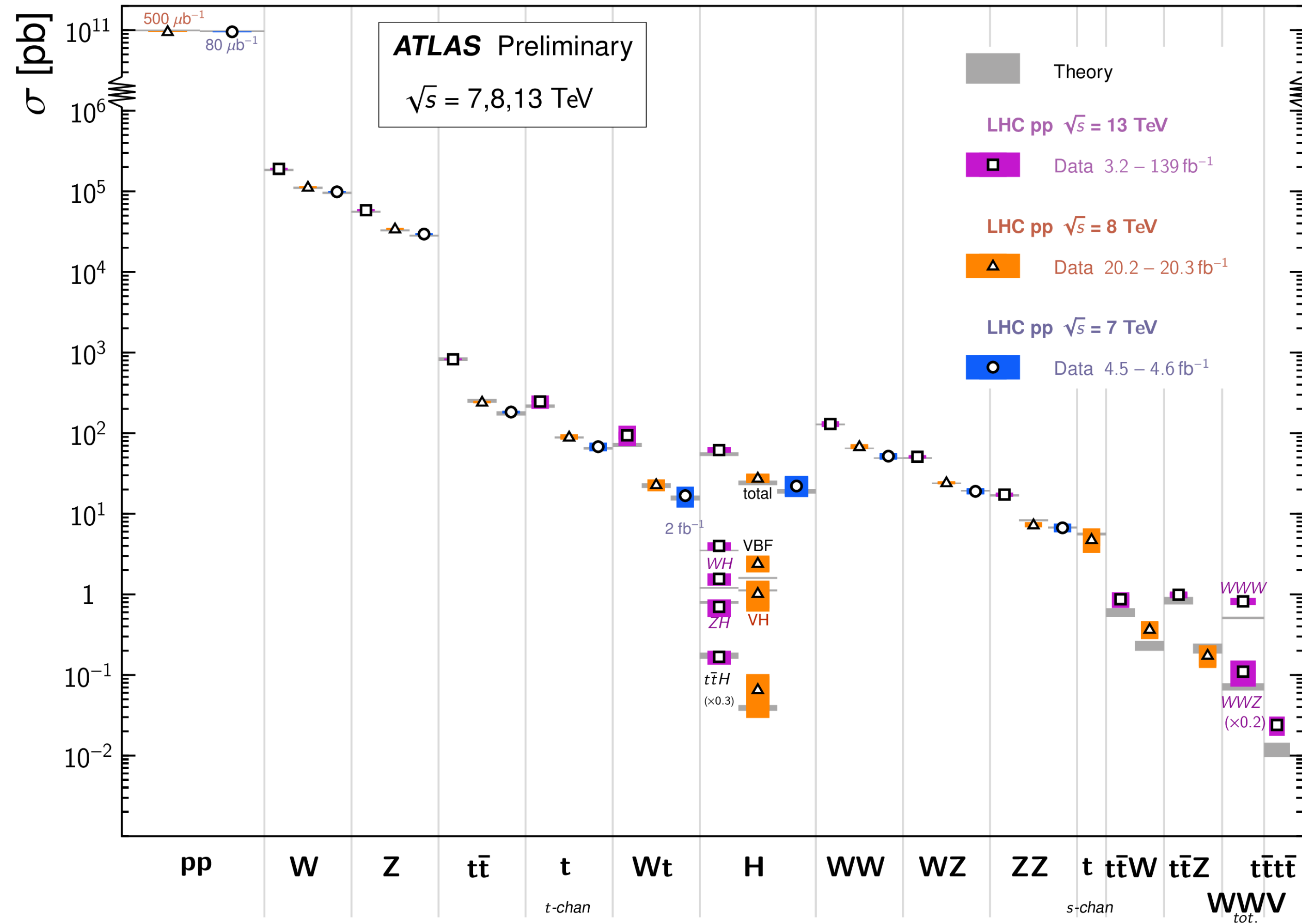
- 1 High Energy Physics Today
- 2 → The Breaking of Symmetry,
- 3 → Multi-scalar production.



Experiment *VS* Theory

Standard Model Total Production Cross Section Measurements

Status: February 2022



Cross Sections
[pb]

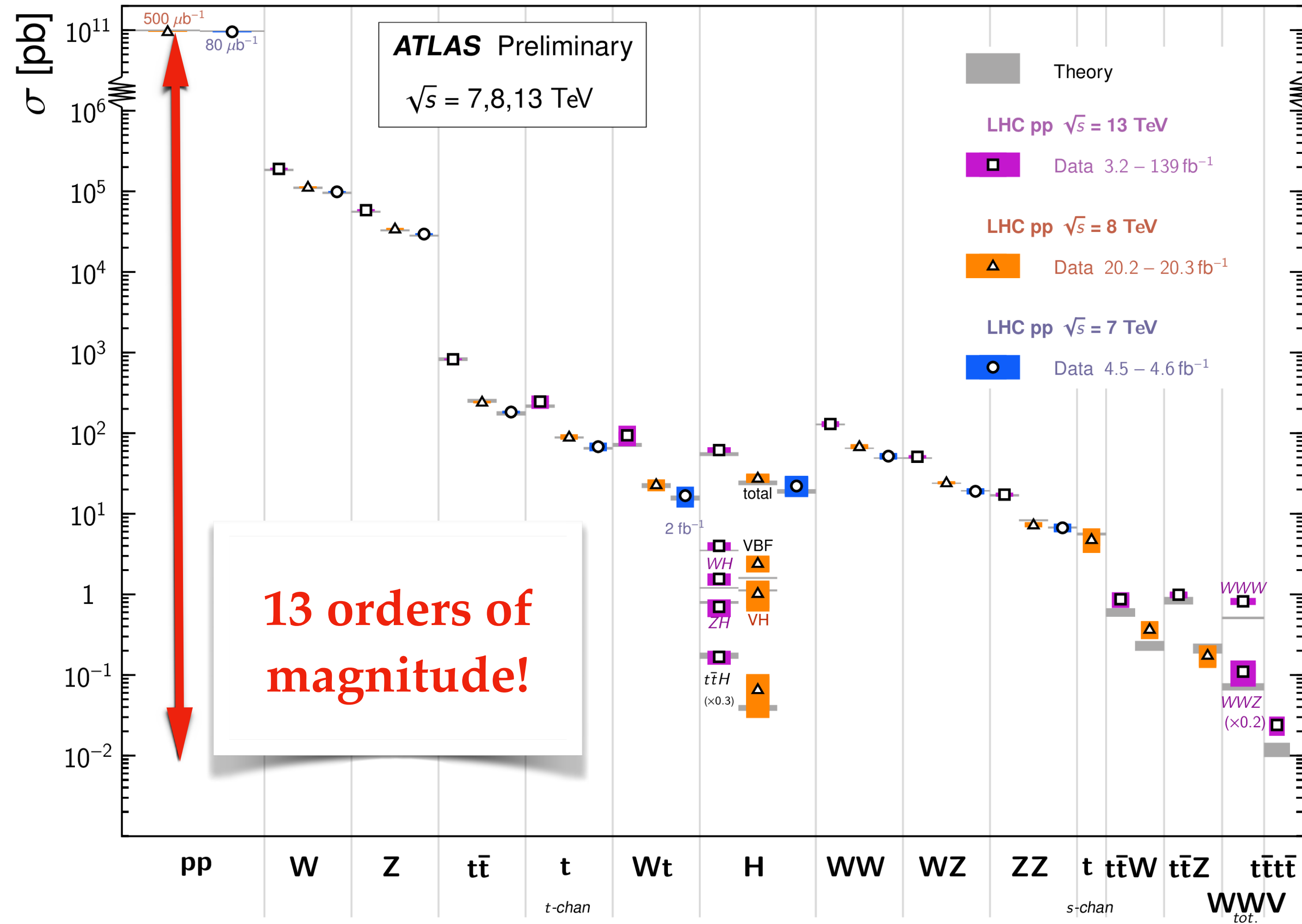
Process Type

Experiment *VS* Theory

Standard Model Total Production Cross Section Measurements

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Cross Sections
[pb]



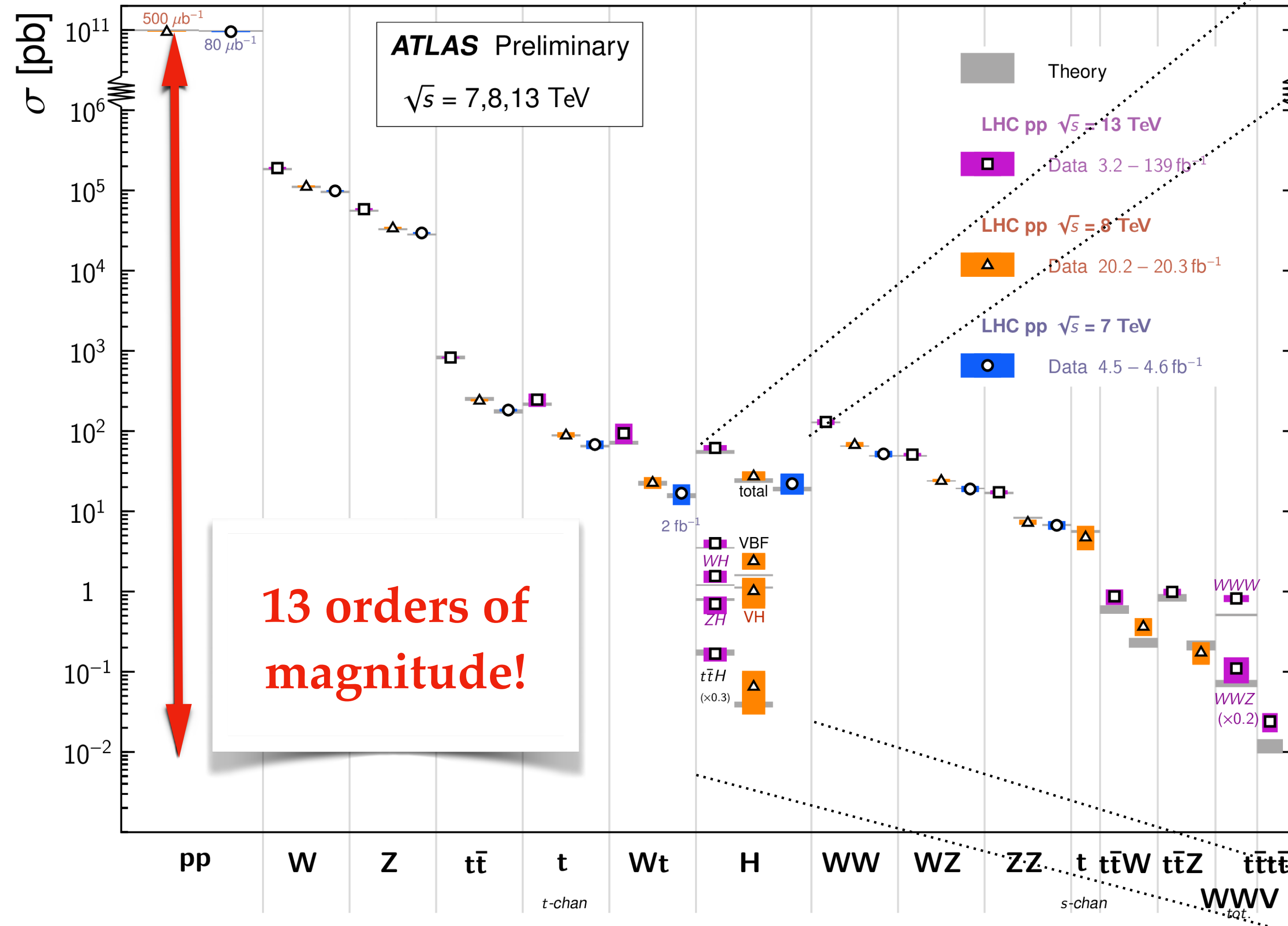
Process Type

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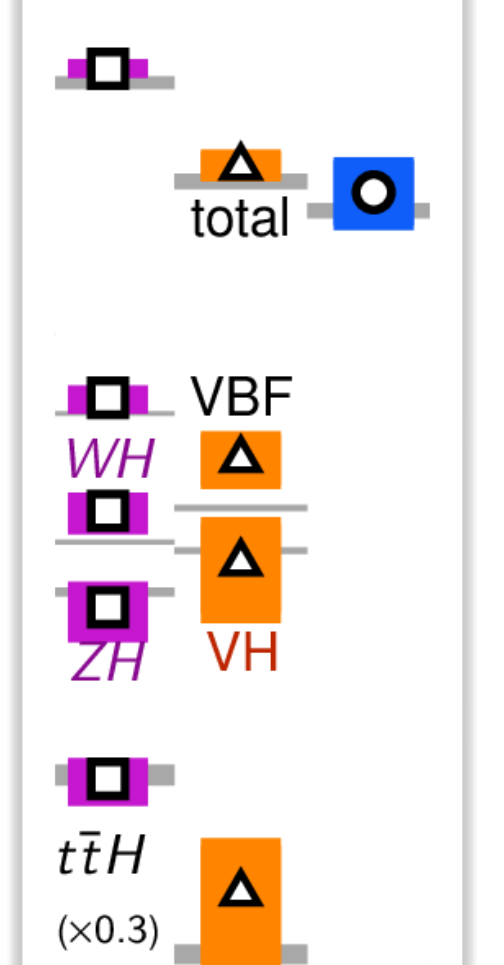
Standard Model Total Production Cross Section Measurements

Status: February 2022

Cross Sections
[pb]



Higgs Boson
Measurements



H

Process Type

Experiment **VS** Exotic New Phenomena

ATLAS Heavy Particle Searches* - 95% CL Upper Exclusion Limits

Status: March 2023

ATLAS Preliminary

$$\int \mathcal{L} dt = (3.6 - 139) \text{ fb}^{-1}$$

$$\sqrt{s} = 13 \text{ TeV}$$

Model	ℓ, γ	Jets [†]	E_T^{miss}	$\int \mathcal{L} dt [\text{fb}^{-1}]$	Limit	Reference		
Extra dimen.	ADD $G_{KK} + g/q$	$0 e, \mu, \tau, \gamma$	$1-4 j$	Yes	139	M_D 11.2 TeV $n=2$	2102.10874	
	ADD non-resonant $\gamma\gamma$	2γ	-	-	36.7	M_S 8.6 TeV $n=3$ HLZ NLO	1707.04147	
	ADD QBH	-	$2 j$	-	139	M_{th} 9.4 TeV $n=6$	1910.08447	
	ADD BH multijet	-	$\geq 3 j$	-	3.6	M_{th} 9.55 TeV $n=6, M_D=3 \text{ TeV, rot BH}$	1512.02586	
	RS1 $G_{KK} \rightarrow \gamma\gamma$	2γ	-	-	139	G_{KK} mass 4.5 TeV $k/\overline{M}_{Pl} = 0.1$	2102.13405	
	Bulk RS $G_{KK} \rightarrow WW/ZZ$	multi-channel	-	-	36.1	G_{KK} mass 2.3 TeV $k/\overline{M}_{Pl} = 1.0$	1808.02380	
	Bulk RS $g_{KK} \rightarrow tt$	$1 e, \mu$	$\geq 1 b, \geq 1 J/2j$	Yes	36.1	g_{KK} mass 3.8 TeV $\Gamma/m = 15\%$	1804.10823	
	2UED / RPP	$1 e, \mu$	$\geq 2 b, \geq 3 j$	Yes	36.1	KK mass 1.8 TeV Tier (1,1), $\mathcal{B}(A^{(1,1)} \rightarrow tt) = 1$	1803.09678	
	Gauge bosons	SSM $Z' \rightarrow \ell\ell$	$2 e, \mu$	-	-	139	Z' mass 5.1 TeV	1903.06248
		SSM $Z' \rightarrow \tau\tau$	2τ	-	-	36.1	Z' mass 2.42 TeV	1709.07242
Leptophobic $Z' \rightarrow bb$		-	$2 b$	-	36.1	Z' mass 2.1 TeV	1805.09299	
Leptophobic $Z' \rightarrow tt$		$0 e, \mu$	$\geq 1 b, \geq 2 J$	Yes	139	Z' mass 4.1 TeV $\Gamma/m = 1.2\%$	2005.05138	
SSM $W' \rightarrow \ell\nu$		$1 e, \mu$	-	Yes	139	W' mass 6.0 TeV	1906.05609	
SSM $W' \rightarrow \tau\nu$		1τ	-	Yes	139	W' mass 5.0 TeV	ATLAS-CONF-2021-025	
SSM $W' \rightarrow tb$		-	$\geq 1 b, \geq 1 J$	-	139	W' mass 4.4 TeV	ATLAS-CONF-2021-043	
HVT $W' \rightarrow WZ$ model B		$0-2 e, \mu$	$2 j / 1 J$	Yes	139	W' mass 4.3 TeV $g_V = 3$	2004.14636	
HVT $W' \rightarrow WZ \rightarrow \ell\nu \ell'\ell'$ model C		$3 e, \mu$	$2 j$ (VBF)	Yes	139	W' mass 340 GeV $g_V c_H = 1, g_F = 0$	2207.03925	
HVT $Z' \rightarrow WW$ model B		$1 e, \mu$	$2 j / 1 J$	Yes	139	Z' mass 3.9 TeV $g_V = 3$	2004.14636	
LRSM $W_R \rightarrow \mu N_R$	2μ	$1 J$	-	80	W_R mass 5.0 TeV $m(N_R) = 0.5 \text{ TeV, } g_L = g_R$	1904.12679		
CI	CI $qqqq$	-	$2 j$	-	37.0	Λ 21.8 TeV η_{LL}	1703.09127	
	CI $\ell\ell qq$	$2 e, \mu$	-	-	139	Λ 35.8 TeV η_{LL}	2006.12946	
	CI $e e b s$	$2 e$	$1 b$	-	139	Λ 1.8 TeV $g_* = 1$	2105.13847	
	CI $\mu\mu b s$	2μ	$1 b$	-	139	Λ 2.0 TeV $g_* = 1$	2105.13847	
	CI $t t t t$	$\geq 1 e, \mu$	$\geq 1 b, \geq 1 j$	Yes	36.1	Λ 2.57 TeV $ C_{4t} = 4\pi$	1811.02305	
DM	Axial-vector med. (Dirac DM)	-	$2 j$	-	139	m_{med} 3.8 TeV $g_q=0.25, g_\ell=1, m(\chi)=10 \text{ TeV}$	ATL-PHYS-PUB-2022-036	
	Pseudo-scalar med. (Dirac DM)	$0 e, \mu, \tau, \gamma$	$1-4 j$	Yes	139	m_{med} 376 GeV $g_q=1, g_\ell=1, m(\chi)=1 \text{ GeV}$	2102.10874	
	Vector med. Z' -2HDM (Dirac DM)	$0 e, \mu$	$2 b$	Yes	139	$m_{Z'}$ 3.0 TeV $\tan\beta=1, g_Z=0.8, m(\chi)=100 \text{ GeV}$	2108.13391	
	Pseudo-scalar med. 2HDM+a	multi-channel	-	-	139	m_a 800 GeV $\tan\beta=1, g_\ell=1, m(\chi)=10 \text{ GeV}$	ATLAS-CONF-2021-036	
LQ	Scalar LQ 1 st gen	$2 e$	$\geq 2 j$	Yes	139	LQ mass 1.8 TeV $\beta = 1$	2006.05872	
	Scalar LQ 2 nd gen	2μ	$\geq 2 j$	Yes	139	LQ mass 1.7 TeV $\beta = 1$	2006.05872	
	Scalar LQ 3 rd gen	1τ	$2 b$	Yes	139	LQ_3^u mass 1.49 TeV $\mathcal{B}(LQ_3^u \rightarrow b\tau) = 1$	2303.01294	
	Scalar LQ 3 rd gen	$0 e, \mu$	$\geq 2 j, \geq 2 b$	Yes	139	LQ_3^d mass 1.24 TeV $\mathcal{B}(LQ_3^d \rightarrow \tau\nu) = 1$	2004.14060	
	Scalar LQ 3 rd gen	$\geq 2 e, \mu, \geq 1 \tau, \geq 1 j, \geq 1 b$	-	-	139	LQ_3^s mass 1.43 TeV $\mathcal{B}(LQ_3^s \rightarrow t\tau) = 1$	2101.11582	
	Scalar LQ 3 rd gen	$0 e, \mu, \geq 1 \tau, 0-2 j, 2 b$	Yes	139	LQ_3^s mass 1.26 TeV $\mathcal{B}(LQ_3^s \rightarrow b\nu) = 1$	2101.12527		
	Vector LQ mix gen	multi-channel	$\geq 1 j, \geq 1 b$	Yes	139	LQ_3^v mass 2.0 TeV $\mathcal{B}(\tilde{U}_1 \rightarrow t\mu) = 1, \text{Y-M coupl.}$	ATLAS-CONF-2022-052	
	Vector LQ 3 rd gen	$2 e, \mu, \tau$	$\geq 1 b$	Yes	139	LQ_3^v mass 1.96 TeV $\mathcal{B}(LQ_3^v \rightarrow b\tau) = 1, \text{Y-M coupl.}$	2303.01294	
Vector-like fermions	VLQ $TT \rightarrow Zt + X$	$2e/2\mu \geq 3e, \mu \geq 1 b, \geq 1 j$	-	-	139	T mass 1.46 TeV SU(2) doublet	2210.15413	
	VLQ $BB \rightarrow Wt/Zb + X$	multi-channel	-	-	36.1	B mass 1.34 TeV SU(2) doublet	1808.02343	
	VLQ $T_{5/3} T_{5/3} / T_{5/3} \rightarrow Wt + X$	$2(SS)/\geq 3 e, \mu \geq 1 b, \geq 1 j$	Yes	36.1	$T_{5/3}$ mass 1.64 TeV $\mathcal{B}(T_{5/3} \rightarrow Wt) = 1, c(T_{5/3} Wt) = 1$	1807.11883		
	VLQ $T \rightarrow Ht/Zt$	$1 e, \mu \geq 1 b, \geq 3 j$	Yes	139	T mass 1.8 TeV SU(2) singlet, $\kappa_T = 0.5$	ATLAS-CONF-2021-040		
	VLQ $Y \rightarrow Wb$	$1 e, \mu \geq 1 b, \geq 1 j$	Yes	36.1	Y mass 1.85 TeV $\mathcal{B}(Y \rightarrow Wb) = 1, c_R(Wb) = 1$	1812.07343		
	VLQ $B \rightarrow Hb$	$0 e, \mu \geq 2b, \geq 1j, \geq 1J$	-	139	B mass 2.0 TeV SU(2) doublet, $\kappa_B = 0.3$	ATLAS-CONF-2021-018		
VLL $\tau' \rightarrow Z\tau/H\tau$	multi-channel	$\geq 1 j$	Yes	139	τ' mass 898 GeV SU(2) doublet	2303.05441		
Exctd ferm.	Excited quark $q^* \rightarrow qg$	-	$2 j$	-	139	q^* mass 6.7 TeV only u^* and $d^*, \Lambda = m(q^*)$	1910.08447	
	Excited quark $q^* \rightarrow q\gamma$	1γ	$1 j$	-	36.7	q^* mass 5.3 TeV only u^* and $d^*, \Lambda = m(q^*)$	1709.10440	
	Excited quark $b^* \rightarrow bg$	-	$1 b, 1 j$	-	139	b^* mass 3.2 TeV $\Lambda = 4.6 \text{ TeV}$	1910.08447	
	Excited lepton τ^*	2τ	$\geq 2 j$	-	139	τ^* mass 4.6 TeV	2303.09444	
Other	Type III Seesaw	$2,3,4 e, \mu$	$\geq 2 j$	Yes	139	N^0 mass 910 GeV	2202.02039	
	LRSM Majorana ν	2μ	$2 j$	Yes	36.1	N_R mass 3.2 TeV $m(W_R) = 4.1 \text{ TeV, } g_L = g_R$	1809.11105	
	Higgs triplet $H^{\pm\pm} \rightarrow W^\pm W^\pm$	$2,3,4 e, \mu$ (SS)	various	Yes	139	$H^{\pm\pm}$ mass 350 GeV DY production	2101.11961	
	Higgs triplet $H^{\pm\pm} \rightarrow \ell\ell$	$2,3,4 e, \mu$ (SS)	-	-	139	$H^{\pm\pm}$ mass 1.08 TeV DY production	2211.07505	
	Multi-charged particles	-	-	-	139	multi-charged particle mass 1.59 TeV DY production, $ q = 5e$	ATLAS-CONF-2022-034	
	Magnetic monopoles	-	-	-	34.4	monopole mass 2.37 TeV DY production, $ g = 1g_D, \text{spin } 1/2$	1905.10130	

$\sqrt{s} = 13 \text{ TeV}$ partial data
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10⁻¹ 1 10 Mass scale [TeV]



Kinds of New Physics

*Only a selection of the available mass limits on new states or phenomena is shown.
[†]Small-radius (large-radius) jets are denoted by the letter j (J).

Experiment **VS** Exotic New Phenomena

ATLAS Heavy Particle Searches* - 95% CL Upper Exclusion Limits

Status: March 2023

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2UED / RPP	$1 e, \mu$	$\geq 2 b, \geq 3 j$	Yes	36.1	KK mass 1.8 TeV Tier (1,1), $\mathcal{B}(A^{(1,1)} \rightarrow tt) = 1$	1803.09678	
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	Leptophobic $Z' \rightarrow bb$	-	$2 b$	-	36.1	Z' mass 2.1 TeV	1805.09299
	Leptophobic $Z' \rightarrow tt$	$0 e, \mu$	$\geq 1 b, \geq 2 J$	Yes	139	Z' mass 4.1 TeV $\Gamma/m = 1.2\%$	2005.05138
	SSM $W' \rightarrow \ell\nu$	$1 e, \mu$	-	Yes	139	W' mass 6.0 TeV	1906.05609
	SSM $W' \rightarrow \tau\nu$	1τ	-	Yes	139	W' mass 5.0 TeV	ATLAS-CONF-2021-025
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	HVT $W' \rightarrow WZ$ model B	$0-2 e, \mu$	$2 j / 1 J$	Yes	139	W' mass 4.3 TeV $g_V = 3$	2004.14636
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	CI $\ell\ell qq$	$2 e, \mu$	-	-	139	Λ 35.8 TeV η_{LL}	2006.12946
	CI $eebs$	$2 e$	$1 b$	-	139	Λ 1.8 TeV $g_s = 1$	2105.13847
	CI $\mu\mu bs$	2μ	$1 b$	-	139	Λ 2.0 TeV $g_s = 1$	2105.13847
	CI $tttt$	$\geq 1 e, \mu$	$\geq 1 b, \geq 1 j$	Yes	36.1	Λ 2.57 TeV $ C_{4t} = 4\pi$	1811.02305
DM	Axial-vector med. (Dirac DM)	-	$2 j$	-	139	m_{med} 3.8 TeV $g_q = 0.25, g_\ell = 1, m(\chi) = 10 \text{ TeV}$	ATL-PHYS-PUB-2022-036
	Pseudo-scalar med. (Dirac DM)	$0 e, \mu, \tau, \gamma$	$1 - 4 j$	Yes	139	m_{med} 376 GeV $g_q = 1, g_\ell = 1, m(\chi) = 1 \text{ GeV}$	2102.10874
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	Scalar LQ 3 rd gen	$0 e, \mu$	$\geq 2 j, \geq 2 b$	Yes	139	LQ_3^d mass 1.24 TeV $\mathcal{B}(LQ_3^d \rightarrow \tau\nu) = 1$	2004.14060
	Scalar LQ 3 rd gen	$\geq 2 e, \mu, \geq 1 \tau, \geq 1 j, \geq 1 b$	-	-	139	LQ_3^d mass 1.43 TeV $\mathcal{B}(LQ_3^d \rightarrow t\tau) = 1$	2101.11582
	Scalar LQ 3 rd gen	$0 e, \mu, \geq 1 \tau, 0 - 2 j, 2 b$	Yes	139	LQ_3^d mass 1.26 TeV $\mathcal{B}(LQ_3^d \rightarrow b\nu) = 1$	2101.12527	
	Vector LQ mix gen	multi-channel	$\geq 1 j, \geq 1 b$	Yes	139	LQ_3^u mass 2.0 TeV $\mathcal{B}(\tilde{U}_1 \rightarrow t\mu) = 1, \text{Y-M coupl.}$	ATLAS-CONF-2022-052
	Vector LQ 3 rd gen	$2 e, \mu, \tau$	$\geq 1 b$	Yes	139	LQ_3^d mass 1.96 TeV $\mathcal{B}(LQ_3^d \rightarrow b\tau) = 1, \text{Y-M coupl.}$	2303.01294
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	VLQ $BB \rightarrow Wt/Zb + X$	multi-channel	-	-	36.1	B mass 1.34 TeV SU(2) doublet	1808.02343
	VLQ $T_{5/3} T_{5/3} T_{5/3} \rightarrow Wt + X$	$2(SS)/\geq 3 e, \mu$	$\geq 1 b, \geq 1 j$	Yes	36.1	$T_{5/3}$ mass 1.64 TeV $\mathcal{B}(T_{5/3} \rightarrow Wt) = 1, c(T_{5/3} Wt) = 1$	1807.11883
	VLQ $T \rightarrow Ht/Zt$	$1 e, \mu$	$\geq 1 b, \geq 3 j$	Yes	139	T mass 1.8 TeV SU(2) singlet, $\kappa_T = 0.5$	ATLAS-CONF-2021-040
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	Excited quark $b^* \rightarrow bg$	-	$1 b, 1 j$	-	139	b^* mass 3.2 TeV $\Lambda = 4.6 \text{ TeV}$	1910.08447
	Excited lepton τ^*	2τ	$\geq 2 j$	-	139	τ^* mass 4.6 TeV	2303.09444
Other	Type III Seesaw	$2,3,4 e, \mu$	$\geq 2 j$	Yes	139	N^0 mass 910 GeV $m(W_R) = 4.1 \text{ TeV, } g_L = g_R$	2202.02039
	LRSM Majorana ν	2μ	$2 j$	Yes	36.1	N_R mass 3.2 TeV DY production	1809.11105
	Higgs triplet $H^{\pm\pm} \rightarrow W^\pm W^\pm$	$2,3,4 e, \mu$ (SS)	various	Yes	139	$H^{\pm\pm}$ mass 350 GeV DY production	2101.11961
	Higgs triplet $H^{\pm\pm} \rightarrow \ell\ell$	$2,3,4 e, \mu$ (SS)	-	-	139	$H^{\pm\pm}$ mass 1.08 TeV DY production	2211.07505
	Multi-charged particles	-	-	-	139	multi-charged particle mass 1.59 TeV DY production, $ q = 5e$	ATLAS-CONF-2022-034
	Magnetic monopoles	-	-	-	34.4	monopole mass 2.37 TeV DY production, $ g = 1g_D, \text{spin } 1/2$	1905.10130

$\sqrt{s} = 13 \text{ TeV}$ partial data
 $\sqrt{s} = 13 \text{ TeV}$ full data

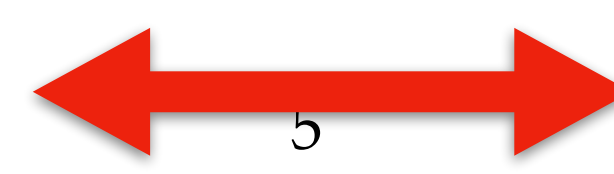
10⁻¹ 1 10 Mass scale [TeV]



Mass Scale limits:
 $\mathcal{O}(1 - 10 \text{ TeV})$

*Only a selection of the available mass limits on new states or phenomena is shown.

†Small-radius (large-radius) jets are denoted by the letter j (J).



STANDARD MODEL WINS!



STANDARD MODEL WINS!



STANDARD MODEL WINS!



⇒ NO NEW PHENOMENA?



Questions



Questions



Answers



Questions



Answers



Questions

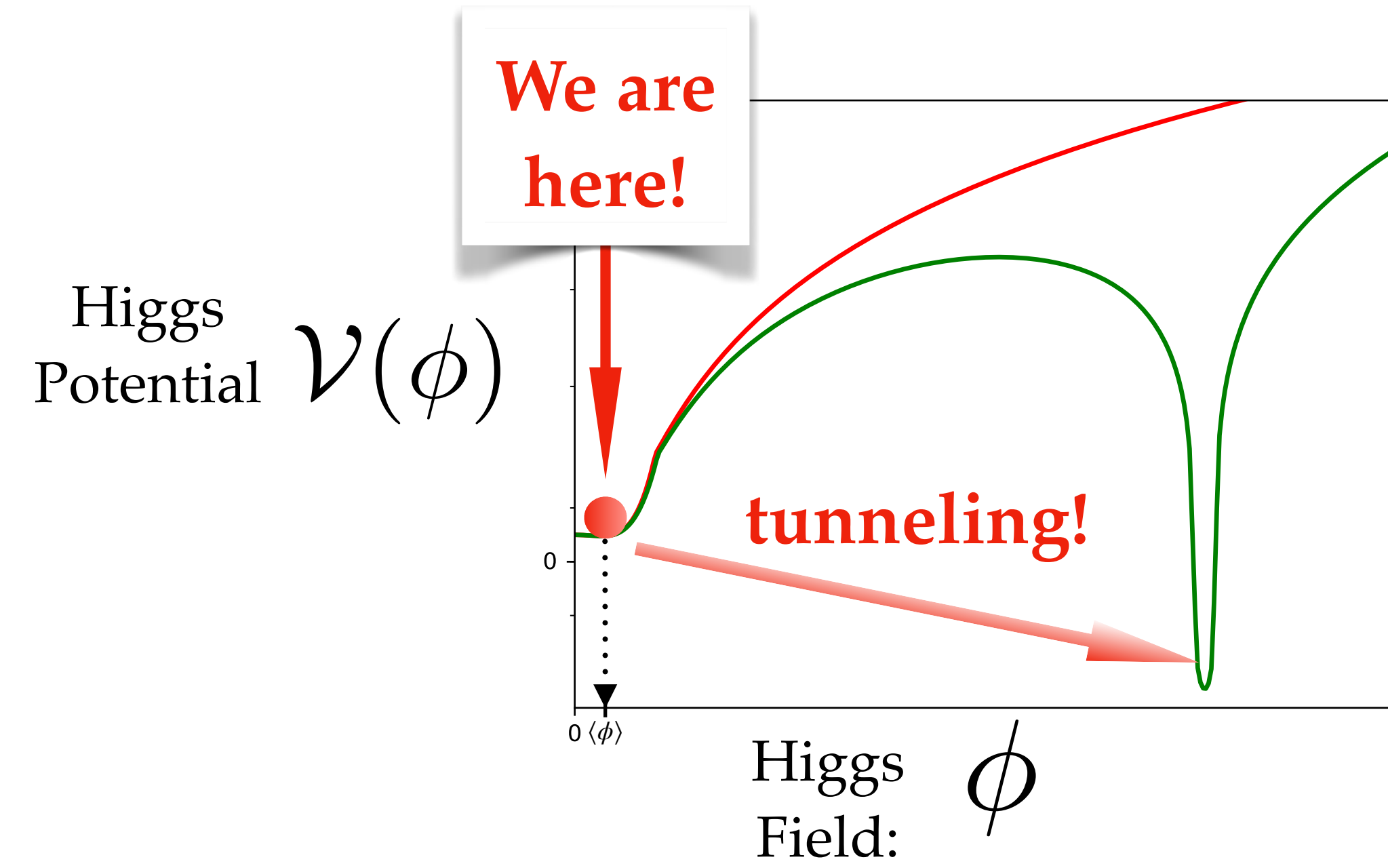


Answers

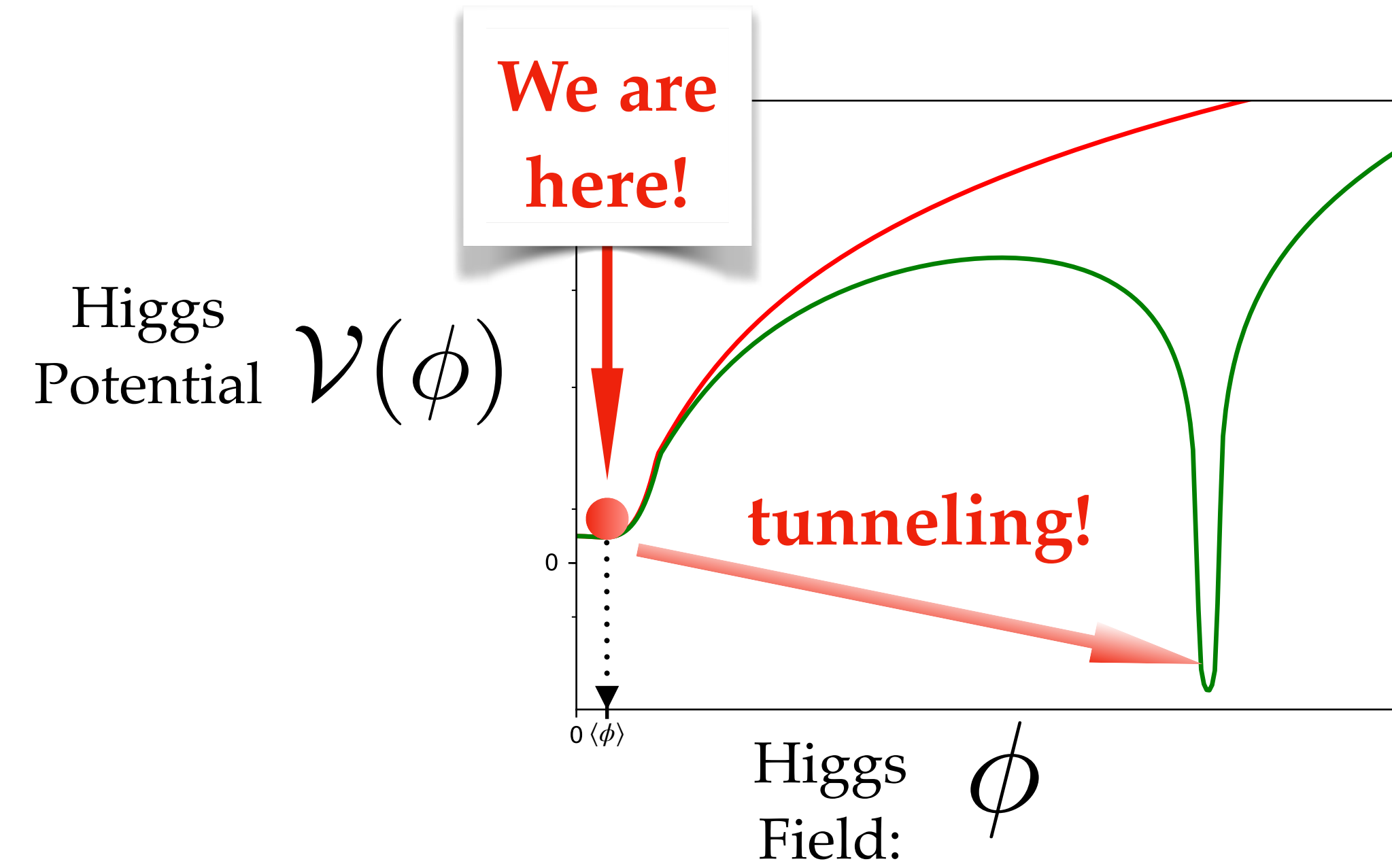


Questions

Questions

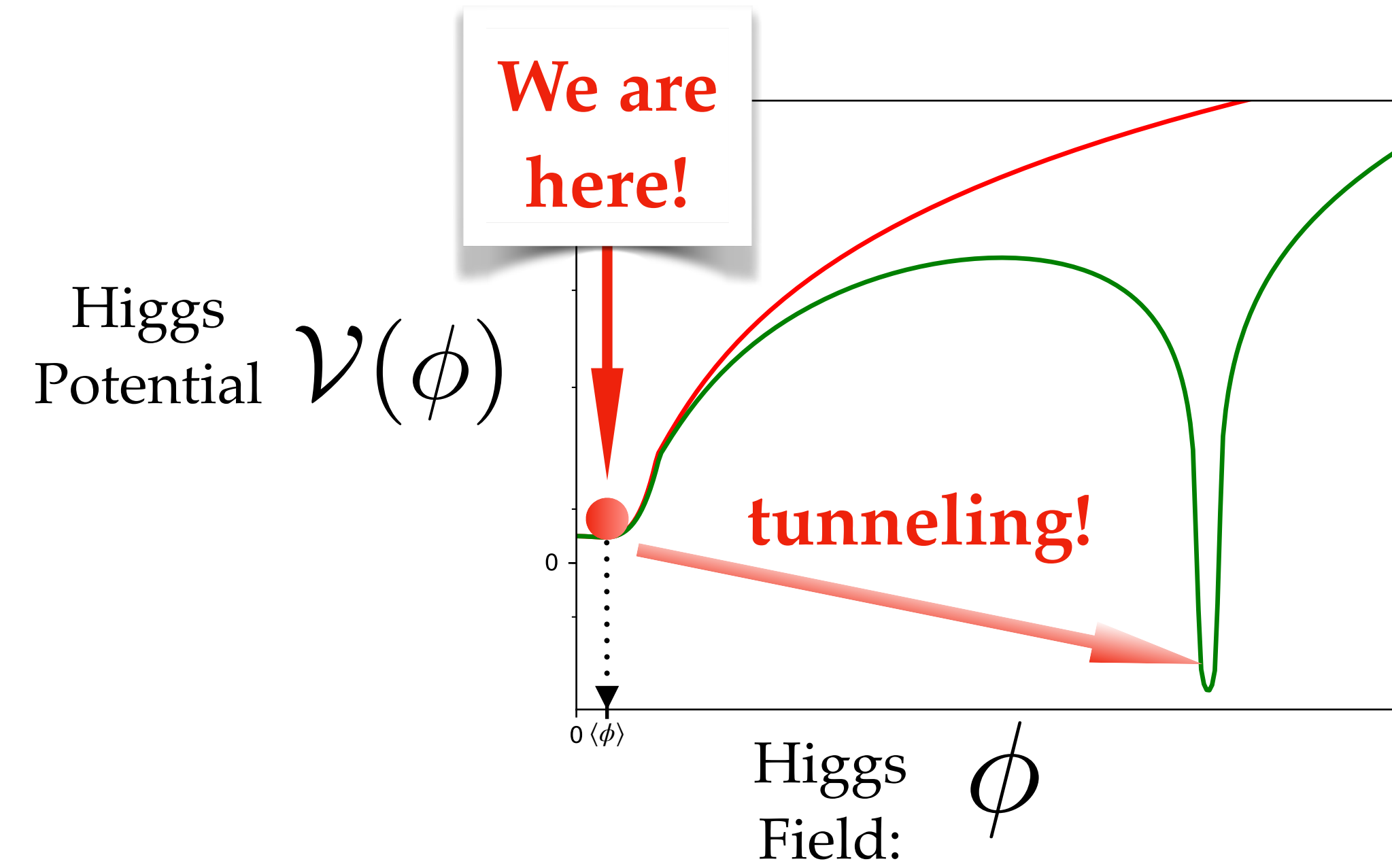


Questions



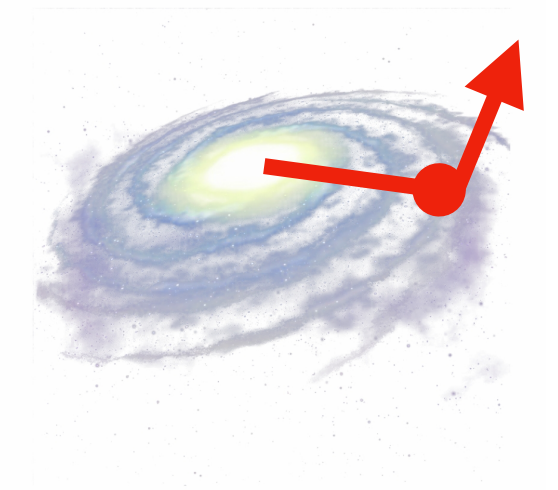
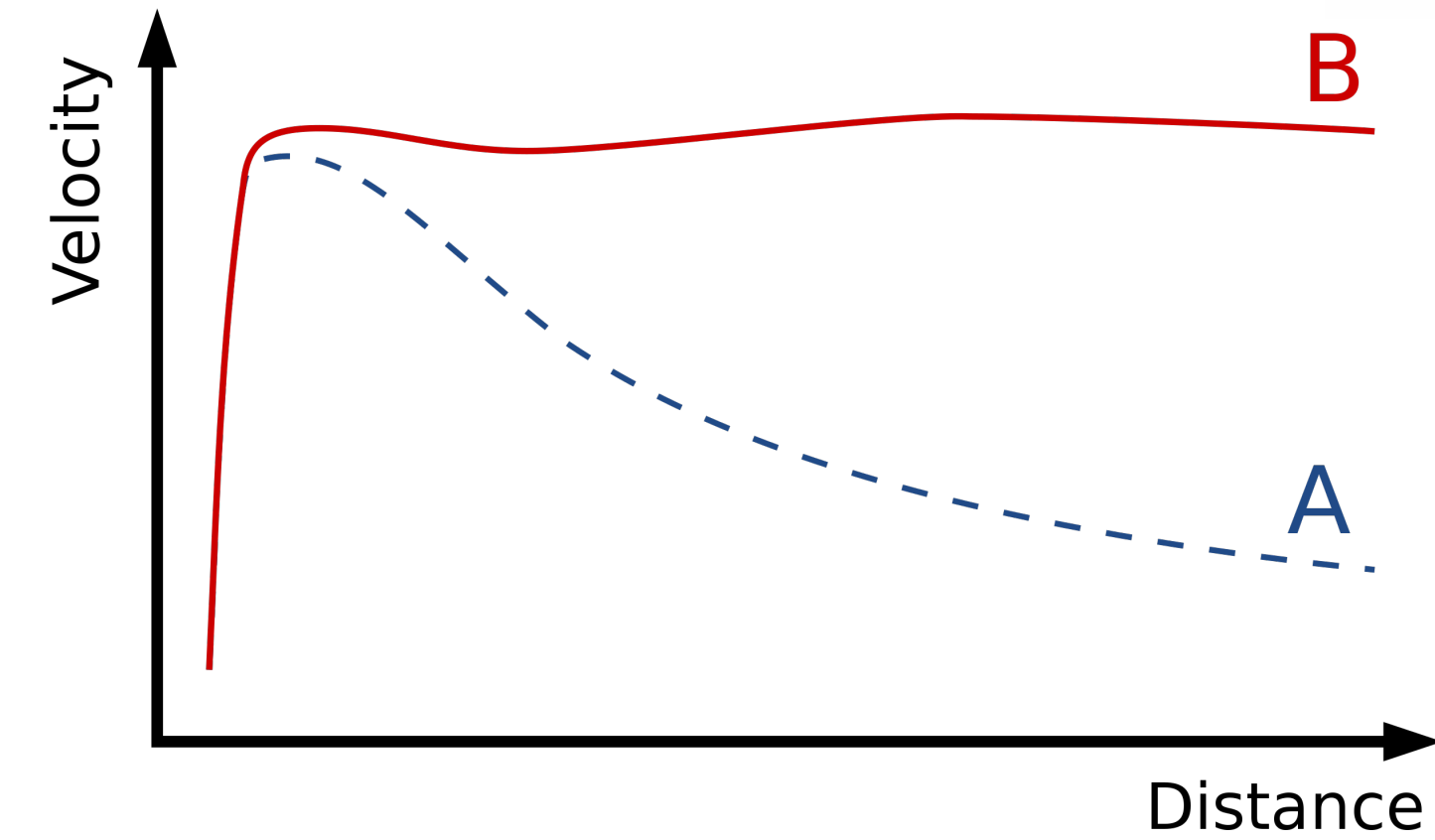
Vacuum Stability

Questions



Vacuum Stability

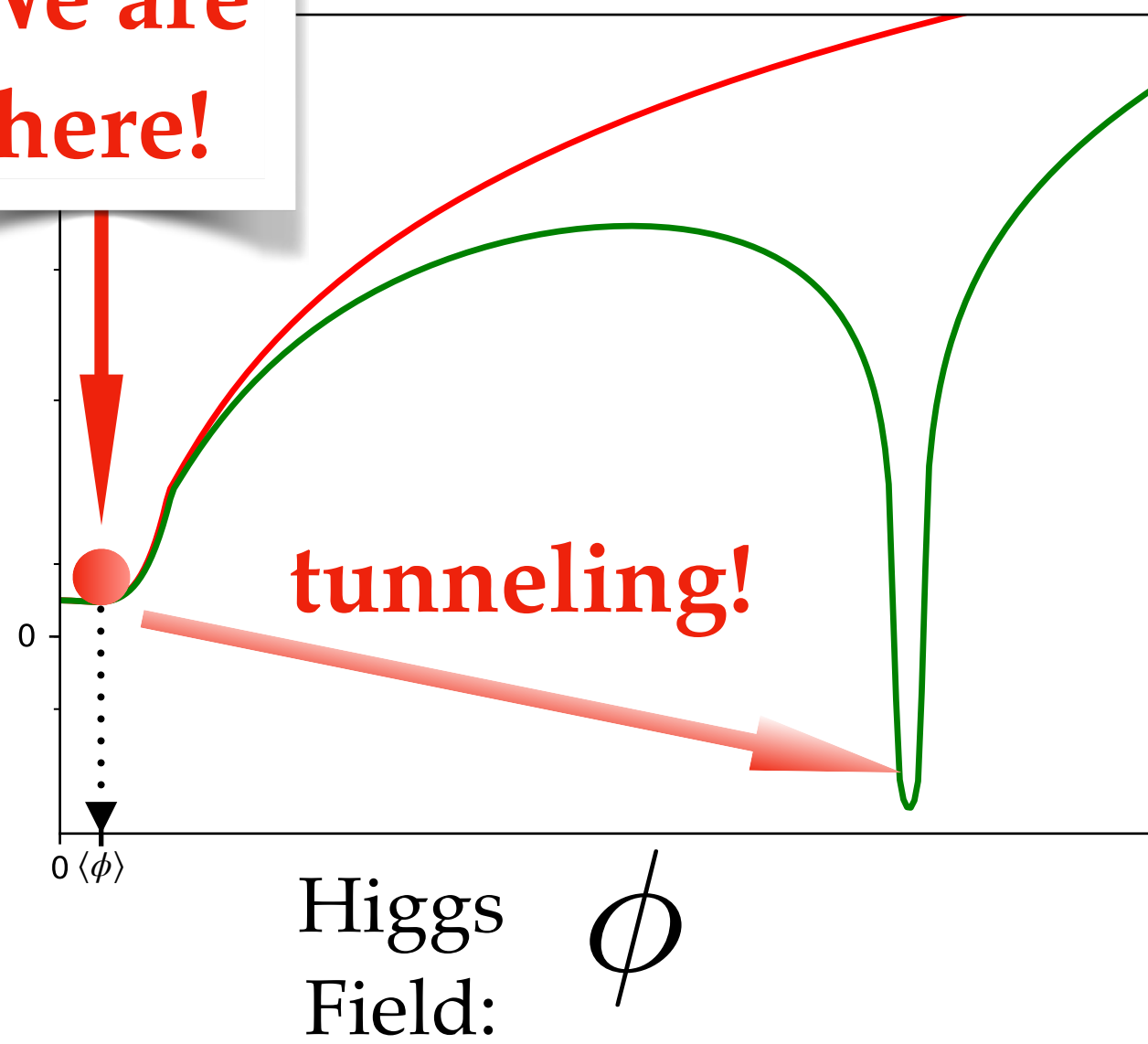
Rotation curve of a typical spiral galaxy:
predicted (A) and observed (B).



Questions

We are here!

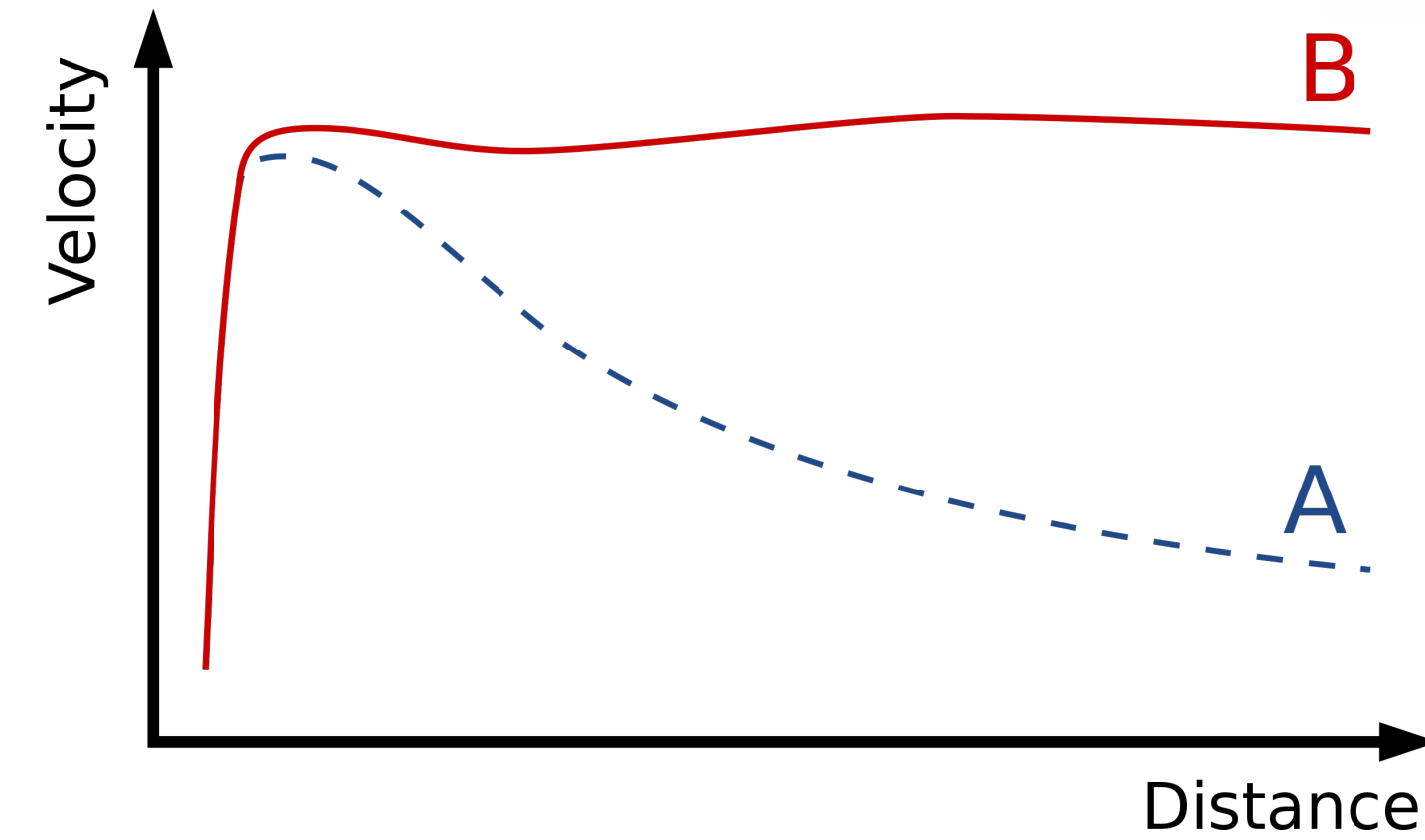
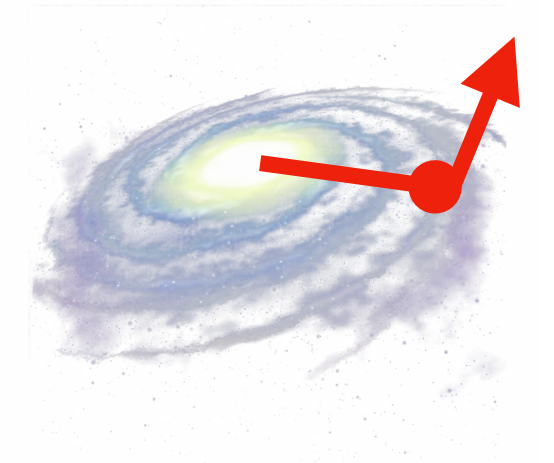
Higgs Potential $\mathcal{V}(\phi)$



Vacuum Stability

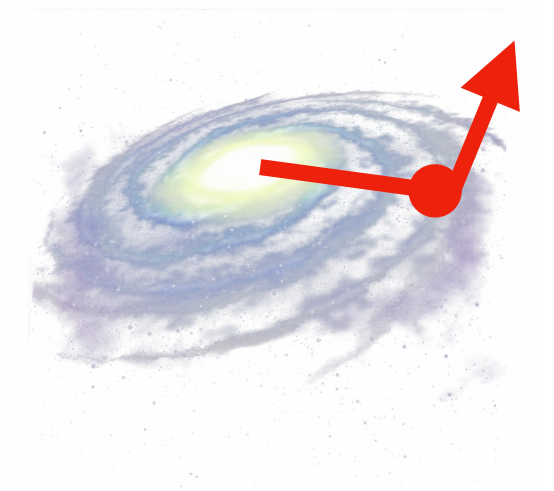
Dark Matter

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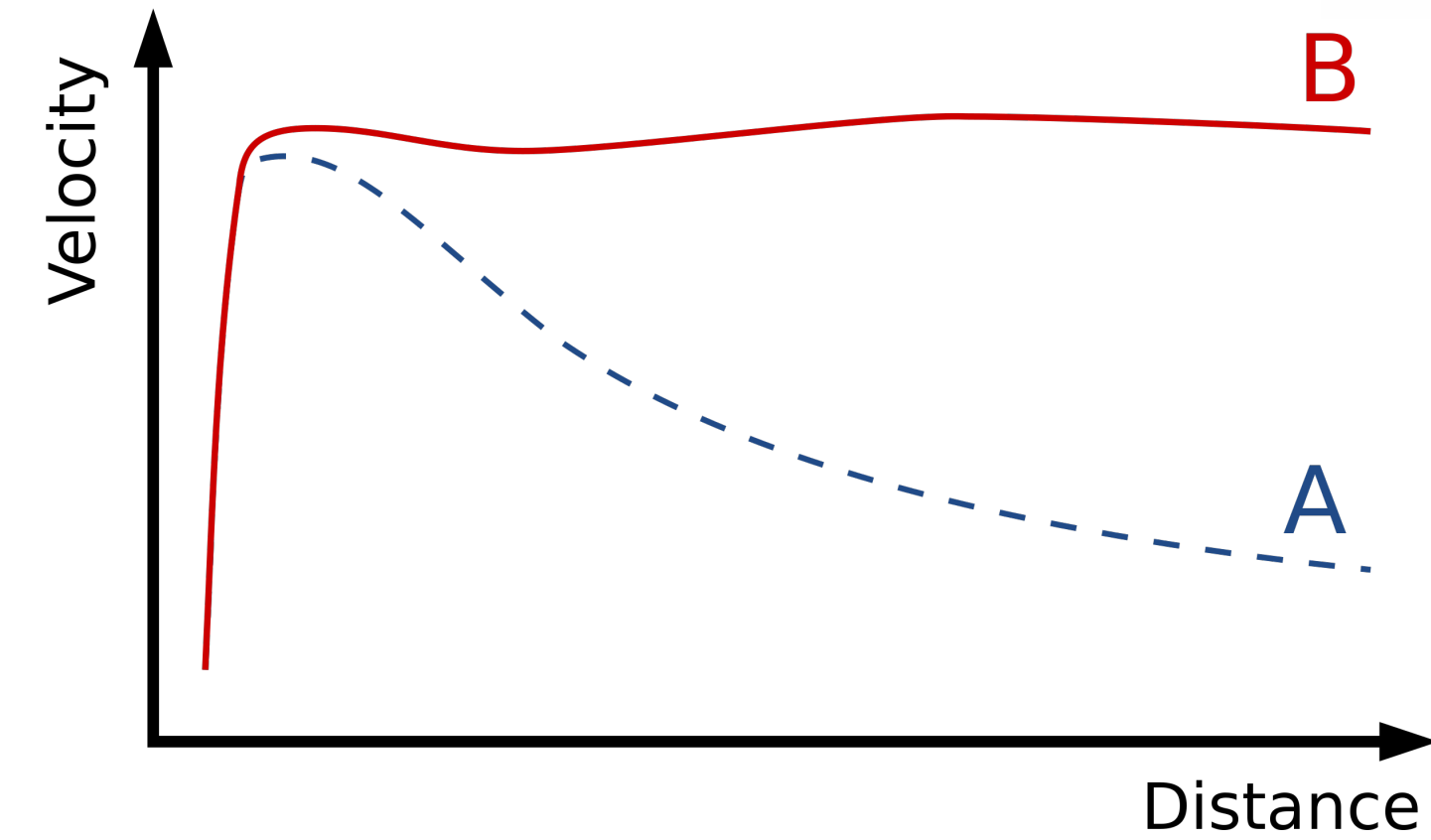


Questions

Dark Matter

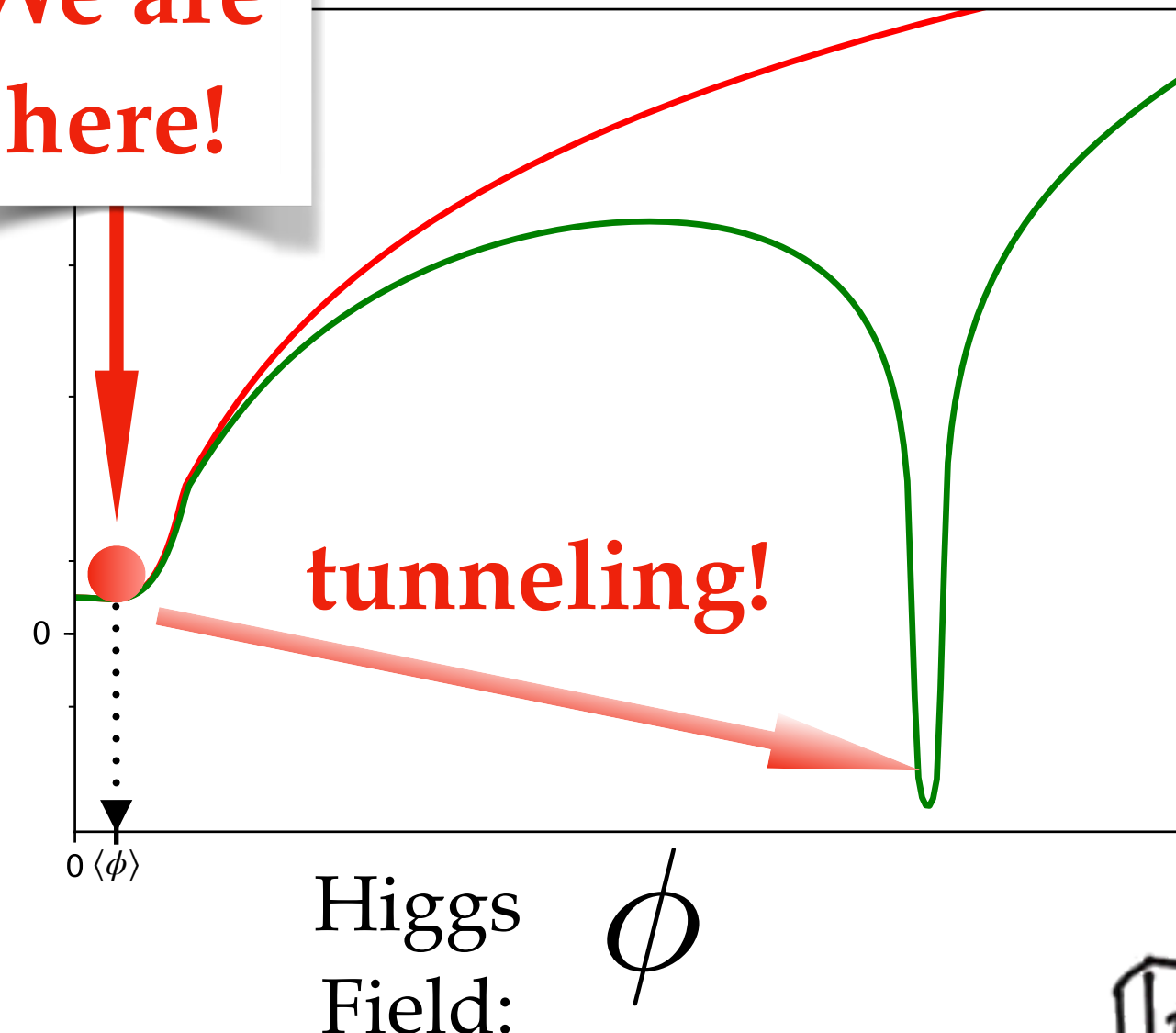


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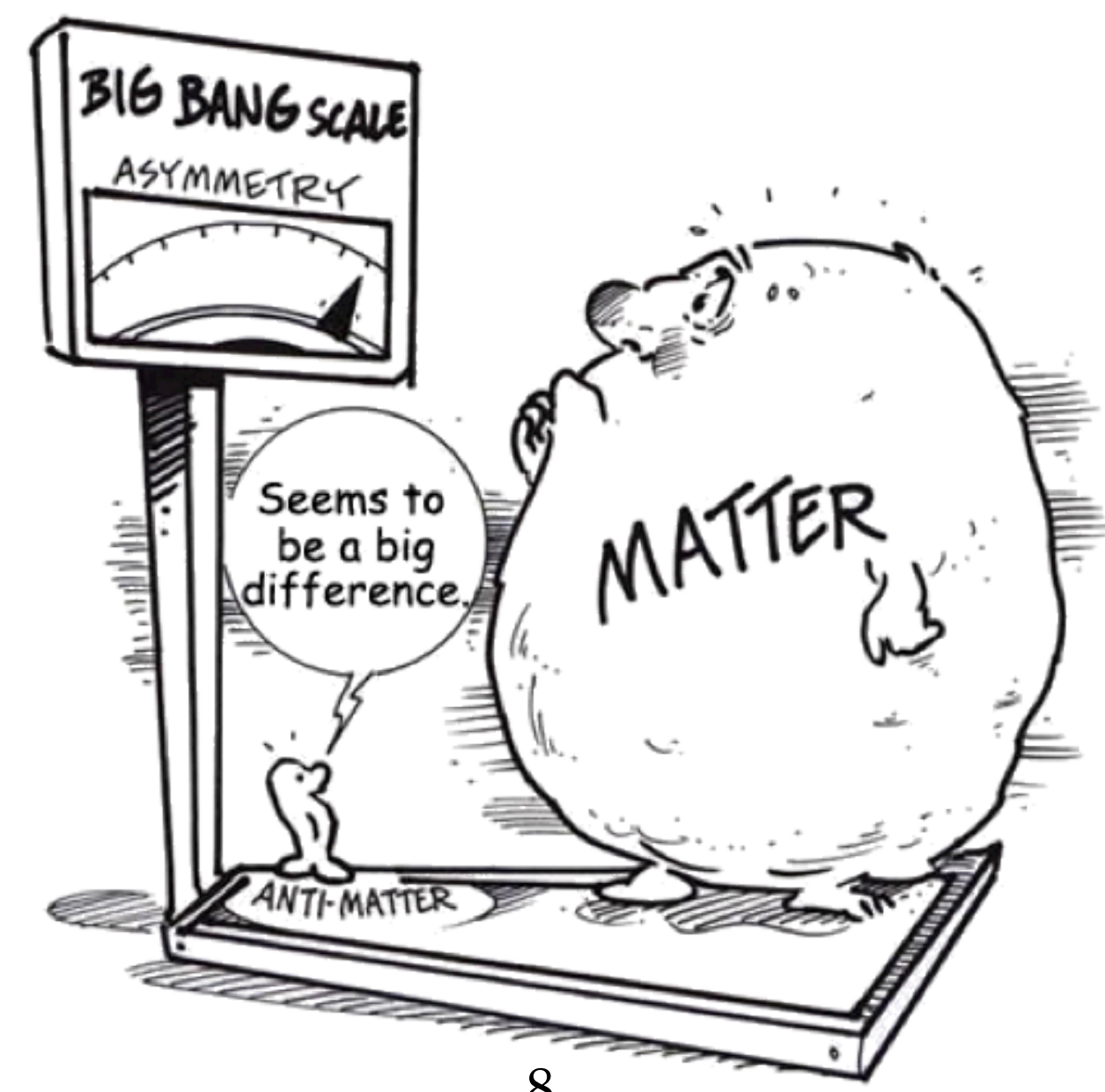


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Higgs Potential $V(\phi)$

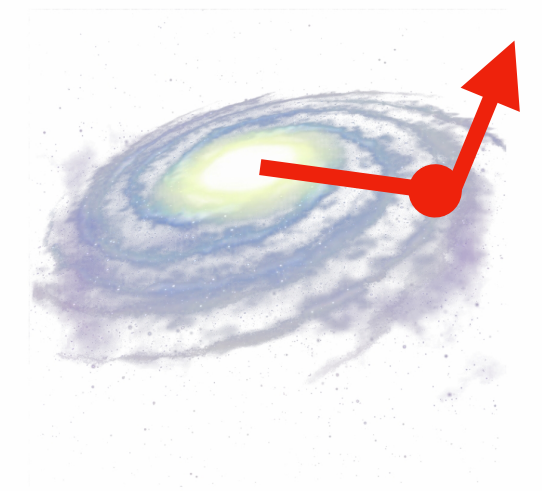


Vacuum Stability

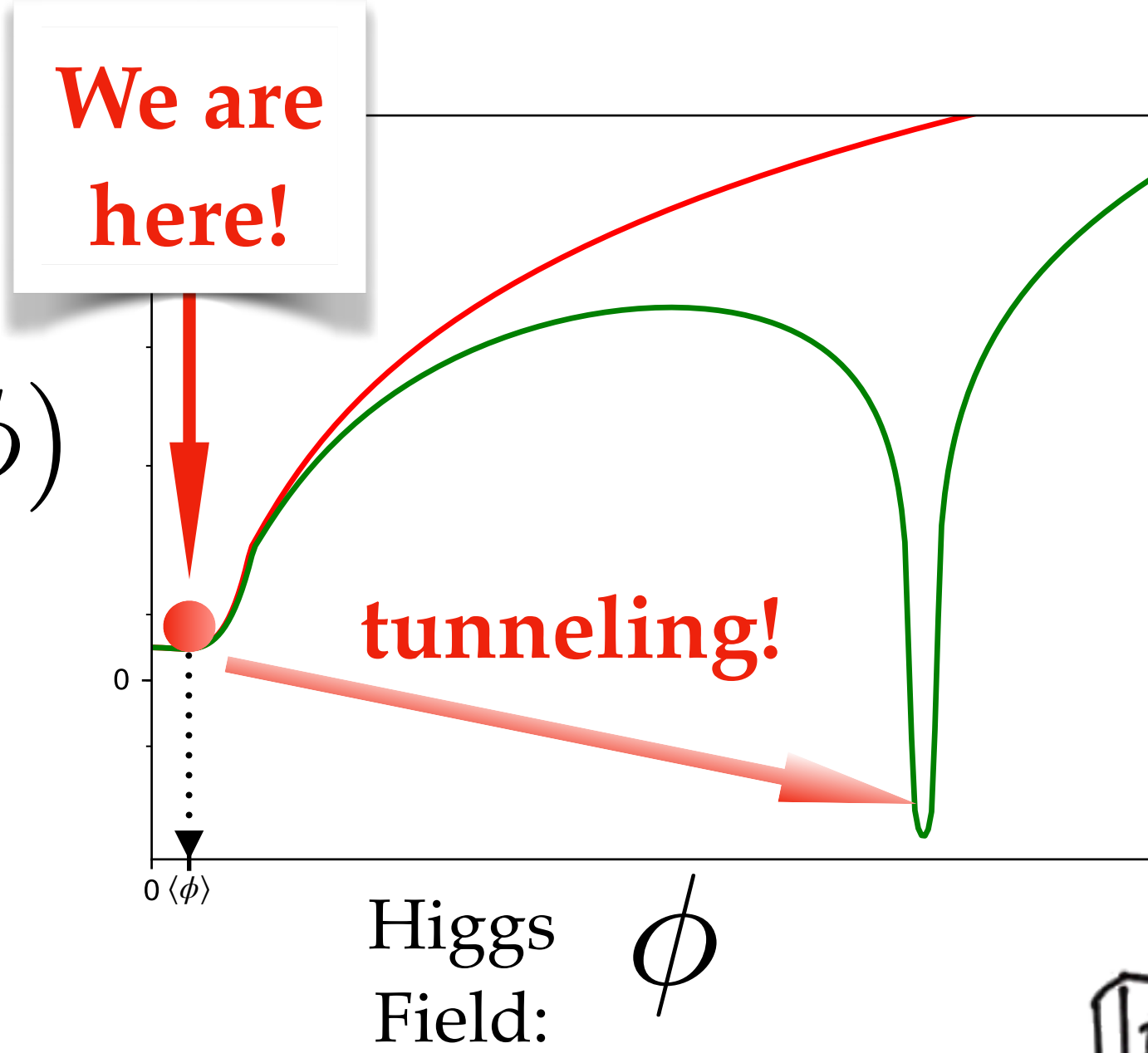
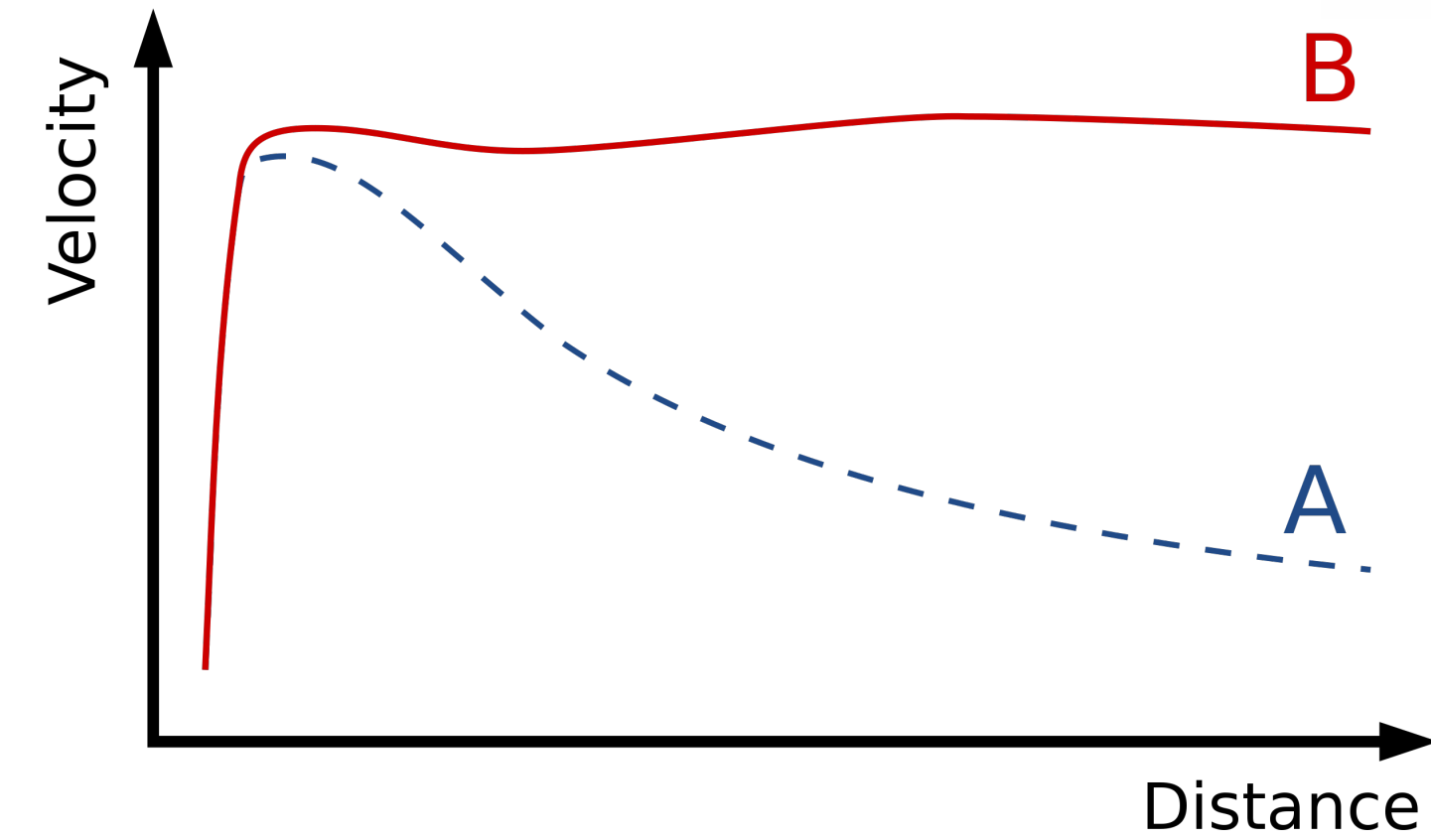


Questions

Dark Matter



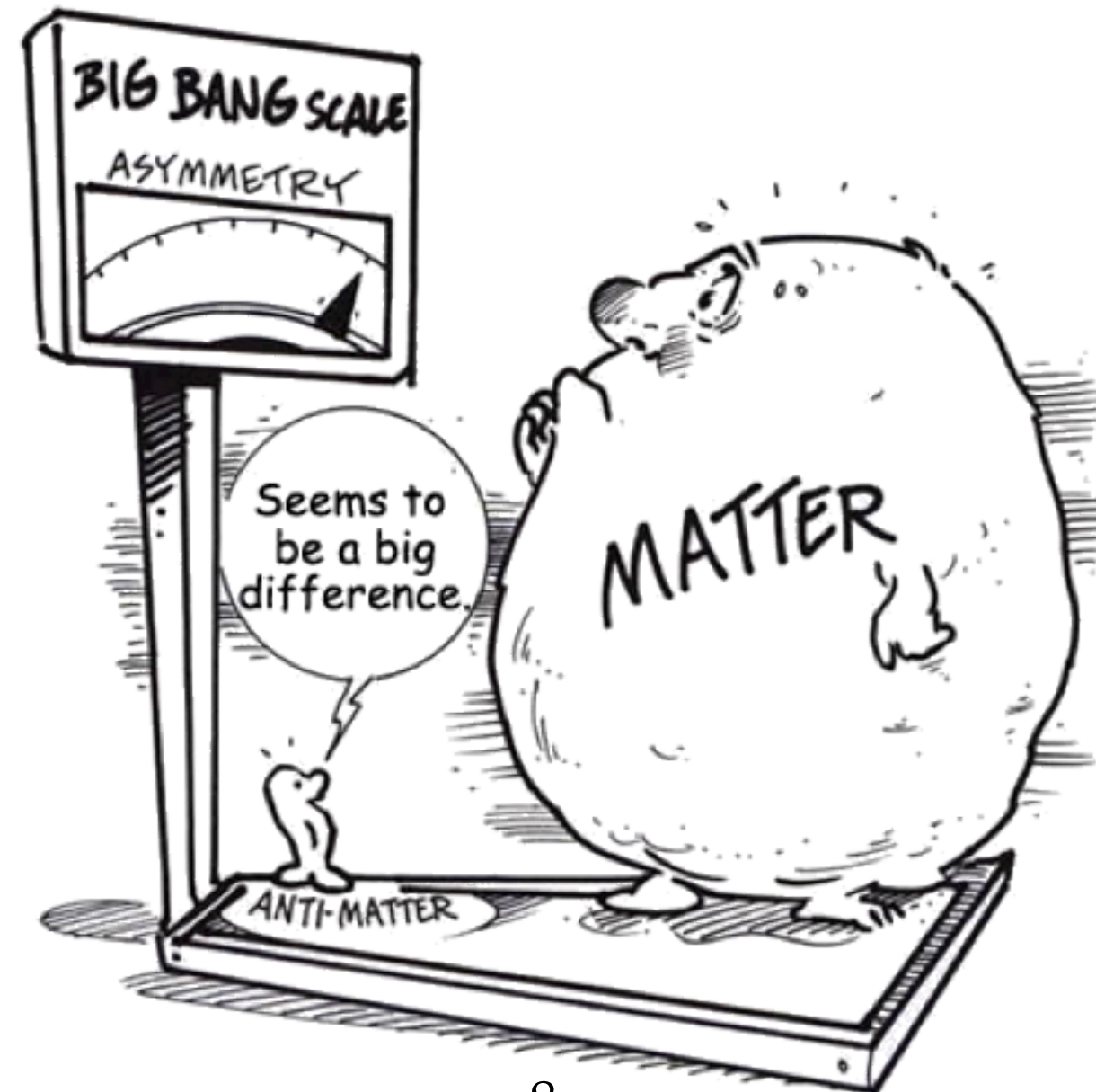
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Higgs Potential $V(\phi)$

Higgs Field: ϕ

Vacuum Stability



Matter-anti-matter Asymmetry



Dark Matter

Vacuum Stability

**Matter-anti-matter
Asymmetry**



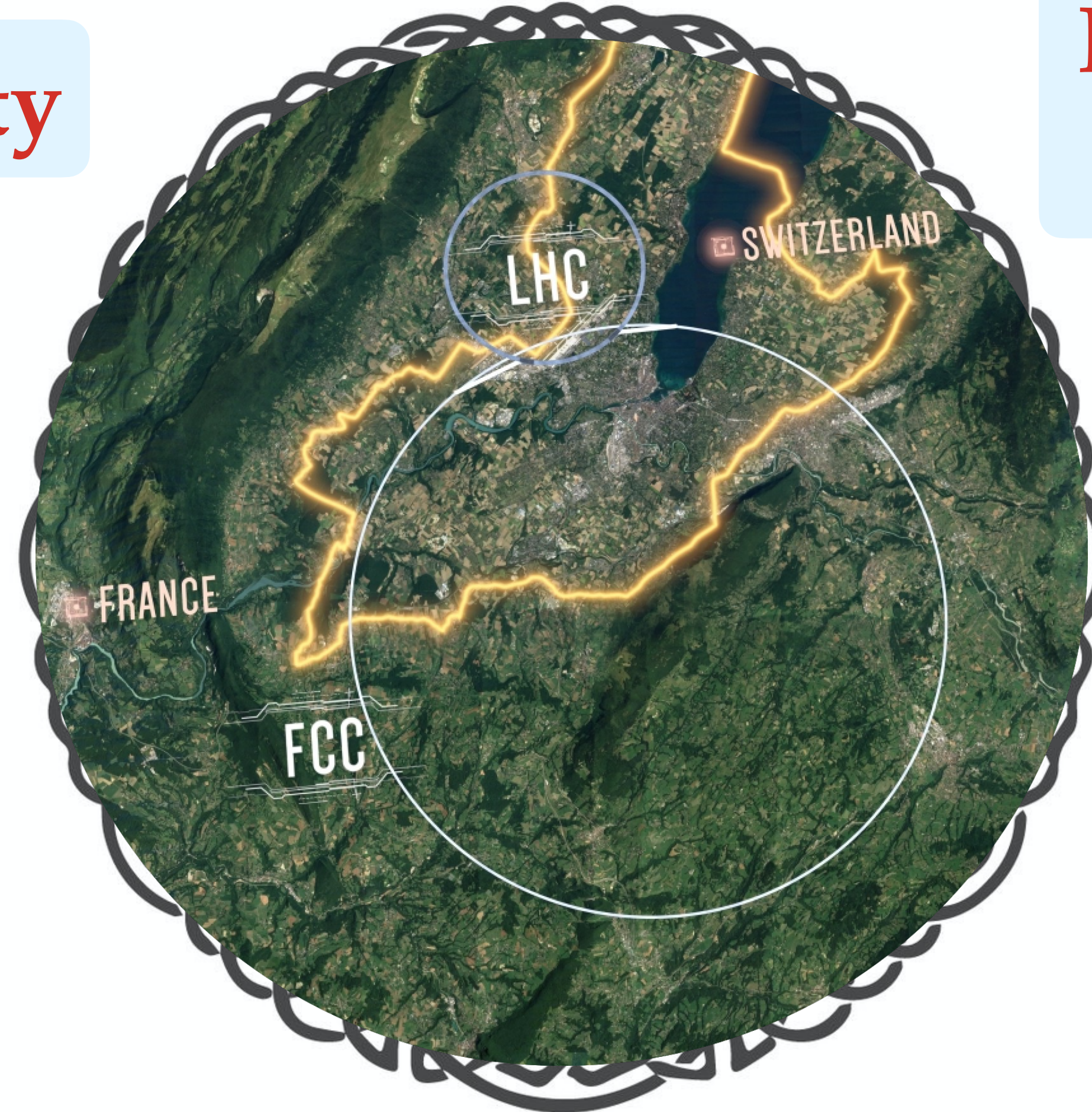
The Higgs Field & Symmetry Breaking



Dark Matter

Vacuum Stability

**Matter-anti-matter
Asymmetry**



e.g. Future Circular Collider:
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e.g. "High-Energy" LHC:
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e.g. Muon Collider.

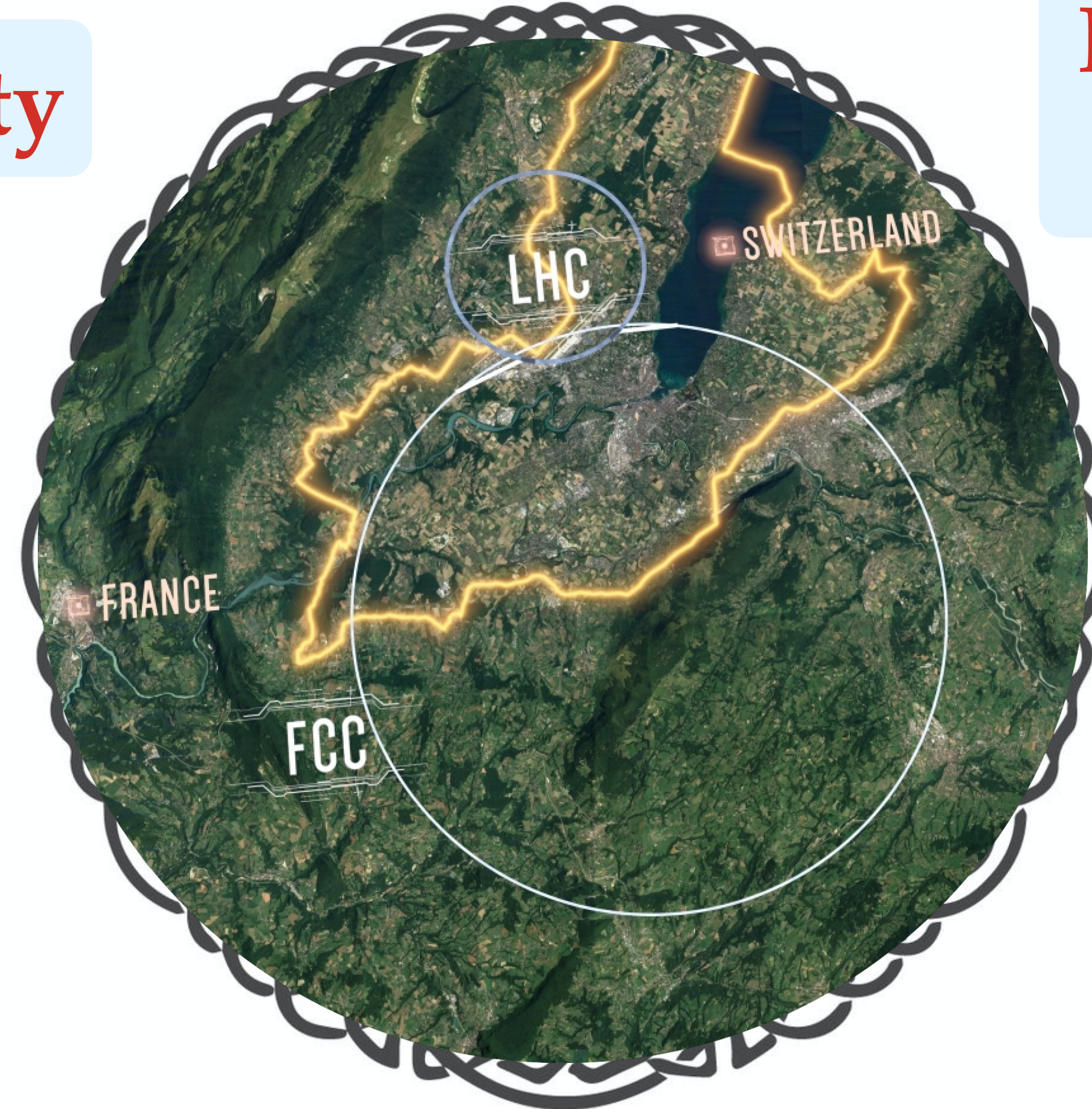
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Andreas Papaefstathiou

Dark Matter

Vacuum Stability



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The Higgs Field & Symmetry Breaking



Breaking the Symmetry



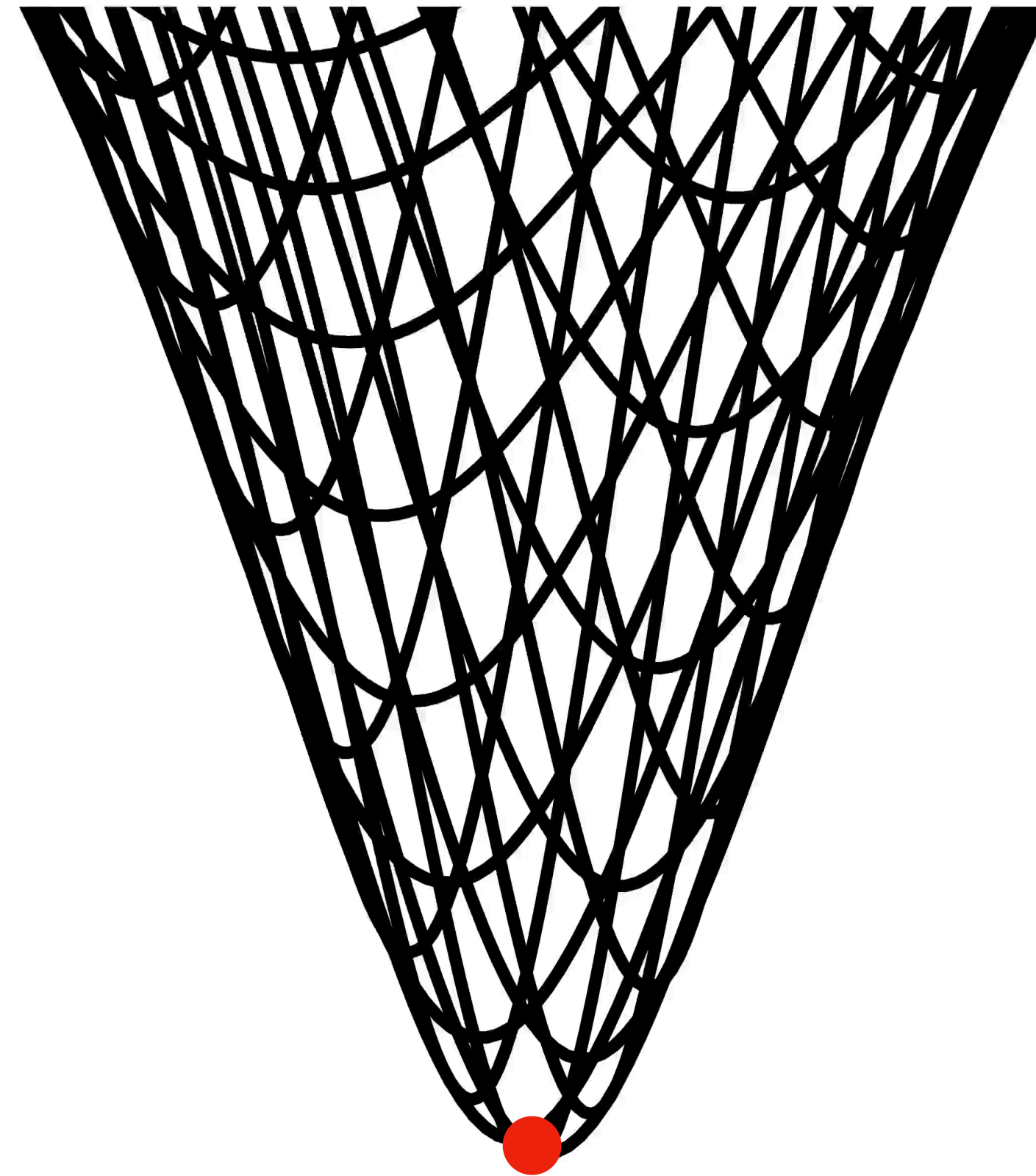
Breaking the Symmetry in the Standard Model

The potential of the Higgs field (ϕ), a complex doublet:

$$\phi = \begin{pmatrix} \phi_1 + i\phi_3 \\ \phi_2 + i\phi_4 \end{pmatrix}$$

(Arbitrarily) Set $\phi_3 = \phi_4 = 0$
to illustrate potential in
 (ϕ_1, ϕ_2) plane.

An example of the evolution of $\mathcal{V}(\phi)$,
Early universe \rightarrow Today:



$\mathcal{V}(\phi)$

Higgs Field Potential

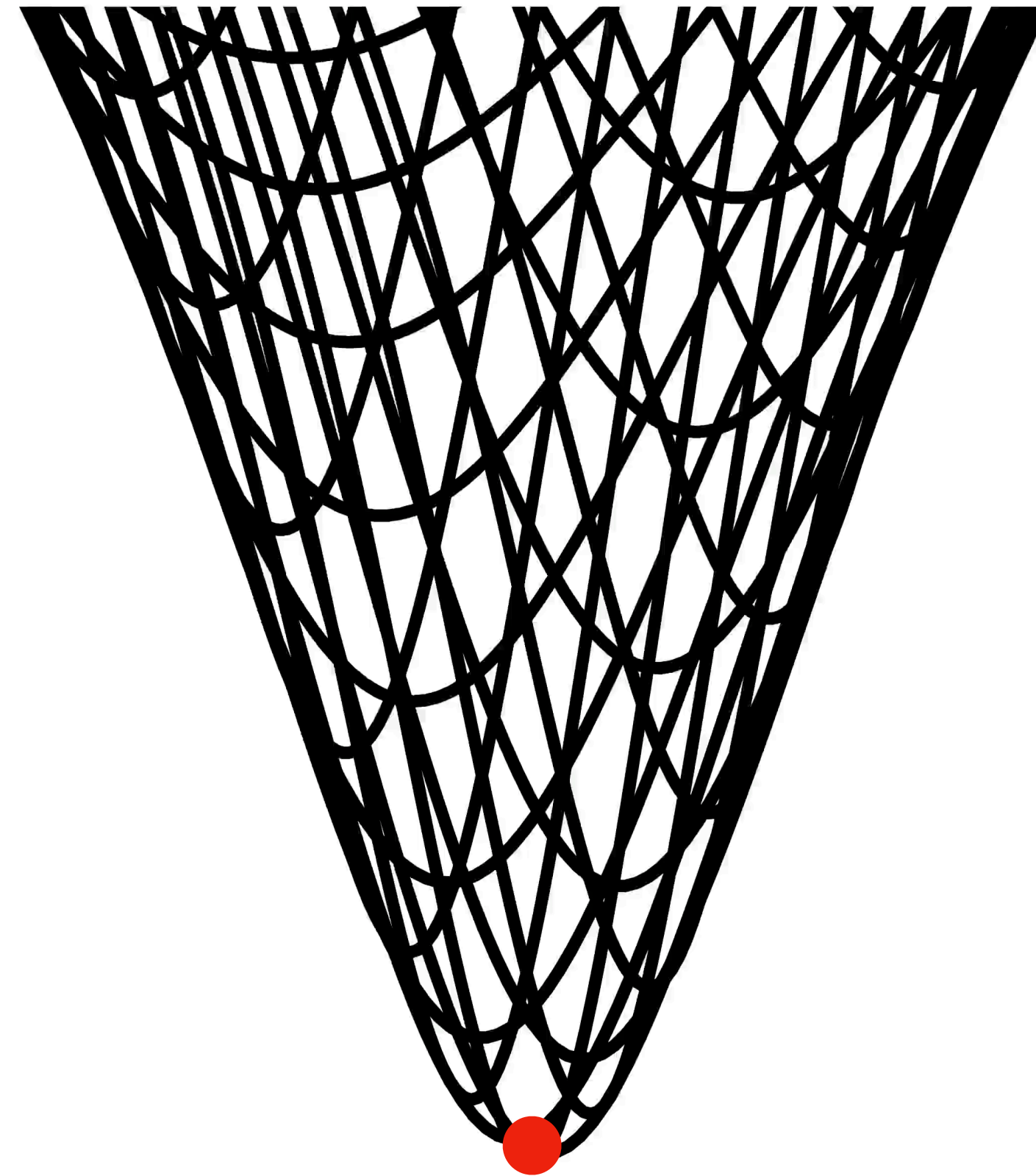
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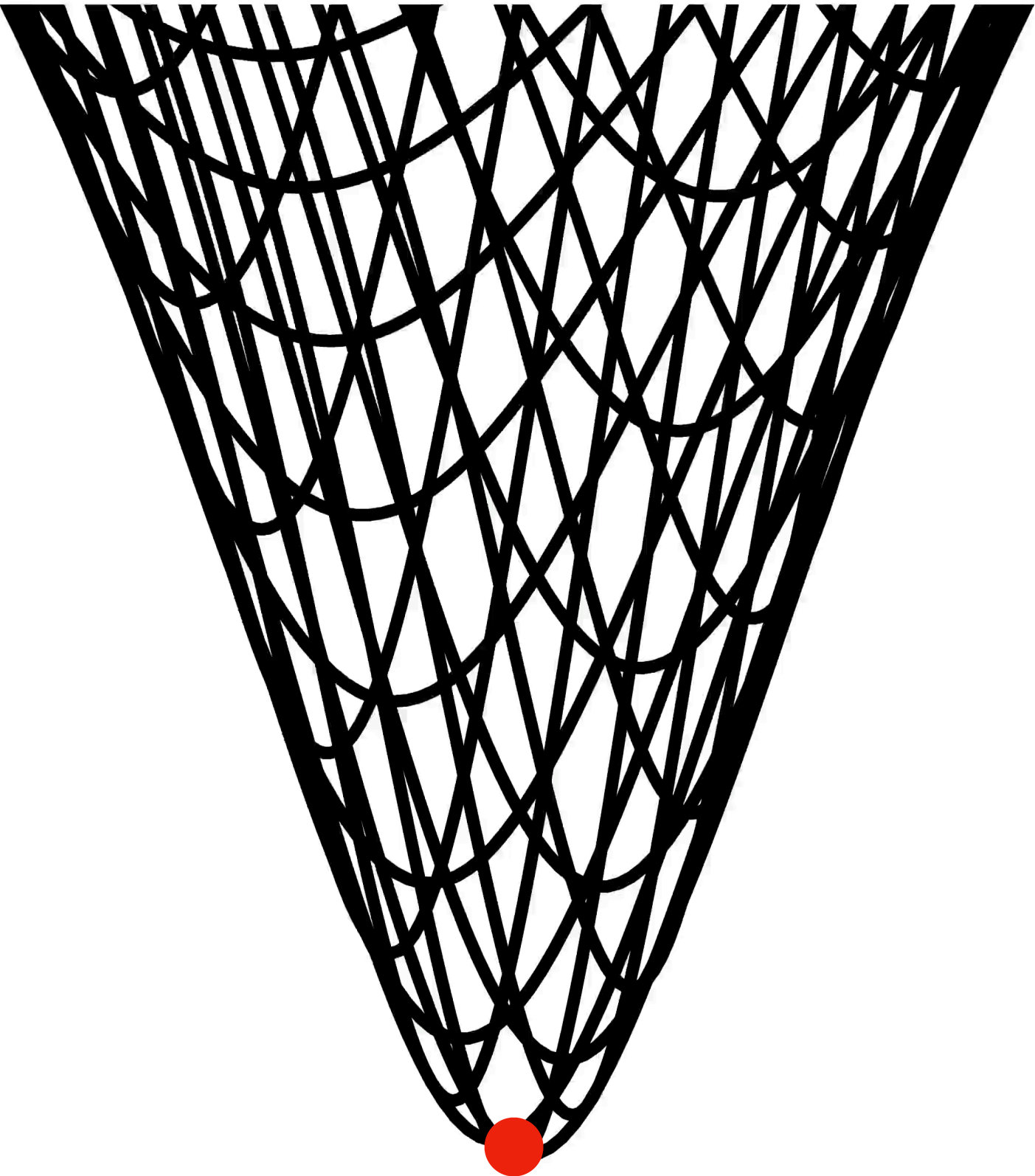
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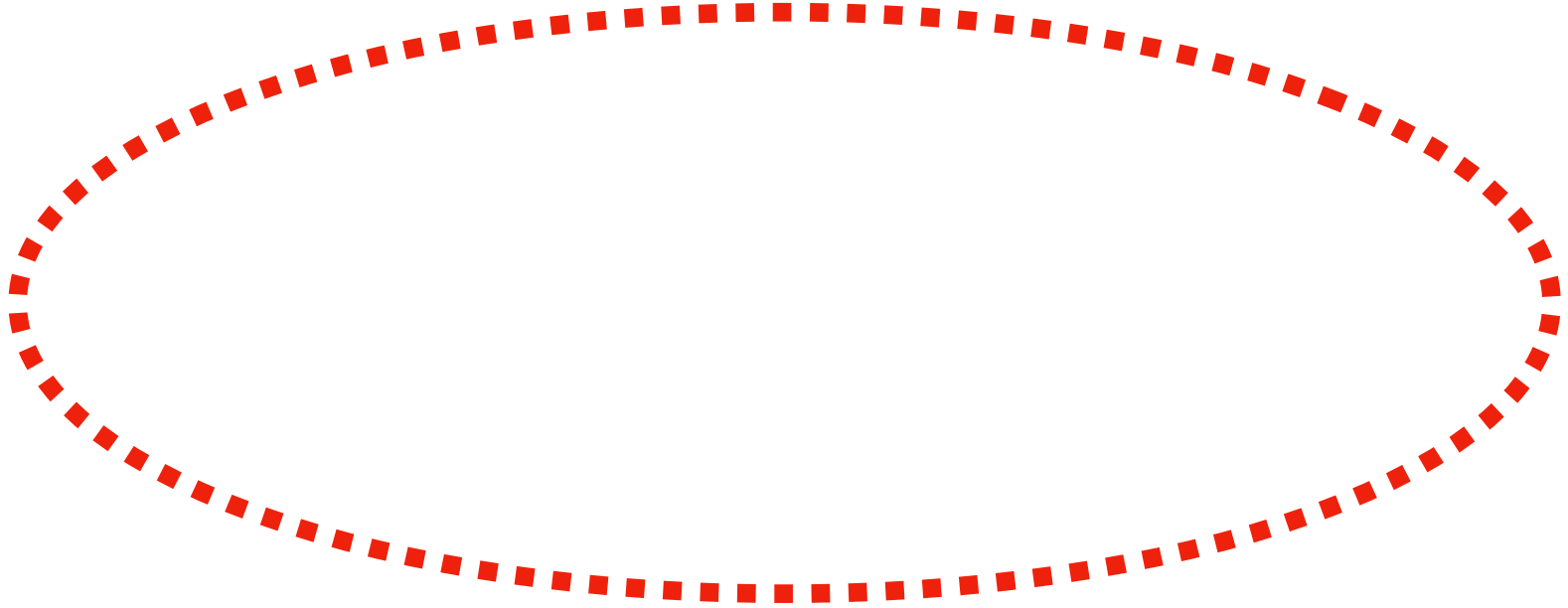
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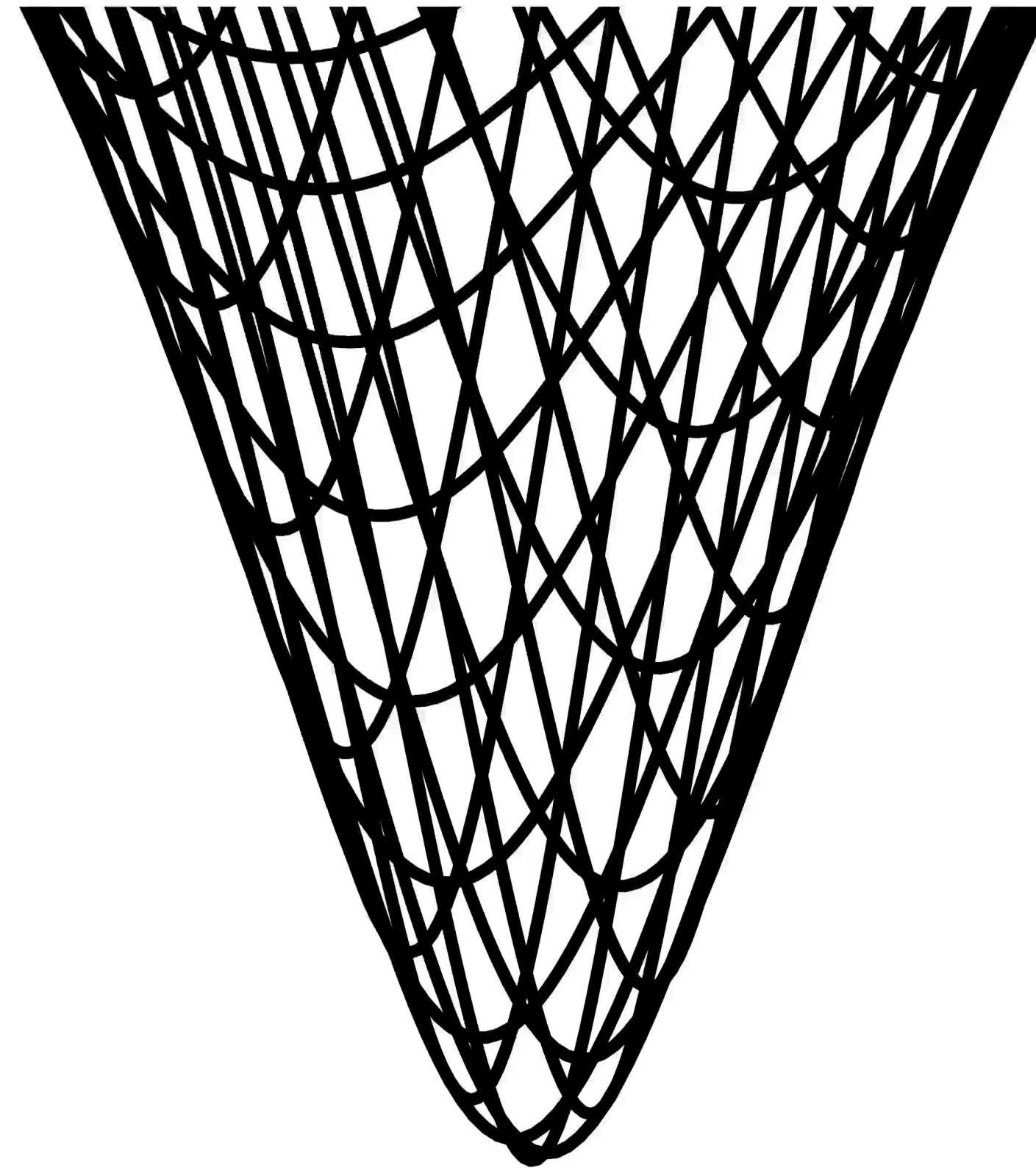
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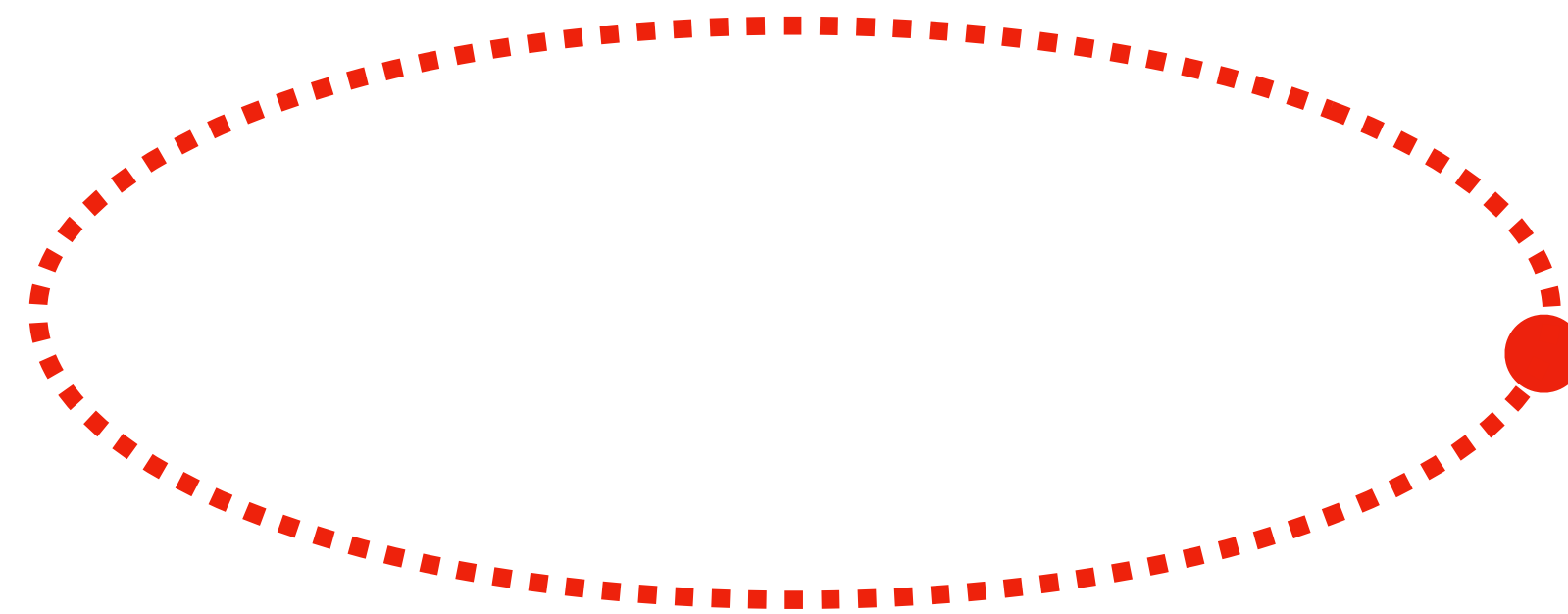
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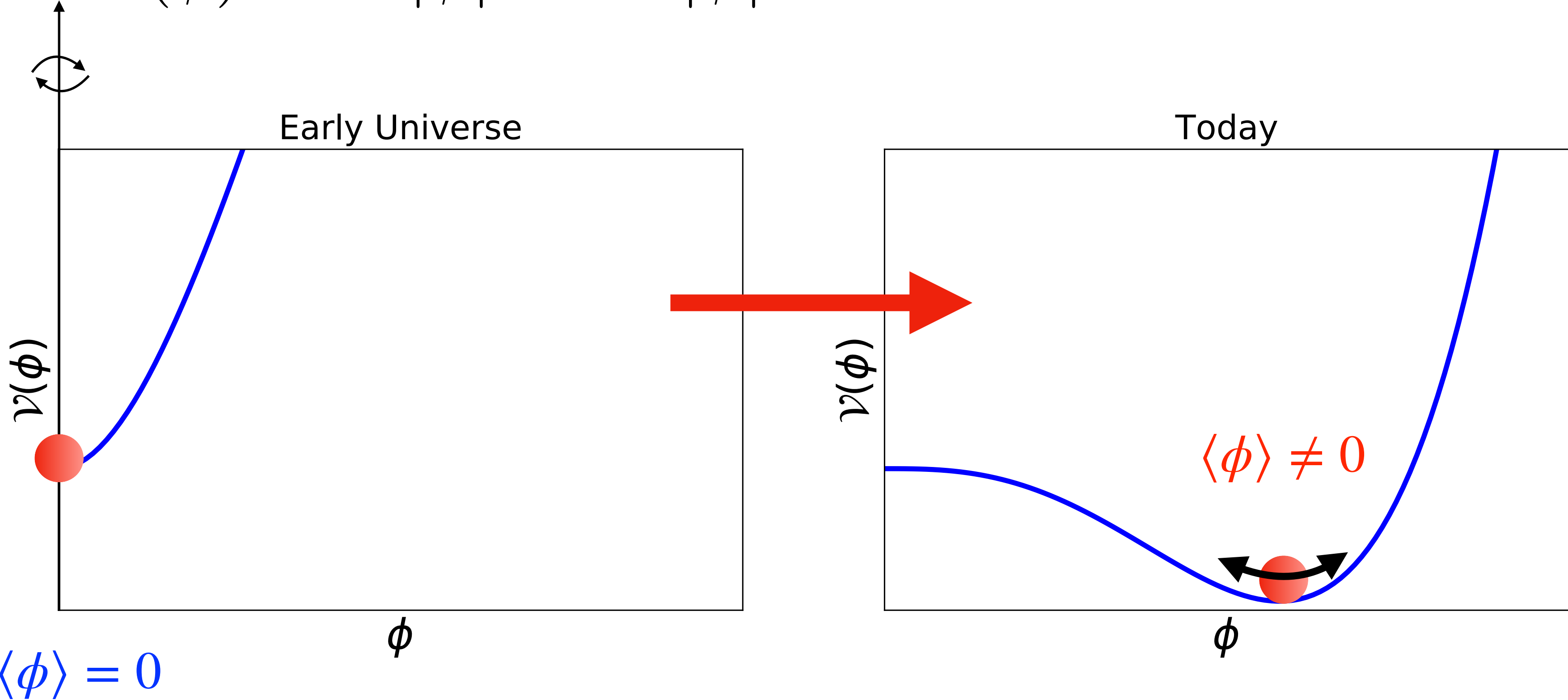
Higgs Field Potential



Breaking the Symmetry in the Standard Model

$$\mathcal{V}(\phi) = \text{●} |\phi|^2 + \text{■} |\phi|^4$$

Higgs field (ϕ) potential

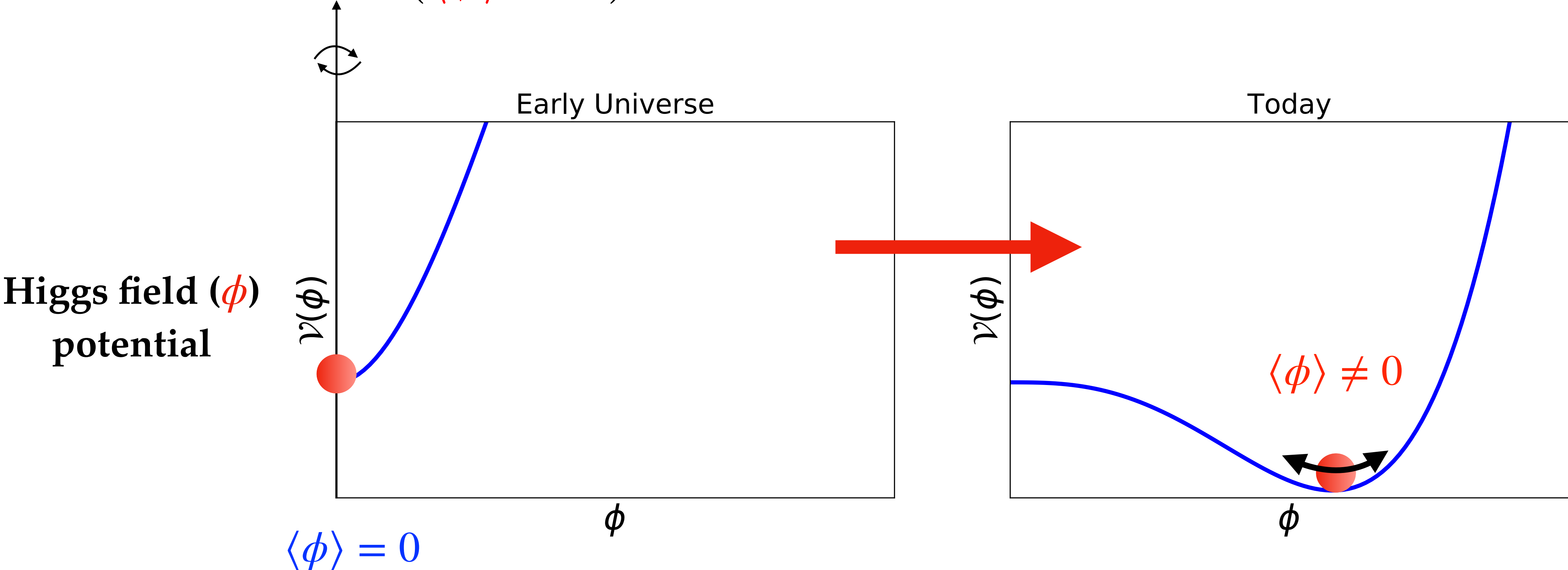


More Symmetry \rightarrow Less (obvious) Symmetry



Breaking the Symmetry in the Standard Model

$$\mathcal{V}(\langle\phi\rangle + h) = \bullet h^2 + \blacktriangle h^3 + \blacksquare h^4 \rightarrow h \text{ is the Higgs boson! (LHC, 2012)}$$



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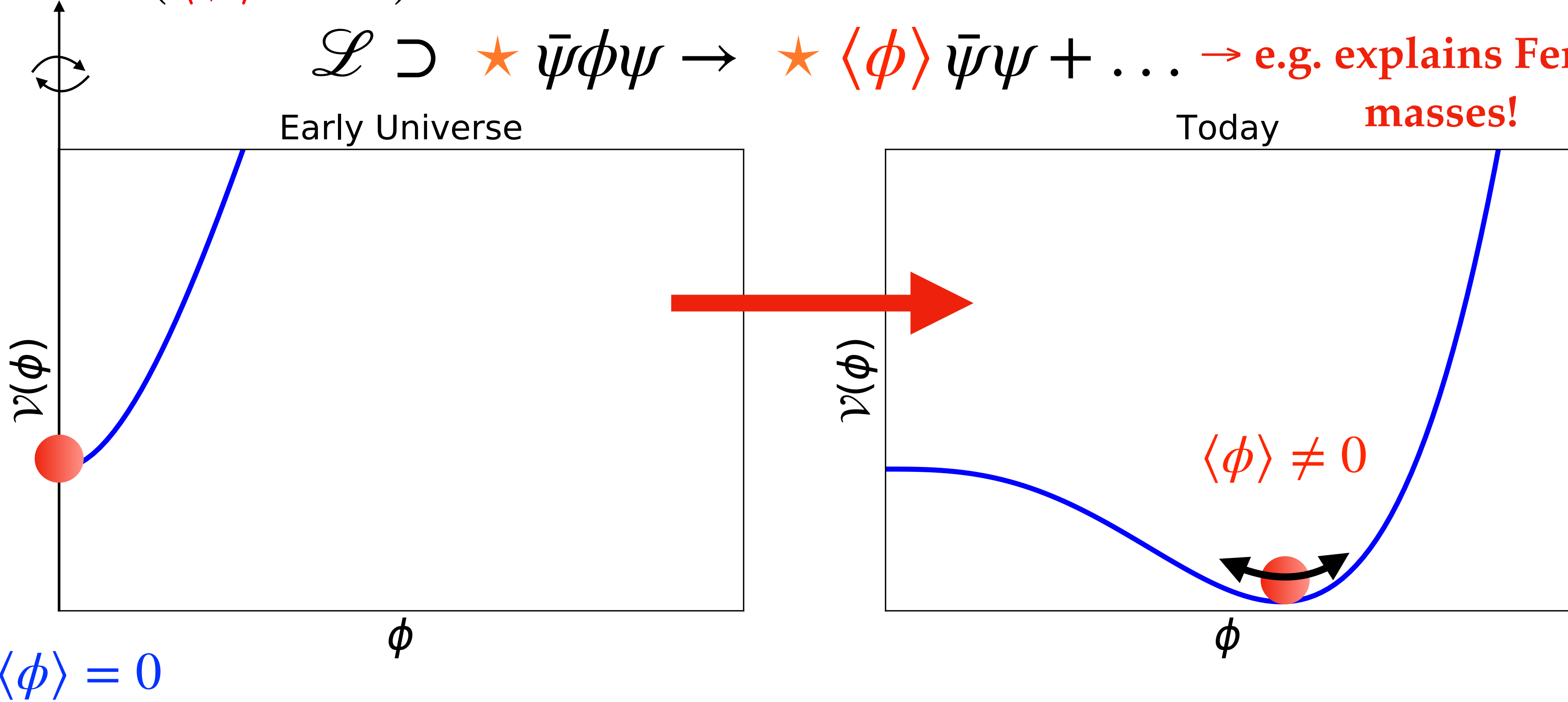


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$\mathcal{L} \supset \star \bar{\psi}\phi\psi \rightarrow \star \langle\phi\rangle \bar{\psi}\psi + \dots \rightarrow$ e.g. explains Fermion masses!

Higgs field (ϕ) potential



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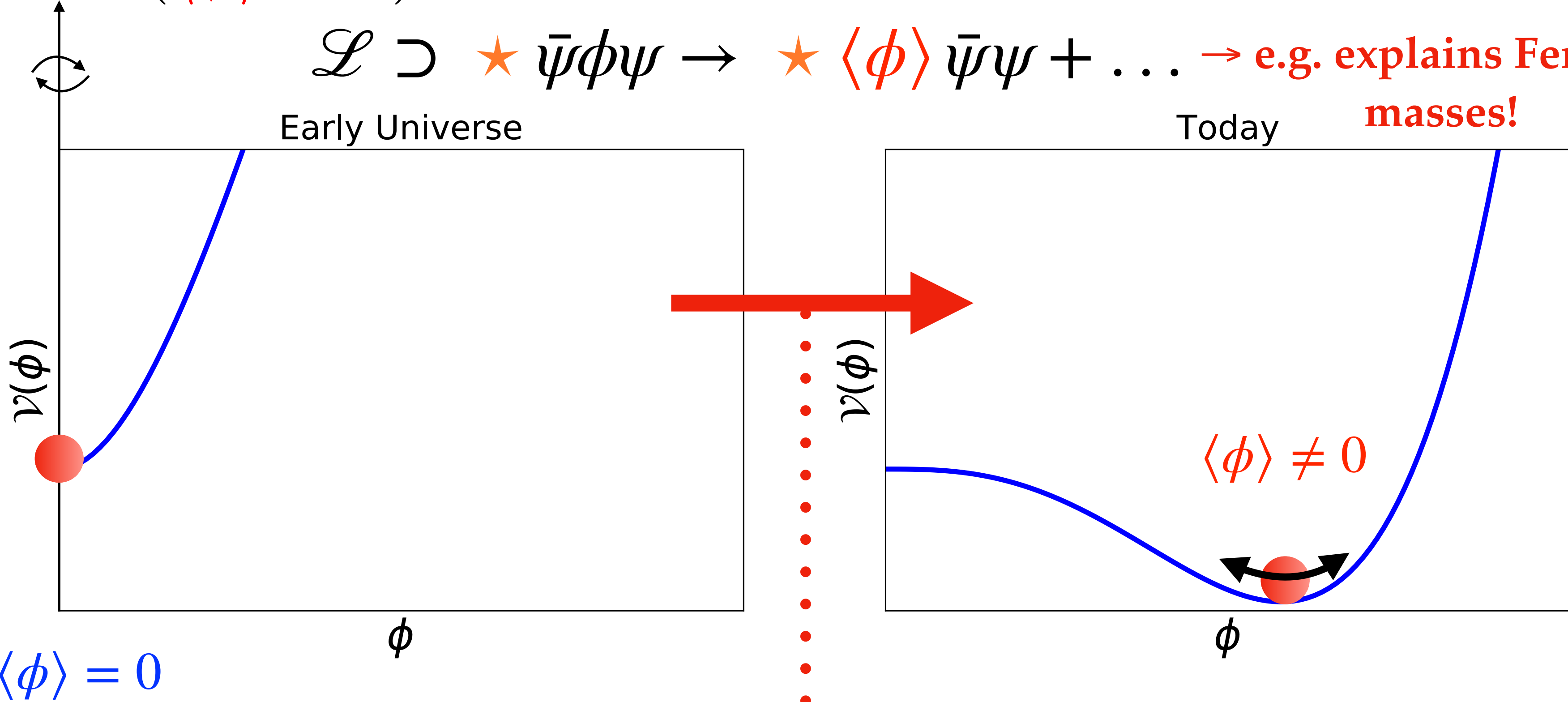


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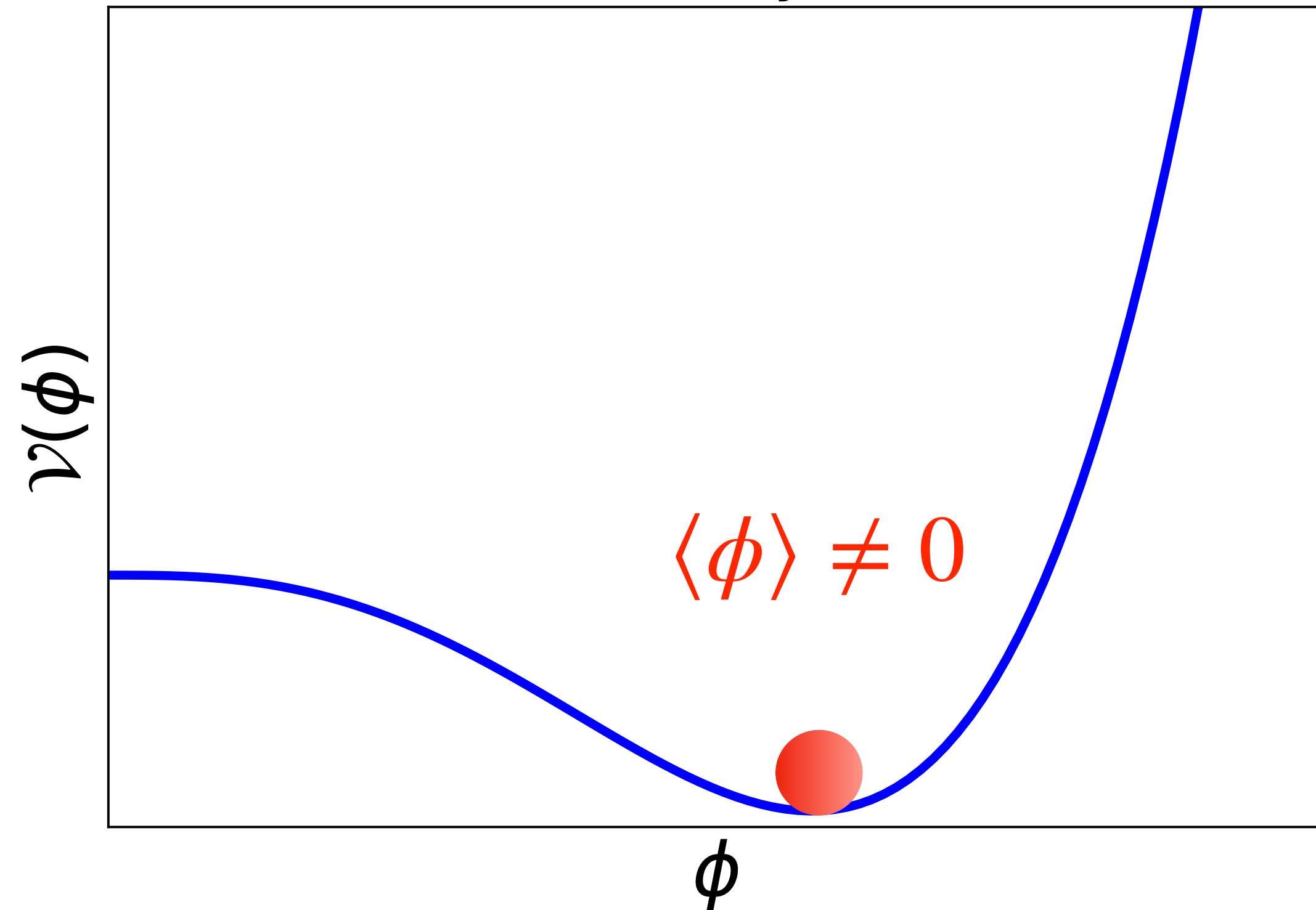
\equiv Electro-Weak Phase Transition (EWPT)



The Higgs Boson's Potential

$$\mathcal{V}(\langle\phi\rangle + h) = \bullet h^2 + \blacktriangle h^3 + \blacksquare h^4 \rightarrow \text{the Higgs boson's self-interactions.}$$

Today



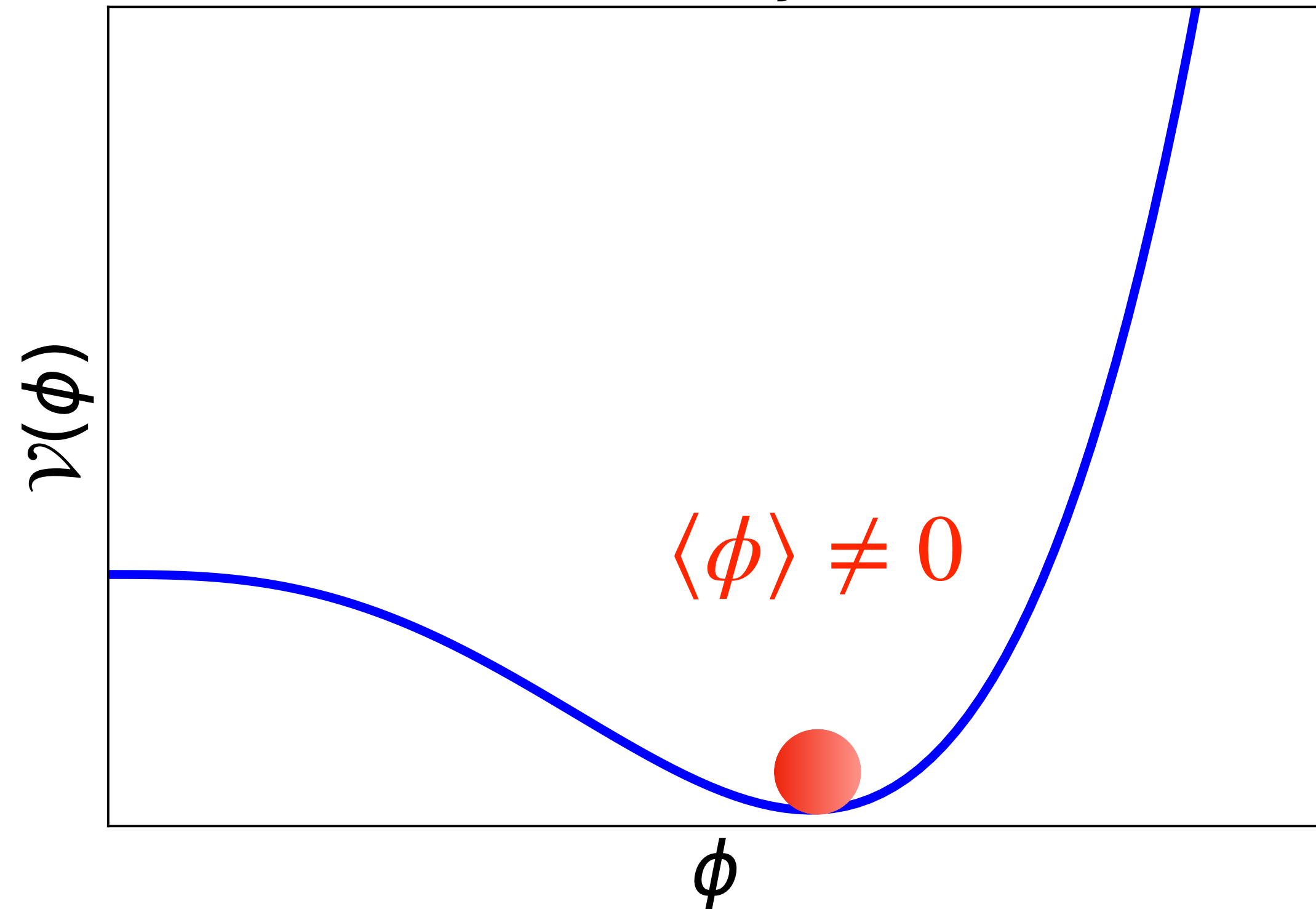
→ Determine shape of potential by measuring:

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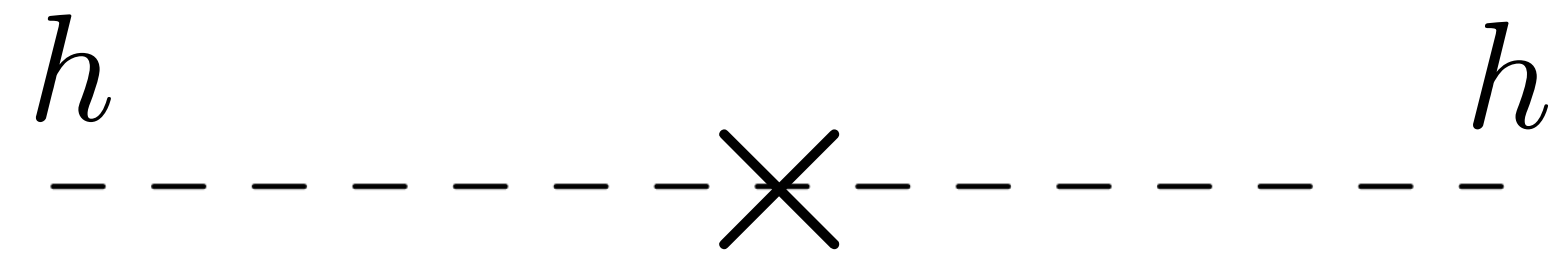
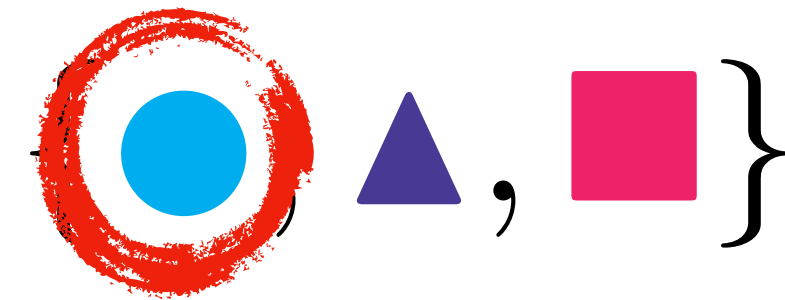
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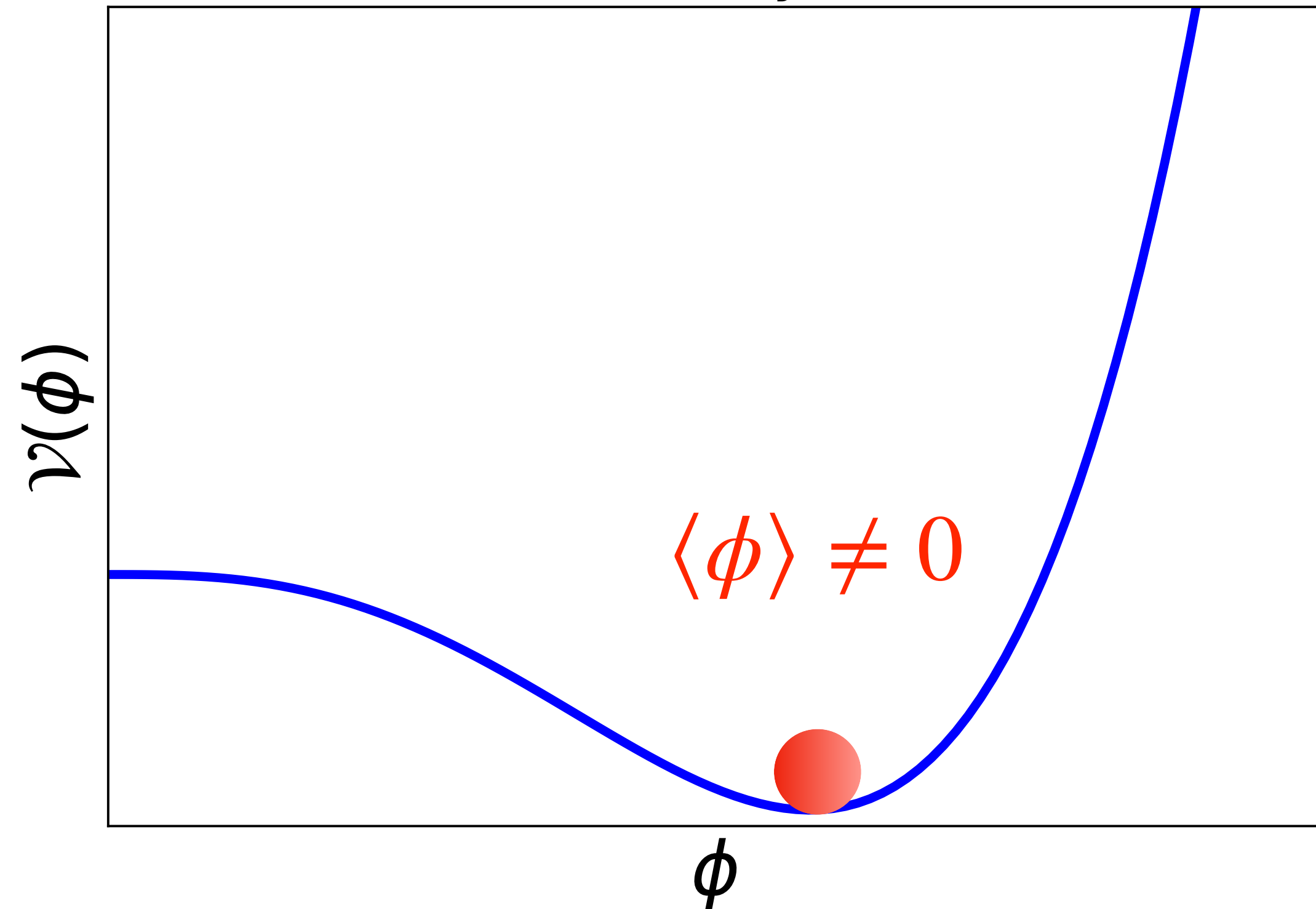
Higgs boson discovery @ LHC, 2012



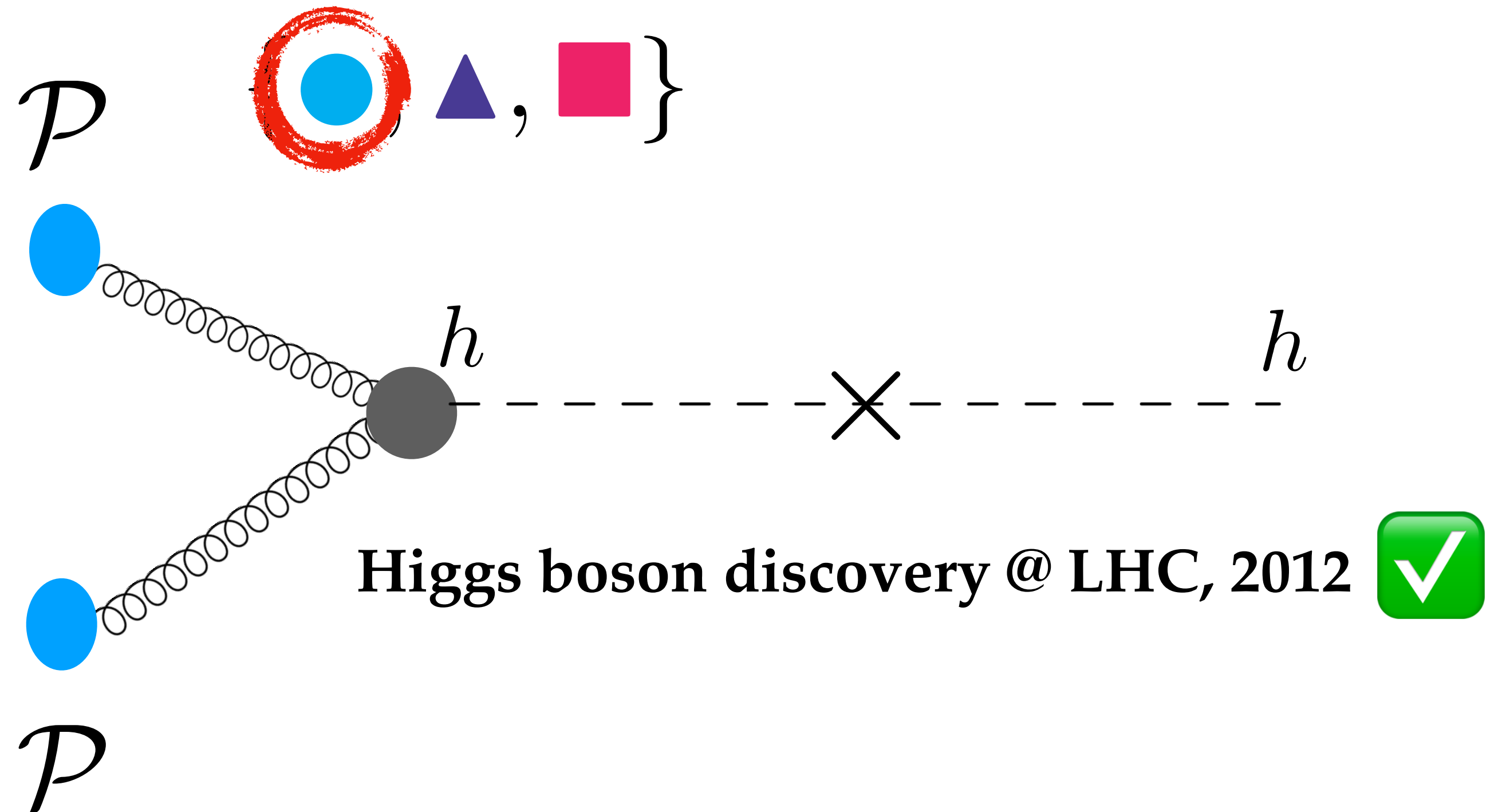
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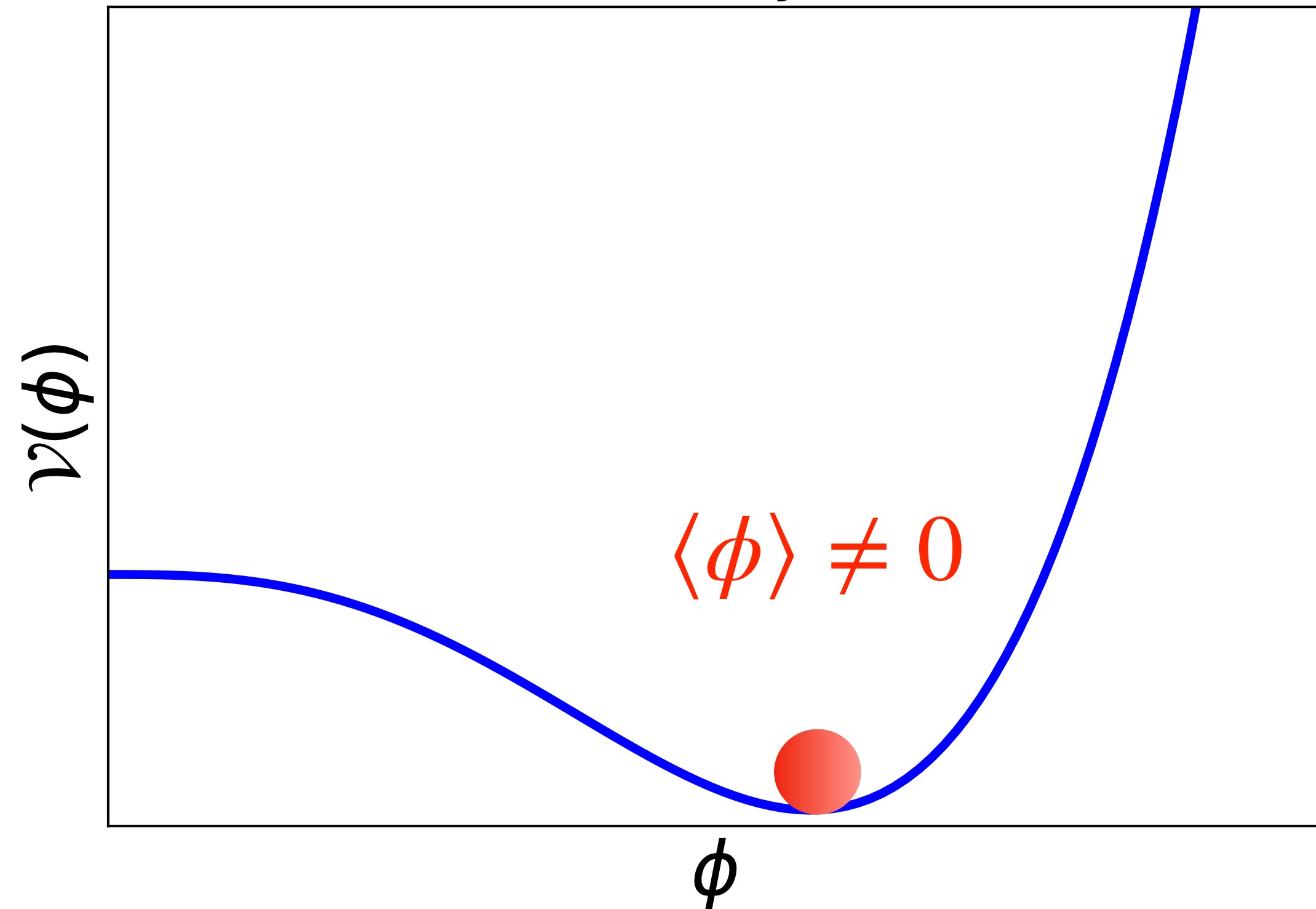
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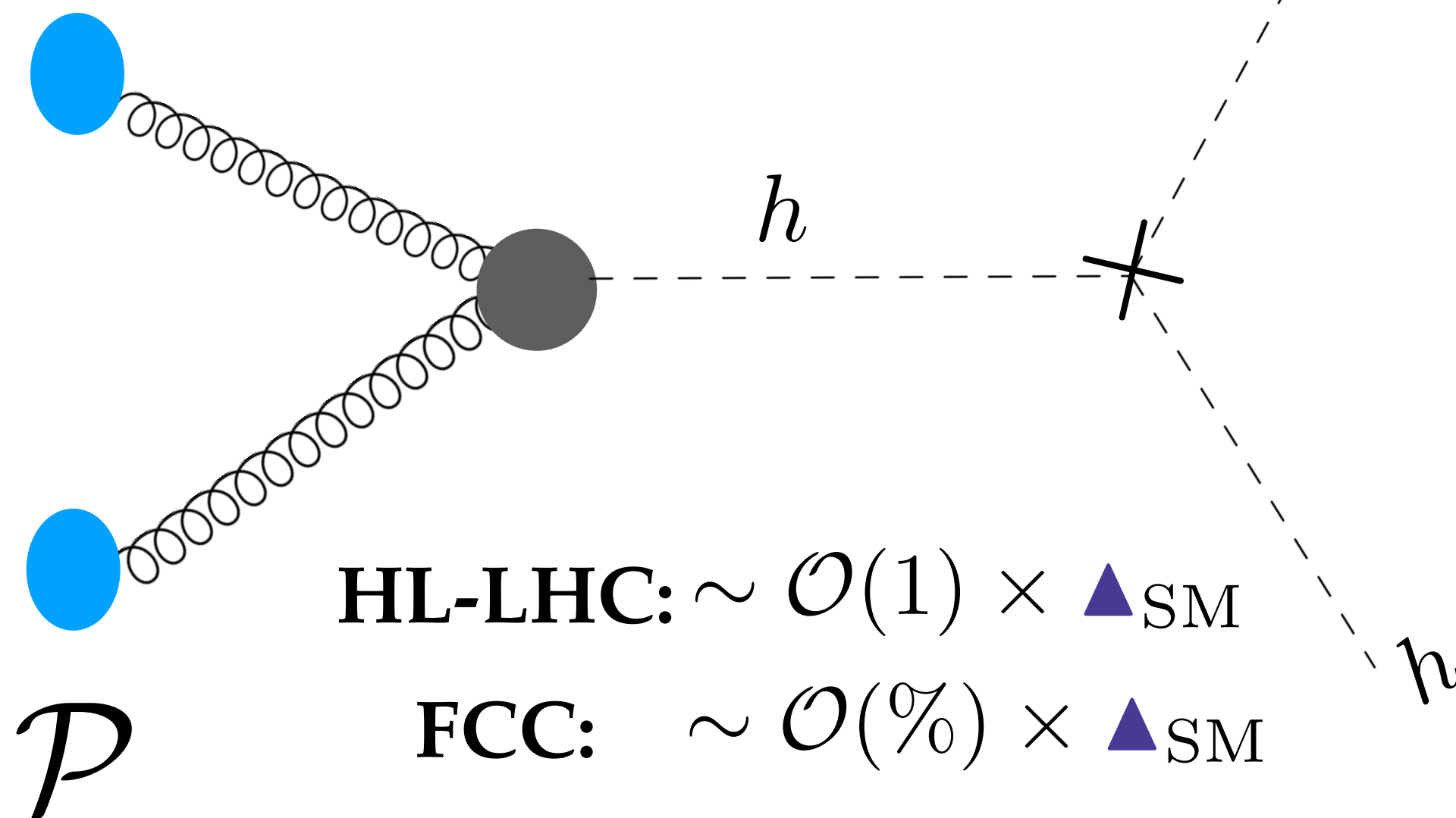
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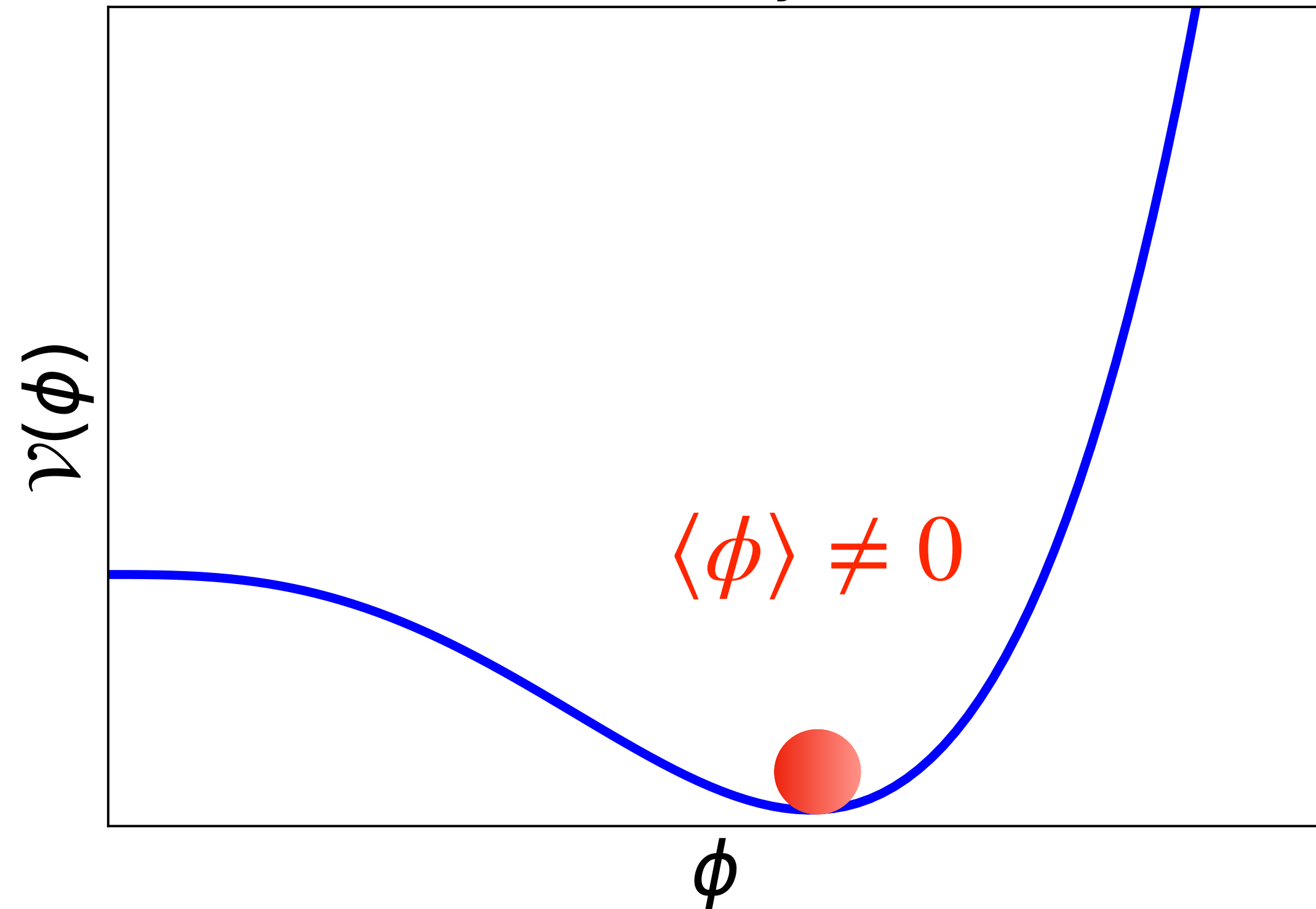
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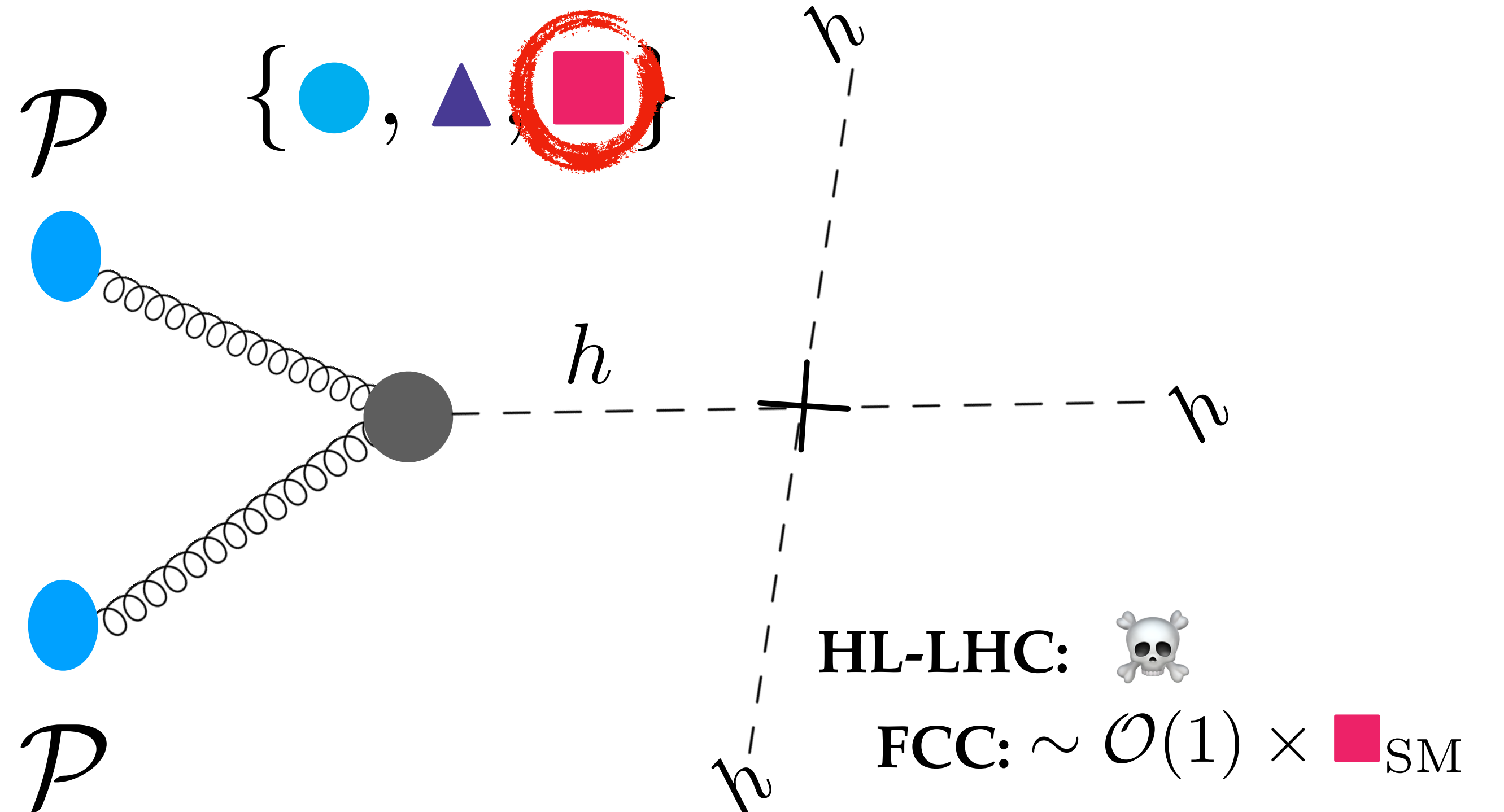
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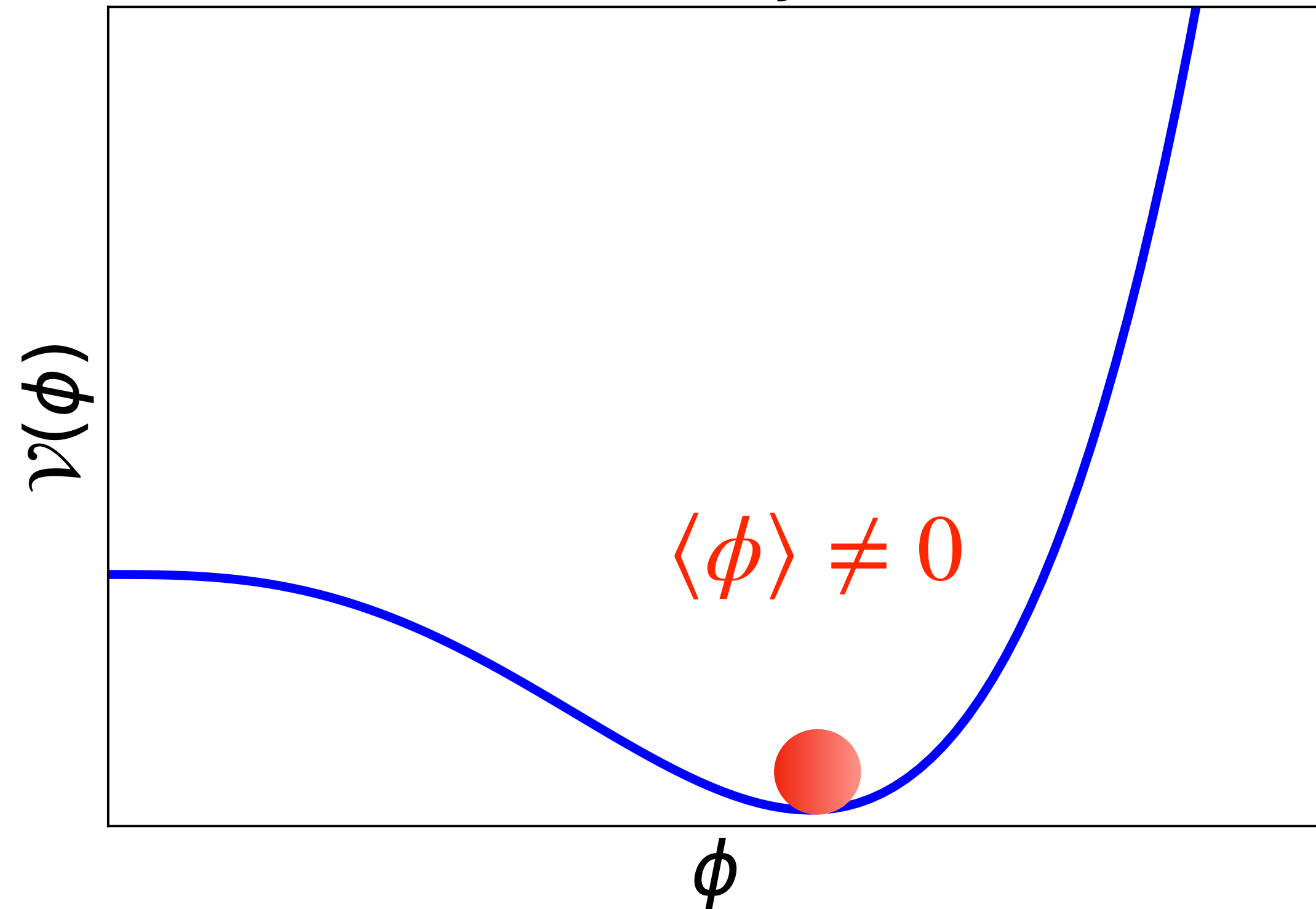


[e.g. **AP**, Sakurai, arXiv:1508.06524, **AP**,
Tetlalmatzi-Xolocotzi, Zaro,
arXiv:1909.09166]

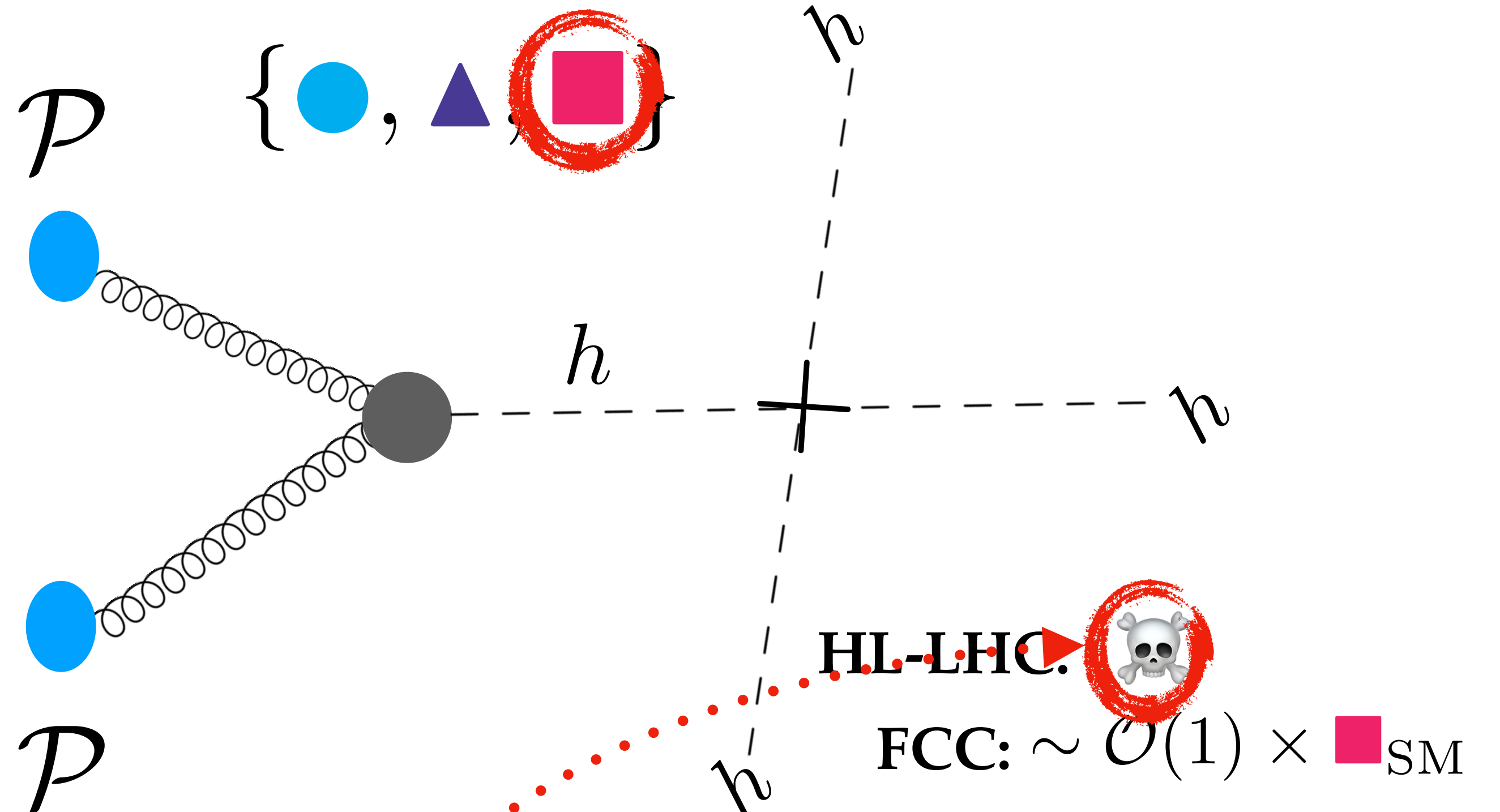
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[AP, Robens, Tetlalmatzi-Xolocotzi, arXiv:2101.00037 + Karkout, AP, Postma, Tetlalmatzi-Xolocotzi, van de Vis, du Pree, arXiv:2404.12425, AP, Tetlalmatzi-Xolocotzi, arXiv:2312.13562]

SEE LATER!

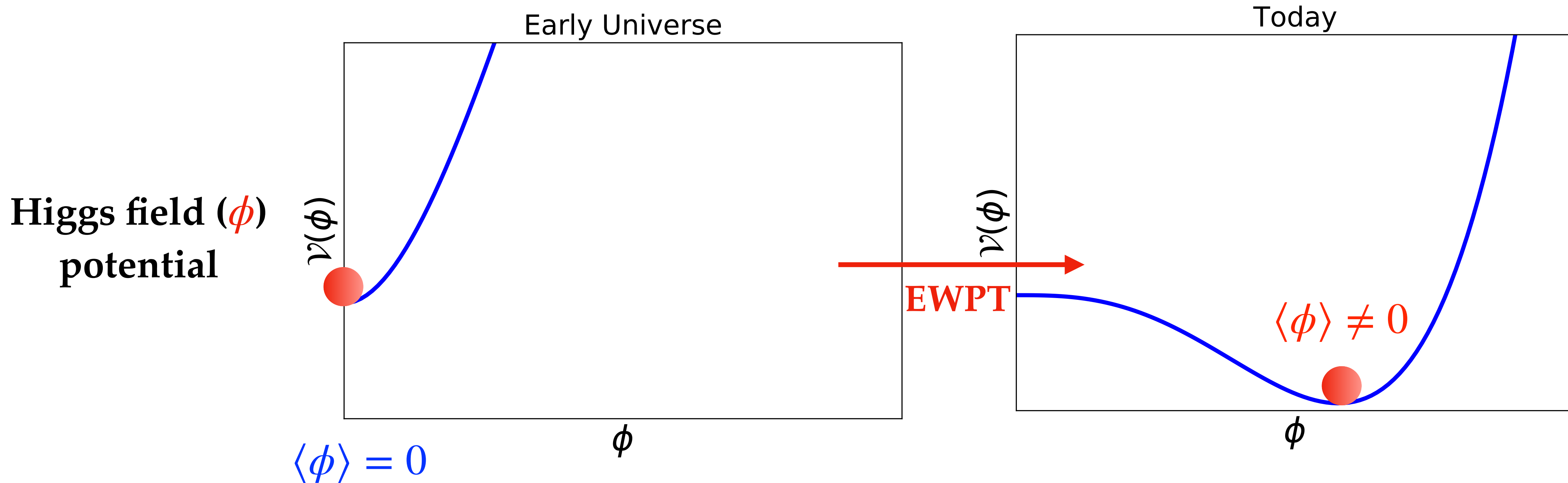
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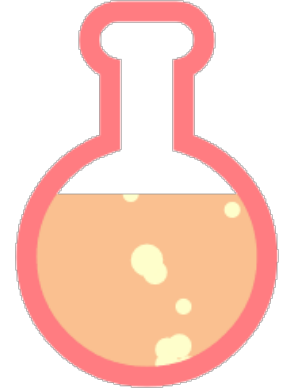
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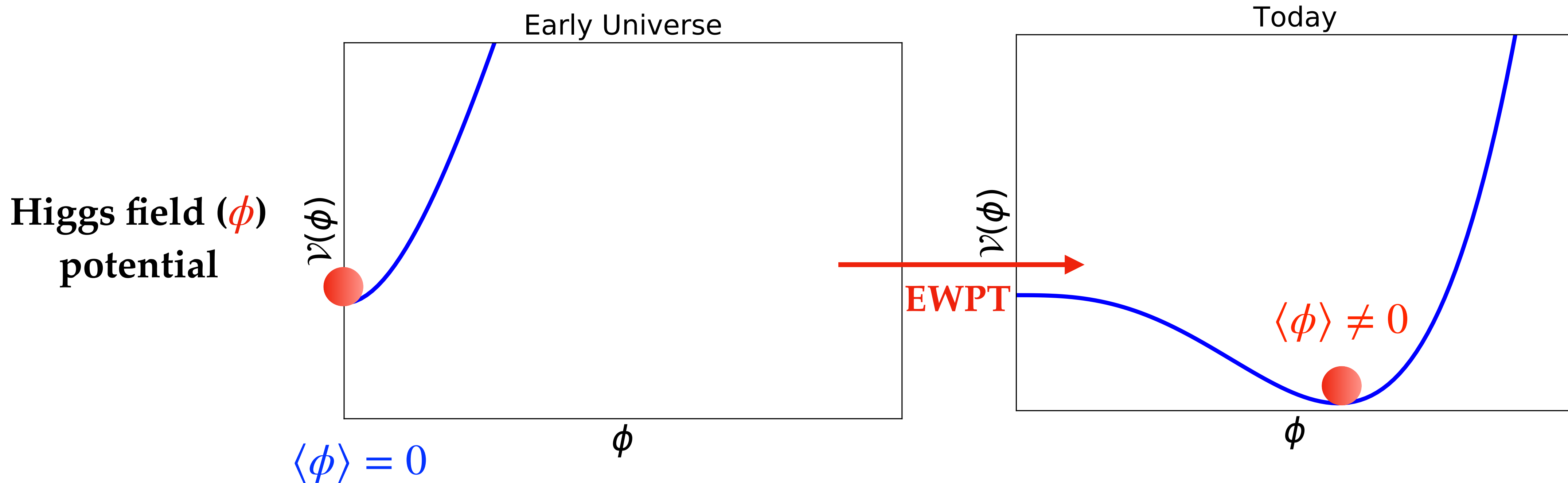
**TO BE
CONTINUED** 

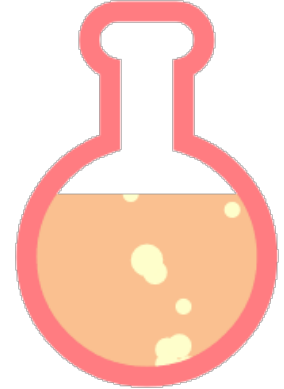
Breaking the Symmetry in the SM



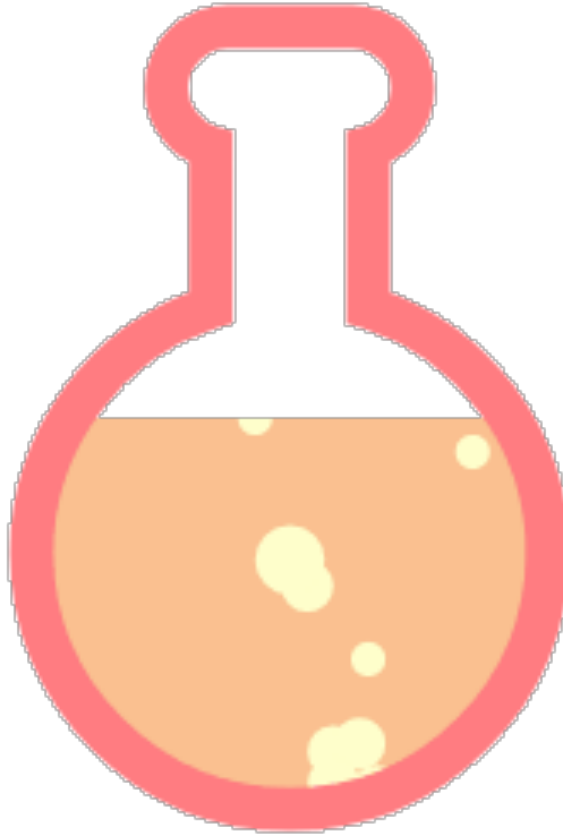
- **Nature** of EWPT \rightarrow Important open question, e.g. **its order**:
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Breaking the Symmetry in the SM



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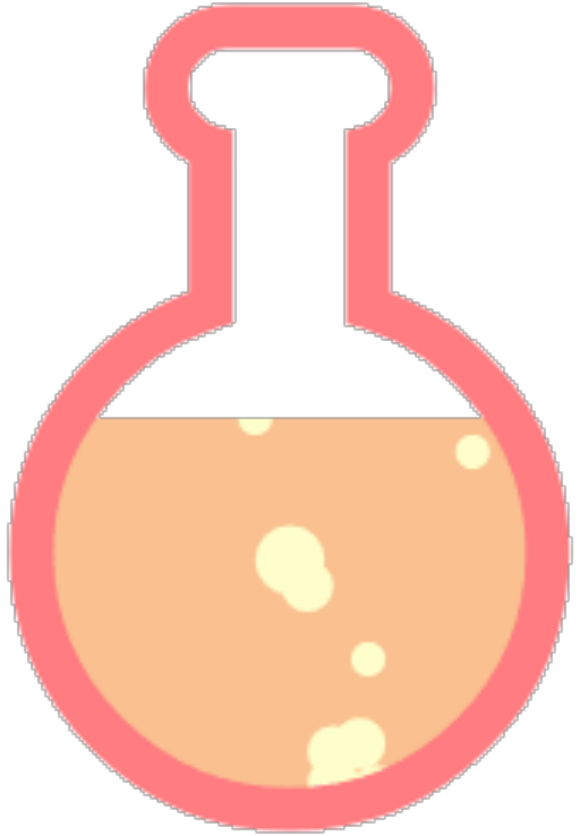
The Nature of the Phase Transition

- Clues to the origin of **matter-anti-matter asymmetry**.
- Was the asymmetry created **during the EWPT?**
 - “Electro-Weak Baryogenesis” (**EWBG**).
- Pre-requisite: **a First-Order transition**. → 
- *Note:* This **does not** occur in the SM!

[Kajantie, Laine, Rummukainen, Shaposhnikov hep-ph/9605288]



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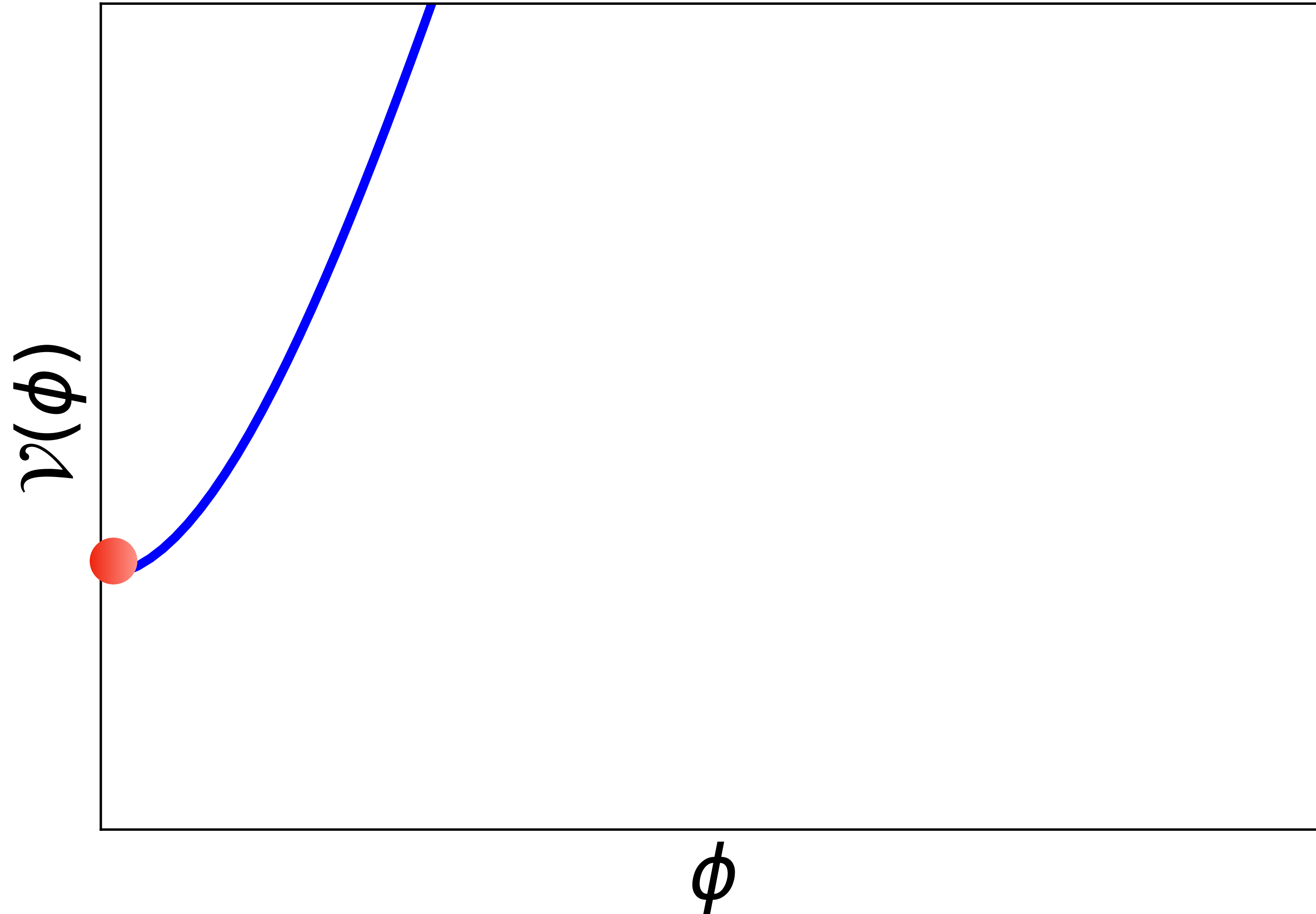
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A First-Order Electro-Weak Phase Transition: As the Universe Cools Down!

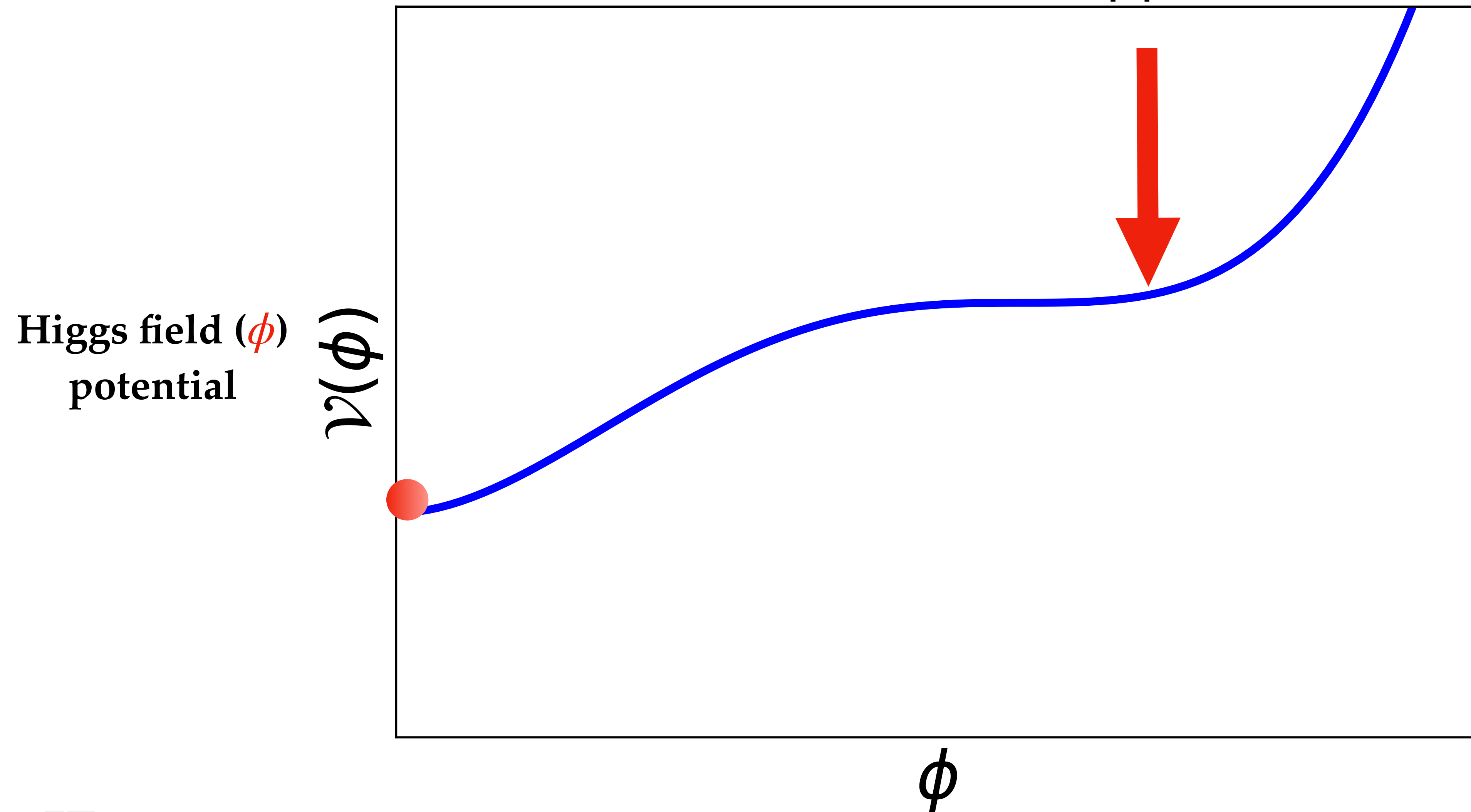
Early Universe

Higgs field (ϕ)
potential



A First-Order Electro-Weak Phase Transition: As the Universe Cools Down!

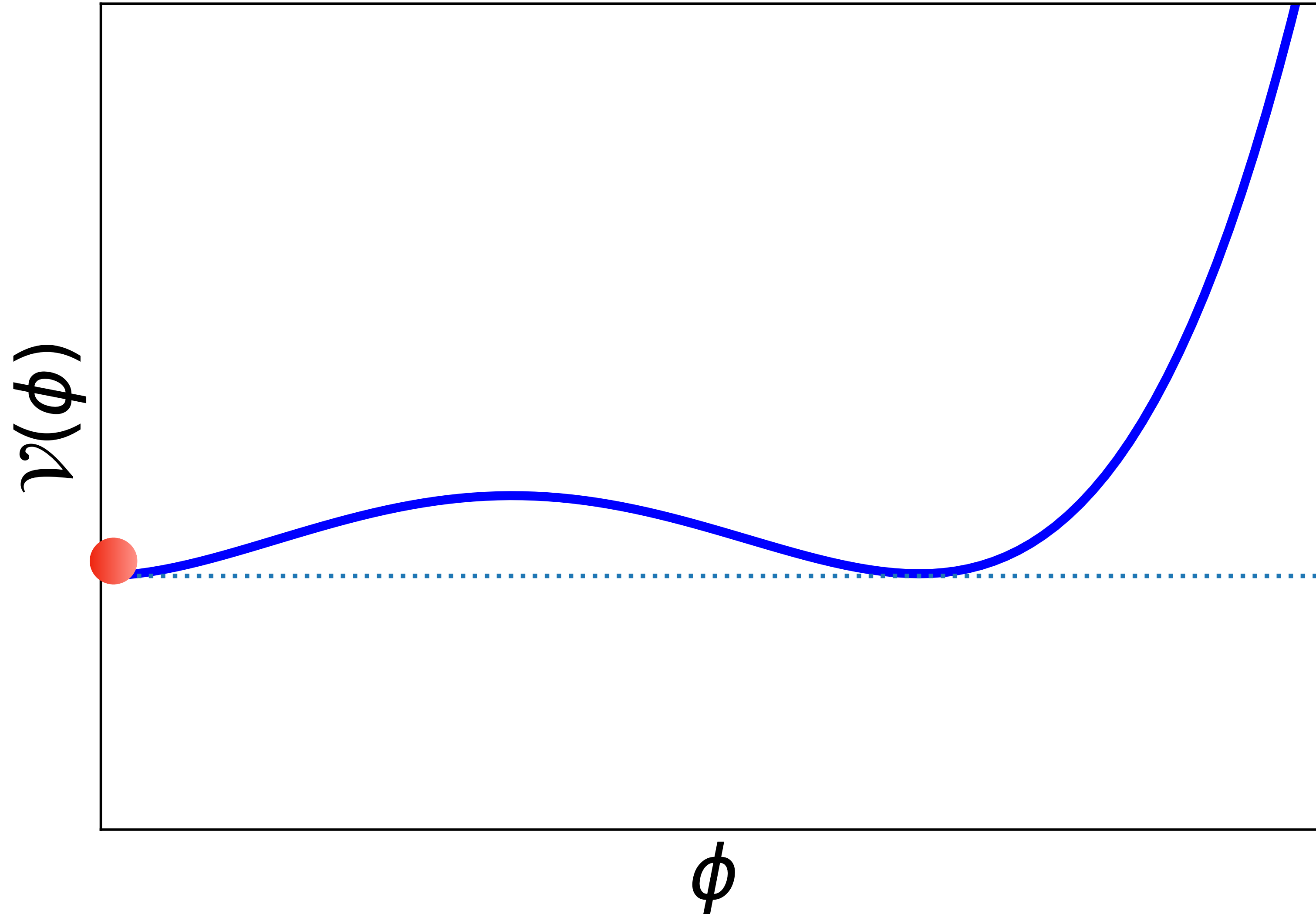
Second minimum appears



A First-Order Electro-Weak Phase Transition: As the Universe Cools Down!

Critical temperature reached

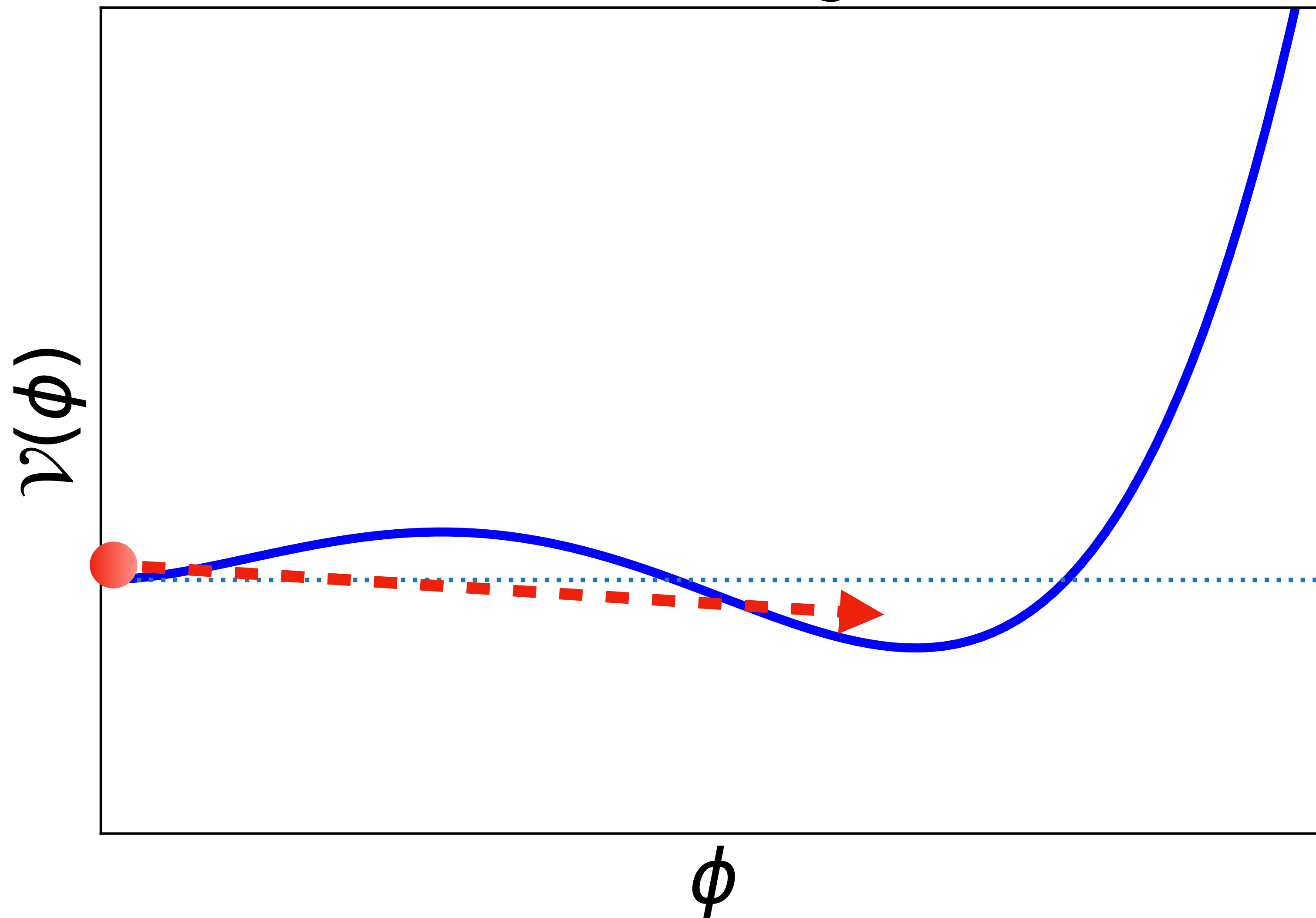
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Tunneling!

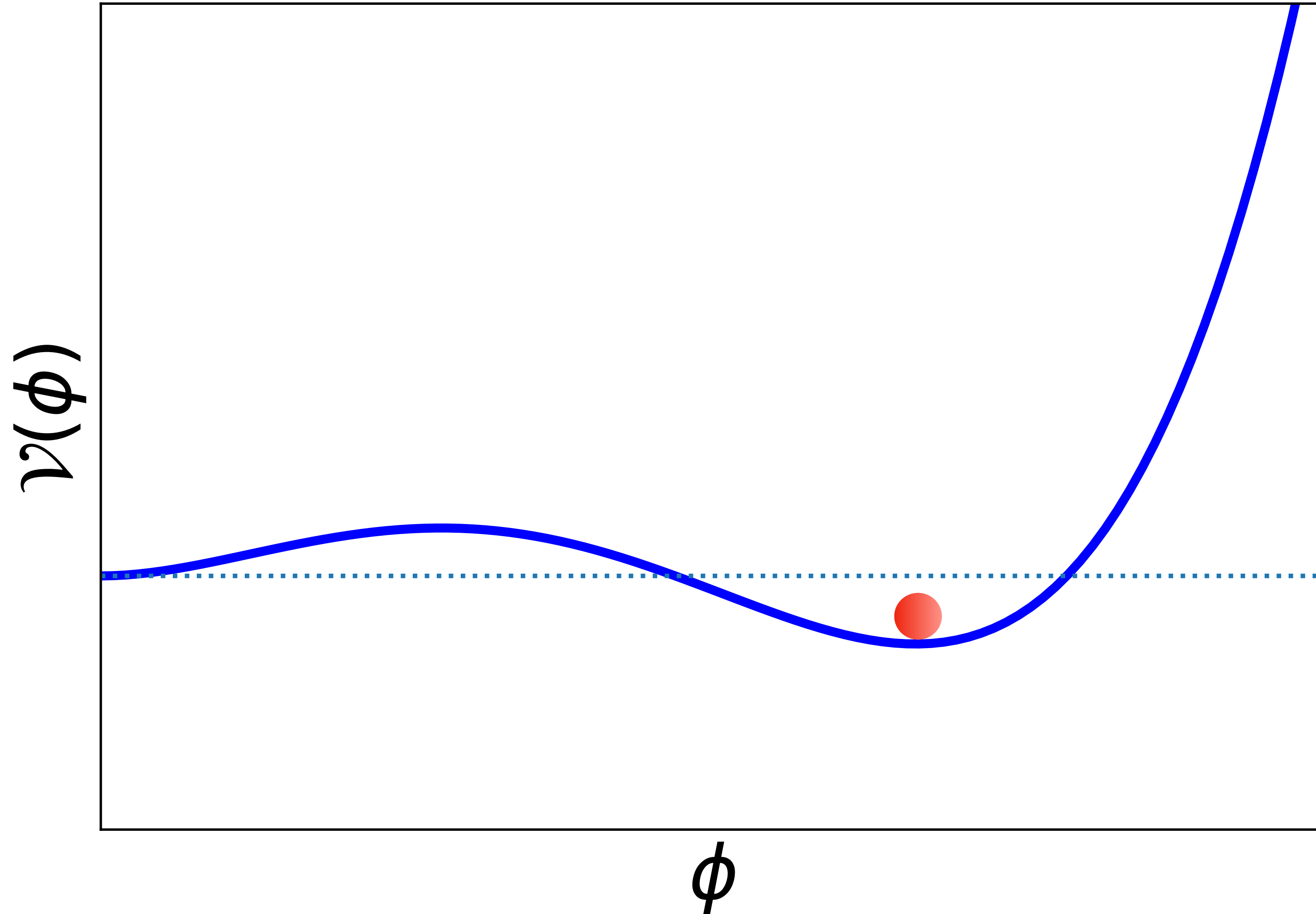
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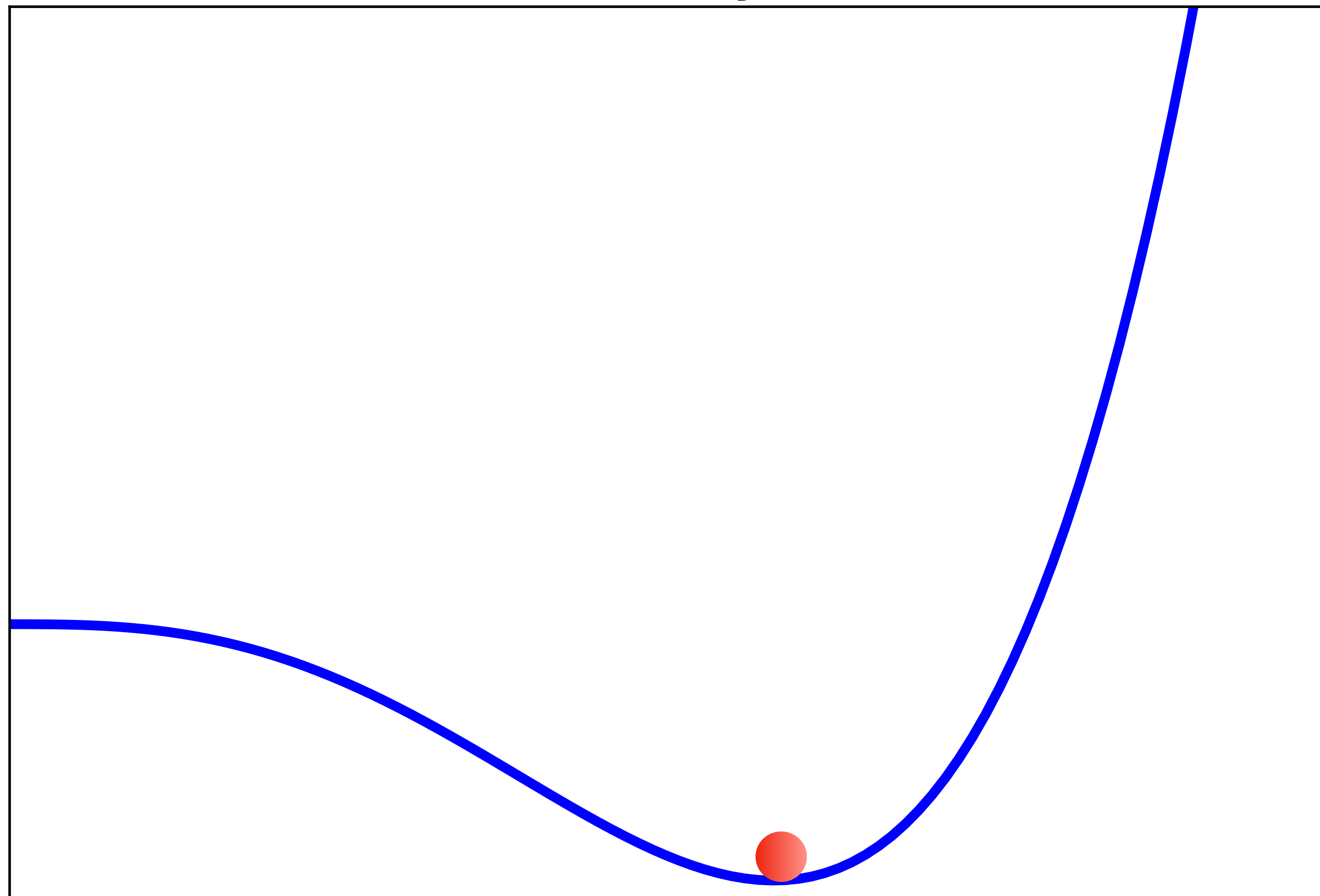


A First-Order Electro-Weak Phase Transition: As the Universe Cools Down!

Today

Higgs field (ϕ)
potential

$V(\phi)$



ϕ

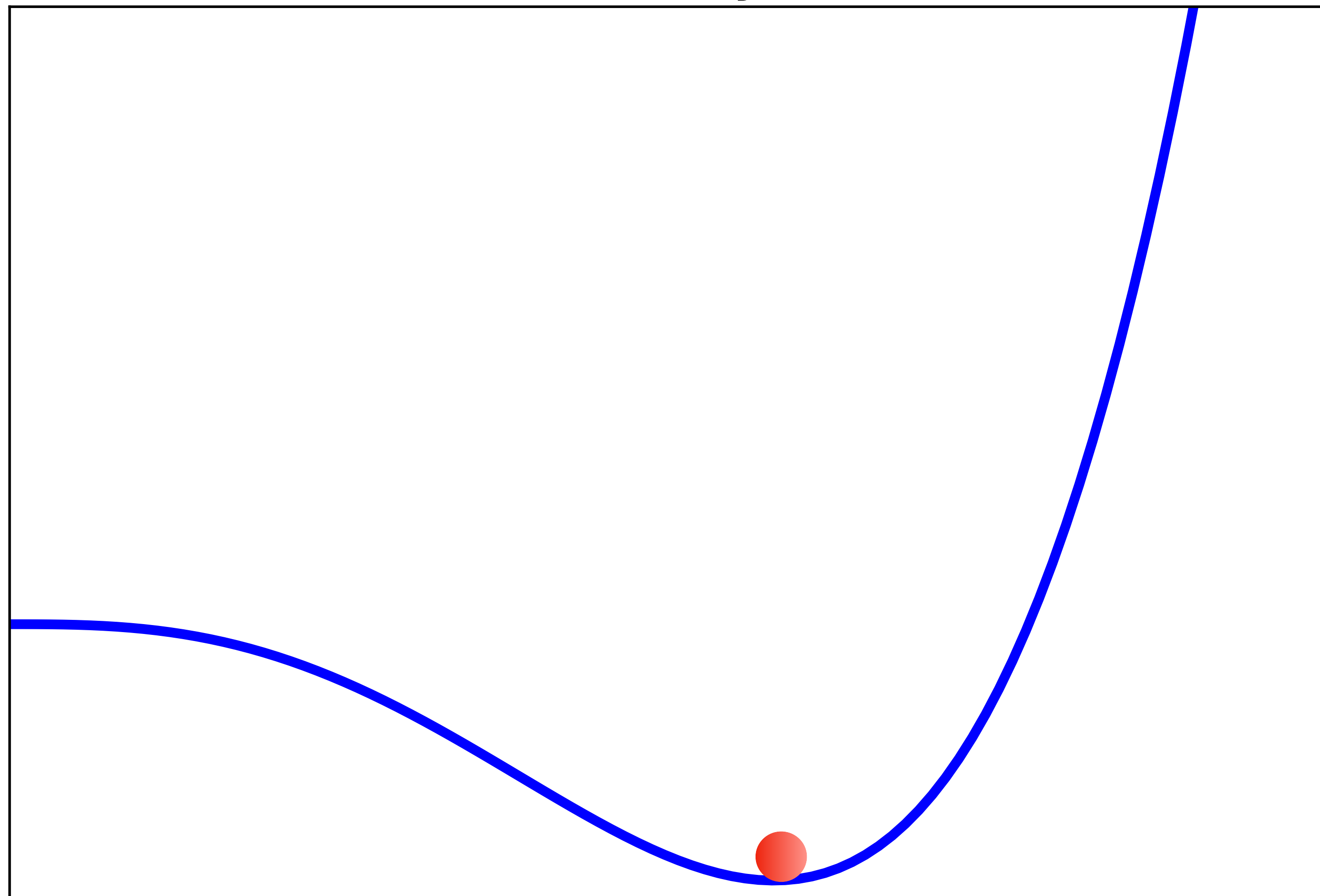


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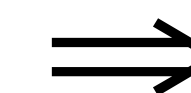
$V(\phi)$



ϕ

$$\langle \phi \rangle \neq 0$$

VEV



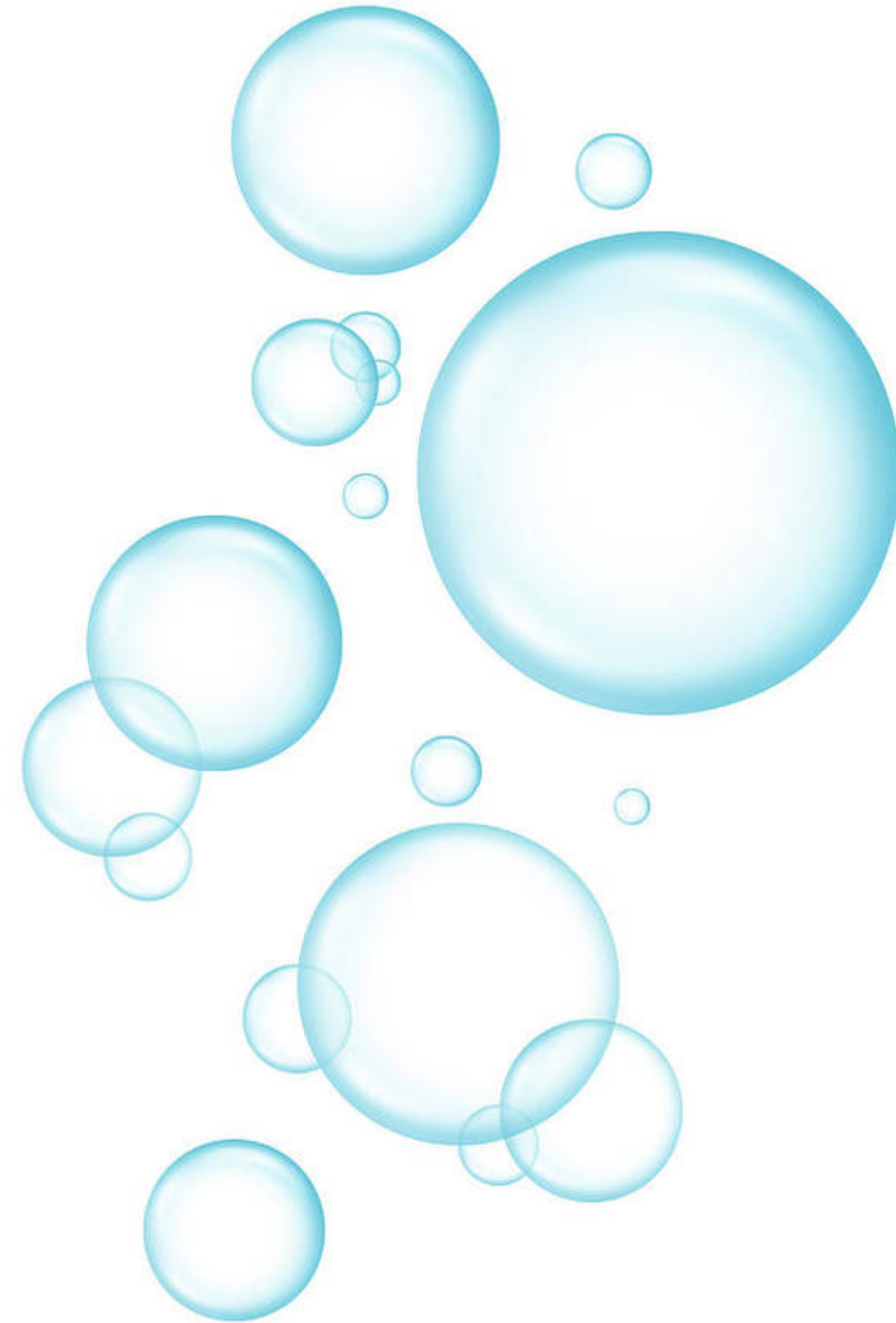
Symmetry
Breaking.



A First-Order EWPT

Some time after critical temperature is reached:

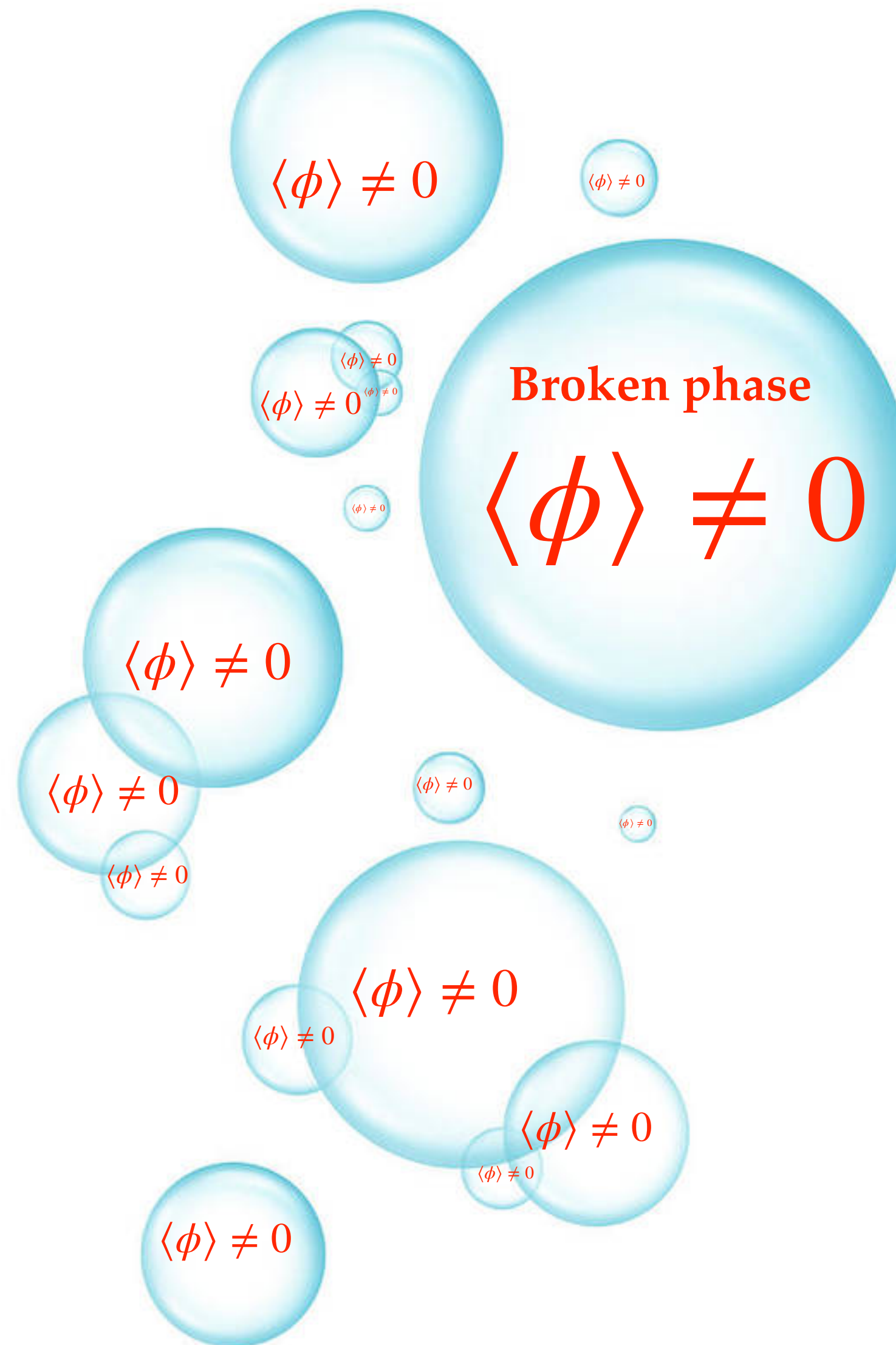
→ **Bubbles** of the broken phase nucleate and expand.



A First-Order EWPT

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$$\langle \phi \rangle = 0$$

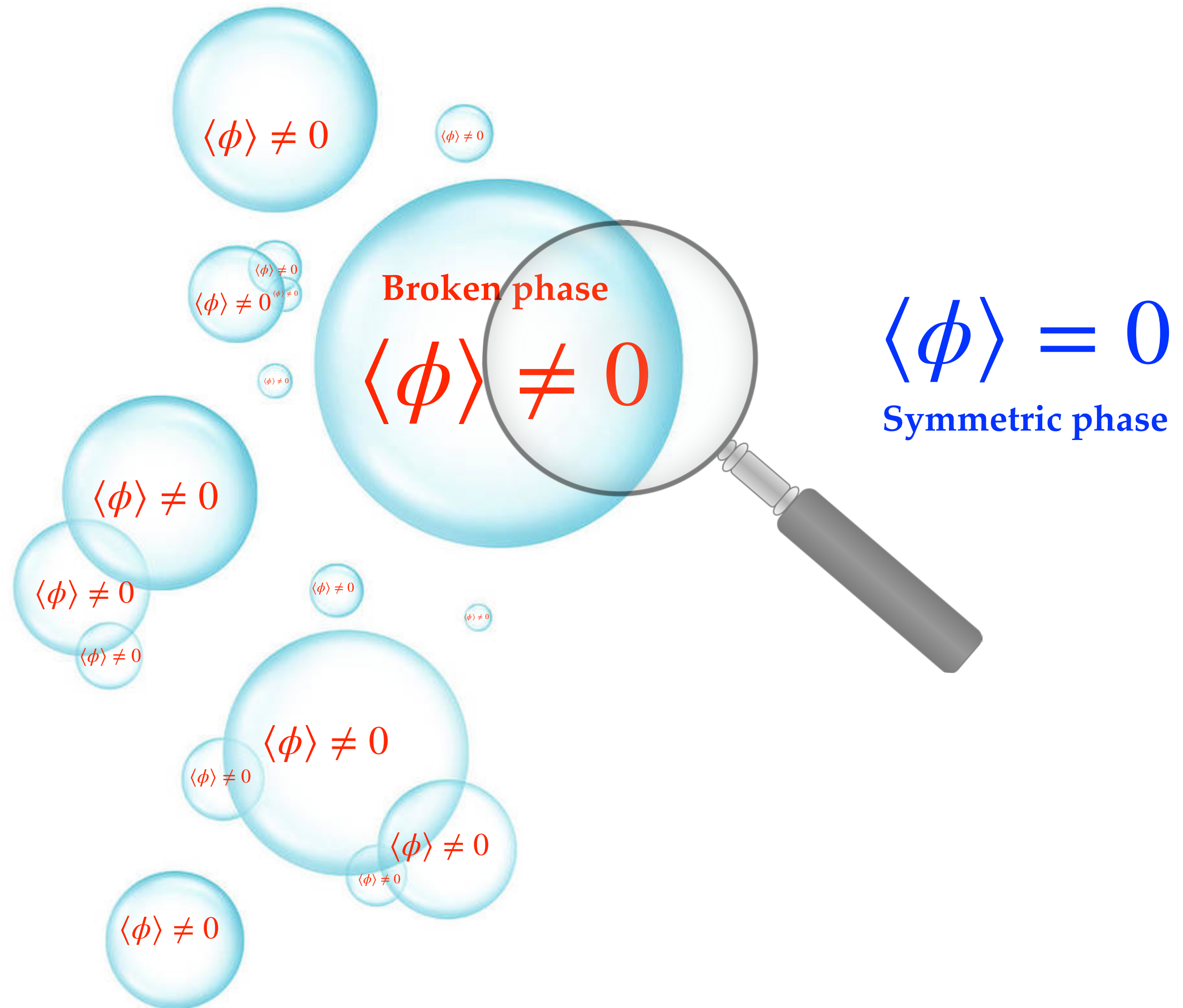
Symmetric phase



A First-Order EWPT

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Electro-Weak Baryogenesis

$\langle \phi \rangle \neq 0$
Broken phase

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Electro-Weak Baryogenesis

Left/Right-Handed Fermions

$$\psi_L + \psi_R$$

$$\langle \phi \rangle \neq 0$$

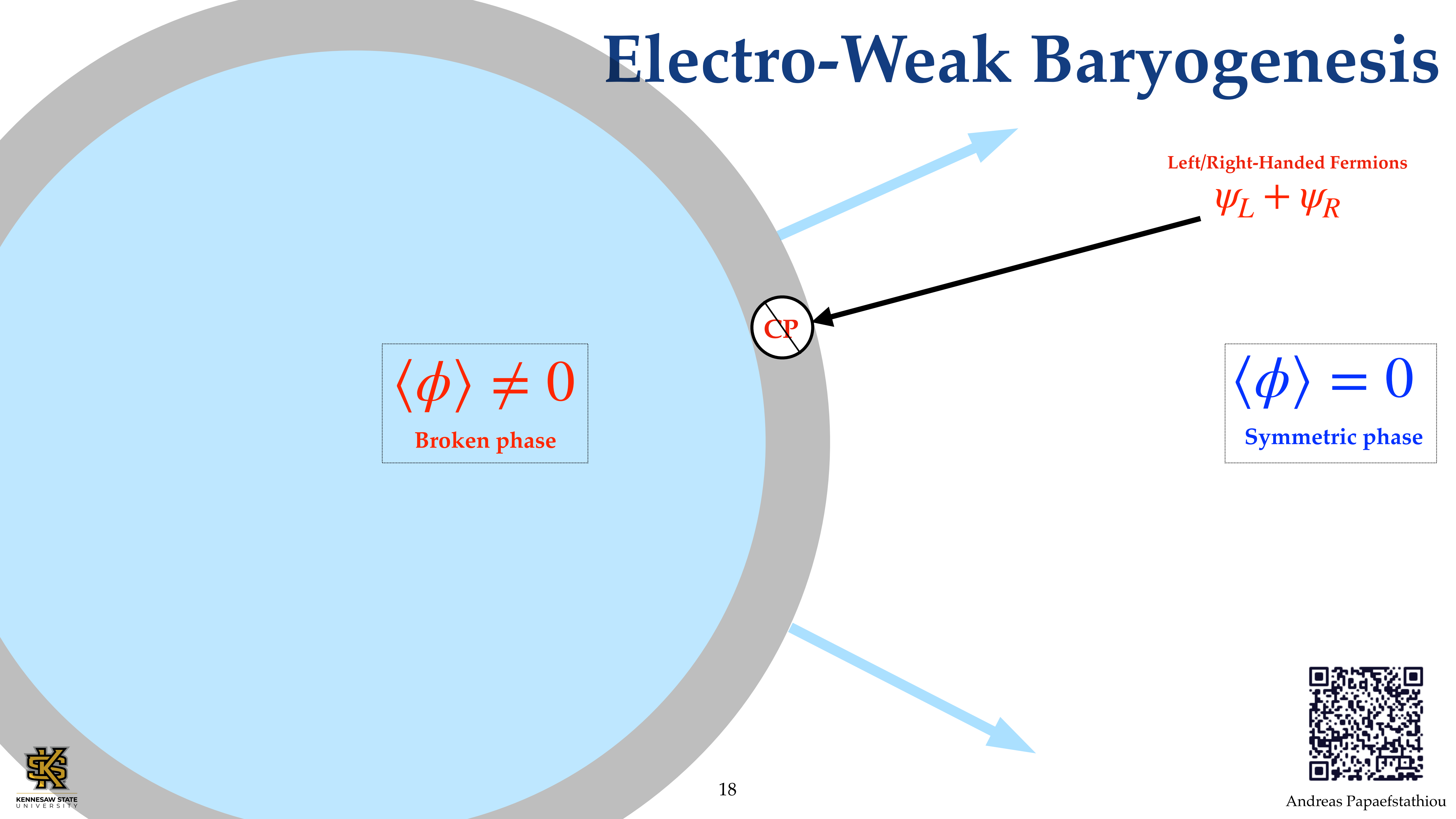
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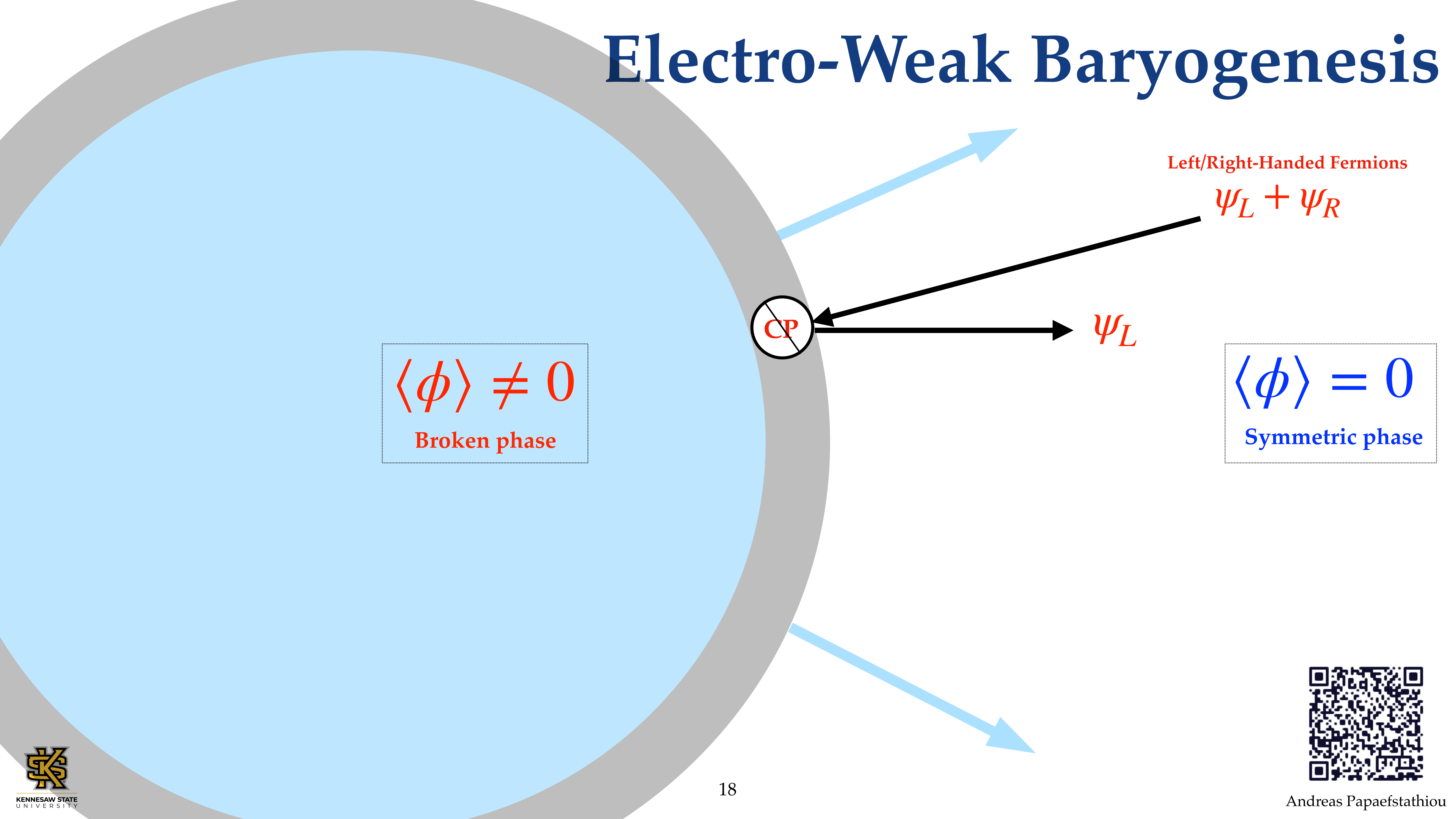
CP

$\langle \phi \rangle \neq 0$
Broken phase

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Symmetric phase



Electro-Weak Baryogenesis



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CP

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ψ_L



Electro-Weak Baryogenesis

~~Sphaleron~~

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Broken phase

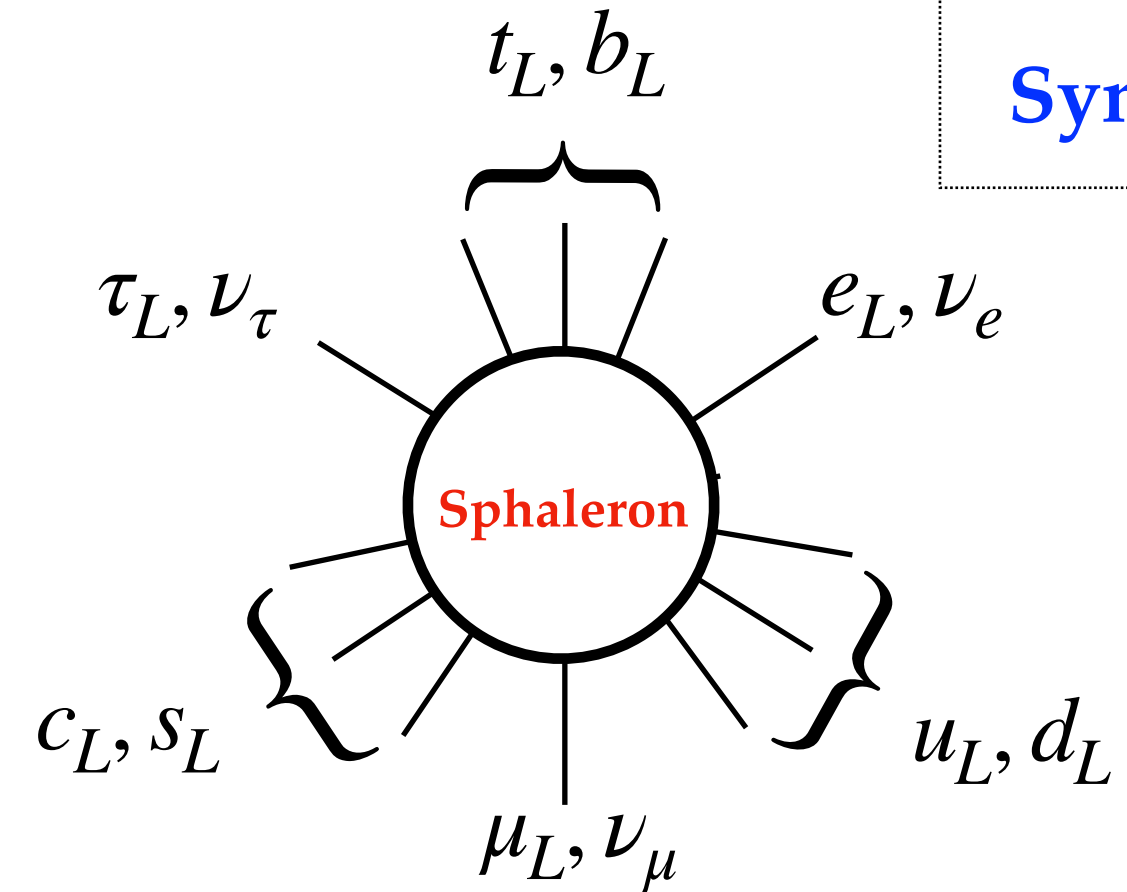
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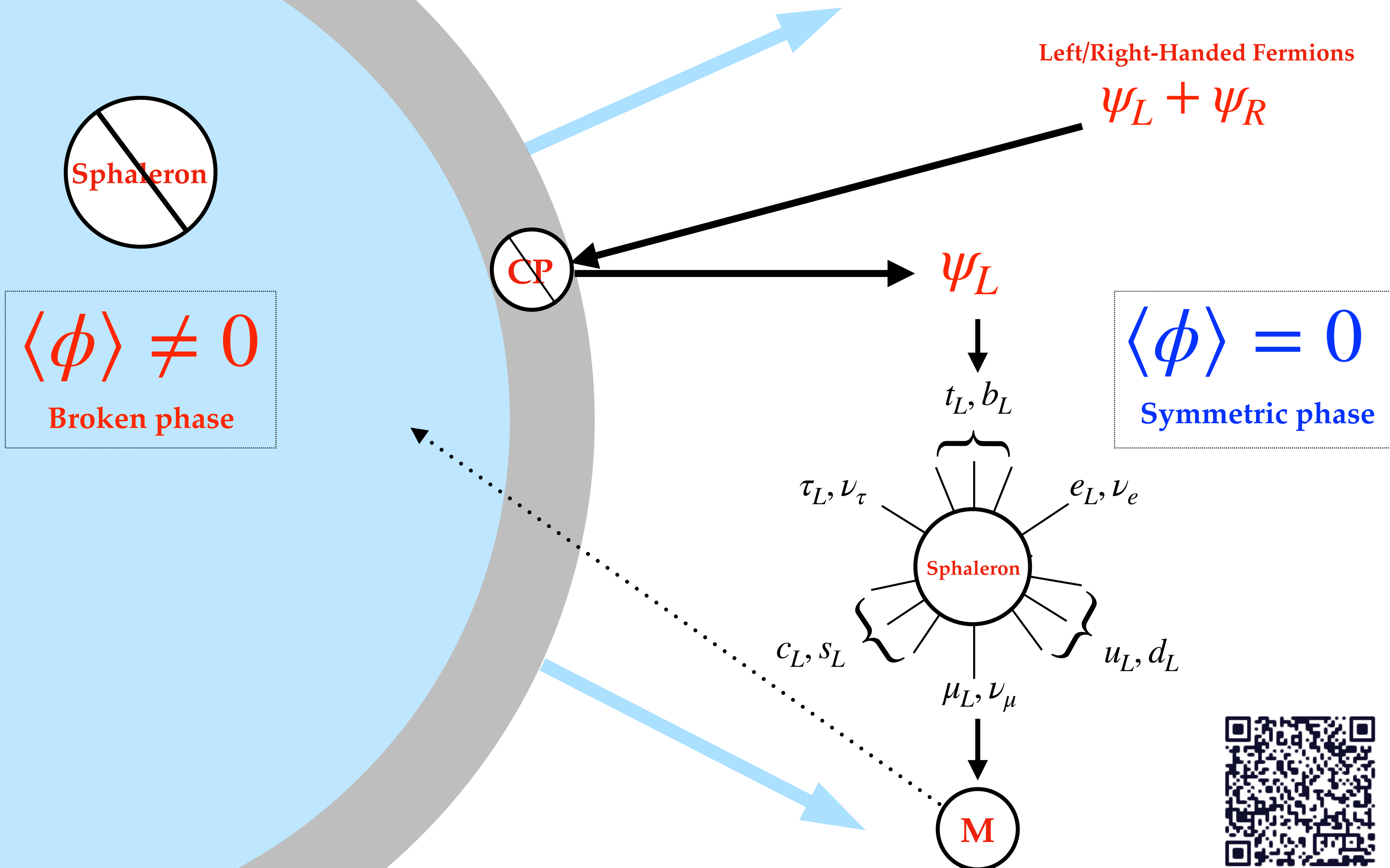
$\psi_L + \psi_R$

ψ_L

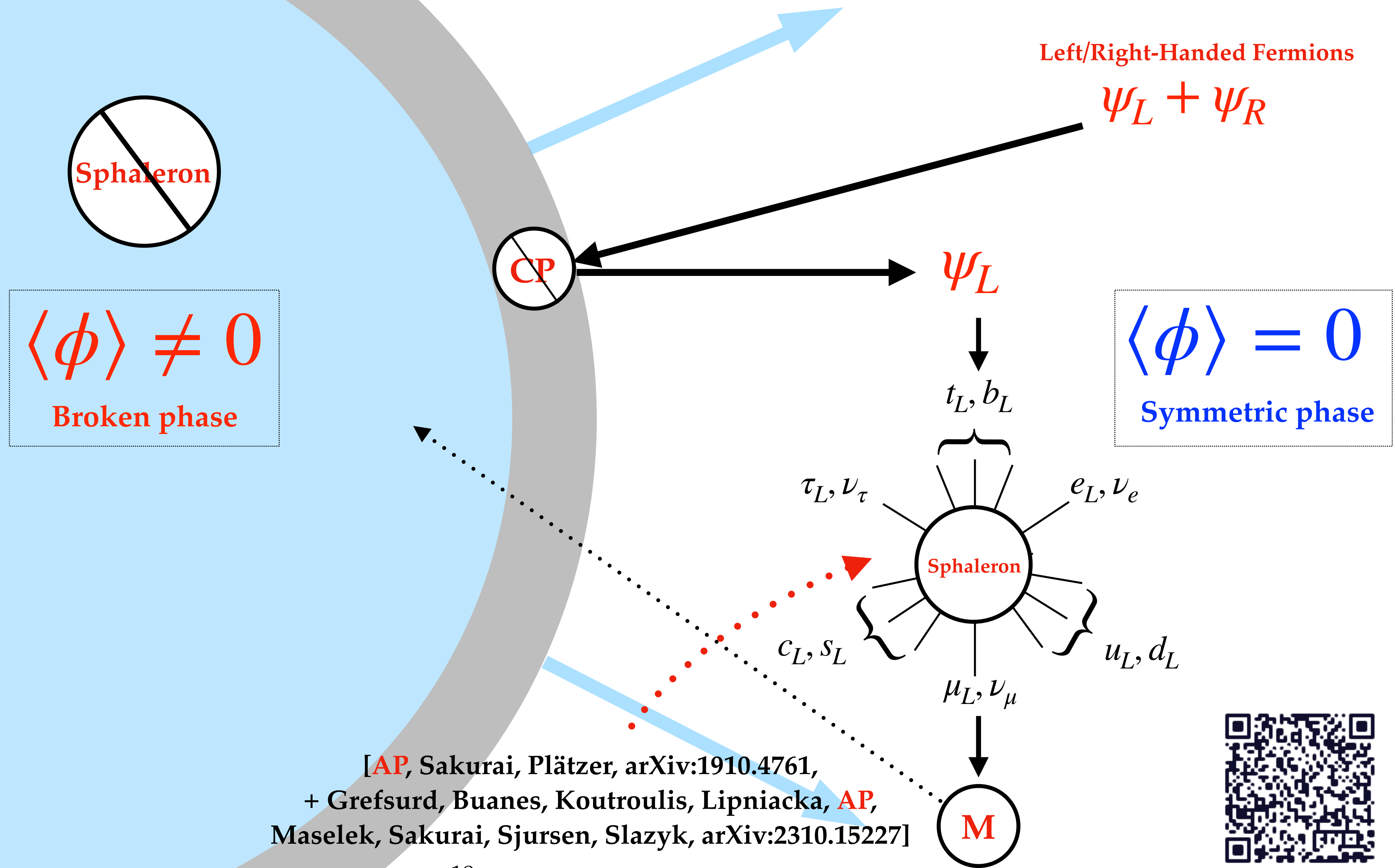
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Electro-Weak Baryogenesis



Electro-Weak Baryogenesis



[AP, Sakurai, Plätzer, arXiv:1910.4761,
 + Grefsurd, Buanes, Koutroulis, Lipniacka, AP,
 Maselek, Sakurai, Sjursen, Slazyk, arXiv:2310.15227]



Electro-Weak Baryogenesis

~~Sphaleron~~

~~CP~~

Left/Right-Handed Fermions

$$\psi_L + \psi_R$$

$$\psi_L$$

$$\langle \phi \rangle = 0$$

Symmetric phase

$$t_L, b_L$$

$$\tau_L, \nu_\tau$$

$$e_L, \nu_e$$

Sphaleron

$$c_L, s_L$$

$$u_L, d_L$$

$$\mu_L, \nu_\mu$$

M

**A First-Order
Transition requires
New Phenomena
beyond the SM!**

[AP, Sakurai, Plätzer, arXiv:1910.4761,
+ Grefsurd, Buanes, Koutroulis, Lipniacka, AP,
Maselek, Sakurai, Sjursen, Slazyk, arXiv:2310.15227]



A Note on Sphaleron Suppression

- Sphaleron rate inside bubble $\sim \exp[-\langle \phi(T_C) \rangle / T_C \times \dots]$,

[T_C = the critical temperature.]

\Rightarrow Require: $\langle \phi(T_C) \rangle / T_C \geq 1$,

\Rightarrow a “Strong” First-Order EWPT \equiv SFO-EWPT.



Electro-Weak Archaeology



We live here!



$$\langle \phi \rangle \neq 0$$

Broken phase



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→ What are the imprints of Electro-Weak Baryogenesis at Colliders?



Electro-Weak Archaeology



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→ What are the imprints of Electro-Weak Baryogenesis at Colliders?

→ Let's explore this in explicit New Physics models!



Extending the Scalar Sector [AP, White, arXiv:2010.00597]

- A First-Order EWPT dictates new phenomena. [Kajantie, Laine, Rummukainen, Shaposhnikov hep-ph/9605288]
- Consider first the **simplest possible extension to the SM!**

$$\mathcal{V}(\phi, S) = \bullet |\phi|^2 + \blacksquare |\phi|^4$$

Add: S , a new scalar field,
No SM “charges” \equiv Singlet.

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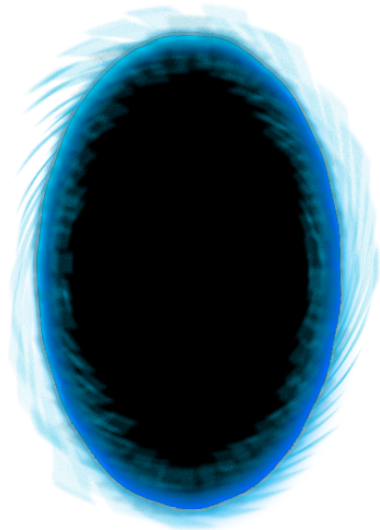
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$$+ \bullet S^2 + \blacktriangle S^3 + \blacksquare S^4$$

$$+ \blacktriangle |\phi|^2 S + \blacksquare |\phi|^2 S^2 \leftarrow \text{“Portal” interactions.}$$

[$|\phi|^2$ is also an SM singlet!]



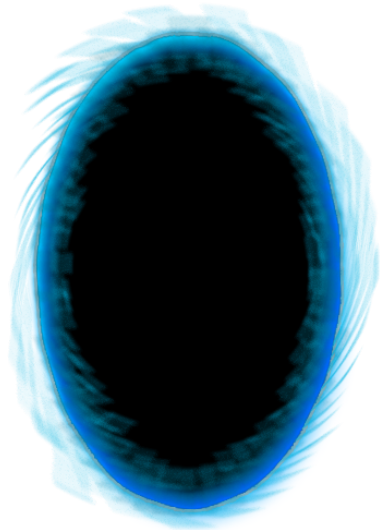
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$$+ \blacktriangle |\phi|^2 S + \blacksquare |\phi|^2 S^2 \leftarrow \text{“Portal” interactions.}$$

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$$+ S \times (\text{Hidden Sector}) + \dots \leftarrow \text{Dark Matter?}$$

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EWSB \leftrightarrow VEVs:

$$\phi \rightarrow \langle \phi \rangle + h$$

$$S \rightarrow \langle S \rangle + \chi$$

Extending the Scalar Sector [AP, White, arXiv:2010.00597]

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$$S \rightarrow \langle S \rangle + \chi$$

\Rightarrow Mass (squared) matrix:

$$M^2 = \begin{pmatrix} \frac{\partial^2 \mathcal{V}}{\partial h^2} & \frac{\partial^2 \mathcal{V}}{\partial h \partial \chi} \\ \frac{\partial^2 \mathcal{V}}{\partial h \partial \chi} & \frac{\partial^2 \mathcal{V}}{\partial \chi^2} \end{pmatrix}$$

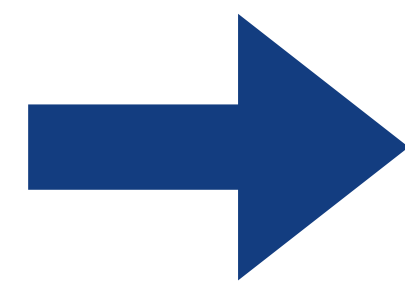
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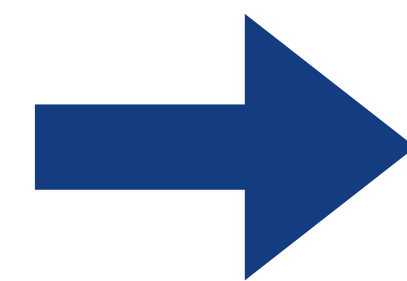
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Diagonalize!

\Rightarrow Mass (squared) matrix:

Mass Eigenstates

$$M^2 = \begin{pmatrix} \frac{\partial^2 \mathcal{V}}{\partial h^2} & \frac{\partial^2 \mathcal{V}}{\partial h \partial \chi} \\ \frac{\partial^2 \mathcal{V}}{\partial h \partial \chi} & \frac{\partial^2 \mathcal{V}}{\partial \chi^2} \end{pmatrix}$$

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} h \\ \chi \end{pmatrix}$$

θ : mixing angle

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$h_1 \rightarrow$ "SM-like" Higgs boson.

$h_2 \rightarrow$ new scalar resonance.

i.e. choose: $|\theta| \gtrsim 0$, and:

$$h_1 = h \cos \theta + \chi \sin \theta$$

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Primary targets for collider studies!

i.e. choose: $|\theta| \gtrsim 0$, and:

$$h_1 = h \cos \theta + \chi \sin \theta$$

$$h_2 = -h \sin \theta + \chi \cos \theta$$



SM+Singlet: Current Collider Constraints

[AP, White, arXiv:2010.00597]

(i) LHC searches for **scalar resonances** in $h_2 \rightarrow h_1 h_1, ZZ, W^+ W^-, \dots$, since:

$$g_{h_2 XX} \sim g_{h XX}^{\text{SM}} \times \sin \theta \text{ and } h_2 \rightarrow h_1 h_1 \text{ (if allowed),}$$

(ii) **Reductions in Higgs boson signal strengths** (LHC), since:

$$g_{h_1 XX} \sim g_{h XX}^{\text{SM}} \times \cos \theta,$$

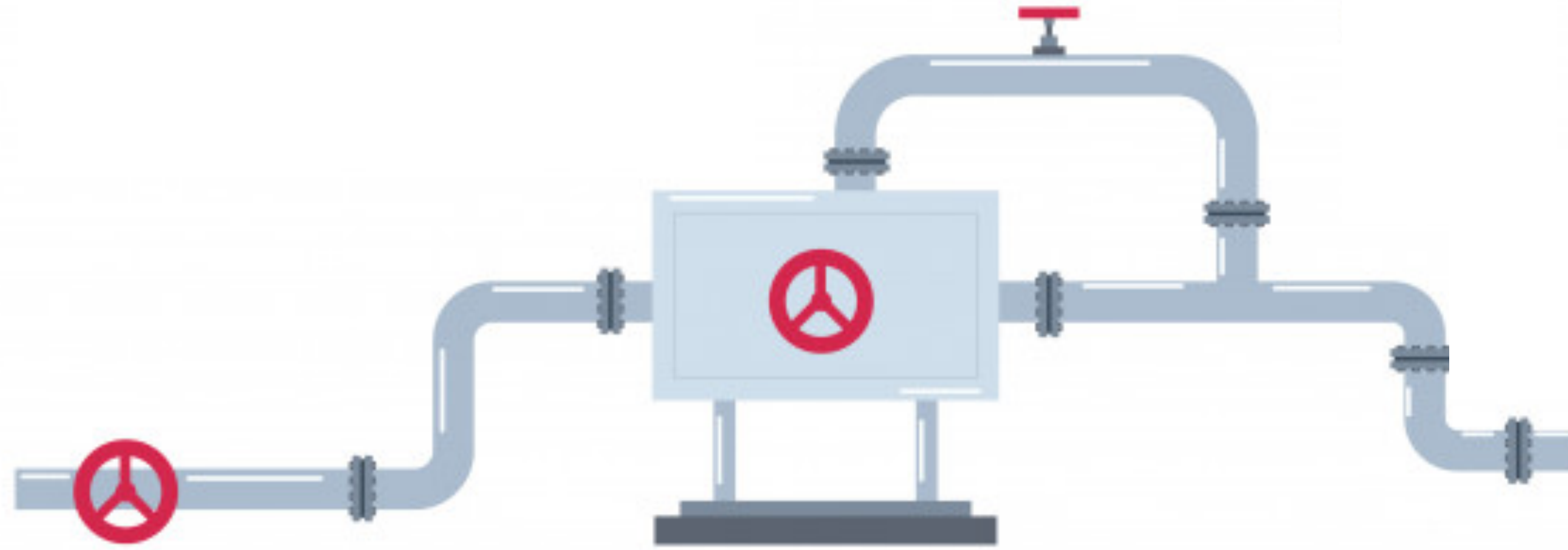
[(i)+(ii) \rightarrow e.g. through HiggsBounds/HiggsSignals = HiggsTools]

(iii) **Electro-Weak Precision Observables** (LEP). [e.g. Profumo, Ramsey-Musolf, Wainwright, Winslow, arXiv:1407.5342]

[+ **W mass constraints**. [e.g. López-Val, Robens arXiv:1406.1043, AP, Robens, White, arXiv:2205.14379]

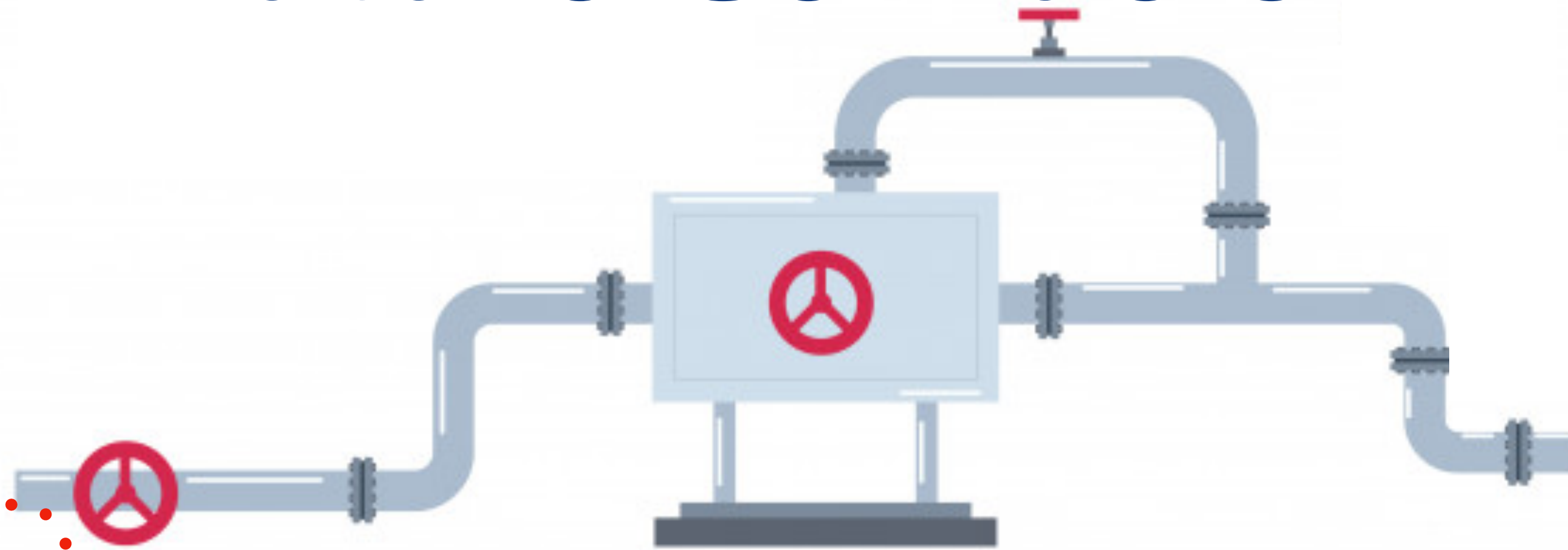


$pp \rightarrow h_2$ @ Future Colliders [AP, White, arXiv:2010.00597]



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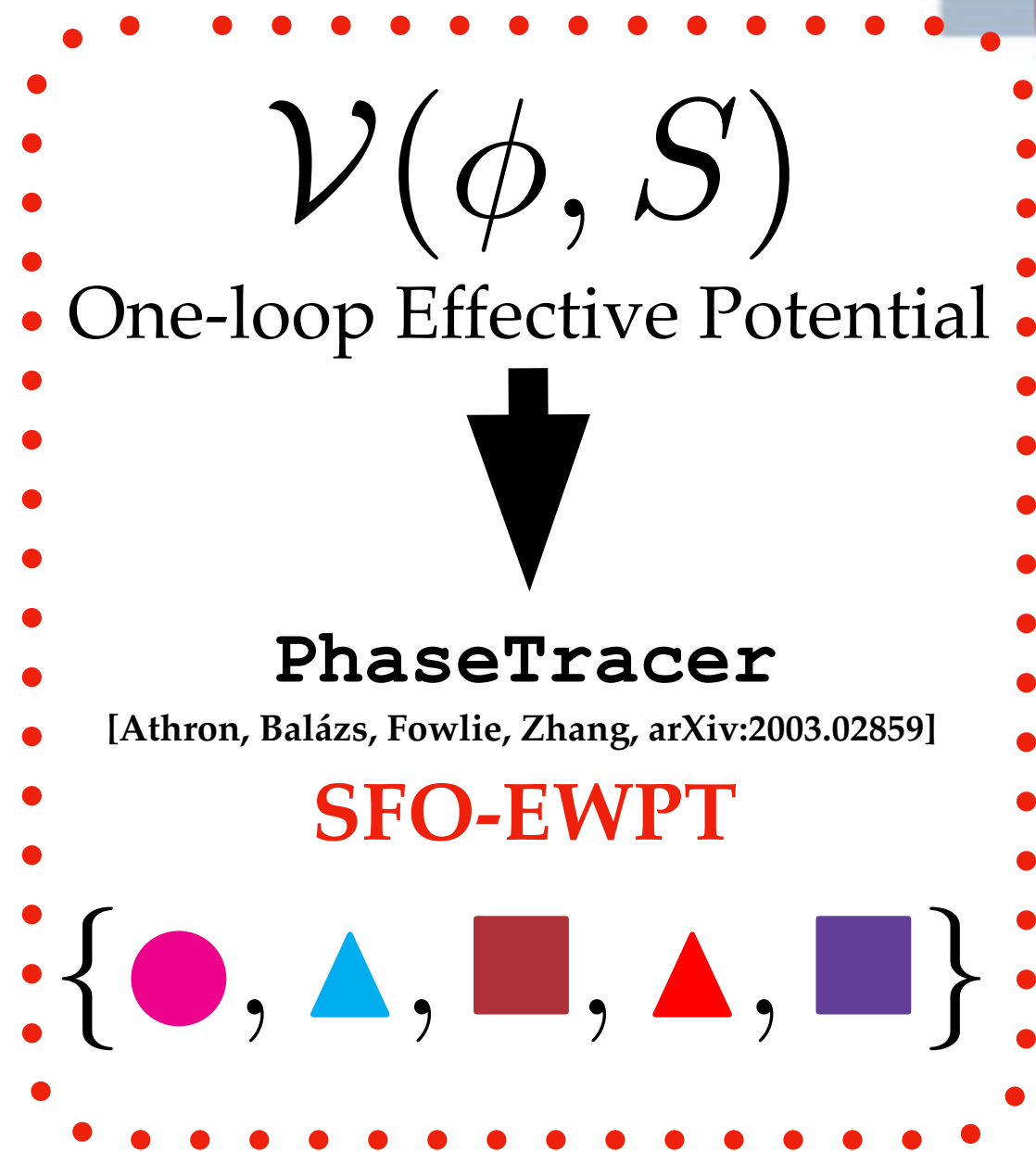
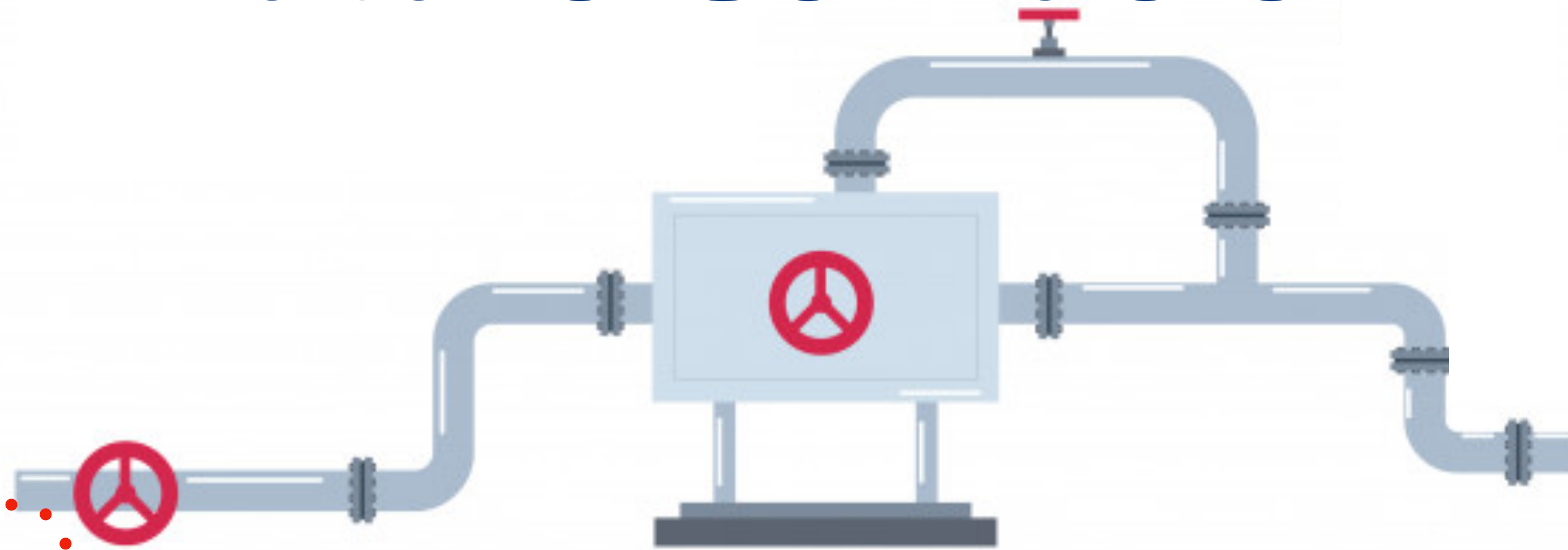
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$\mathcal{V}(\phi, S)$
One-loop Effective Potential
↓

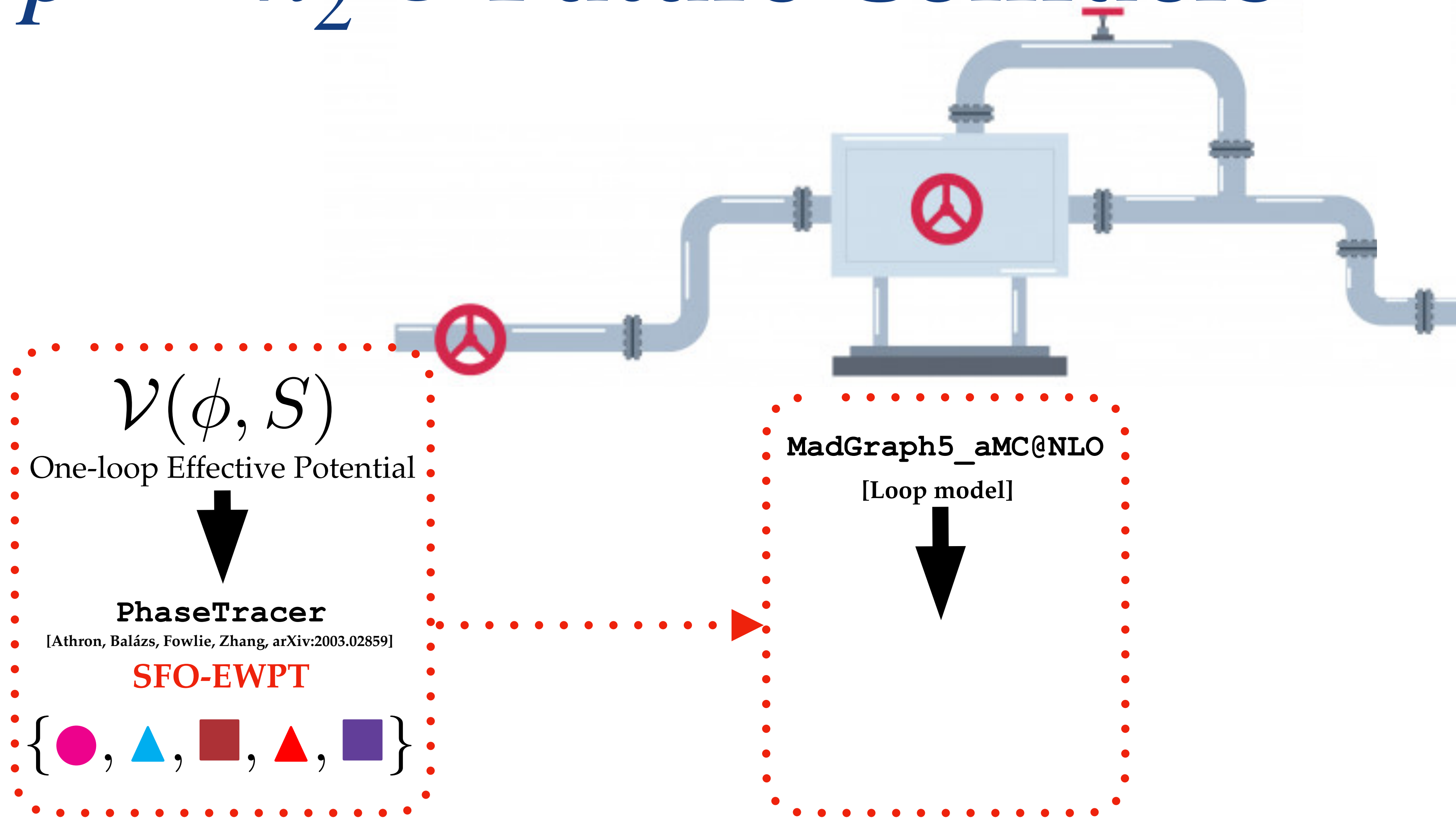
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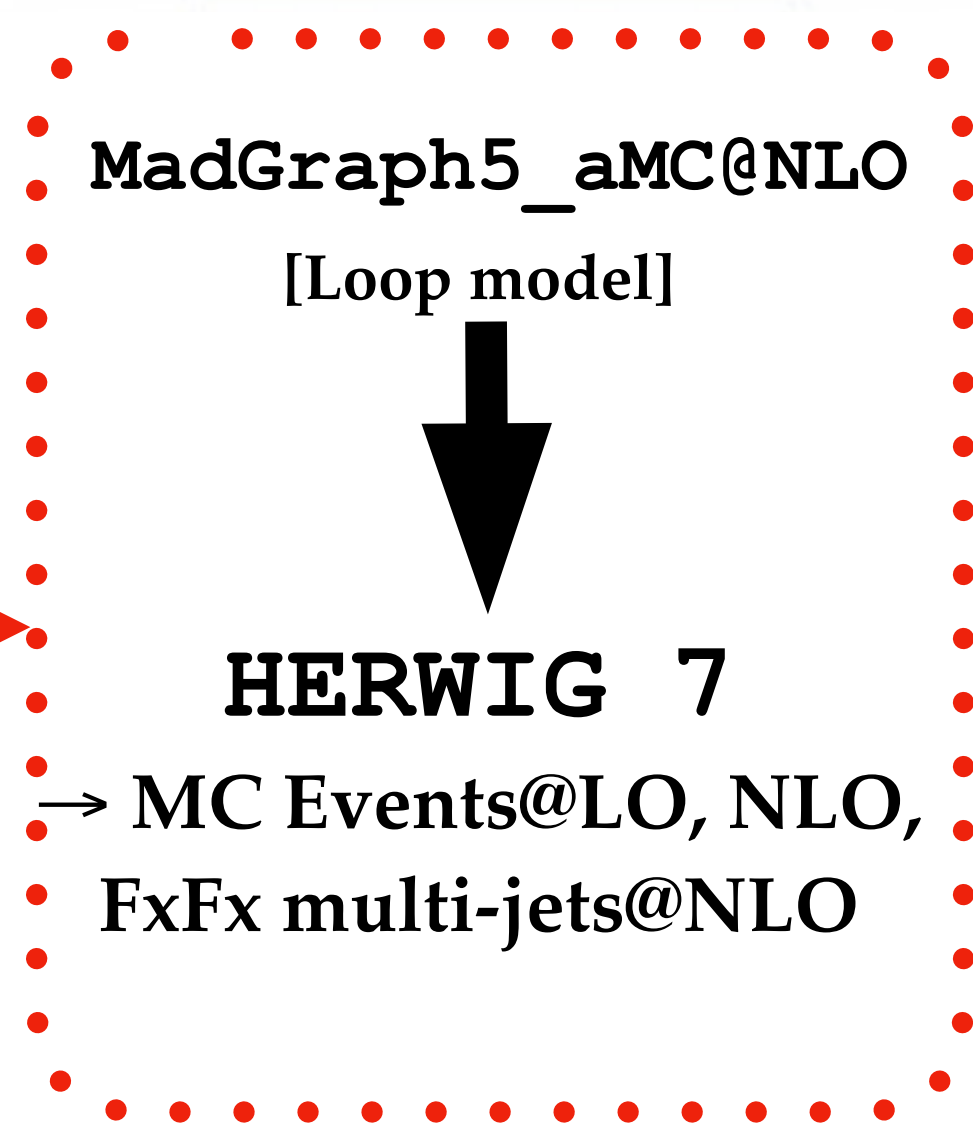
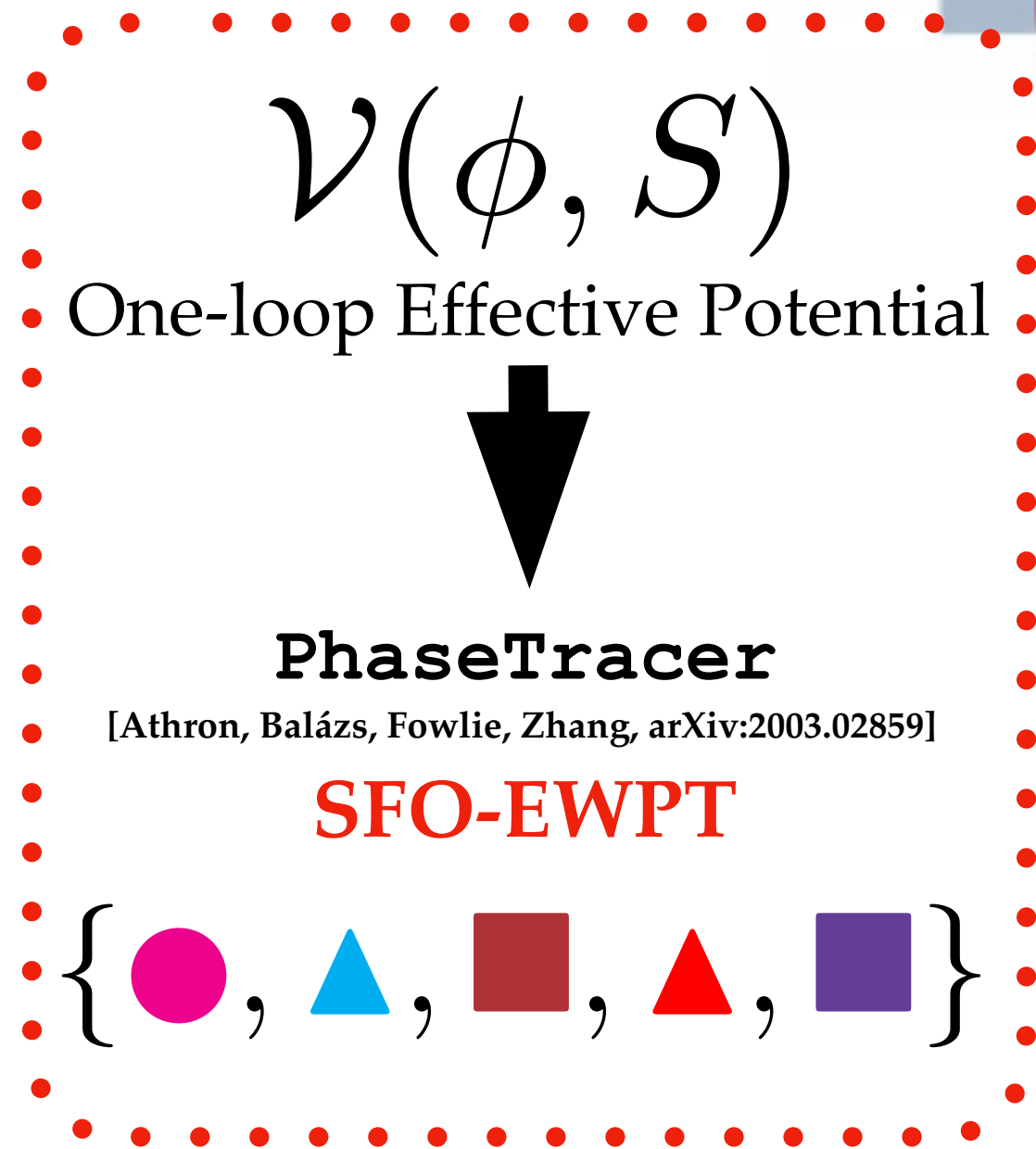
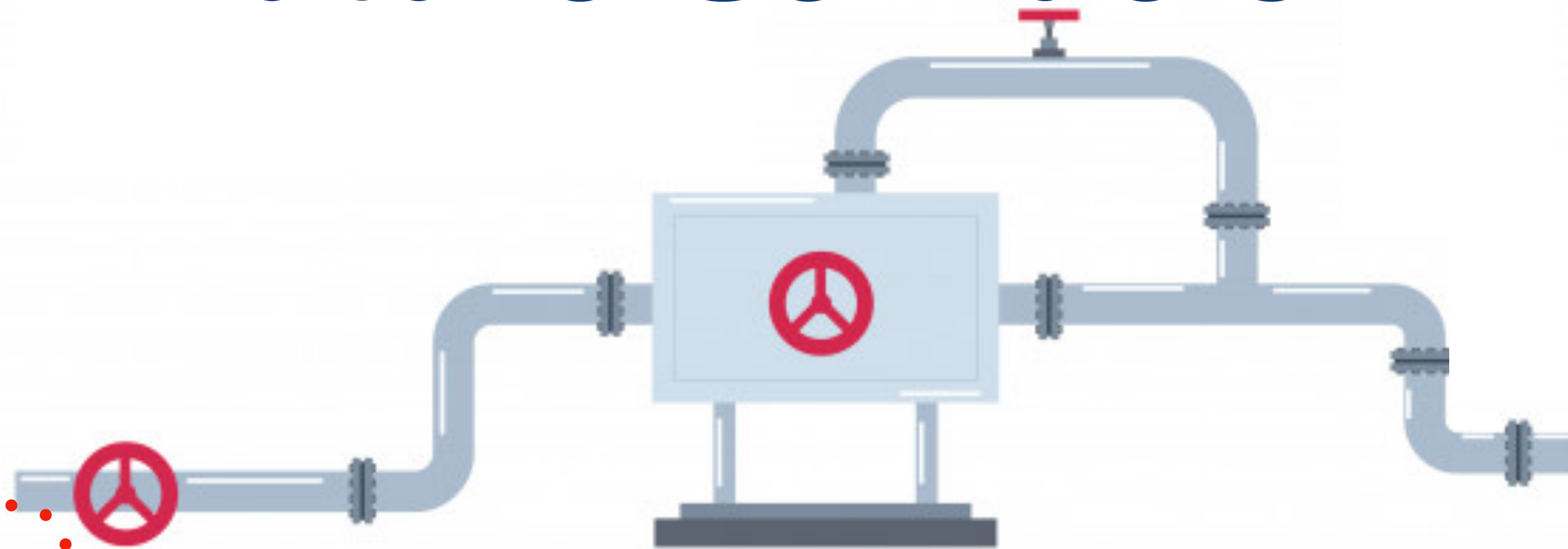
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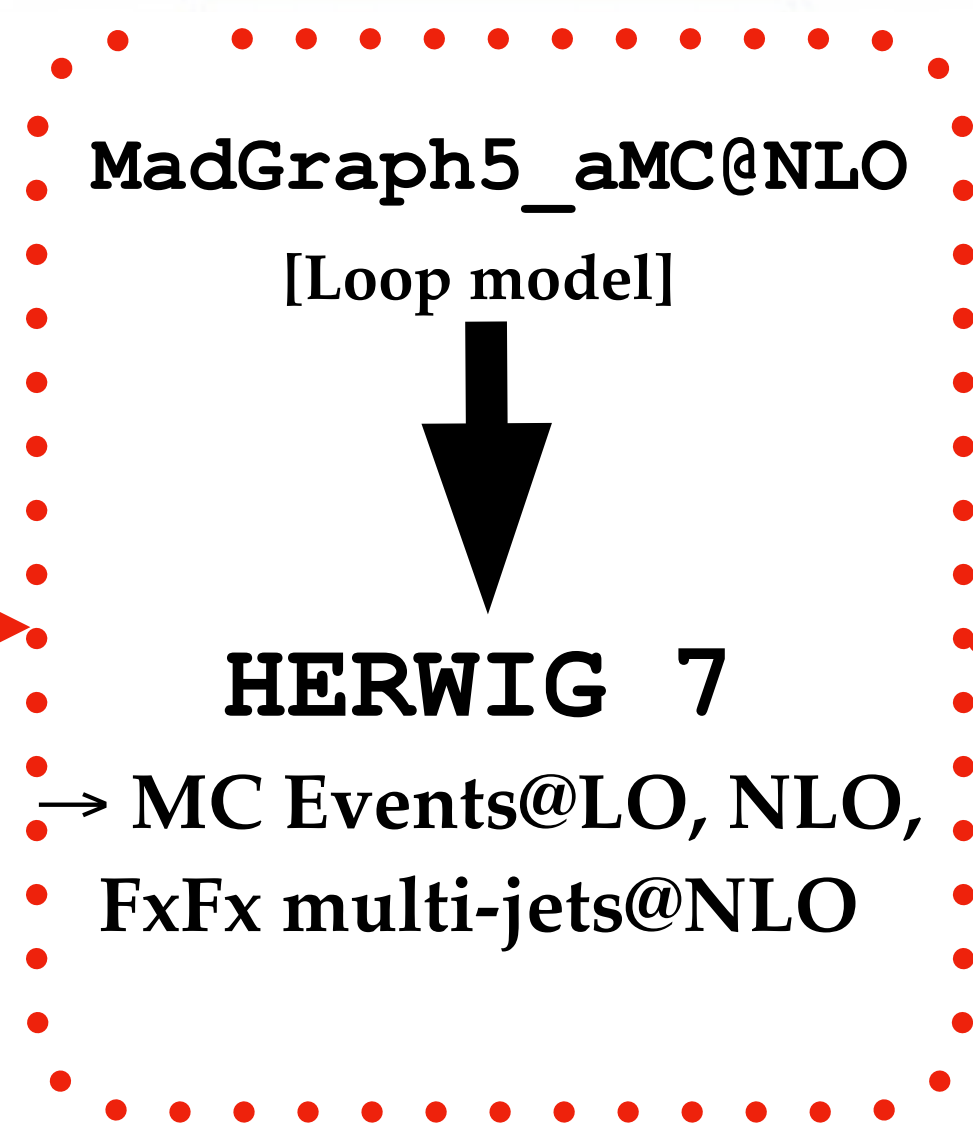
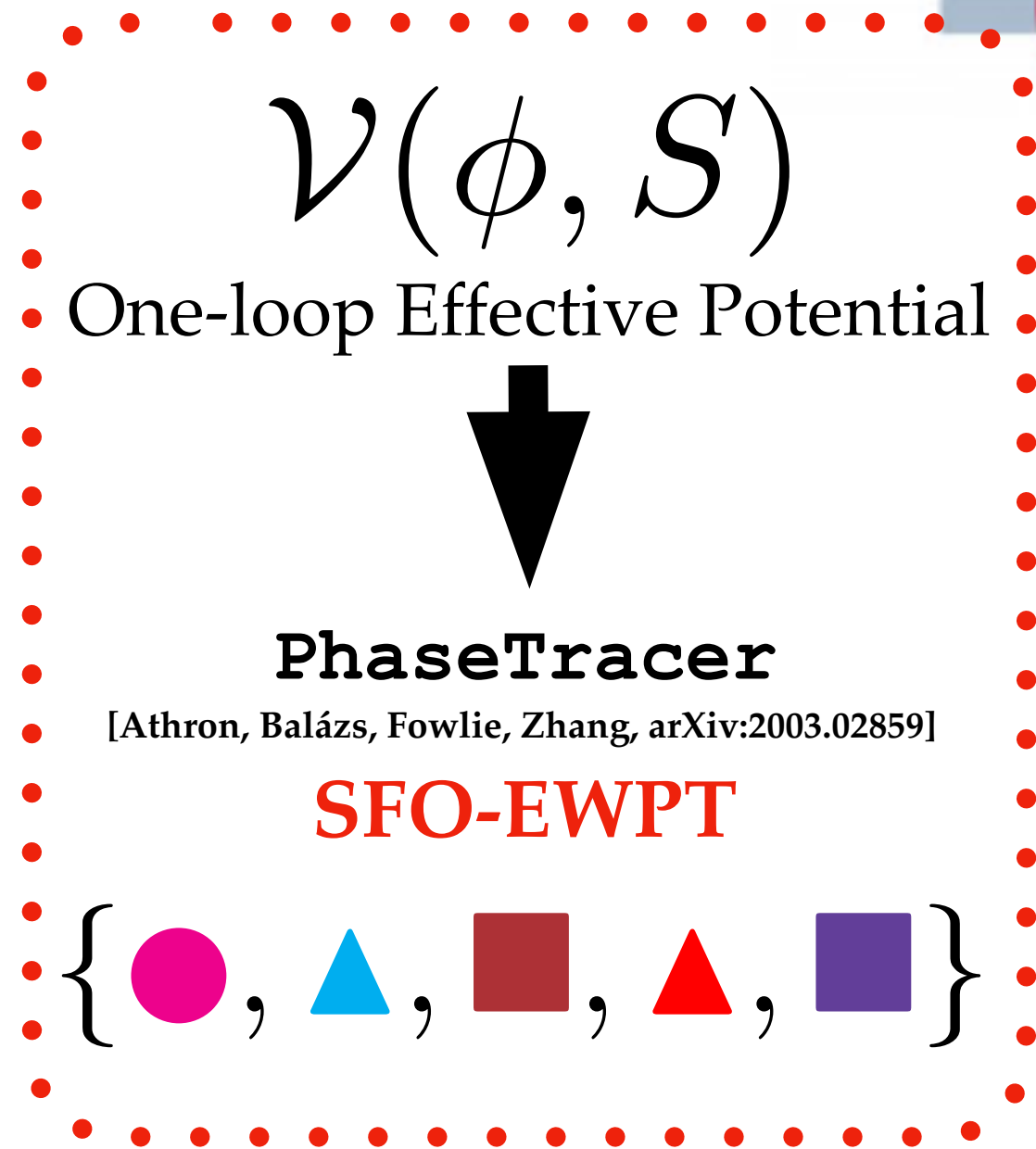
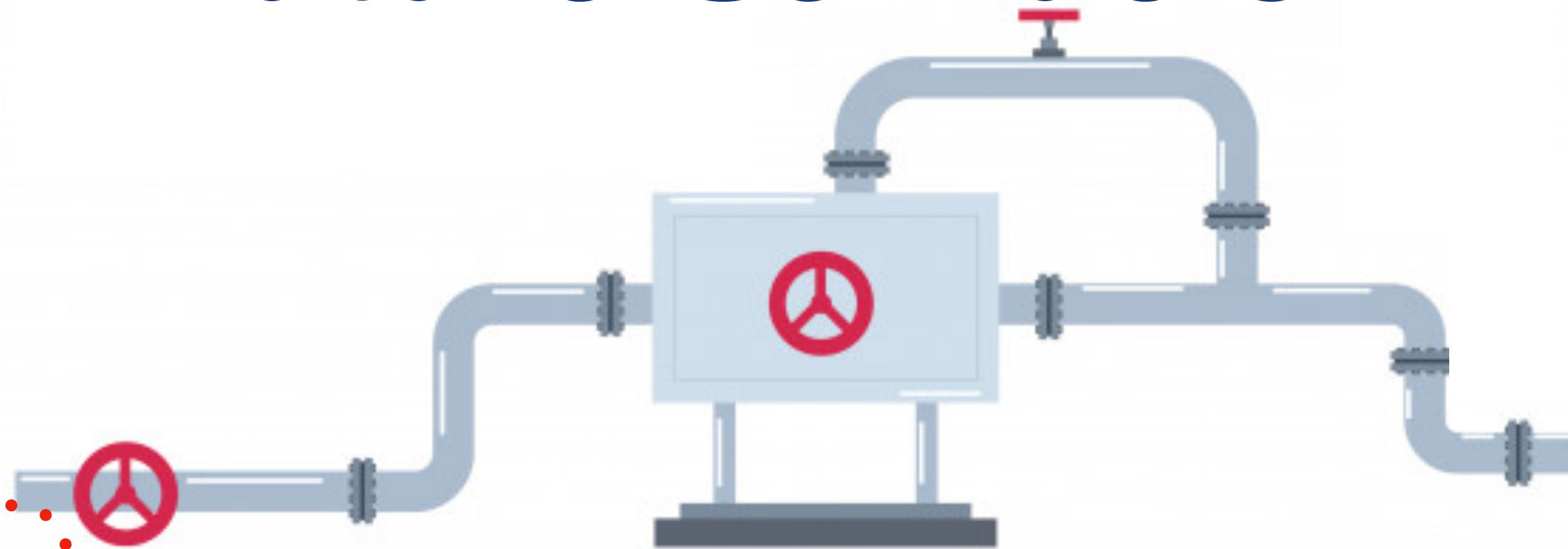
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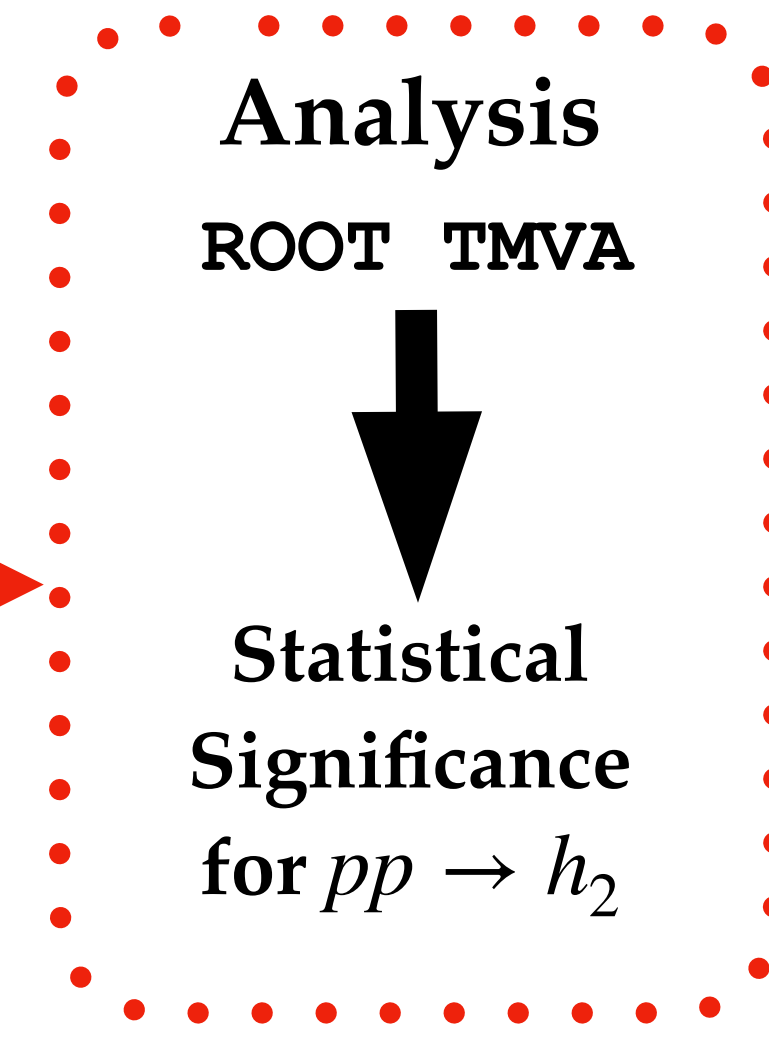


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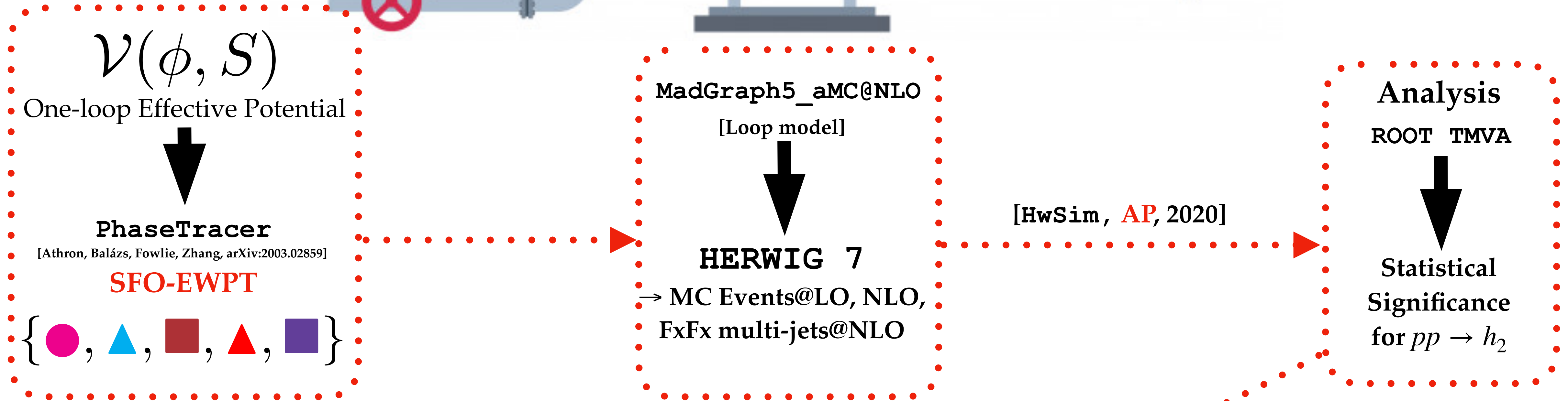
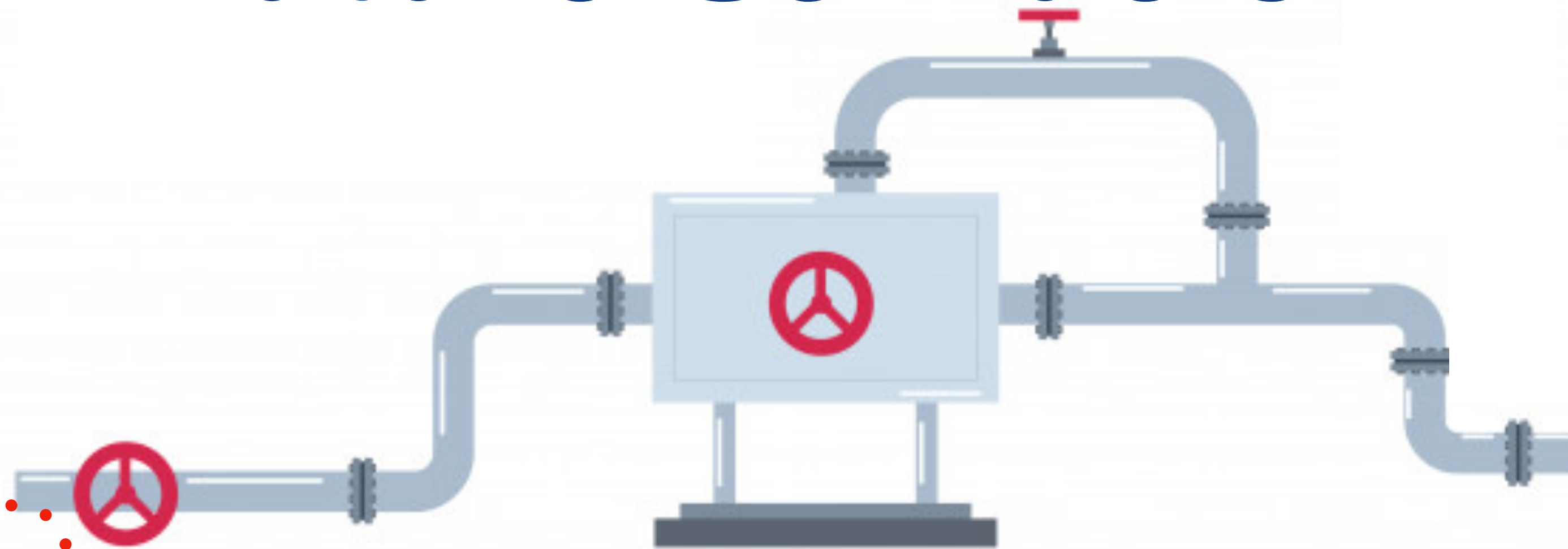


[HwSim, AP, 2020]



$pp \rightarrow h_2$ @ Future Colliders

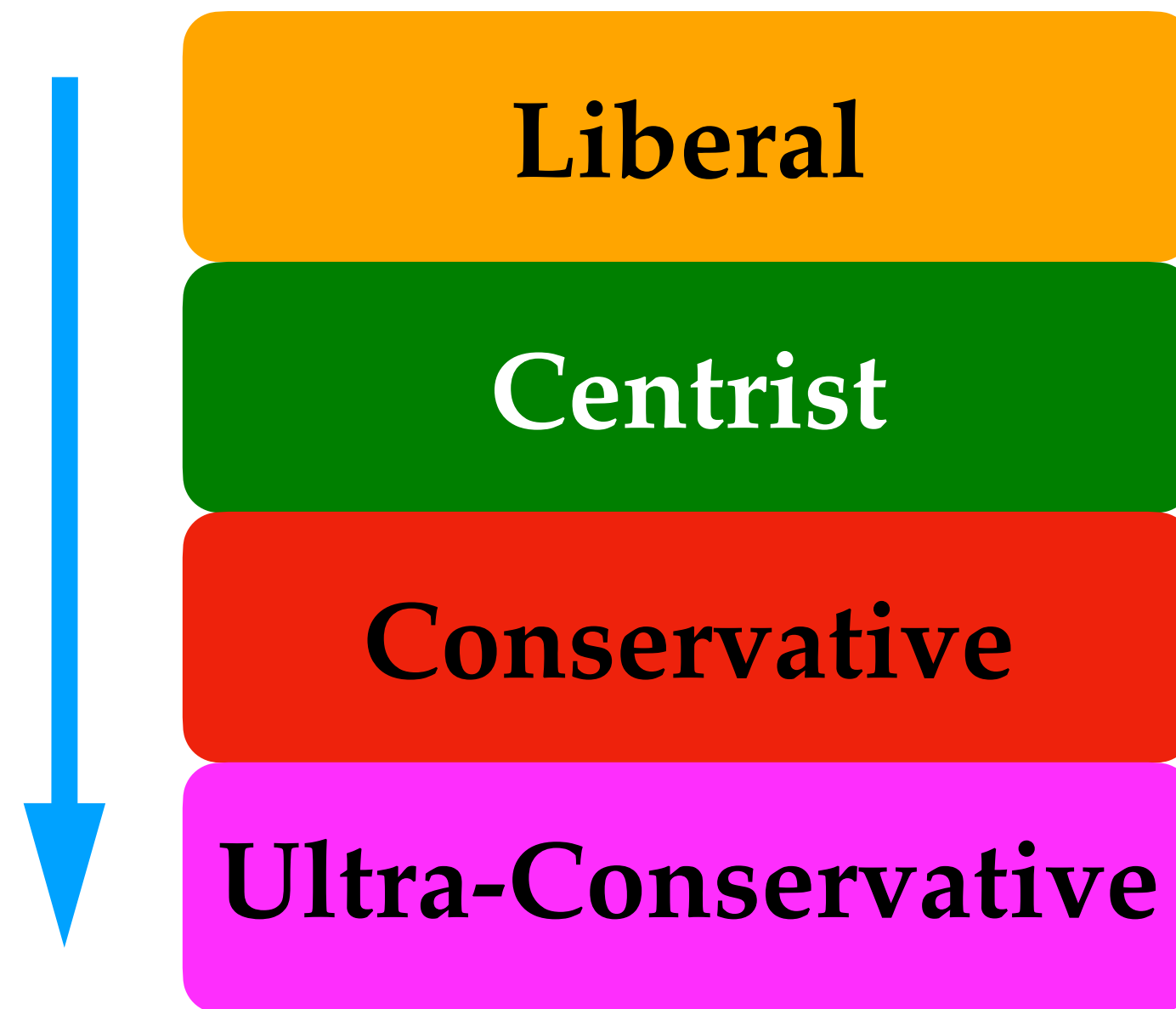
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[HwSim, AP, 2020]

Degrees of Theoretical Uncertainty [AP, White, arXiv:2010.00597]

Lower uncertainty
⇒
SFO-EWPT
more certain to
have occurred.



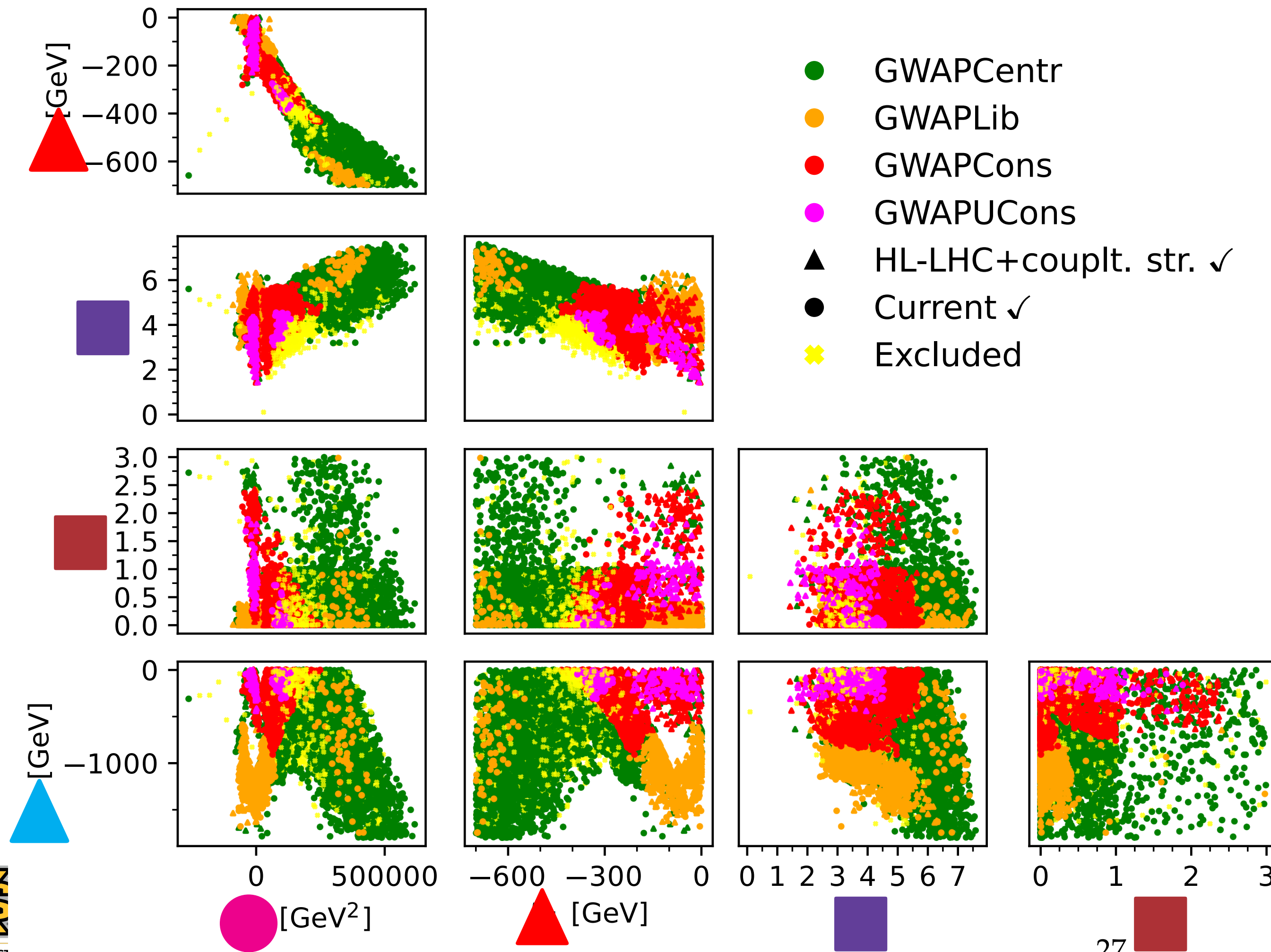
⇒ Colour-coded parameter-space points, denoting **theoretical uncertainty**.

& if: ✕ = Point is excluded today.

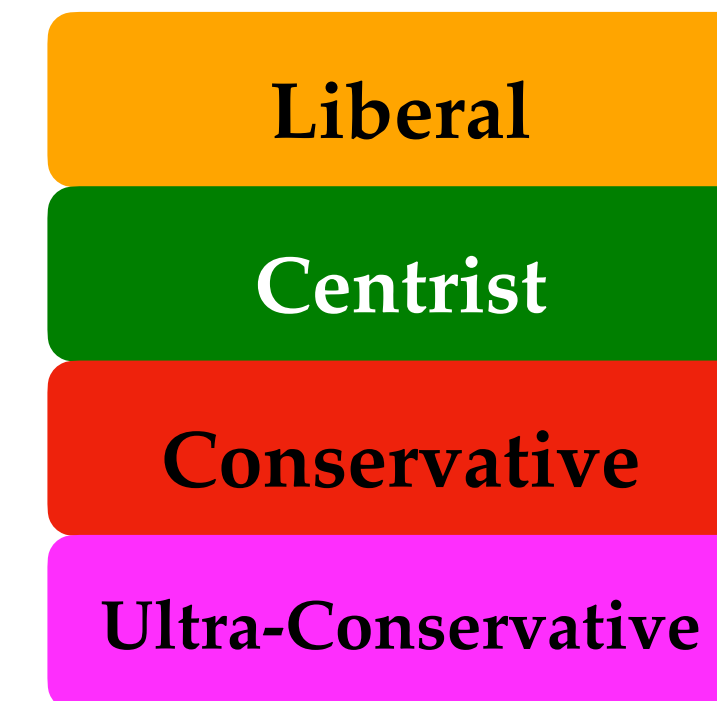


SFO-EWPT Parameter Space

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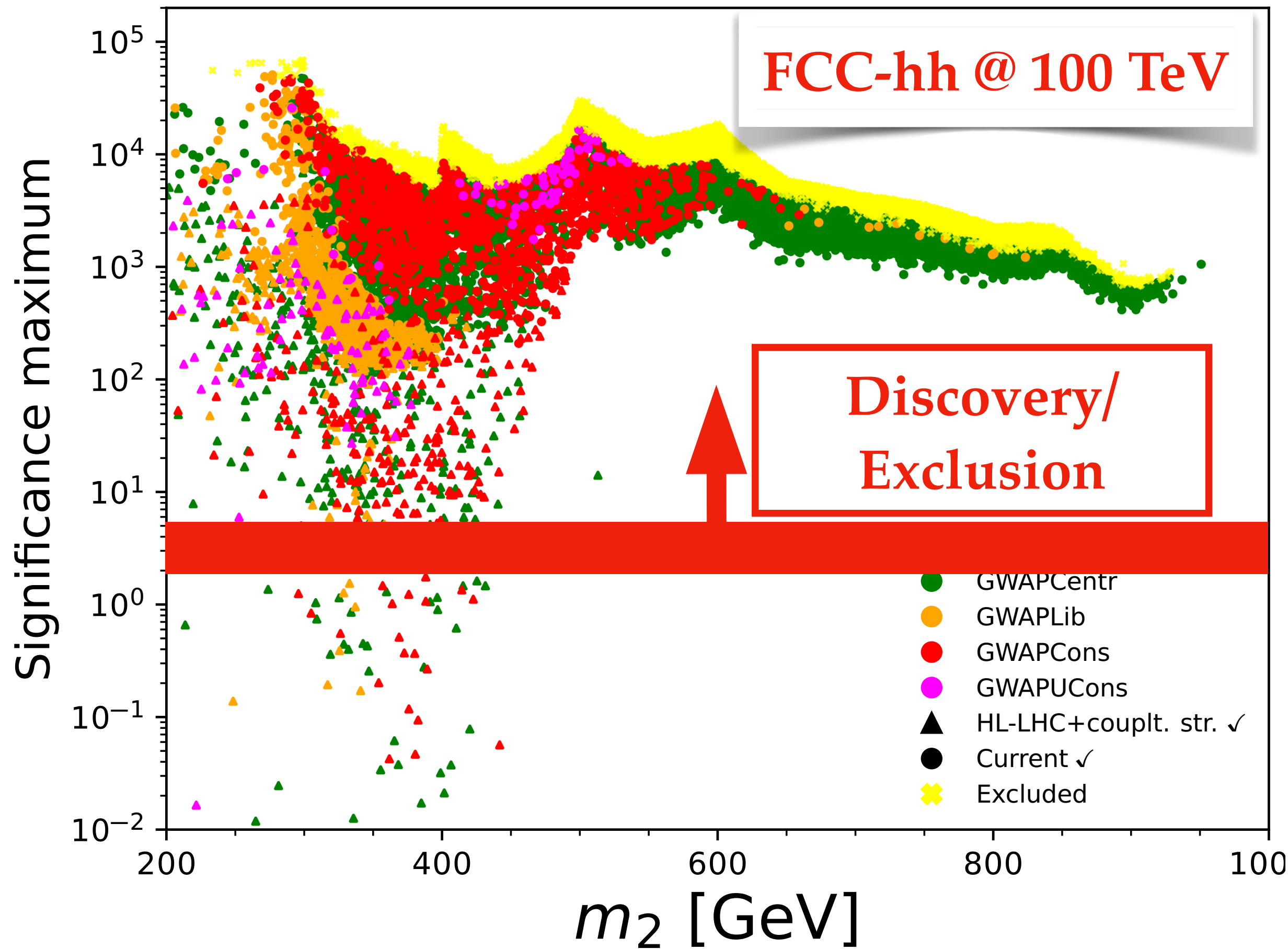
5 free parameters.



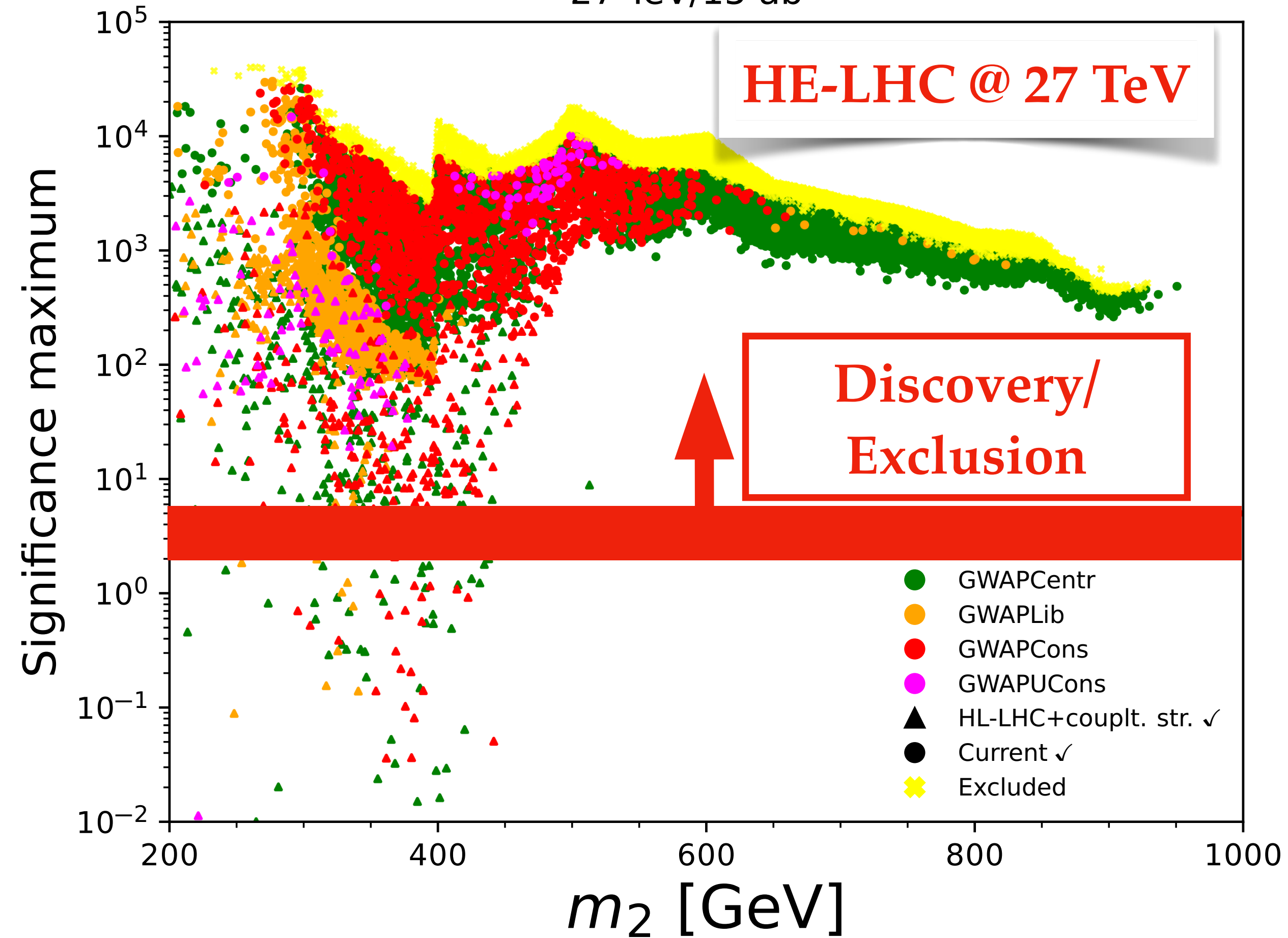
$pp \rightarrow h_2$ Significance @ Future Colliders

Color-coding of parameter-space points denotes **theoretical uncertainty**.

100 TeV/30 ab^{-1}



27 TeV/15 ab^{-1}



✱ = Point is excluded today.

Discovery Post-Mortem

“With 4 parameters I can fit an elephant and with 5 I can make him wiggle his trunk.”
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Can we indeed fit the shape of an elephant with 4 parameters?

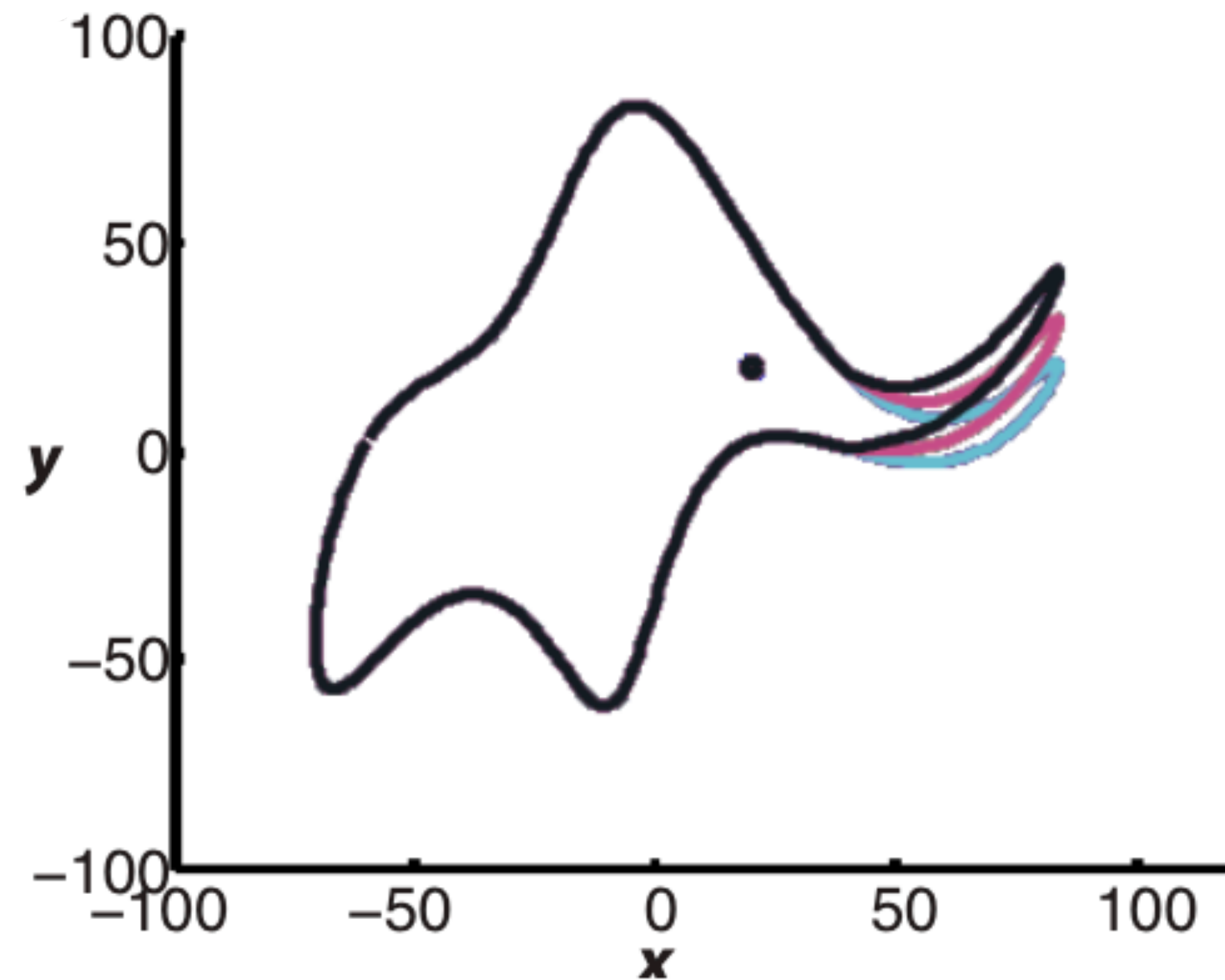
Discovery Post-Mortem

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Can we indeed fit the shape of an elephant with 4 parameters?

→ **Yes!** With **four** complex parameters,
[and with **five** we can make it wiggle its trunk.]

[Mayer, Khairy, Howard, Am. J. Phys., Vol. 78, No. 6, June 2010]



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If we discover e.g. a new scalar particle at colliders,

→ Can we verify that it is indeed the remnant of a singlet field that generates a SFO-EWPT?

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⇒ The **“Inverse Problem”**.

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⇒ The “**Inverse Problem**”.

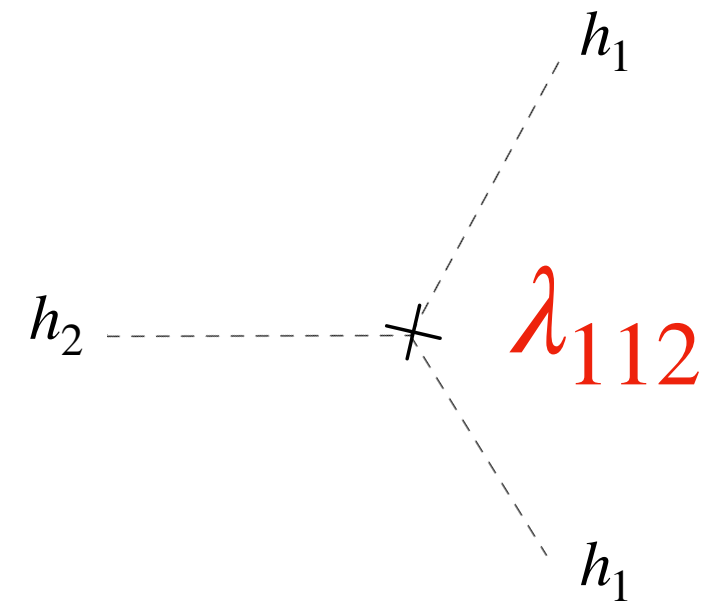
Experimental observations → { , , , ,  }

Discovery Post-Mortem Example [AP, White, arXiv:2108.11394]

Combine possible measurements:

$$pp \rightarrow h_2 \rightarrow ZZ$$

$$pp \rightarrow h_2 \rightarrow h_1 h_1$$

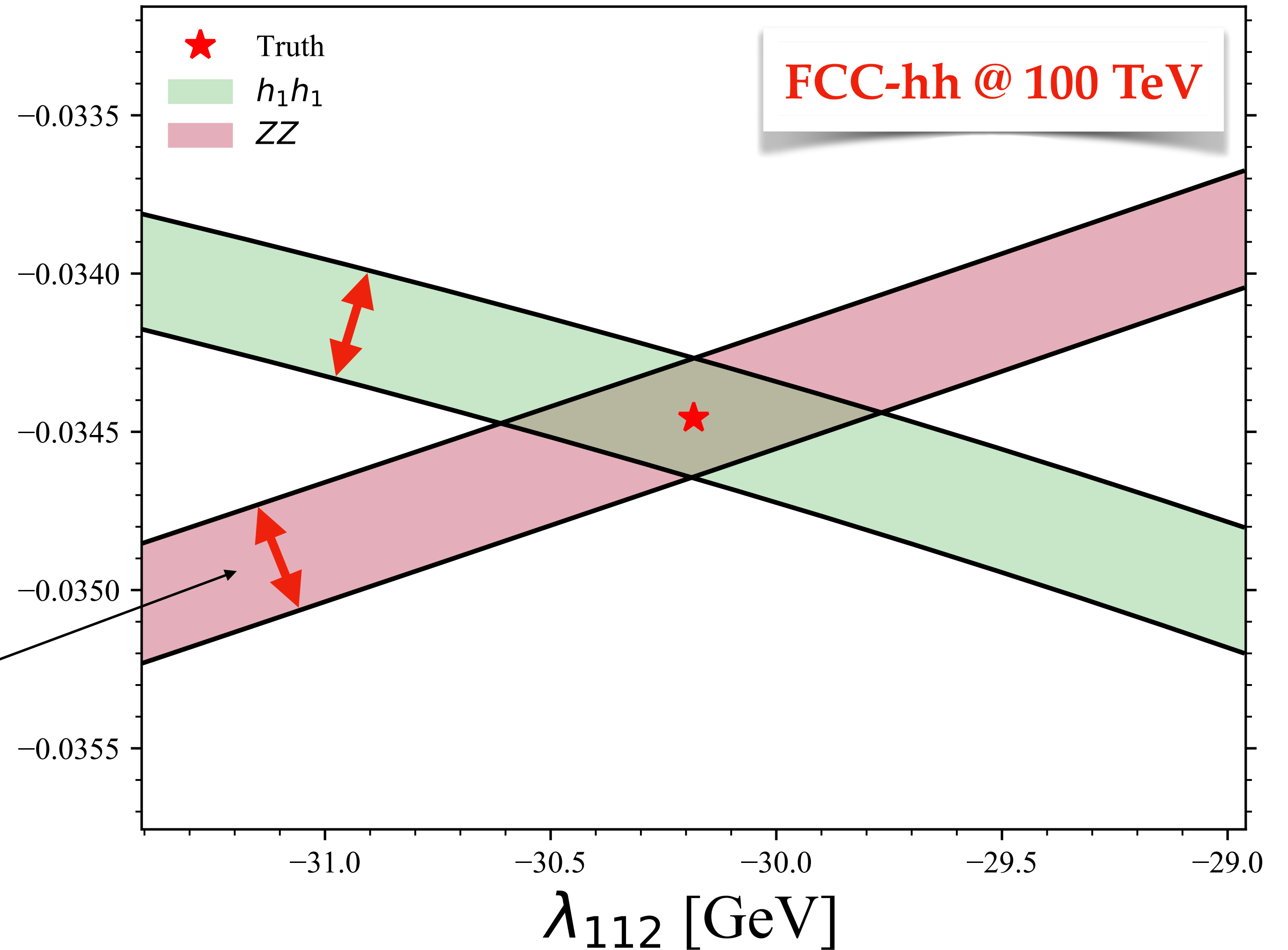


→ both functions of $\sin \theta$ & λ_{112} !

→ in turn f^{ns} of { ●, ▲, ■, ▲, ■ }.

[Width of bands represents expected measurement uncertainty].

pp@100 TeV/30000 fb⁻¹, UCons1

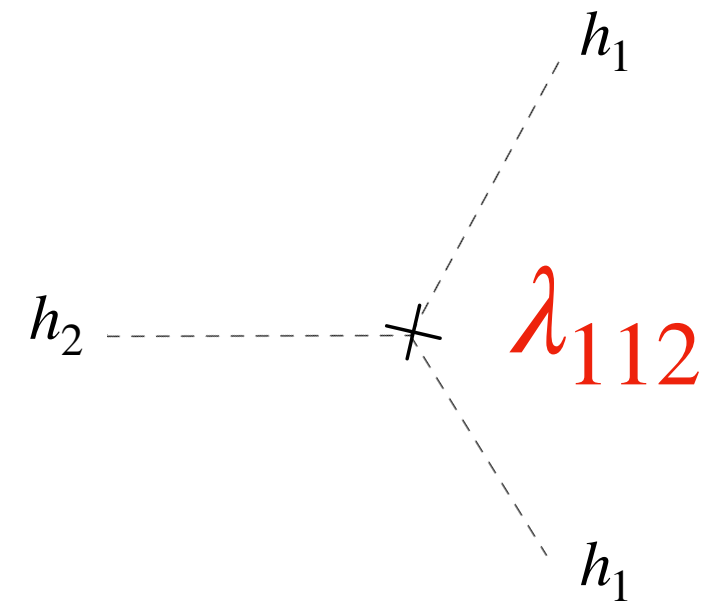


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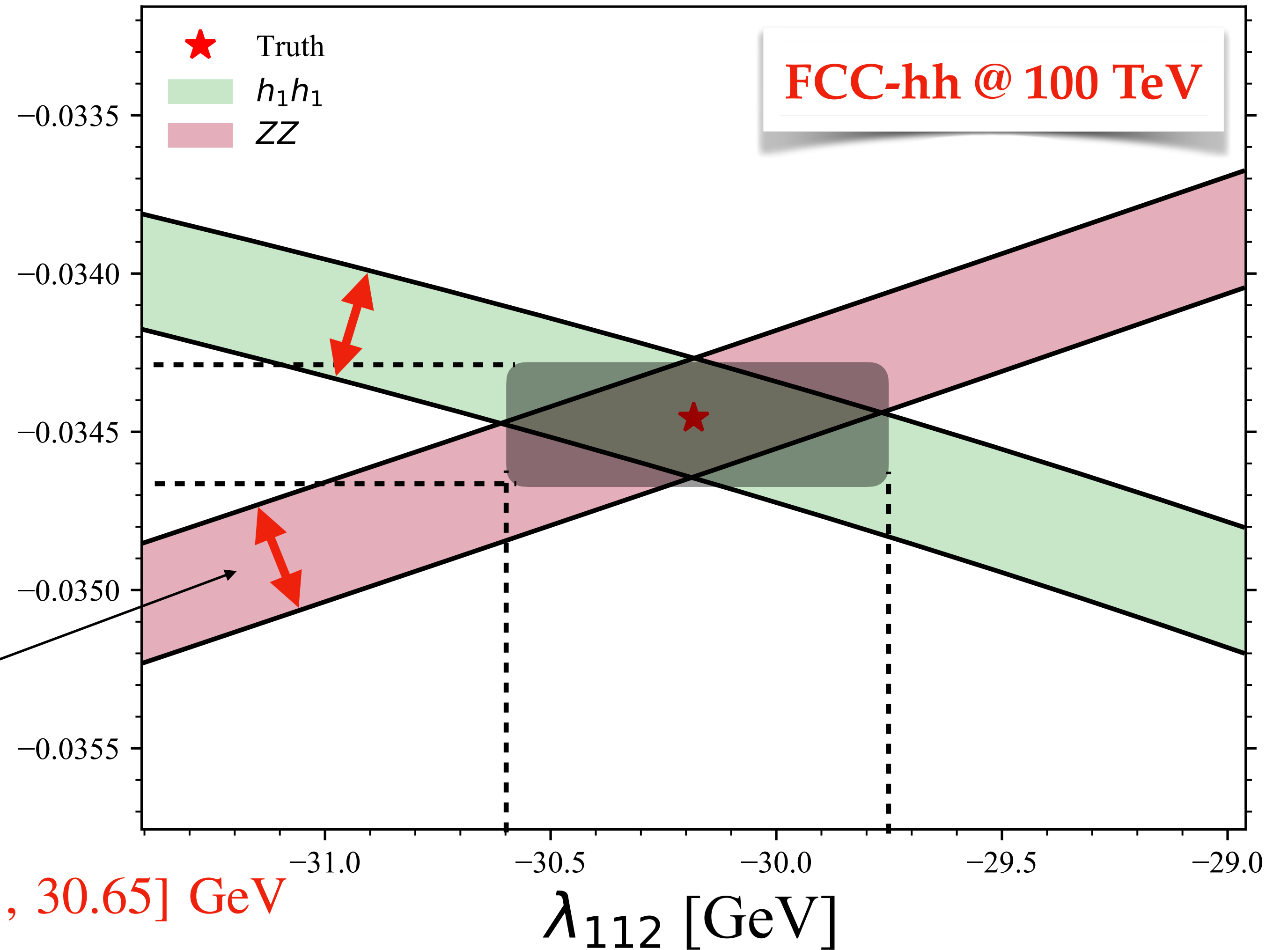


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pp@100 TeV/30000 fb⁻¹, UCons1



$$\lambda_{112} \in [29.81, 30.65] \text{ GeV}$$

$$\sin \theta \in [0.0343, 0.0347]$$

The Inverse Problem in Extended Scalar Sectors:

Multi-scalar processes should play a **crucial rôle**:

$$pp \rightarrow h_1 h_2 \quad | \mathcal{M} |^2 \sim \lambda_{122}^2, \lambda_{112}^2 + \dots$$

$$pp \rightarrow h_2 h_2 \quad | \mathcal{M} |^2 \sim \lambda_{222}^2, \lambda_{122}^2 + \dots$$

$$pp \rightarrow h_1 h_1 h_1 \quad | \mathcal{M} |^2 \sim f[\lambda_{1111}, \lambda_{1112}, \lambda_{111}, \lambda_{112}]$$

[...]

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[...]

pp → *hhh*



VIGEROUS
2/2

BEEF

SM Multi-Higgs Boson Production “Fun” Facts

- \exists factor of $\mathcal{O}(10^{-3})$ each time you “draw” an extra Higgs boson @ pp colliders.



$$\sigma(h) \sim 50 \text{ pb}$$

SM, 14 TeV



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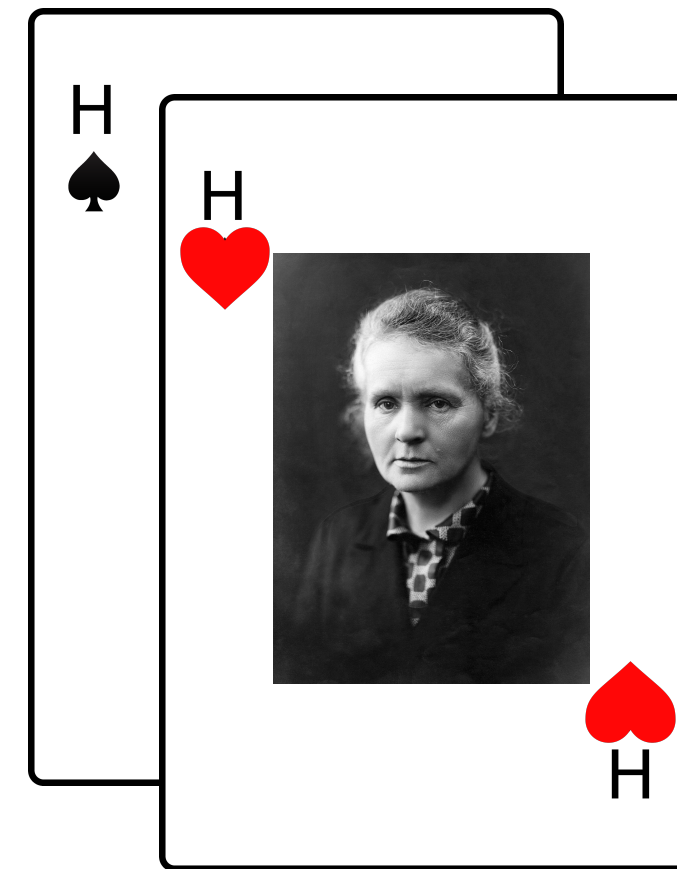
SM, 14 TeV



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$$\times \mathcal{O}(10^{-3})$$



$$\sigma(hh) \sim 40 \text{ fb}$$



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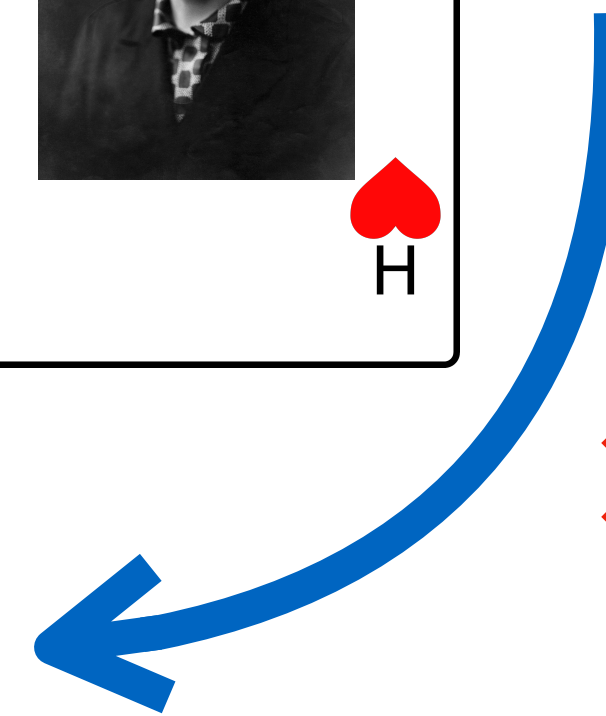
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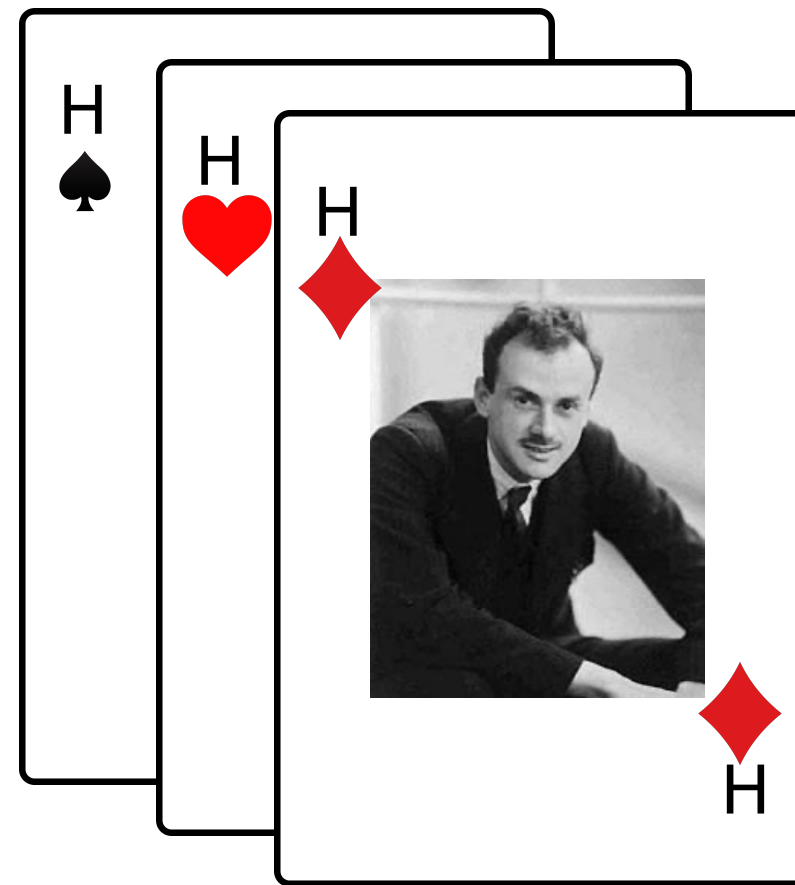
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$$\sigma(hh) \sim 40 \text{ fb}$$



$$\times \mathcal{O}(10^{-3})$$

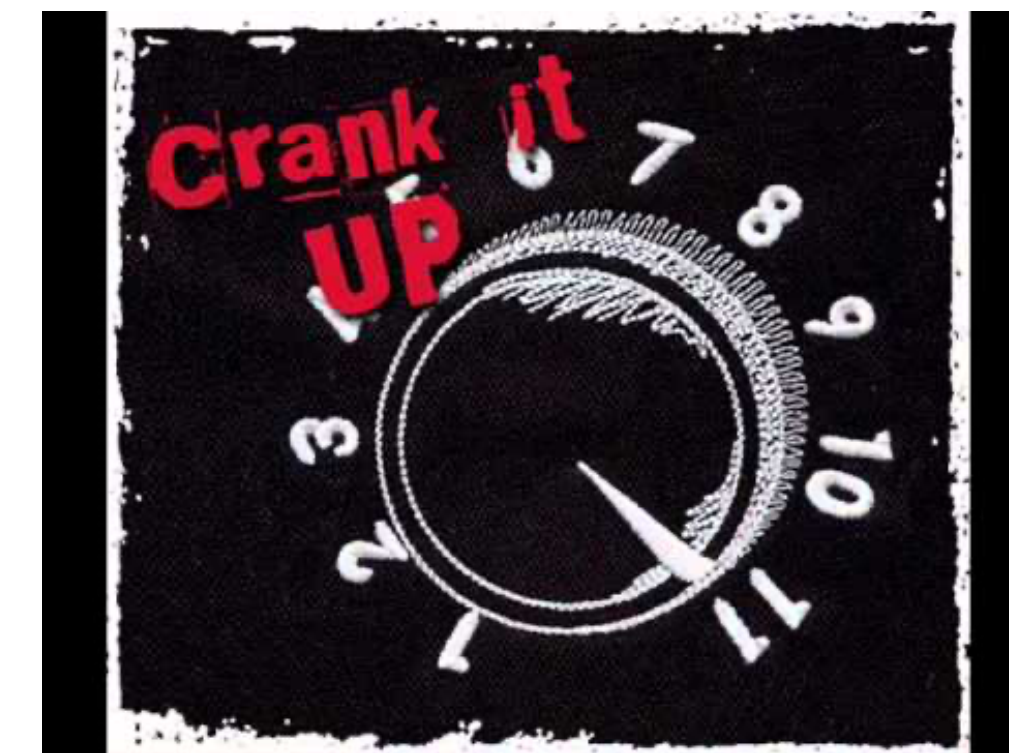
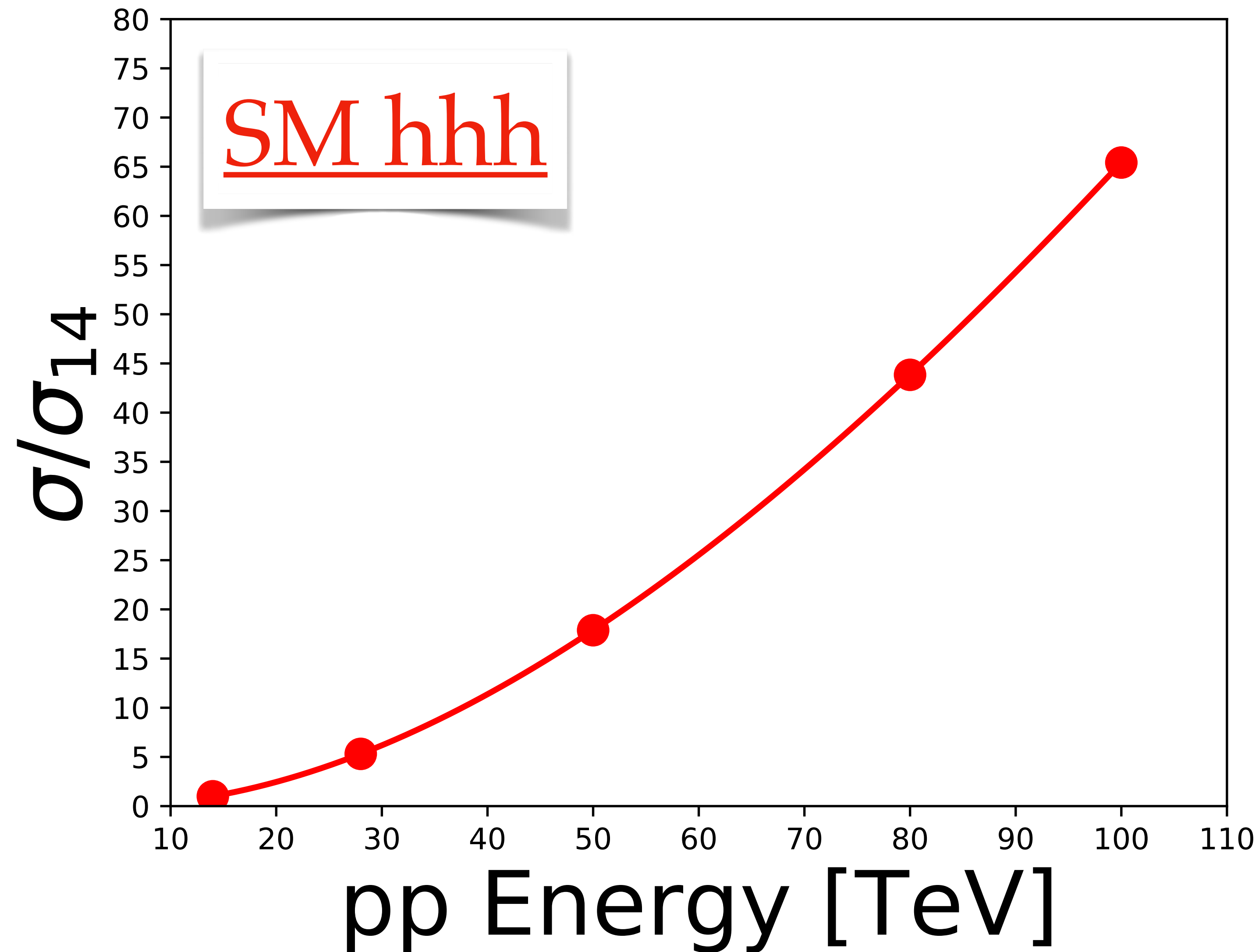


$$\sigma(hhh) \sim 0.1 \text{ fb}$$



SM Multi-Higgs Boson Production “Fun” Facts

- Cranking up the pp energy could help!

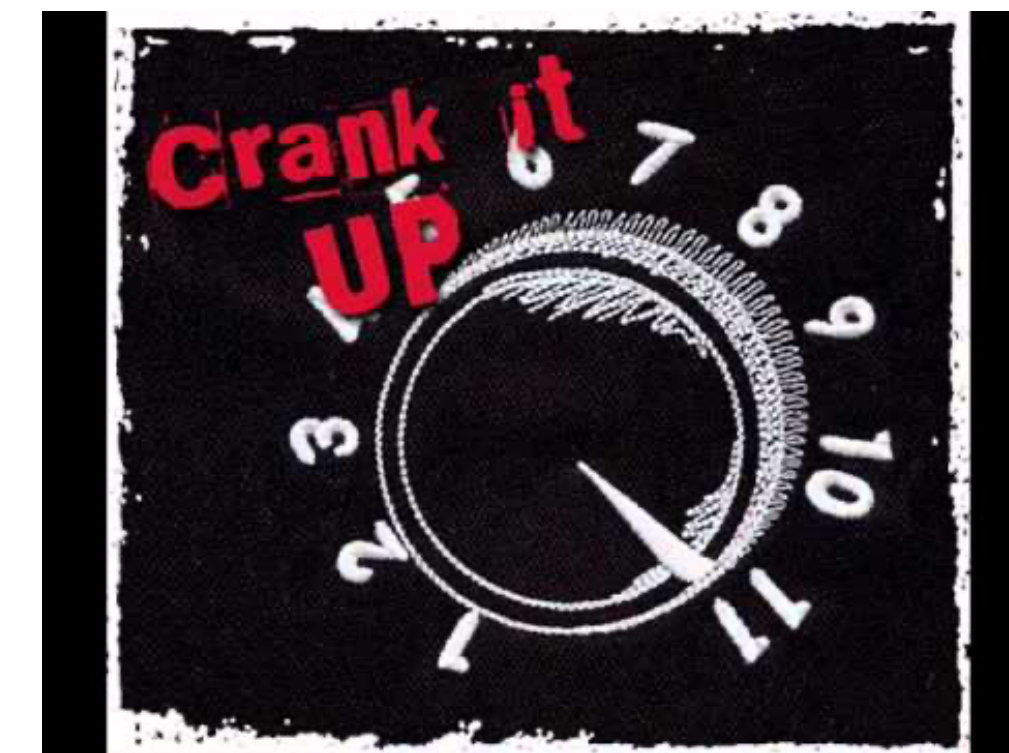
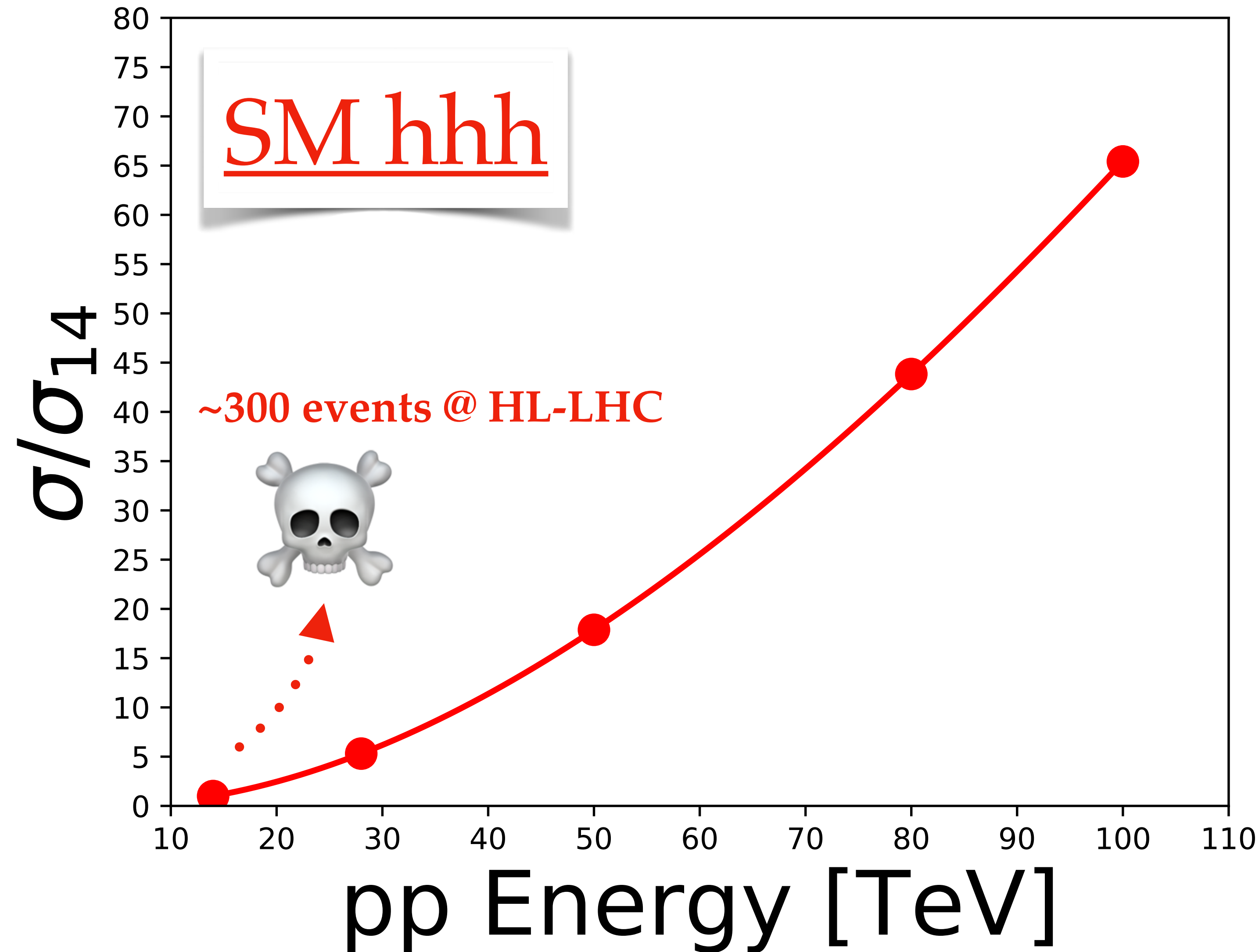


~ $\times 60$ increase in
cross section
14 TeV \rightarrow 100 TeV.



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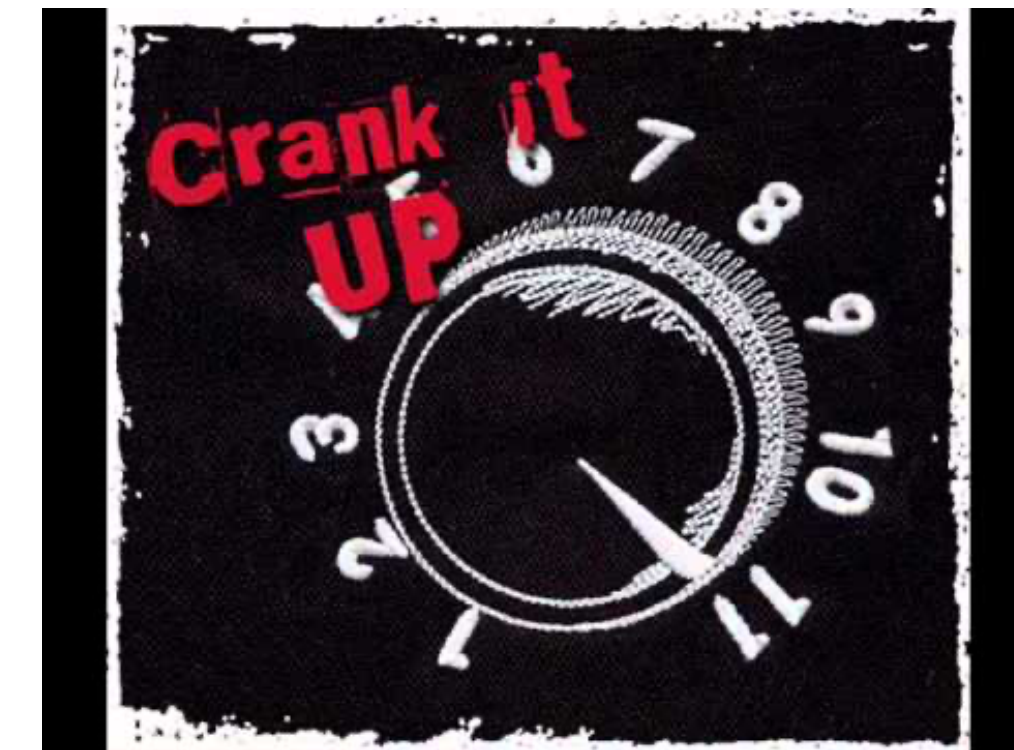
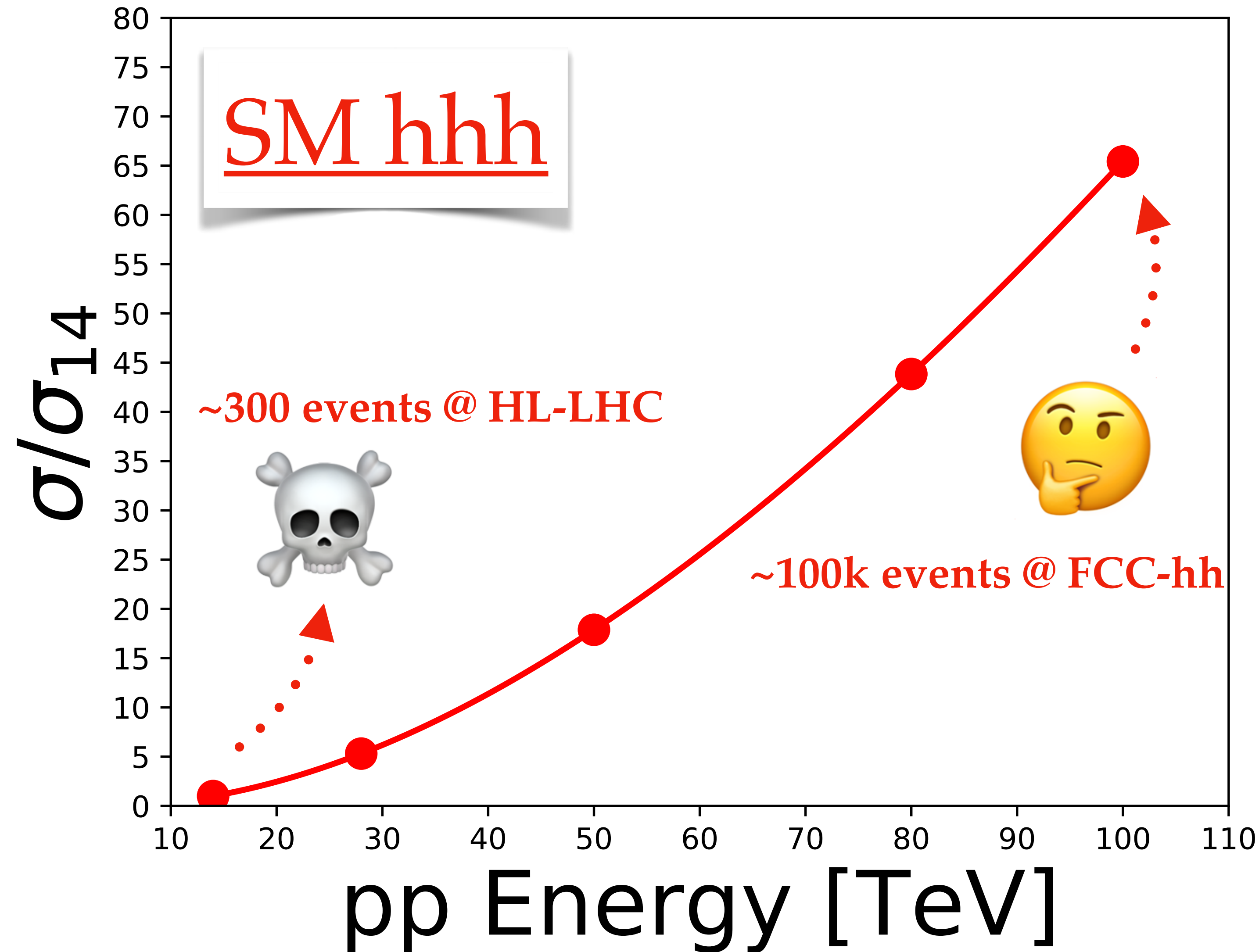


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SM Multi-Higgs Boson Production “Fun” Facts

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THE SECRET
iNGREDiENT
is ALWAYS
LOVE



THE SECRET
iNGREDiENT
is ALWAYS

~~LOVE~~

NEW PHYSICS



THE SECRET
INGREDIENT
IS ALWAYS

~~LOVE~~

NEW PHYSICS

Here:

- A. $h h h$ in SM+2 singlet scalar fields,
- B. $h h h$ with anomalous couplings.



A. hhh in SM+2 singlet scalar fields,



SM + Two Real Singlet Scalars [= TRSM]

- Consider adding two real singlet scalar fields $S, X \rightarrow$ the **TRSM**.
- And: impose discrete \mathcal{Z}_2 symmetries:
 $\mathcal{Z}_2^S : S \rightarrow -S, X \rightarrow X$
 $\mathcal{Z}_2^X : X \rightarrow -X, S \rightarrow S$

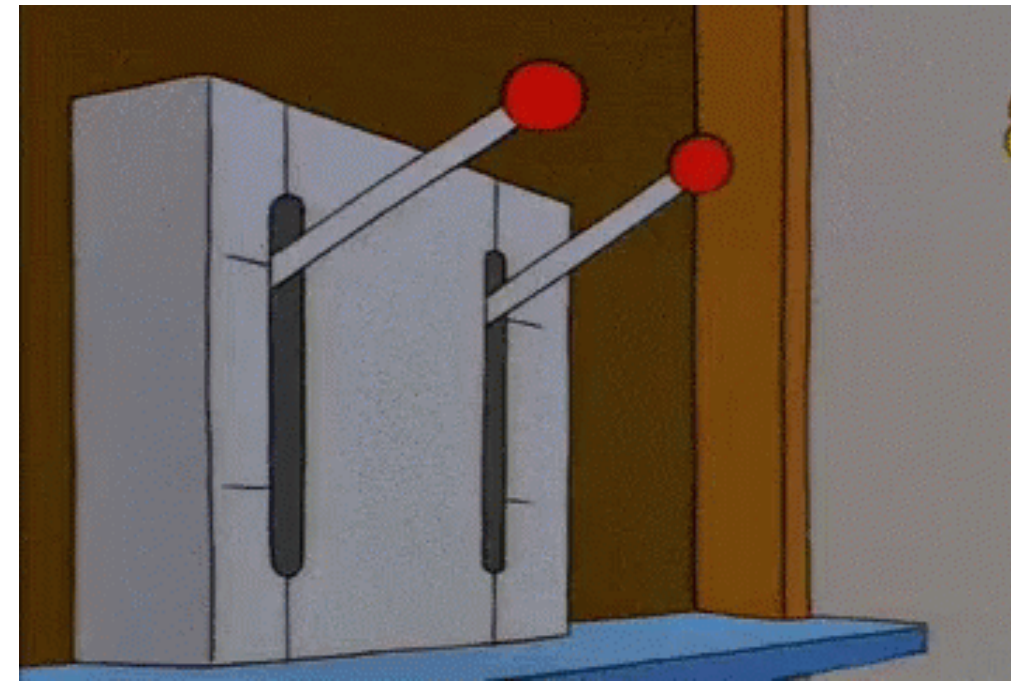
\Rightarrow **TRSM** scalar potential:

$$\begin{aligned} \mathcal{V}(\phi, S, X) = & \bullet |\phi|^2 + \blacksquare |\phi|^4 + \bullet S^2 + \blacksquare S^4 + \bullet X^2 + \blacksquare X^4 \\ & + \blacksquare S^2 X^2 \\ & + \blacksquare |\phi|^2 S^2 + \blacksquare |\phi|^2 X^2 \end{aligned}$$



SM + Two Real Singlet Scalars [= TRSM]

- Go through **EWSB**...



⇒ Get **three** scalar bosons: $h_1, h_2, h_3 \rightarrow h_1 \approx$ SM-like Higgs boson.

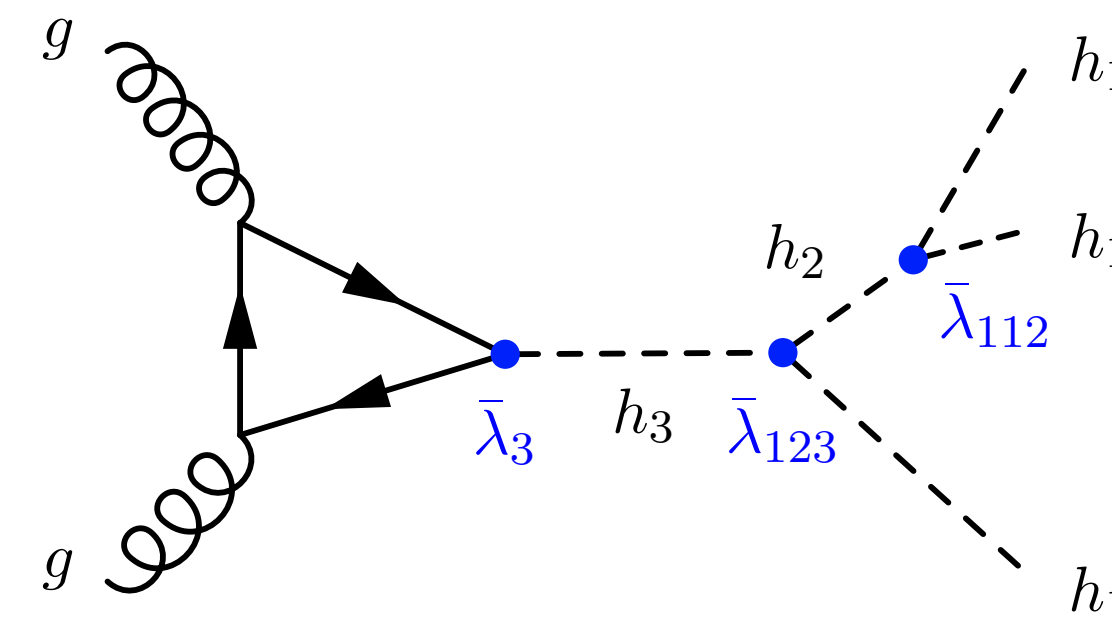
⇒ **Seven** independent parameters: M_2, M_3 + three mixing angles + two VEVs.

⇒ Modified / Additional interactions between scalars.

⇒ **hhh** that may even be detectable at the LHC! [AP, Robens, Tetlalmatzi-Xolocotzi, arXiv:2101.00037]

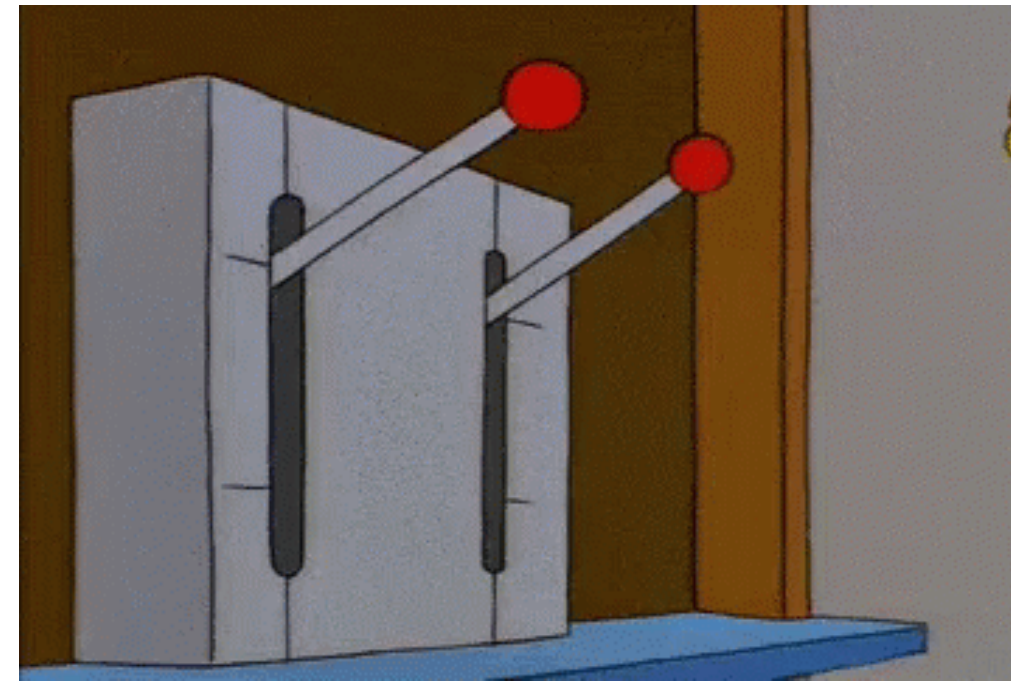
e.g.: $pp \rightarrow h_3 \rightarrow h_2 h_1 \rightarrow h_1 h_1 h_1$

→ Double-resonant enhancement!



SM + Two Real Singlet Scalars [= TRSM]

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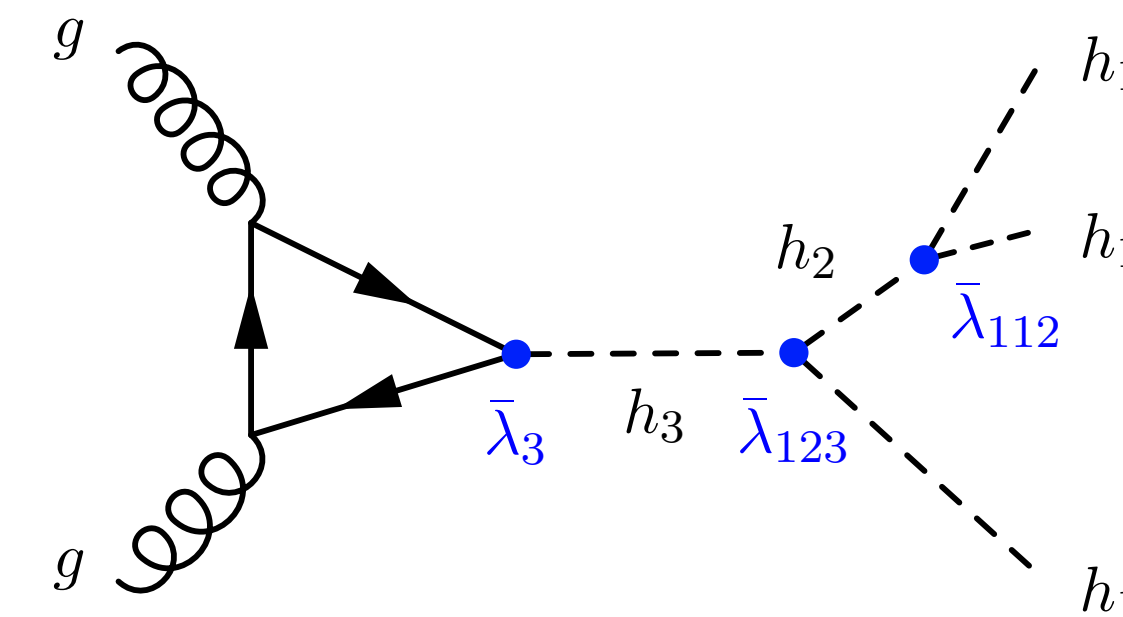
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e.g.: $pp \rightarrow h_3 \rightarrow h_2 h_1 \rightarrow h_1 h_1 h_1$

→ Double-resonant enhancement!



hhh in the TRSM [14 TeV]

- Focus on a particular family of benchmark points: “Benchmark Plane 3” = “BP3” in [Robens, Stefaniak, Wittbrodt, arXiv:1908.08554].

Label	(M_2, M_3) [GeV]	$\sigma(pp \rightarrow h_1 h_1 h_1)$ [fb]
A	(255, 504)	32.40
B	(263, 455)	50.36
C	(287, 502)	39.61
D	(290, 454)	49.00
E	(320, 503)	35.88
F	(264, 504)	37.67
G	(280, 455)	51.00
H	(300, 475)	43.92
I	(310, 500)	37.90
J	(280, 500)	40.26

- In BP3: All params fixed except M_2, M_3 !

Cross section can be much higher than in the SM! 😲

→ **c.f. SM: $\sigma \sim 0.1$ fb @ 14 TeV.**

[AP, Robens, Tetlalmatzi-Xolocotzi, arXiv:2101.00037]



hhh in the TRSM “BP3” [14 TeV]

- Search for **hhh** via: $pp \rightarrow (b\bar{b})(b\bar{b})(b\bar{b}) \rightarrow$ **6 b-jets**.
- About **20%** of the **hhh** final state!
- Significances **large**, even when including systematic uncert.:

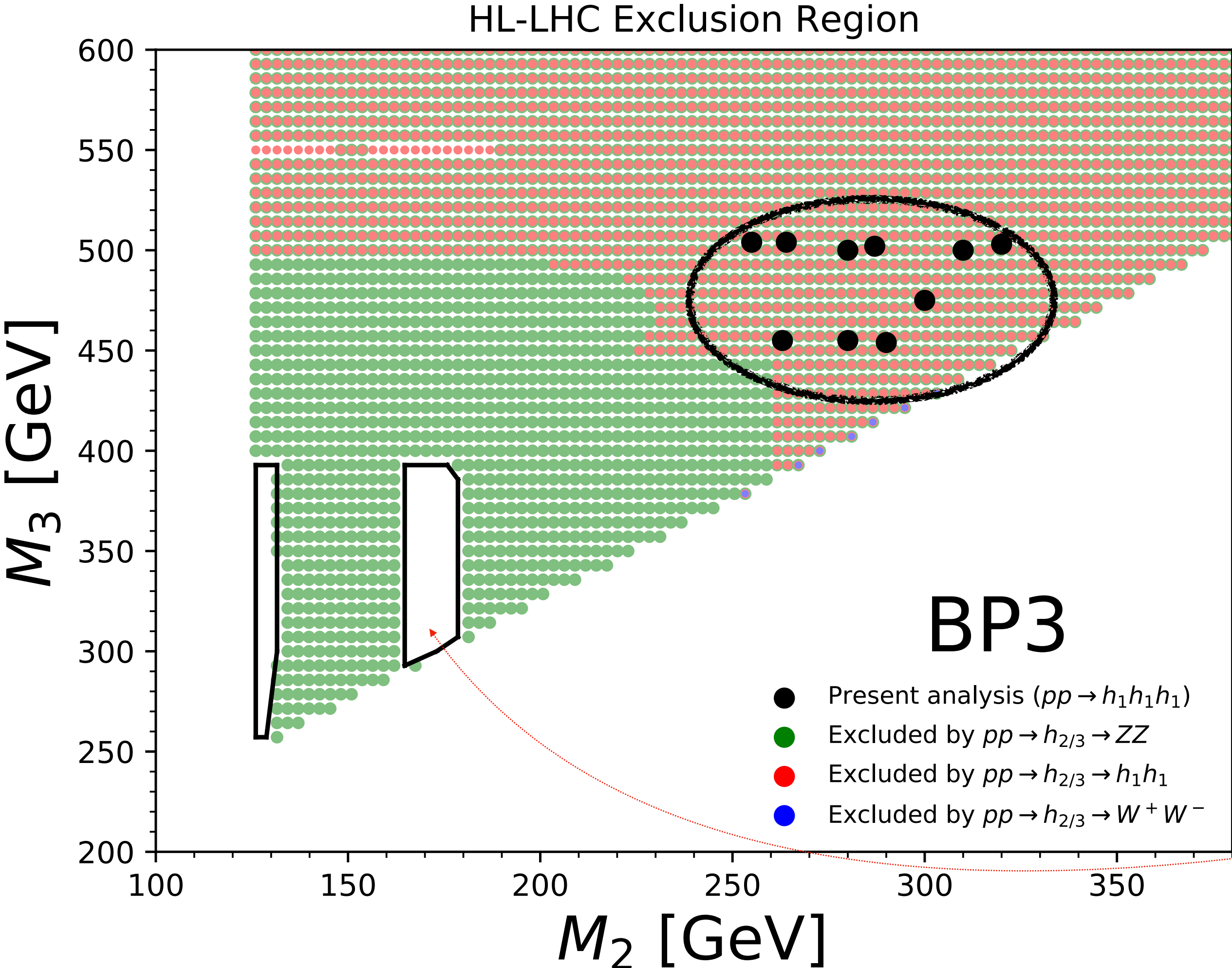
Label	sig <u>300fb</u> ⁻¹ (syst.)	sig <u>3000fb</u> ⁻¹ (syst.)
A	2.92 (2.63)	9.23 (5.07)
B	4.78 (4.50)	15.10 (10.14)
C	4.01 (3.56)	12.68 (6.67)
D	5.02 (4.03)	15.86 (6.25)
E	3.76 (2.87)	11.88 (4.18)
F	3.56 (3.18)	11.27 (5.98)
G	5.18 (4.16)	16.39 (6.45)
H	4.64 (3.47)	14.68 (4.94)
I	4.09 (2.88)	12.94 (3.87)
J	4.00 (3.56)	12.65 (6.66)

[**AP**, Robens, Tetlalmatzi-Xolocotzi, arXiv:2101.00037]



hhh in the TRSM “BP3” [14 TeV]

- hhh will (**probably?**) not be a discovery channel,
- but could be **important in determining the parameters of the model**, if scalars are discovered!



Could help solve the “inverse problem” in the TRSM?

[AP, Robens, Tetlalmatzi-Xolocotzi, arXiv:2101.00037]

Note: regions near $M_2 \sim 130$ GeV and $M_2 \sim 170$ GeV will remain viable at the end of the HL-LHC.



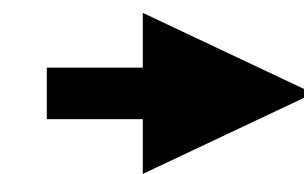
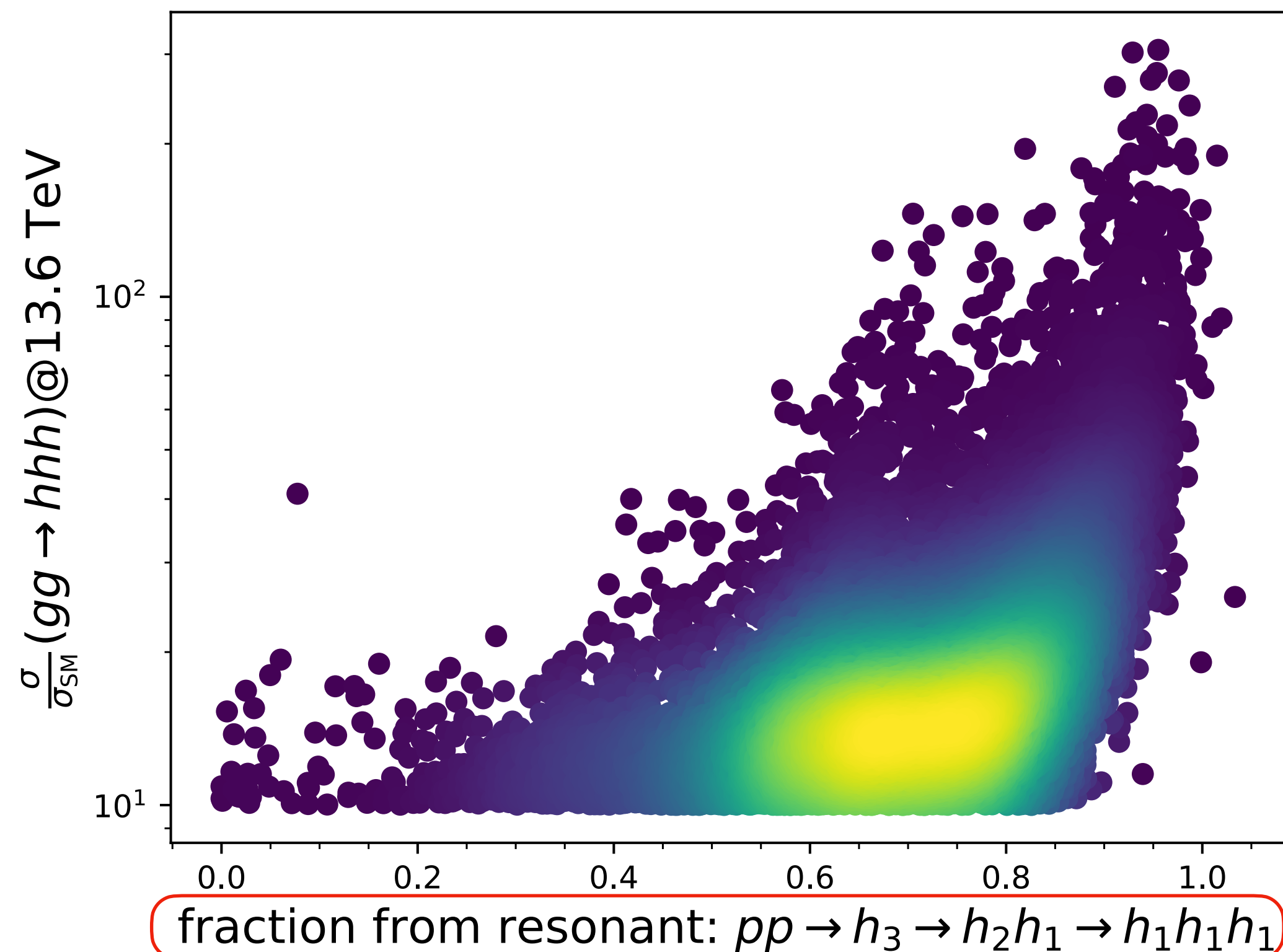
SFO-EWPT and hhh in the TRSM

- **Q:** Can there be a **SFO-EWPT** in the TRSM, related to **electro-weak baryogenesis**?

- and if so, will this lead to **enhanced hhh at the LHC?**

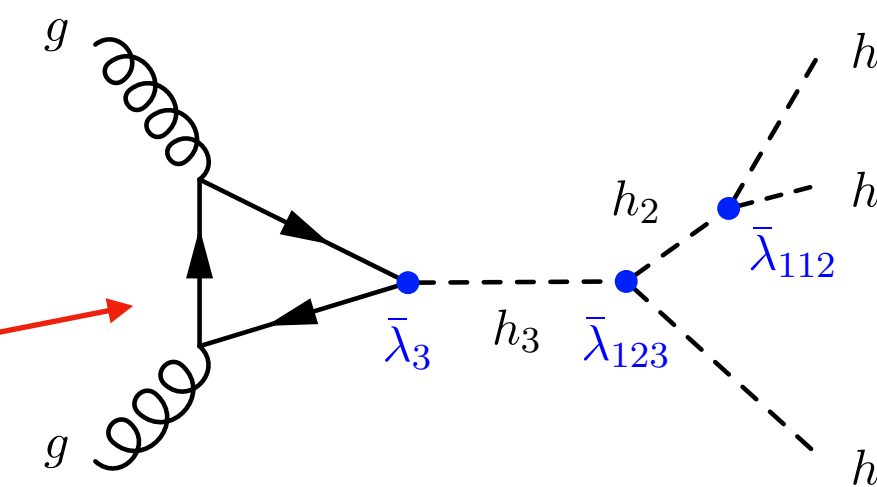
[Karkout, **AP**, Postma, Tetlalmatzi-Xolocotzi, van de Vis, du Pree, arXiv:2404.12425]

Viable points with $\sigma > 10 \times \sigma_{SM}(gg \rightarrow hhh)$ @13.6 TeV



- Updated TRSM scan with **additional TH+EXP constraints.**
- Enhancements $\mathcal{O}(100) \times SM!$

How much of the total cross section comes from... ?

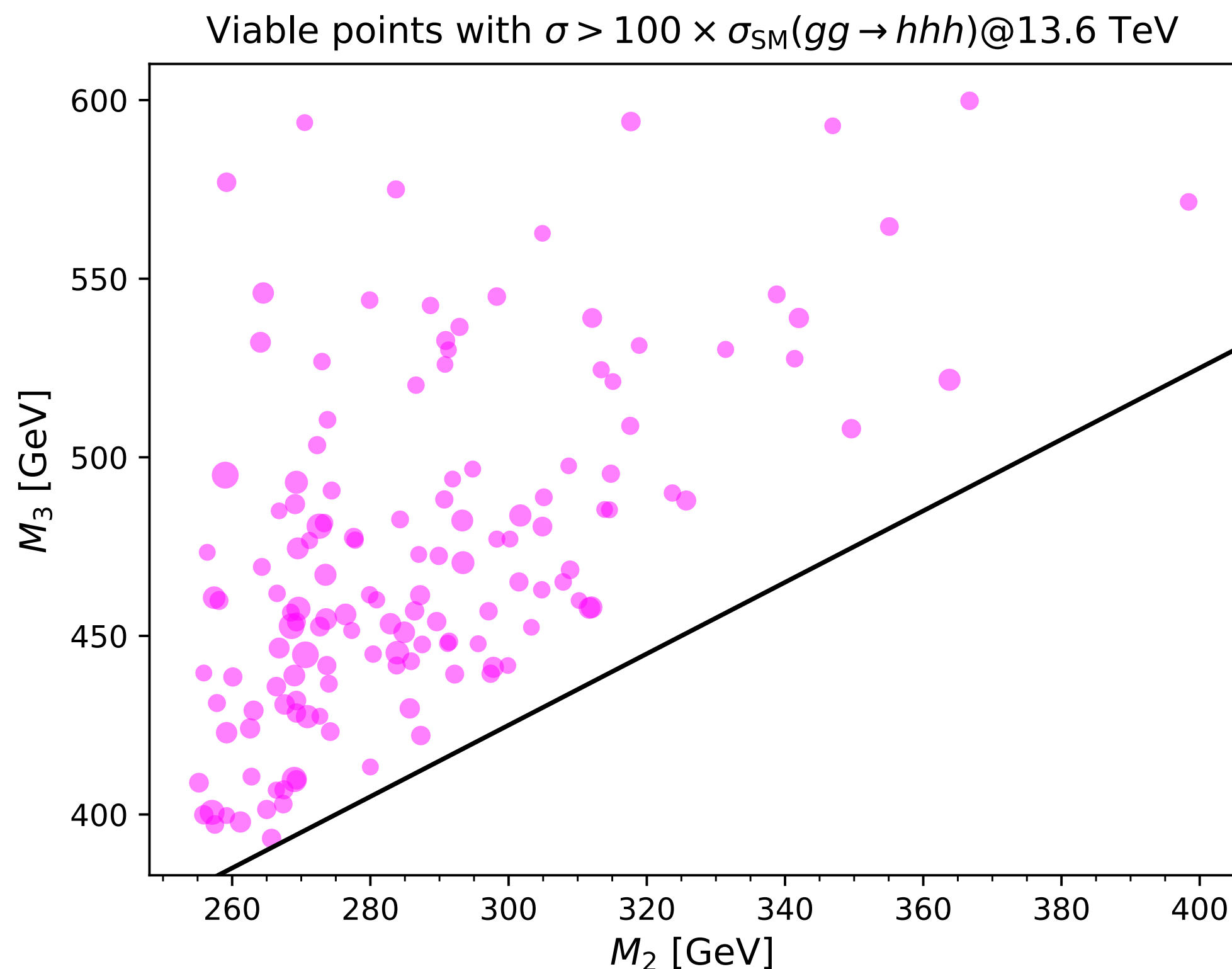


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[Karkout, **AP**, Postma, Tetlalmatzi-Xolocotzi, van de Vis, du Pree, arXiv:2404.12425]



- An updated set of benchmark points,
- they go *beyond* BP3: all params. **varied!**
- Enhancements $\mathcal{O}(100) \times SM @ 13.6 \text{ TeV!}$



SFO-EWPT and hhh in the TRSM

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[Karkout, **AP**, Postma, Tetlalmatzi-Xolocotzi, van de Vis, du Pree, arXiv:2404.12425]

- **Unfortunately...**

- ▶ SFO-EWPT & enhanced hhh are mutually exclusive!

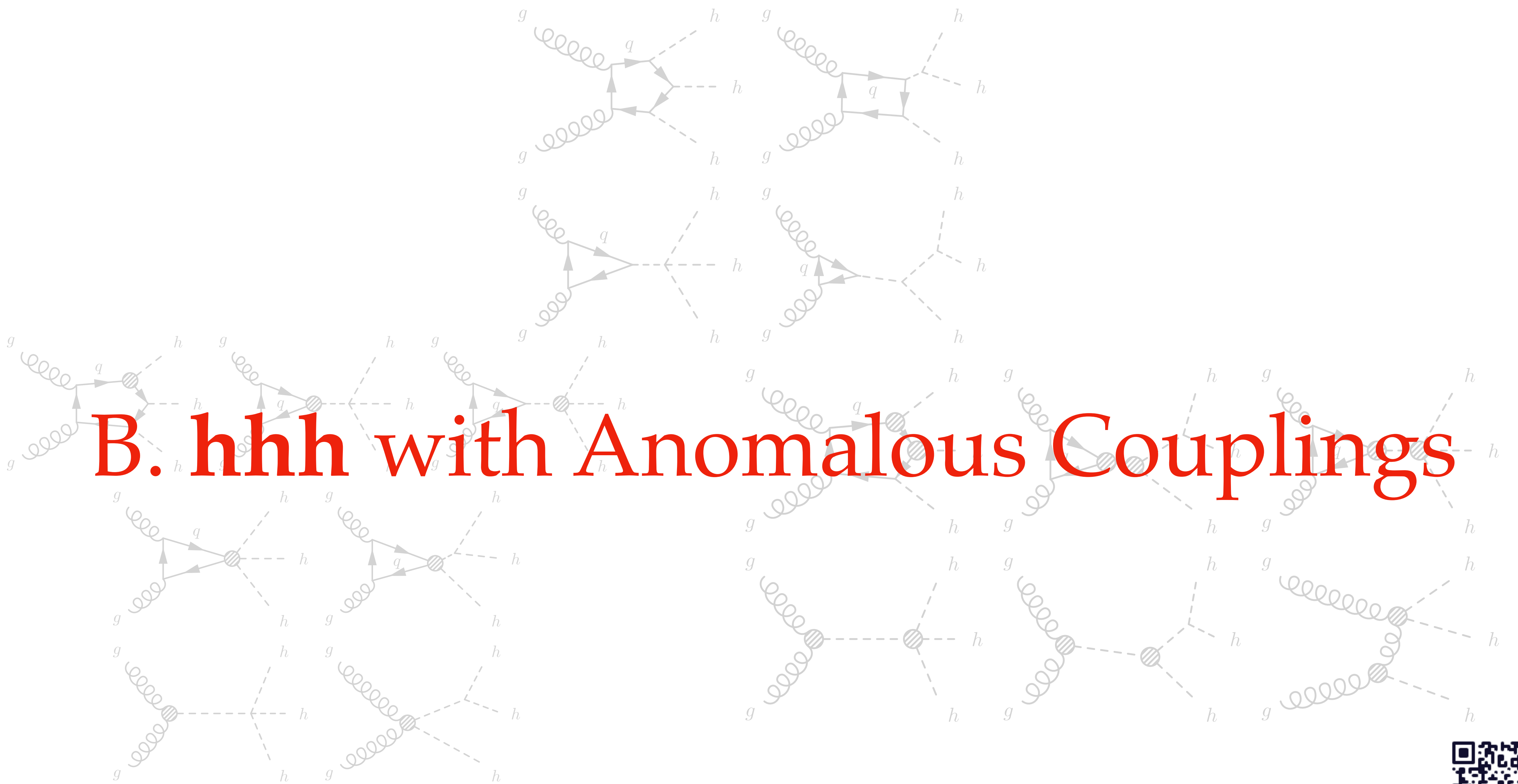
- ▶ barrier not generated if both new scalars attain a non-zero VEV,

- ▶ and non-zero VEVs are necessary for sufficient mixing!

➔ Removing the \mathcal{F}_2 restrictions might help!



B. $hhhh$ with Anomalous Couplings



D=6-Inspired Anomalous Couplings

- Add higher-dimensional operators to the SM Lagrangian!
 → Capture the effects of new particles at scales \gg collision energies.

- e.g. Add **D=6** operators relevant to multi-Higgs boson production, of the form $\frac{\mathcal{O}_6}{\Lambda^2}$:

$$\begin{aligned} \mathcal{L}_{h^n} \supset & -\mu^2 |\phi|^2 - \lambda |\phi|^4 - (y_t \bar{Q}_L \phi^c t_R + y_b \bar{Q}_L \phi b_R + \text{h.c.}) \\ & + \frac{c_H}{2\Lambda^2} (\partial^\mu |\phi|^2)^2 - \frac{c_6}{\Lambda^2} \lambda_{\text{SM}} |\phi|^6 + \frac{\alpha_s c_g}{4\pi\Lambda^2} |\phi|^2 G_{\mu\nu}^a G_a^{\mu\nu} \\ & - \left(\frac{c_t}{\Lambda^2} y_t |\phi|^2 \bar{Q}_L \phi^c t_R + \frac{c_b}{\Lambda^2} y_b |\phi|^2 \bar{Q}_L \phi b_R + \text{h.c.} \right) \end{aligned}$$

[see e.g. Goertz, **AP**, Yang, Zurita, arXiv:1410.3471 for similar $pp \rightarrow hh$ study]

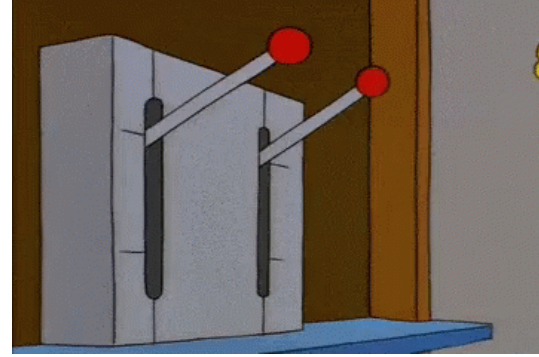
[for 1-loop computations see: smeft@nlo: Degrande, Durieux, Maltoni, Mimasu, Vryonidou, Zhang arXiv:2008.11743]



D=6-Inspired Anomalous Couplings

[AP, Tetlalmatzi-Xolocotzi, arXiv:2312.13562]

- Go through **EWSB**...



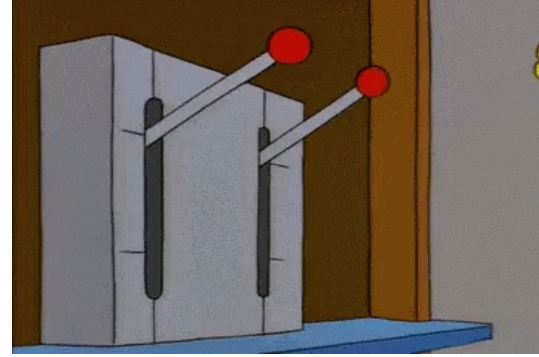
⇒ in terms of the physical scalar Higgs boson h :

$$\begin{aligned}
 \mathcal{L}_{D=6} \supset & -\frac{m_h^2}{2v} (1+c_6) h^3 - \frac{m_h^2}{8v^2} (1+6c_6) h^4 \\
 & + \frac{\alpha_s c_g}{4\pi} \left(\frac{h}{v} + \frac{h^2}{2v^2} \right) G_{\mu\nu}^a G_a^{\mu\nu} \\
 & - \left[\frac{m_t}{v} (1+c_t) \bar{t}_L t_R h + \frac{m_b}{v} (1+c_b) \bar{b}_L b_R h + \text{h.c.} \right] \\
 & - \left[\frac{m_t}{v^2} \left(\frac{3c_t}{2} \right) \bar{t}_L t_R h^2 + \frac{m_b}{v^2} \left(\frac{3c_b}{2} \right) \bar{b}_L b_R h^2 + \text{h.c.} \right] \\
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D=6-Inspired Anomalous Couplings

[AP, Tetlalmatzi-Xolocotzi, arXiv:2312.13562]

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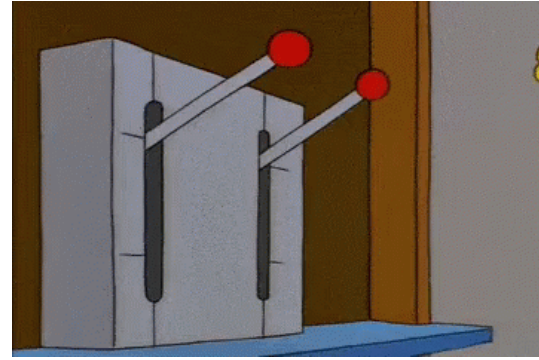
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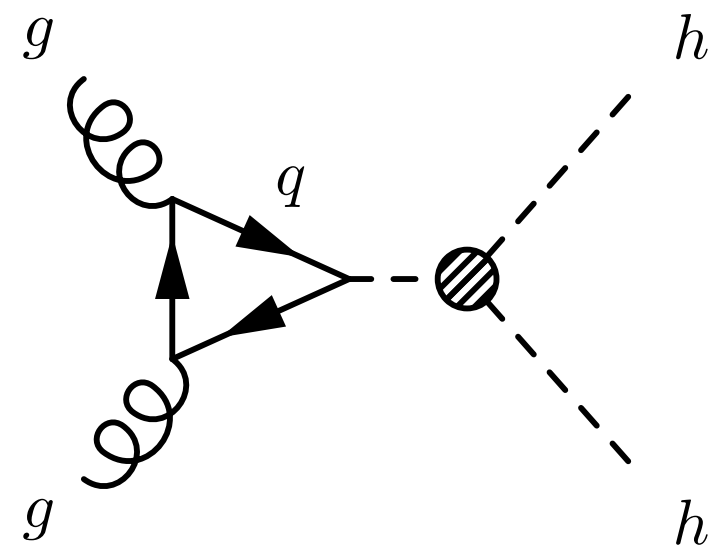
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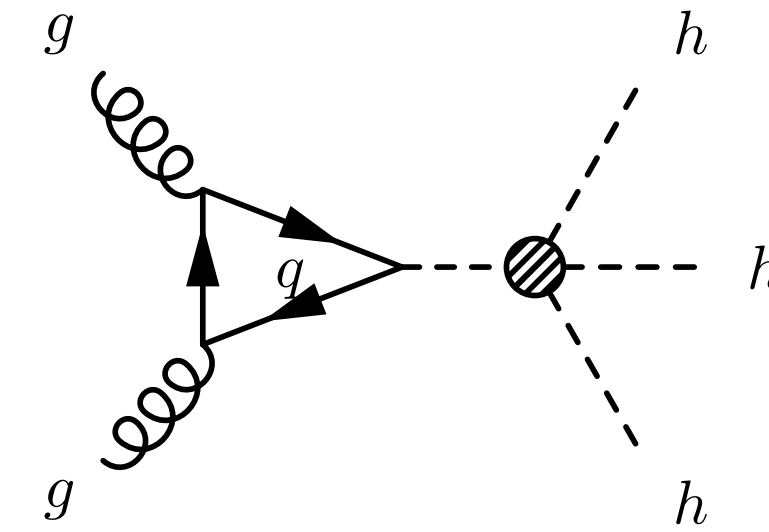
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$$\mathcal{L}_{D=6} \supset \frac{m_h^2}{2v} (1+c_6) h^3 - \frac{m_h^2}{8v^2} (1+6c_6) h^4$$



$$+ \frac{\alpha_s c_g}{4\pi} \left(\frac{h}{v} + \frac{h^2}{2v^2} \right) G_{\mu\nu}^a G_a^{\mu\nu}$$

$$- \left[\frac{m_t}{v} (1+c_t) \bar{t}_L t_R h + \frac{m_b}{v} (1+c_b) \bar{b}_L b_R h + \text{h.c.} \right]$$

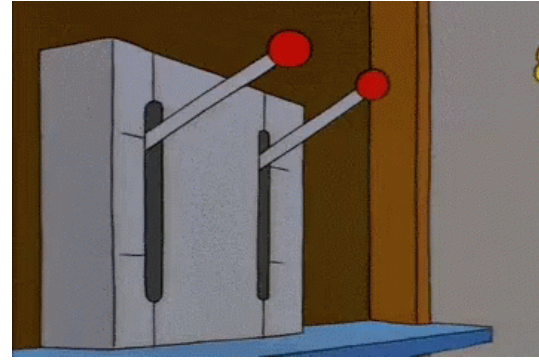
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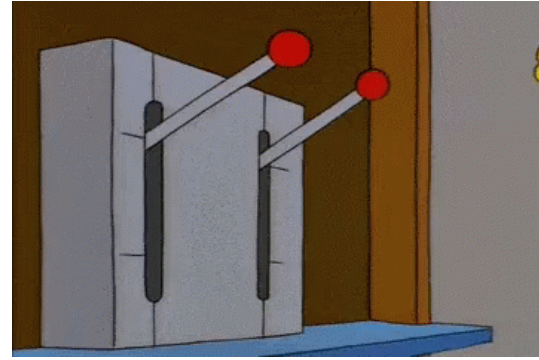
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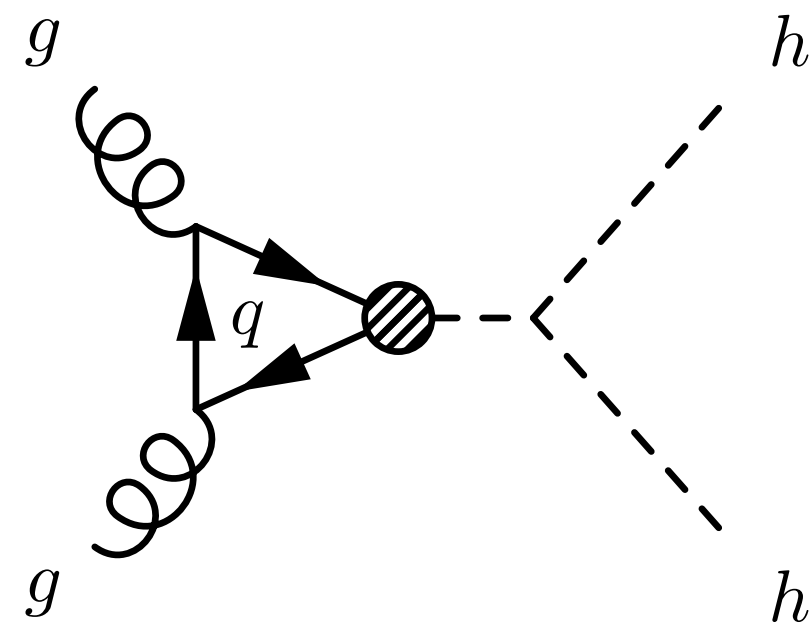
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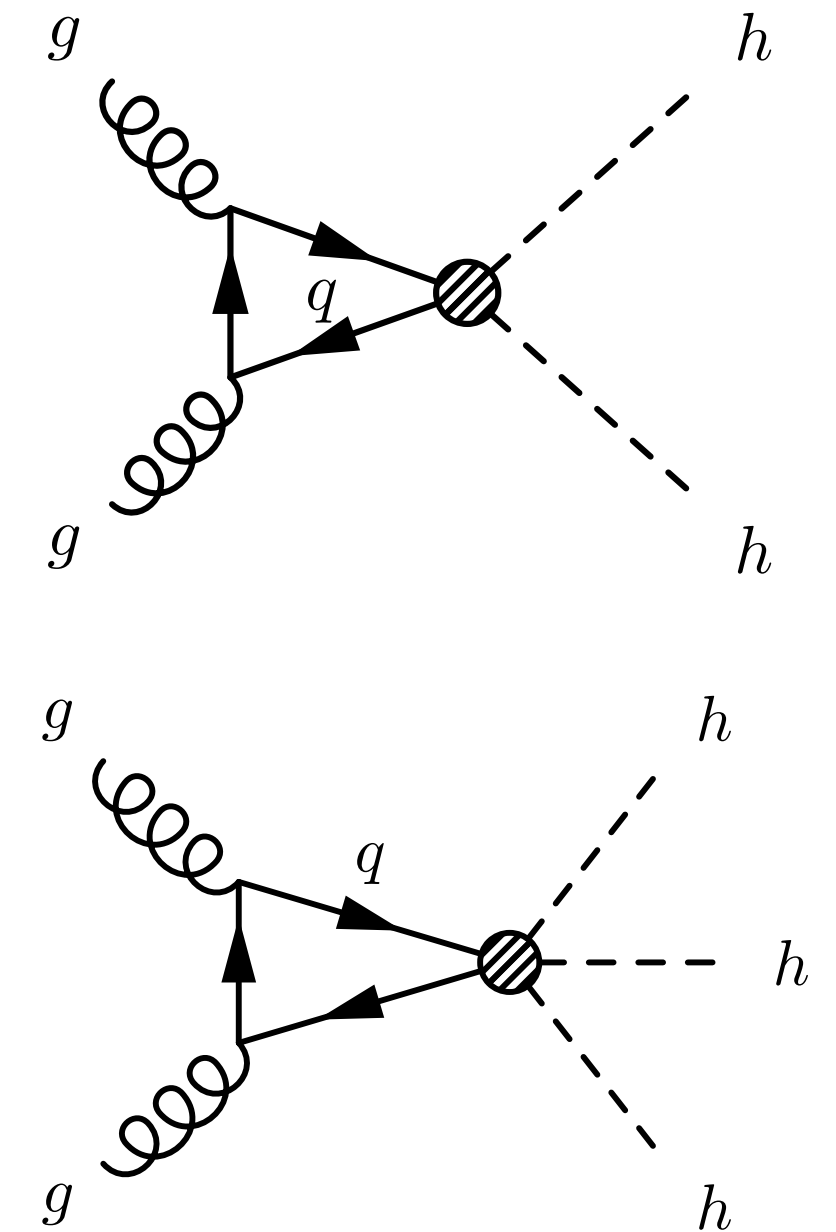
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D=6-Inspired Anomalous Couplings

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- A slightly more “general” picture is obtained by “dissociating” the operators as:

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 \mathcal{L}_{\text{Pheno}} \supset & -\frac{m_h^2}{2v} (1+d_3) h^3 - \frac{m_h^2}{8v^2} (1+d_4) h^4 \\
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 \end{aligned}$$

Recover D=6 by setting:

$$\begin{aligned}
 d_3 &= c_6, \\
 d_4 &= 6c_6, \\
 c_{g1} &= c_{g2} = c_g, \\
 c_{f1} &= c_{f2} = c_{f3} = c_f.
 \end{aligned}$$

Note: This can be also be motivated via the [Electro-weak Chiral Lagrangian](#),

[see e.g. Buchalla, Catá, Krause arXiv:1307.5017]



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 & - \left[\frac{m_t}{v} (1+c_{t1}) \bar{t}_L t_R h + \frac{m_b}{v} (1+c_{b1}) \bar{b}_L b_R h + \text{h.c.} \right] \\
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 c_{g1} &= c_{g2} = c_g, \\
 c_{f1} &= c_{f2} = c_{f3} = c_f.
 \end{aligned}$$

Note: This can be also be motivated via the Electro-weak Chiral Lagrangian,

[see e.g. Buchalla, Catá, Krause arXiv:1307.5017]



D=6-Inspired Anomalous Couplings

[AP, Tetlalmatzi-Xolocotzi, arXiv:2312.13562]

- A slightly more “general” picture is obtained by “dissociating” the operators as:

$$\begin{aligned} \mathcal{L}_{\text{Pheno}} \supset & -\frac{m_h^2}{2v} (1+d_3) h^3 - \frac{m_h^2}{8v^2} (1+d_4) h^4 \\ & + \frac{\alpha_s}{4\pi} \left(c_{g1} \frac{h}{v} + c_{g2} \frac{h^2}{2v^2} \right) G_{\mu\nu}^a G_a^{\mu\nu} \\ & - \left[\frac{m_t}{v} (1+c_{t1}) \bar{t}_L t_R h + \frac{m_b}{v} (1+c_{b1}) \bar{b}_L b_R h + \text{h.c.} \right] \\ & - \left[\frac{m_t}{v^2} \left(\frac{3c_{t2}}{2} \right) \bar{t}_L t_R h^2 + \frac{m_b}{v^2} \left(\frac{3c_{b2}}{2} \right) \bar{b}_L b_R h^2 + \text{h.c.} \right] \\ & - \left[\frac{m_t}{v^3} \left(\frac{c_{t3}}{2} \right) \bar{t}_L t_R h^3 + \frac{m_b}{v^3} \left(\frac{c_{b3}}{2} \right) \bar{b}_L b_R h^3 + \text{h.c.} \right], \end{aligned}$$

Recover D=6 by setting:

$$\begin{aligned} d_3 &= c_6, \\ d_4 &= 6c_6, \\ c_{g1} &= c_{g2} = c_g, \\ c_{f1} &= c_{f2} = c_{f3} = c_f. \end{aligned}$$

instead of c_t

Note: This can be also be motivated via the [Electro-weak Chiral Lagrangian](#),

[see e.g. Buchalla, Catá, Krause arXiv:1307.5017]



D=6-Inspired Anomalous Couplings

[AP, Tetlalmatzi-Xolocotzi, arXiv:2312.13562]

- **Further modify** to match more closely **LHC experiments' definitions**:

$$\begin{aligned}
 \mathcal{L}_{\text{PhenoExp}} \supset & -\lambda_{\text{SM}} v (1+d_3) h^3 - \frac{\lambda_{\text{SM}}}{4} (1+d_4) h^4 \\
 & + \frac{\alpha_s}{12\pi} \left(c_{g1} \frac{h}{v} - c_{g2} \frac{h^2}{2v^2} \right) G_{\mu\nu}^a G_a^{\mu\nu} \\
 & - \left[\frac{m_t}{v} (1+c_{t1}) \bar{t}_L t_R h + \frac{m_b}{v} (1+c_{b1}) \bar{b}_L b_R h + \text{h.c.} \right] \\
 & - \left[\frac{m_t}{v^2} c_{t2} \bar{t}_L t_R h^2 + \frac{m_b}{v^2} c_{b2} \bar{b}_L b_R h^2 + \text{h.c.} \right] \\
 & - \left[\frac{m_t}{v^3} \left(\frac{c_{t3}}{2} \right) \bar{t}_L t_R h^3 + \frac{m_b}{v^3} \left(\frac{c_{b3}}{2} \right) \bar{b}_L b_R h^3 + \text{h.c.} \right],
 \end{aligned}$$

Defined: $\lambda_{\text{SM}} = m_h^2/2v^2$.

Obtain **CMS-like** parametrization by:

$$\begin{aligned}
 \kappa_\lambda &= (1+d_3), \\
 k_t &= c_{t1}, \\
 c_2 &= c_{t2}, \\
 c_g &= c_{g1}, \\
 c_{gg} &= c_{2g}.
 \end{aligned}$$

And **ATLAS-like** parametrization by:

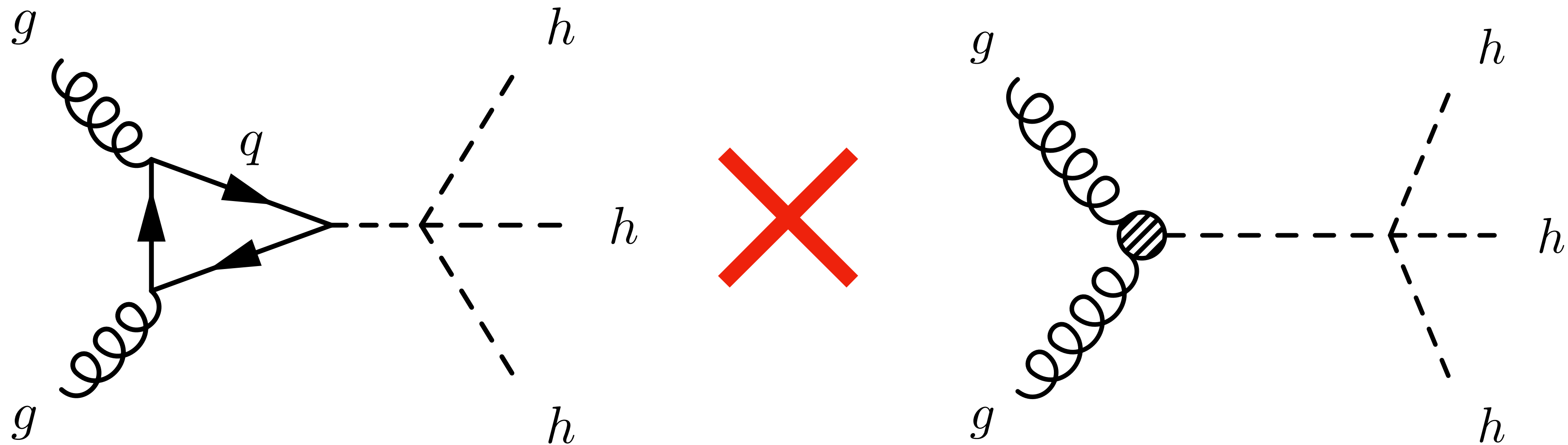
$$\begin{aligned}
 c_{hhh} &= (1+d_3), \\
 c_{ggh} &= 2c_{g1}/3, \\
 c_{gggh} &= -c_{g2}/3.
 \end{aligned}$$

Monte Carlo Implementation of Anomalous Couplings

- We have implemented a MadGraph5_aMC@NLO “loop” model for $\mathcal{L}_{\text{PhenoExp}}$.
- Includes Loop \times Tree level interference between the various diagrams.

[see: Hirschi, <https://cp3.irmp.ucl.ac.be/projects/madgraph/wiki/LoopInducedTimesTree>].

- e.g.:



[AP, Tetlalmatzi-Xolocotzi, arXiv:2312.13562]

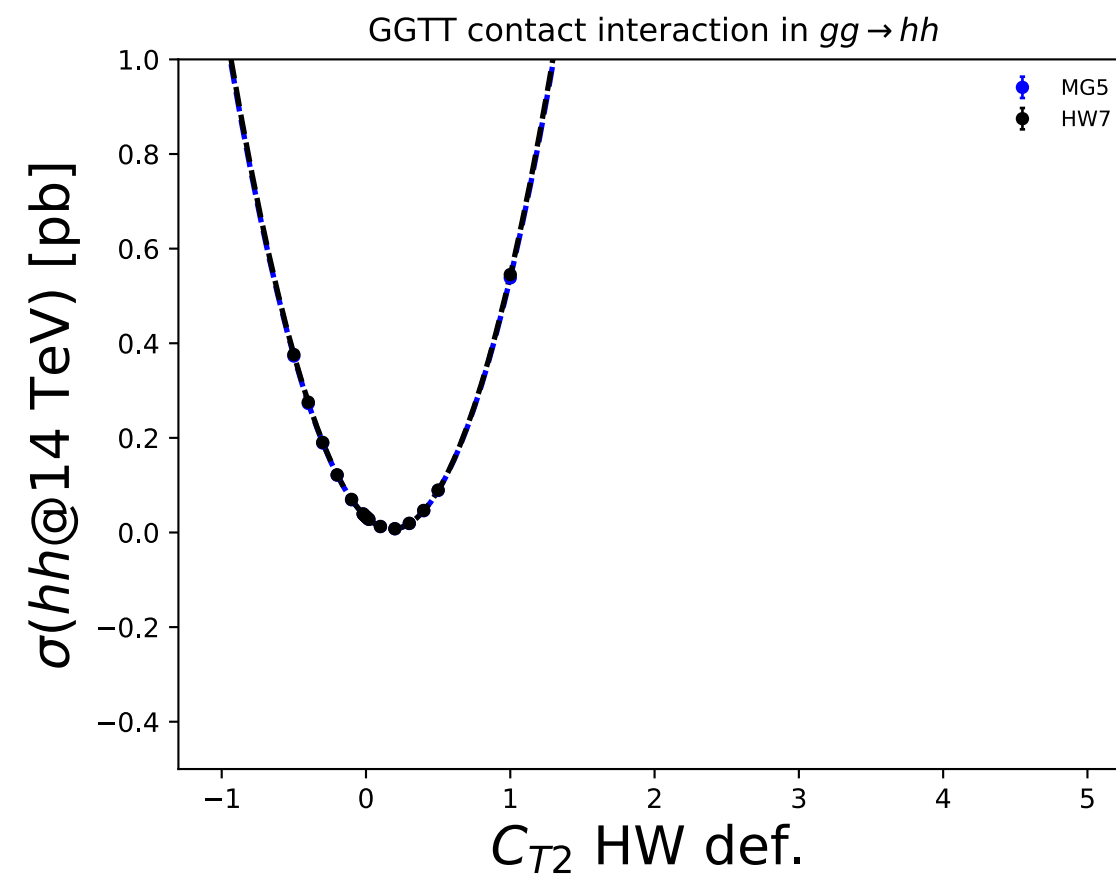
[Get model at: https://gitlab.com/apapaefs/multihiggs_loop_sm]



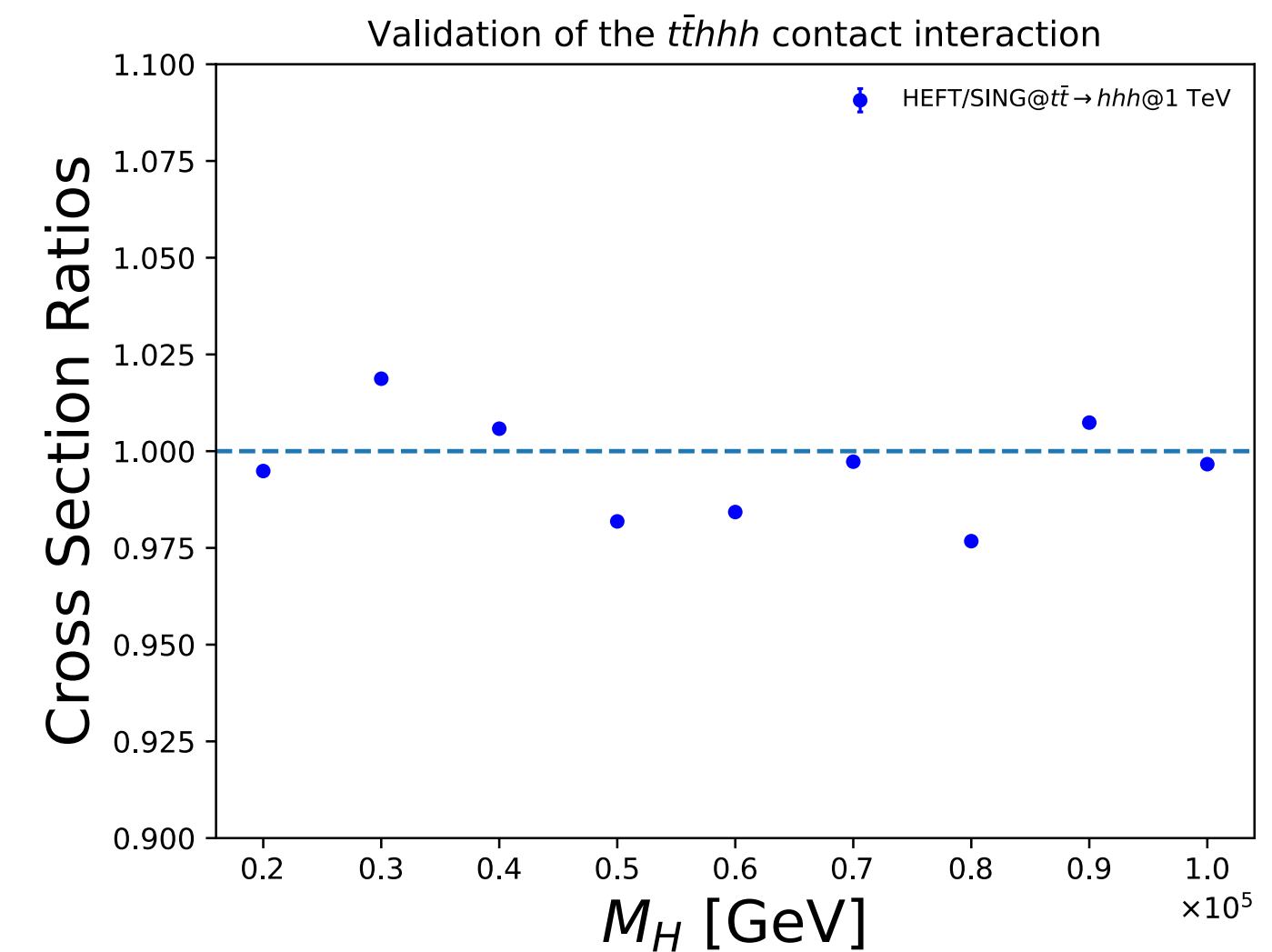
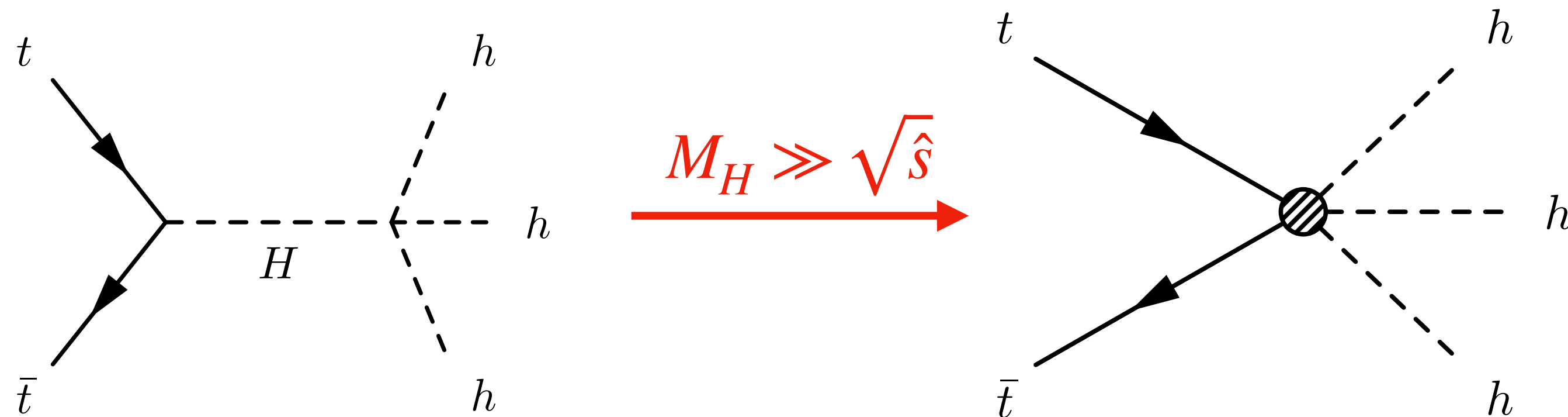
Model Validation

[AP, Tetlalmatzi-Xolocotzi, arXiv:2312.13562]

- Most couplings validated vs. a Herwig 7 $pp \rightarrow hh$ implementation, e.g.:



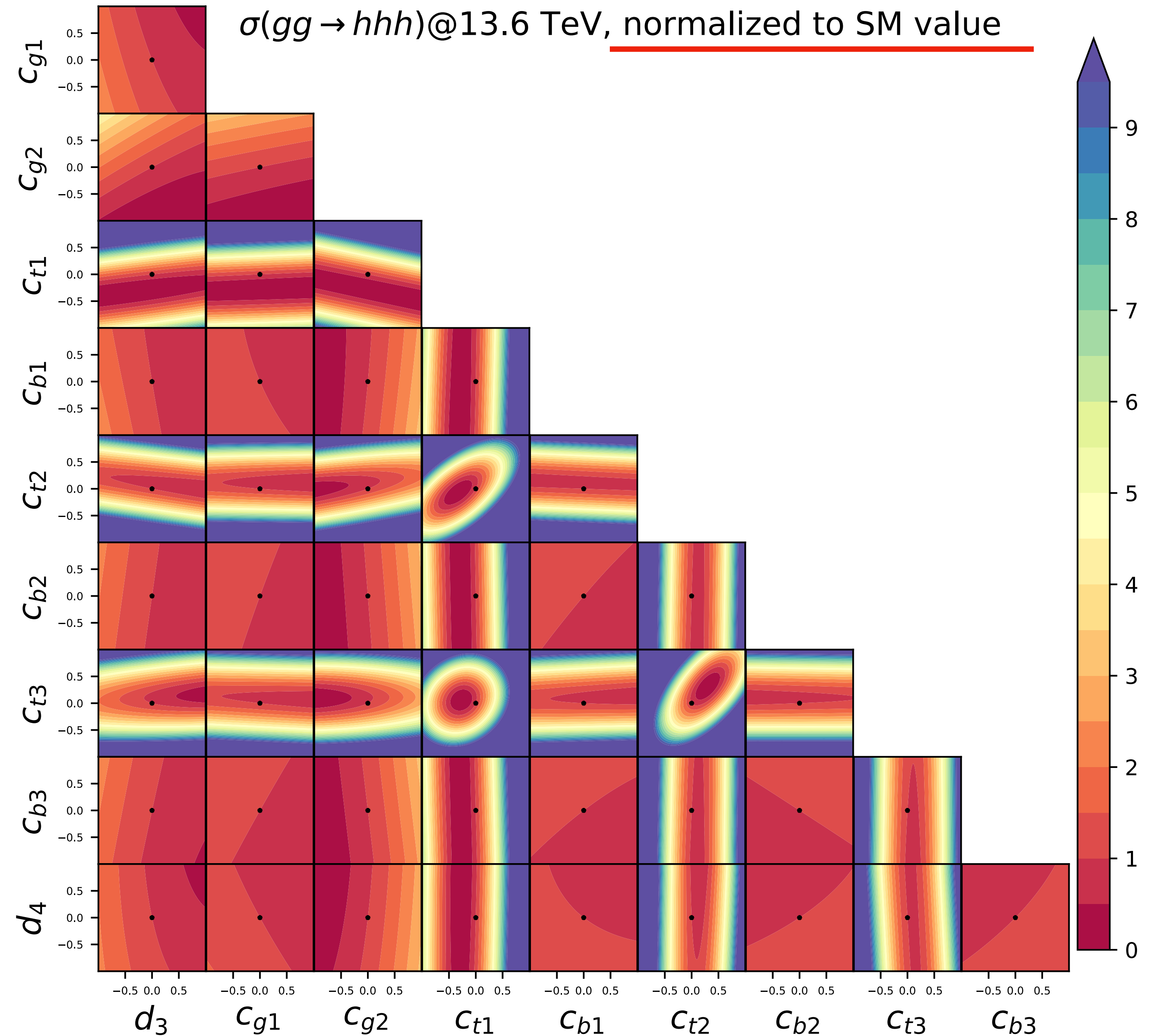
- The one “new” non-trivial coupling that appears, $\propto c_{t3} t\bar{t}h^3$ has been validated via an “EFT” limit, in the $t\bar{t} \rightarrow hhh$ process:



hhh Cross Sections @ 13.6 TeV

[AP, Tetlalmatzi-Xolocotzi, arXiv:2312.13562]

- Cross section as a multiple of the SM
- ($\sigma_{\text{SM}} \sim 0.04$ fb at LO@13.6 TeV).
- In each 2D panel shown: **all other coefficients set to zero!**



Anomalous Couplings Constraints

- Other processes constrain (at LO) all coefficients except $\{c_{t3}, d_4\}$ (**only in hhh**).
- **Projected constraints:**

Percentage uncertainties			
	HL-LHC	FCC-hh	Ref.
$\delta(d_3)$	50	5	[145] (table 12)
$\delta(c_{g1})$	2.3	0.49	[145] (table 3)
$\delta(c_{g2})$	5	1	[140] (Figure 12, right)
$\delta(c_{t1})$	3.3	1.0	[145] (table 3)
$\delta(c_{t2})$	30	10	[140] (Figure 12, right)
$\delta(c_{b1})$	3.6	0.43	[145] (table 3)
$\delta(c_{b2})$	30	10	assumed same as c_{t2}

[See **AP**, Tetlalmatzi-Xolocotzi, arXiv:2312.13562 for the references]



Anomalous Couplings Constraints

[AP, Tetlalmatzi-Xolocotzi, arXiv:2312.13562]

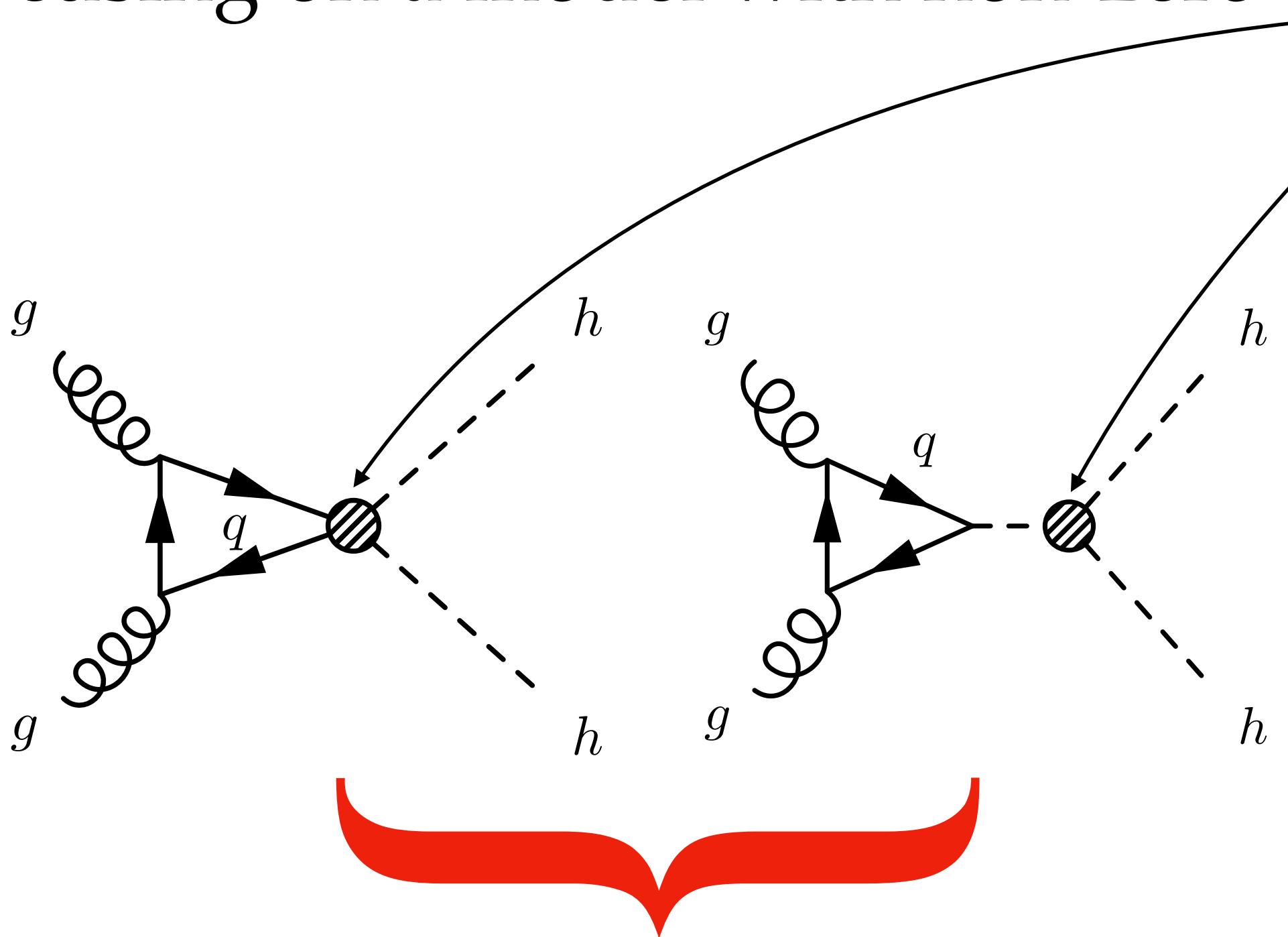
- Focusing on a model with non-zero $\{c_{t2}, d_3, c_{t3}, d_4\}$:



Anomalous Couplings Constraints

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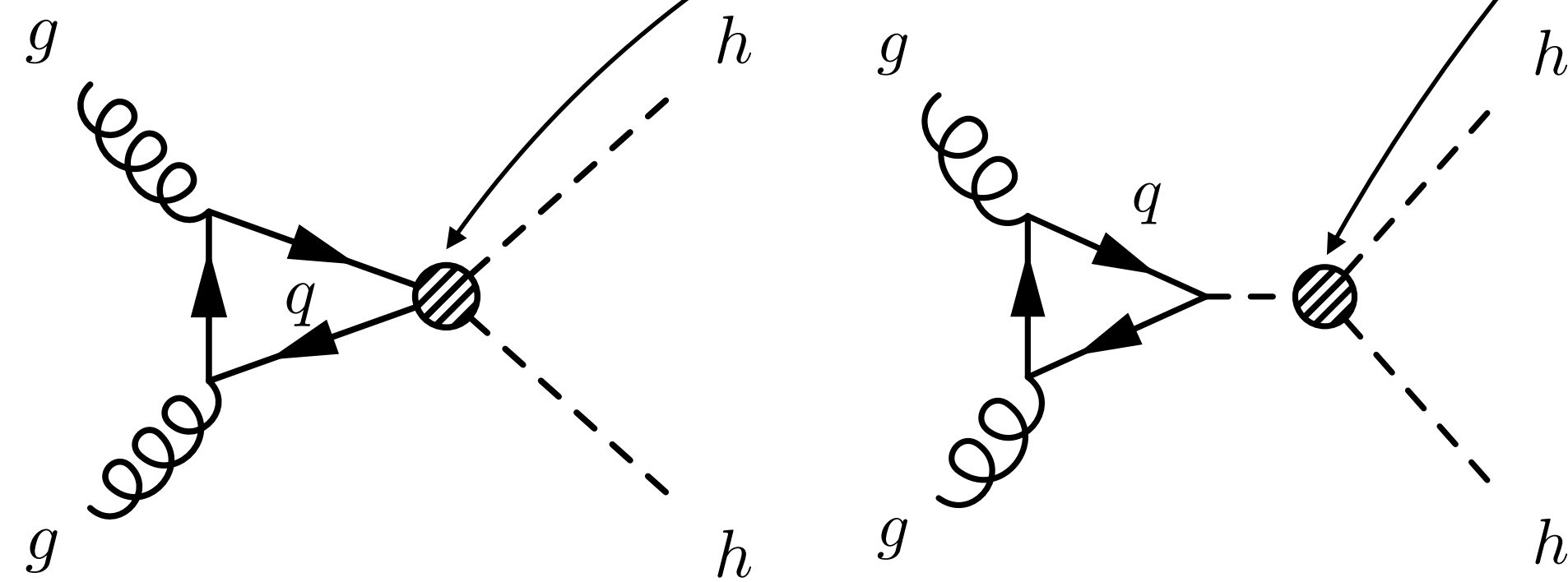
constrained by $pp \rightarrow hh$



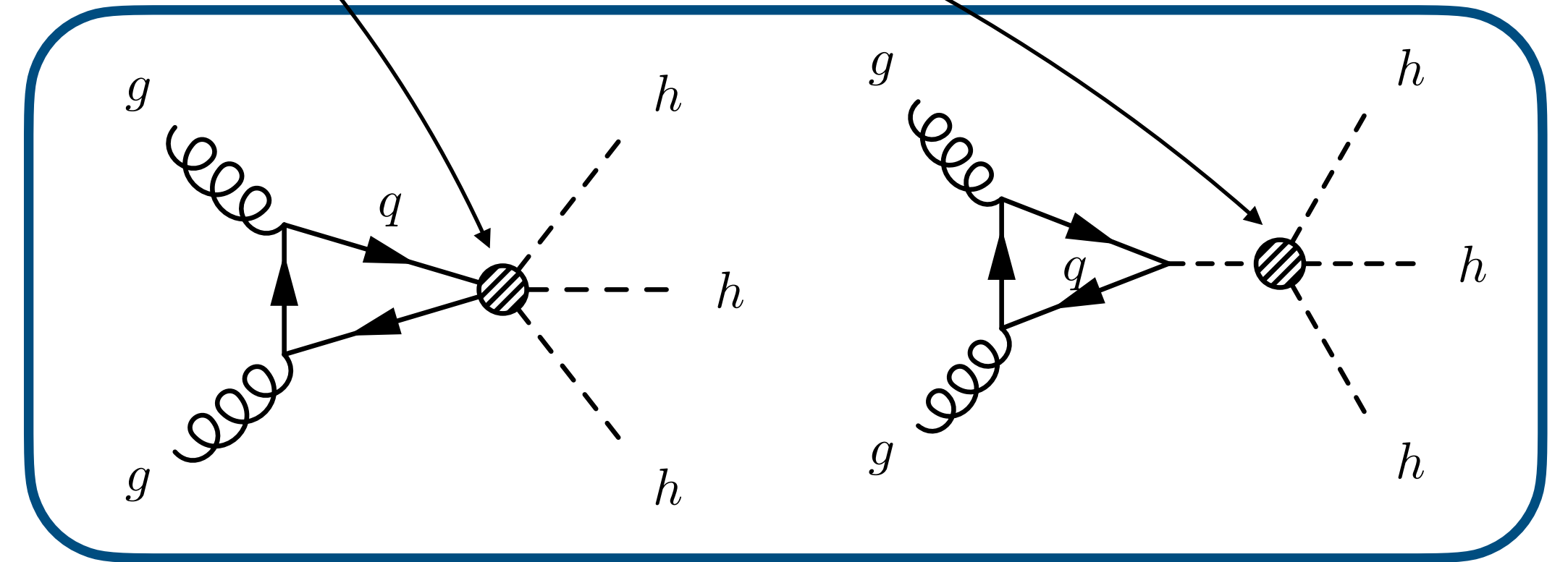
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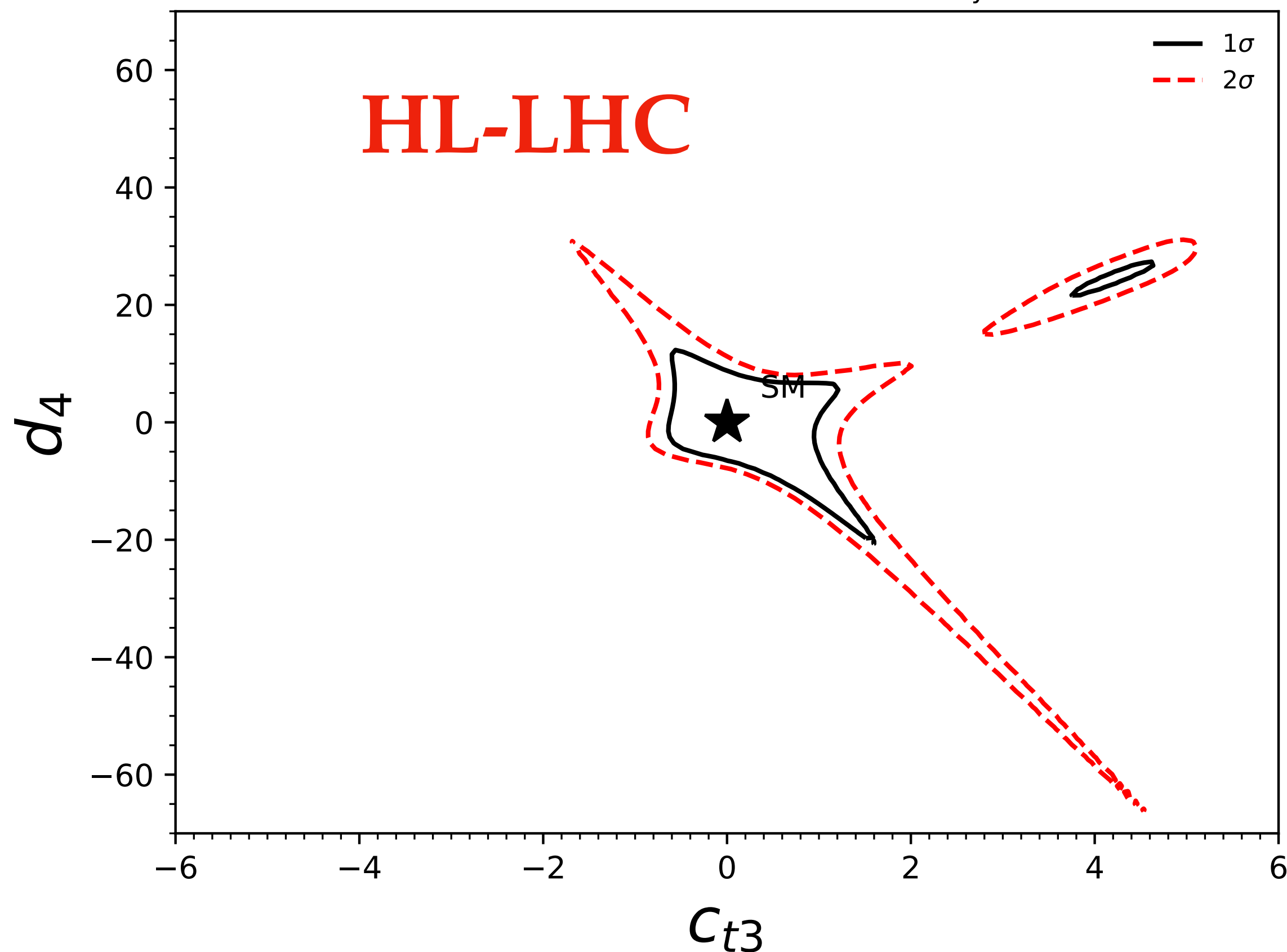


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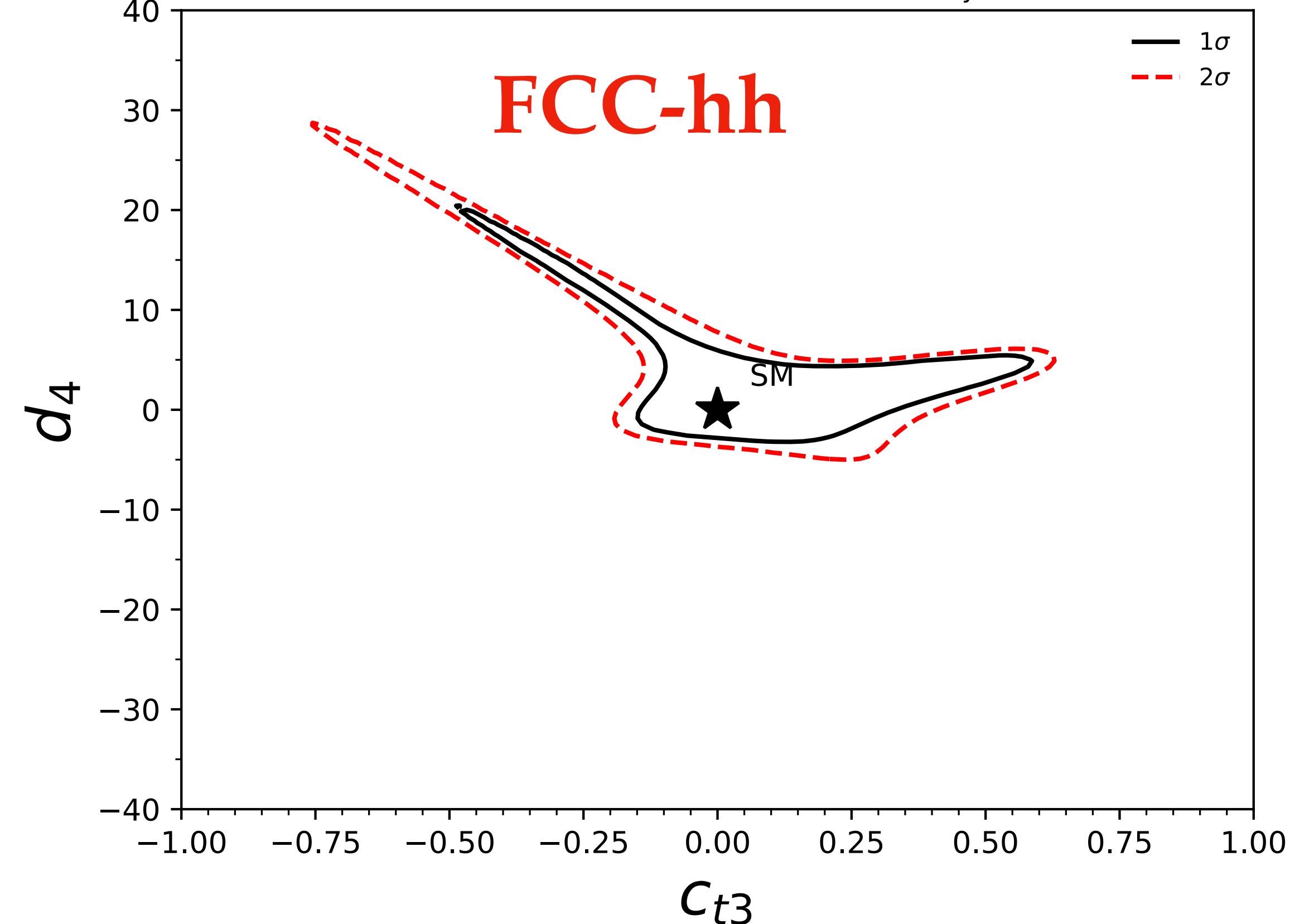
[AP, Tetlalmatzi-Xolocotzi, arXiv:2312.13562]

- Focusing on a model with only $\{c_{t2}, d_3, c_{t3}, d_4\}$,
- Using the **6 b-jet final state**, and marginalizing over $\{c_{t2}, d_3\}$ within projected constraints:

$gg \rightarrow hhh@13.6 \text{ TeV}, L=3000 \text{ fb}^{-1}, \alpha_{\text{syst.}} = 5.0\%$



$gg \rightarrow hhh@100 \text{ TeV}, L=20000 \text{ fb}^{-1}, \alpha_{\text{syst.}} = 5.0\%$

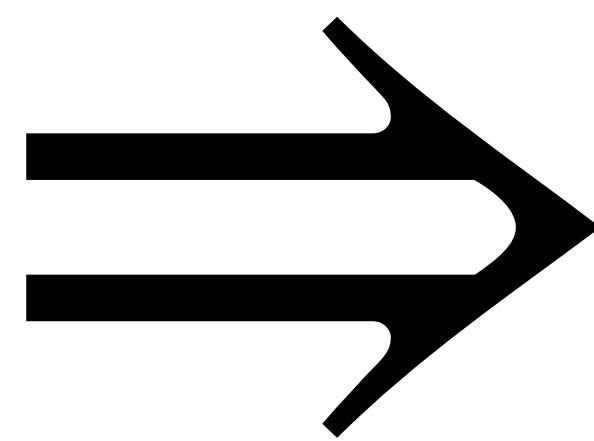


Anomalous Couplings Constraints

[AP, Tetlalmatzi-Xolocotzi, arXiv:2312.13562]

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	HL-LHC 68%	HL-LHC 95%	FCC-hh 68%	FCC-hh 95%
d_4	$[-6.6, 12.4]$	$[-10.0, 21.3]$	$[-3.9, 10.5]$	$[-10.6, 18.8]$
c_{t3}	$[-0.6, 1.1]$	$[-0.9, 3.6]$	$[-0.1, 0.3]$	$[-0.4, 0.6]$



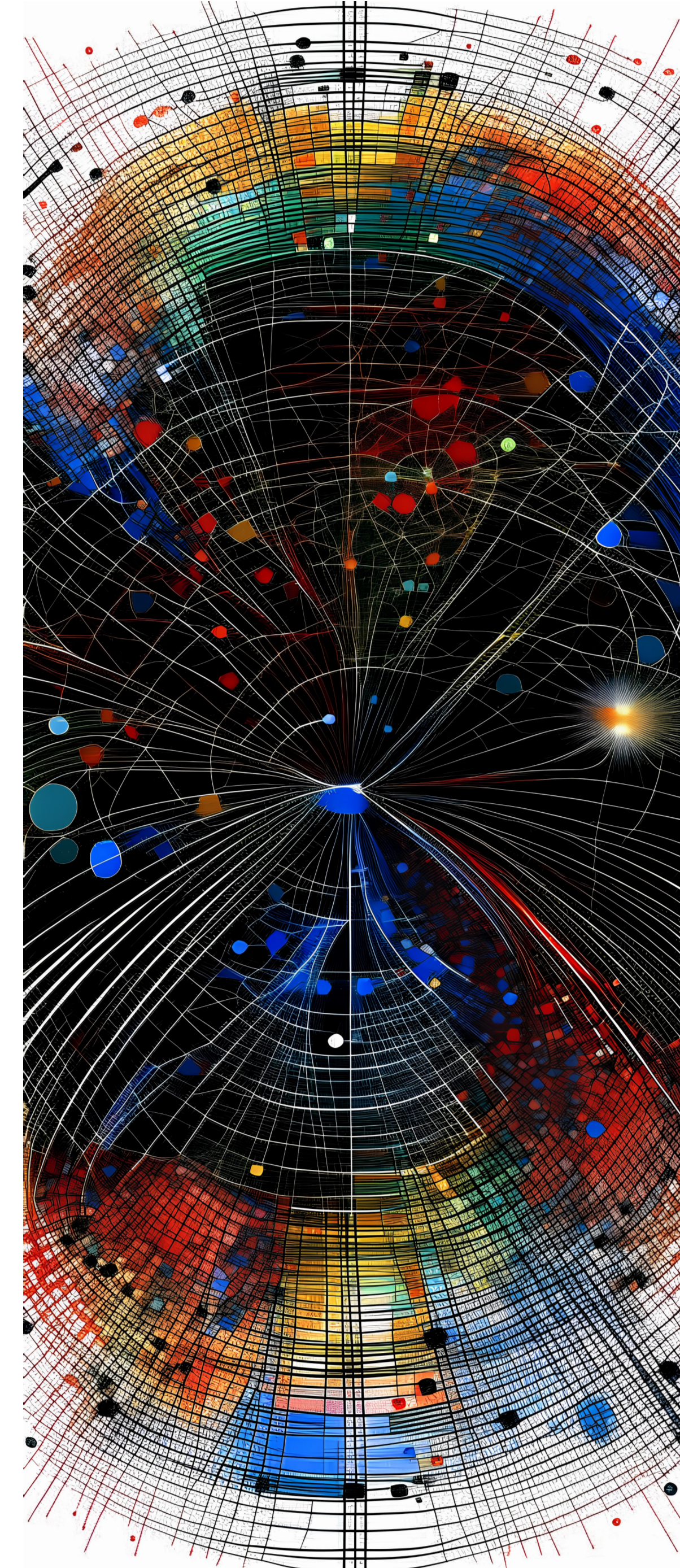
$$c_{t3} \sim \mathcal{O}(0.1 - 1)$$

$$d_4 \sim \mathcal{O}(10)$$



Summary & Outlook (I)

- Despite the successes of the SM, there remains a multitude of **open questions**,
 - ➔ Some may be linked via the **Electro-Weak Phase Transition**.
- The **Nature of the Electro-Weak Phase Transition** is an important scientific enquiry.
 - ▶ **(Strong) First-Order EWPT [not in SM!] → Matter-Anti-Matter asymmetry.**
 - ▶ **Extending the scalar sector of the SM could be the necessary catalyst.**
 - ▶ **Future particle colliders have the potential to probe this mechanism.**



[made with stablecog] Andreas Papaefstathiou

Summary & Outlook (II)

- Following *any* discovery, solving the inverse problem would be the crucial next step.
- Multi-scalar production processes (e.g. hhh production) may play a crucial role in models with extended scalar sectors.
- hhh may be enhanced in models with extended scalar sectors.
Could we see hints at the LHC?
- hhh production will probe modifications to the Higgs quartic self-coupling, within the anomalous coupling picture.
- Questions merit investigation both at the LHC and other future colliders (e.g. FCC, Muon Collider, ...).



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Thanks!

Questions?



Appendices

Theoretical Uncertainties

- **Uncertainties** → Can affect e.g. the strength of the transition, $\langle \phi(T) \rangle / T$.
- Due to:
 - **gauge dependence**, [Patel, Ramsey-Musolf, arXiv:1101.4665]
 - **scale dependence** → Linde's IR problem: expansion parameter is $gn_B \sim gT/m$, (n_B mode occupation), diverges as $m \rightarrow 0 \Rightarrow$ perturbativity breaks down.
[Linde, Phys. Lett. 96B (1980) 289.]
- \Rightarrow **To make reliable and sensible statements on colliders prospects:**
 - **Crucial to take uncertainties into account.**

Theoretical Uncertainty Bands

- Define “uncertainty band” by:

1. Deriving 1-loop effective potential in the covariant gauge,

[Arnold, Espinosa, hep-ph/9212235], [Andreassen, MSc, Norwegian U. Sci. Tech., 2013]

2. Run couplings $\lambda \rightarrow \lambda(\mu)$, μ is RGE scale, [SARAH, Staub, arXiv:0806.0538]

3. Scan parameter space of Lagrangian,

4. Vary $\mu \in [\frac{1}{2} \times m_Z, 5 \times m_Z]$ & gauge params. $\xi_i \in [0,3] \rightarrow$ band of 8 pts.

5. Use **PhaseTracer** for each point in band \rightarrow Get phase transitions, $\langle \phi(T_c) \rangle / T_c$.

[Athron, Balázs, Fowlie, Zhang, arXiv:2003.02859]

Parameter-space Categories

1. Define two conditions:
 - i. VEV at 1-loop: $\langle \phi(T = 0) \rangle = 246 \pm 30 \text{ GeV}$ & deepest minimum.
 - ii. $\langle \phi(T_c) \rangle / T_c > 1$ & no other transition with higher T_c .
2. Define four mutually-exclusive categories*:

*An alternative classification appears in our article: see Appendix.

SFO-EWPT more certain



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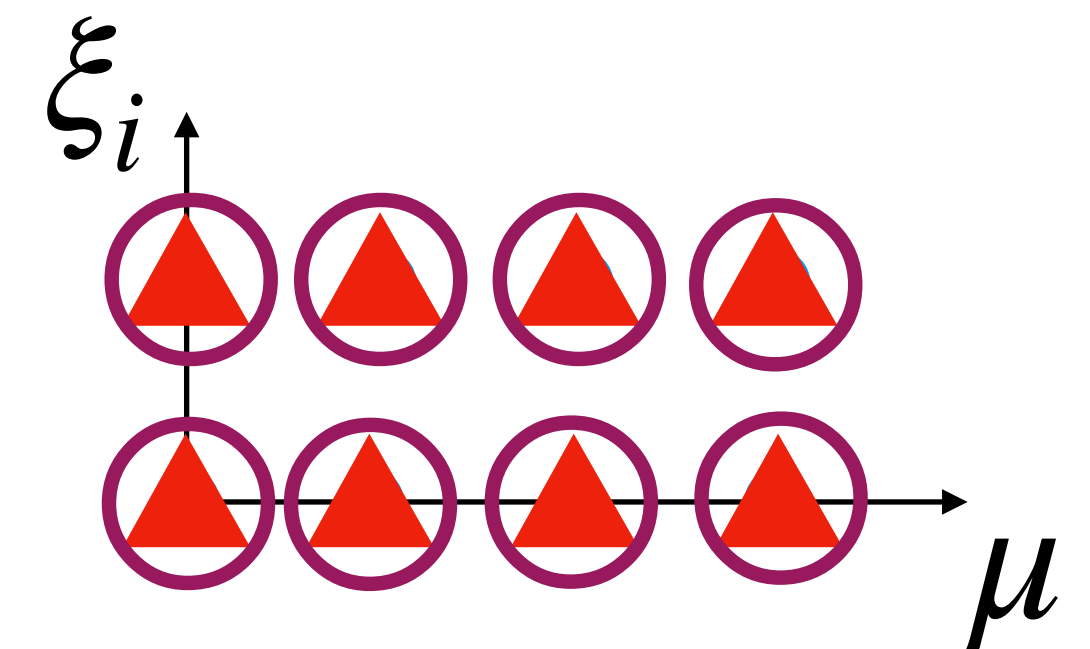
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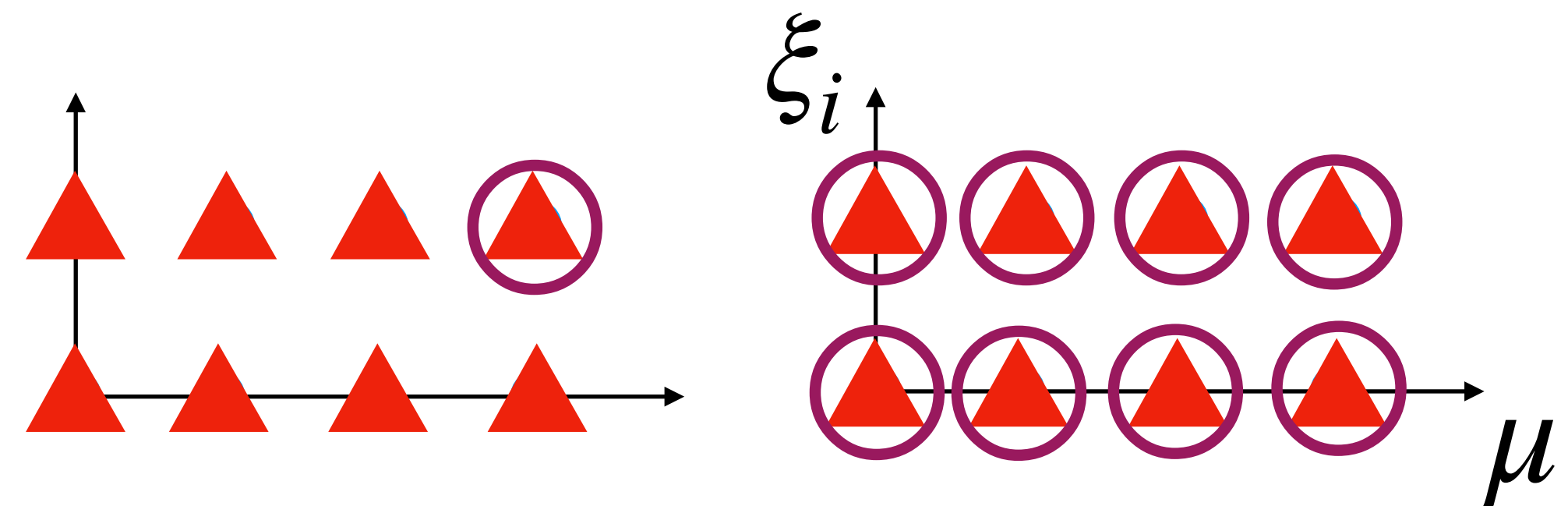
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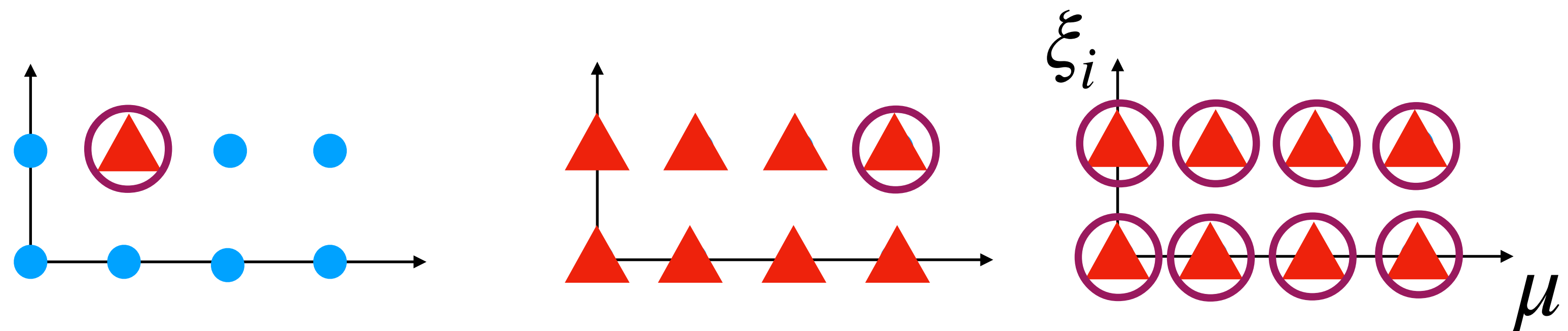
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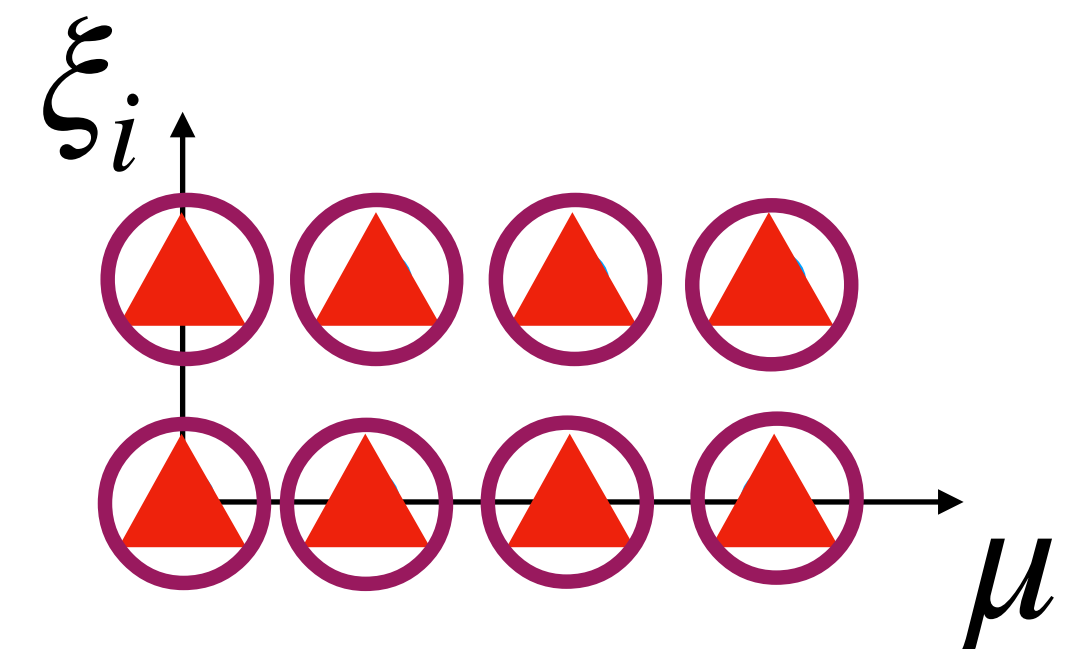
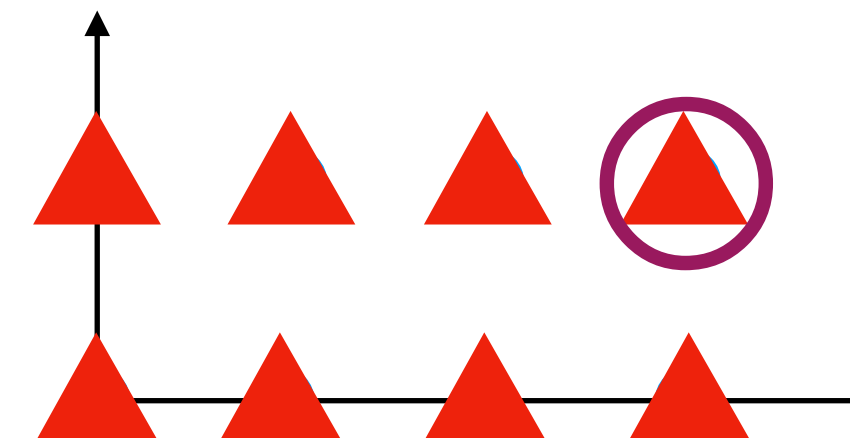
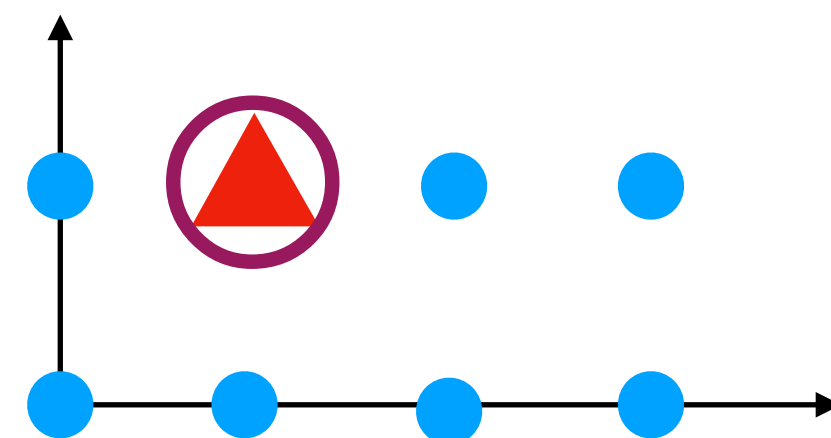
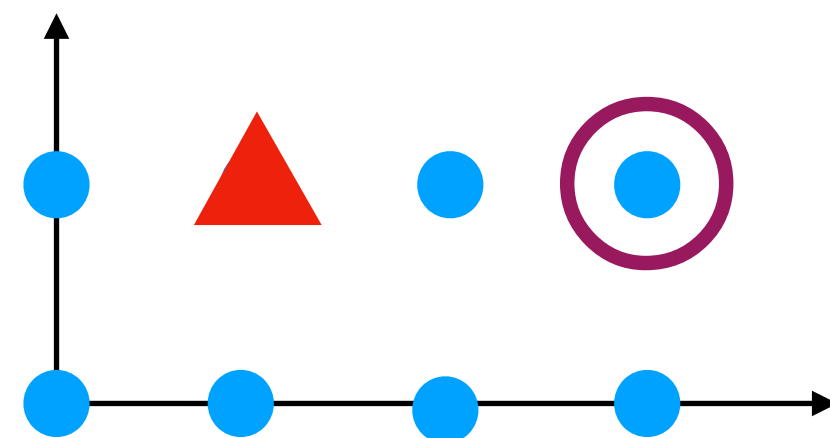
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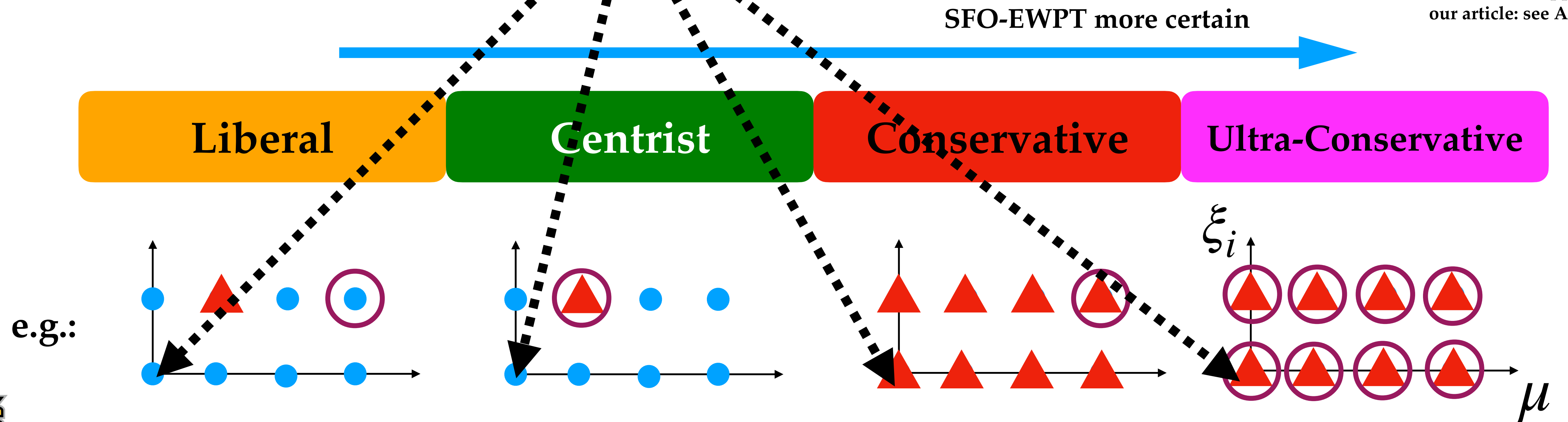
Parameter-space Categories

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Note: For phenomenological analyses, take “central” μ and ξ_i .

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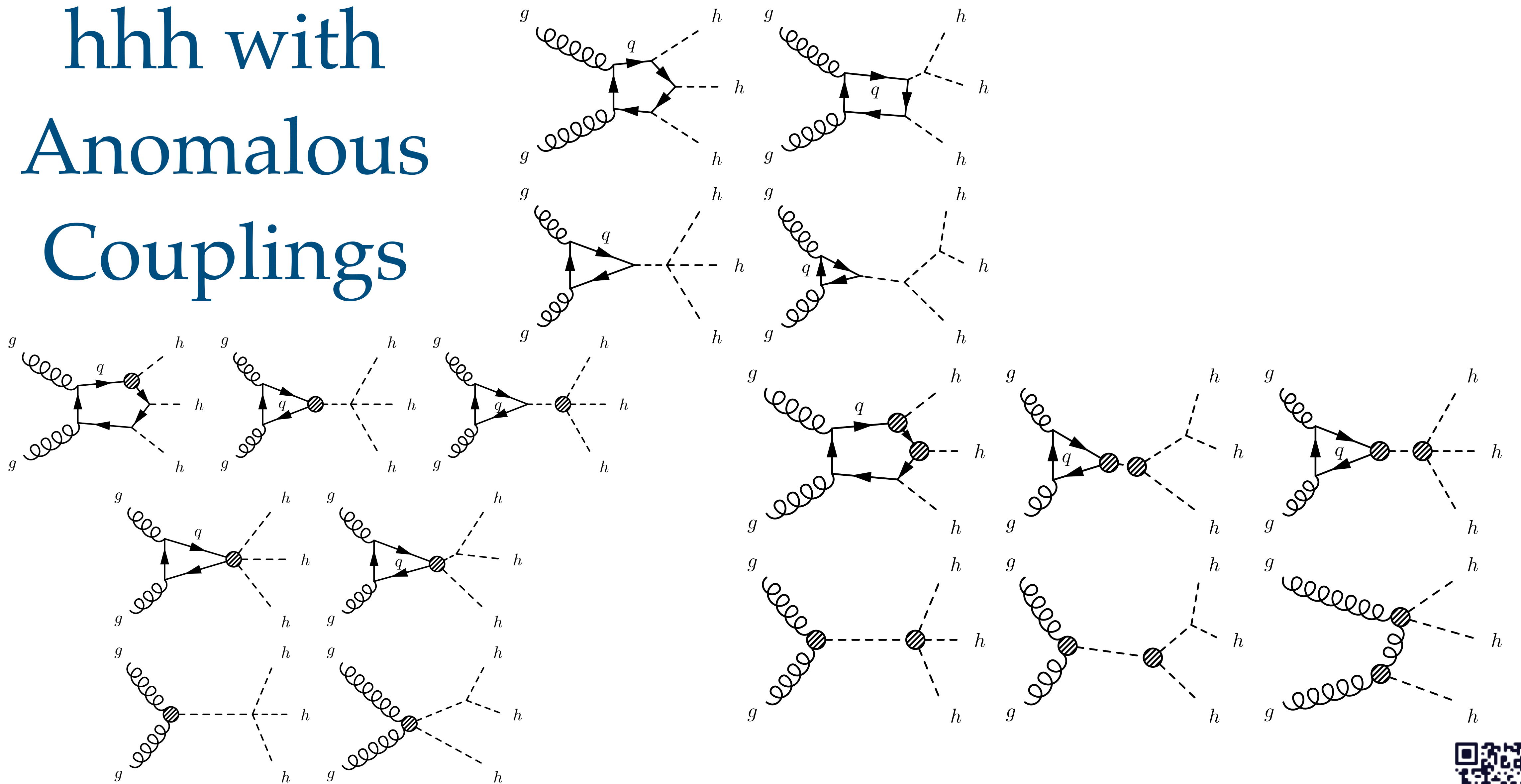
TRSM Monte Carlo Event Generation

- We have implemented a MadGraph5_aMC@NLO (MG5_aMC) “loop” model for the TRSM:
 - **MG5_aMC input parameters:** the three mixing angles, two masses / widths and **all** the scalar couplings (only 7 are independent in TRSM).
 - Comes with a **Python script** that:
 - allows conversion of M_2, M_3 + **three mixing angles** + **two VEVs** to the MG5_aMC model input,
 - calculates several single-production cross sections, branching ratios, widths,
 - and writes associated MG5_aMC parameter card (**param_card.dat**) automatically.
 - **Get it at:** <https://gitlab.com/apapaefs/twosinglet>.

[[AP](#), Tania Robens, Gilberto Tetlalmatzi-Xolocotzi, arXiv:2101.00037]



hhh with Anomalous Couplings



Electro-Weak Precision Observables

- Real singlet scalar field
 - \rightarrow modifies Higgs contributions to diagonal weak gauge boson vacuum polarisation diagrams,
 - & introduces additional contributions.
- Quantify via S, T, U parameters. [Hagiwara, Matsumoto, Haidt, Kim, hep-ph/9409380]
- Change in EWPO \mathcal{O} ($= S, T, U$):

$$\Delta\mathcal{O} = (\mathcal{O}(m_2^2) - \mathcal{O}(m_1^2)) \times \sin^2 \theta$$

\Rightarrow calculate compatibility with experimental measurement $\Delta\mathcal{O}^{\text{EXP}}$.

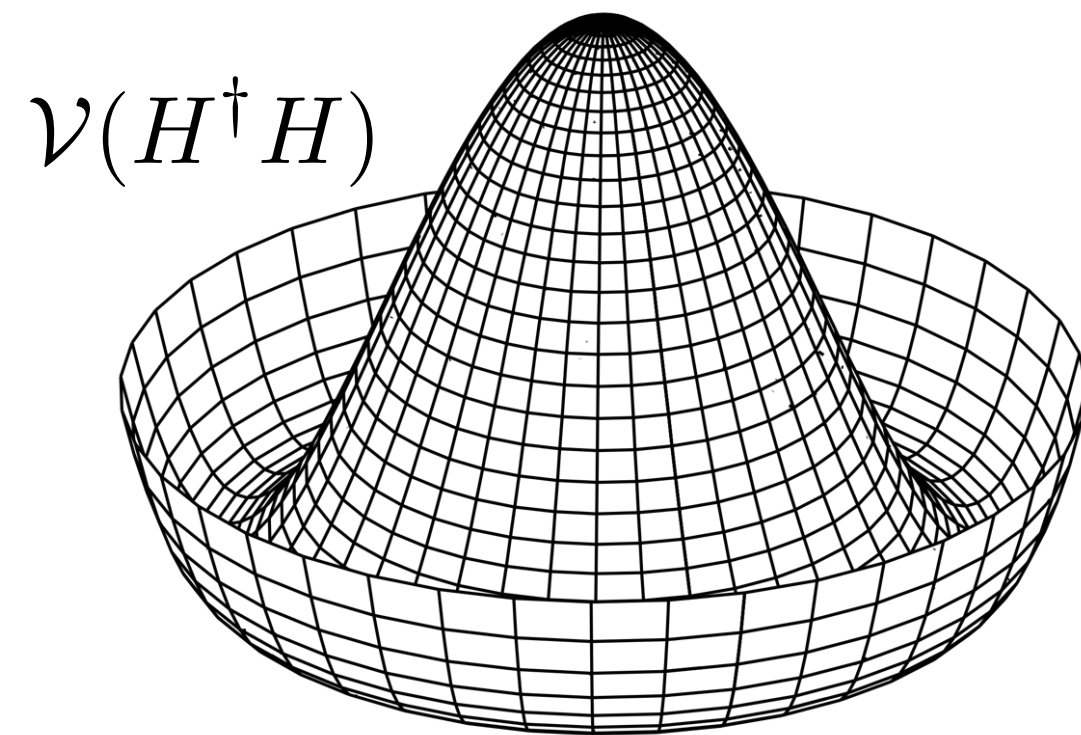


The Higgs Potential & Vacuum Stability

The Importance of the Higgs sector

- the Higgs boson: the central protagonist of EWSB:

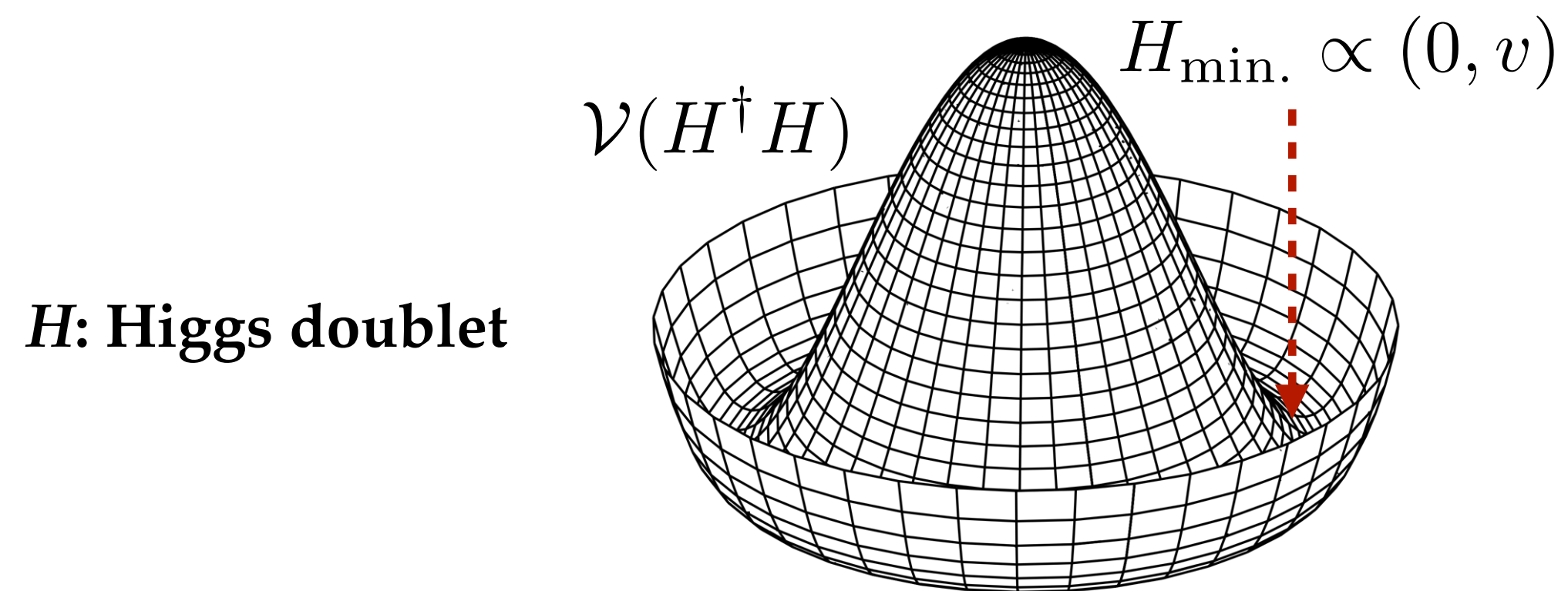
H: Higgs doublet



- an important characteristic of the Higgs boson is the way it couples to itself:

The Importance of the Higgs sector

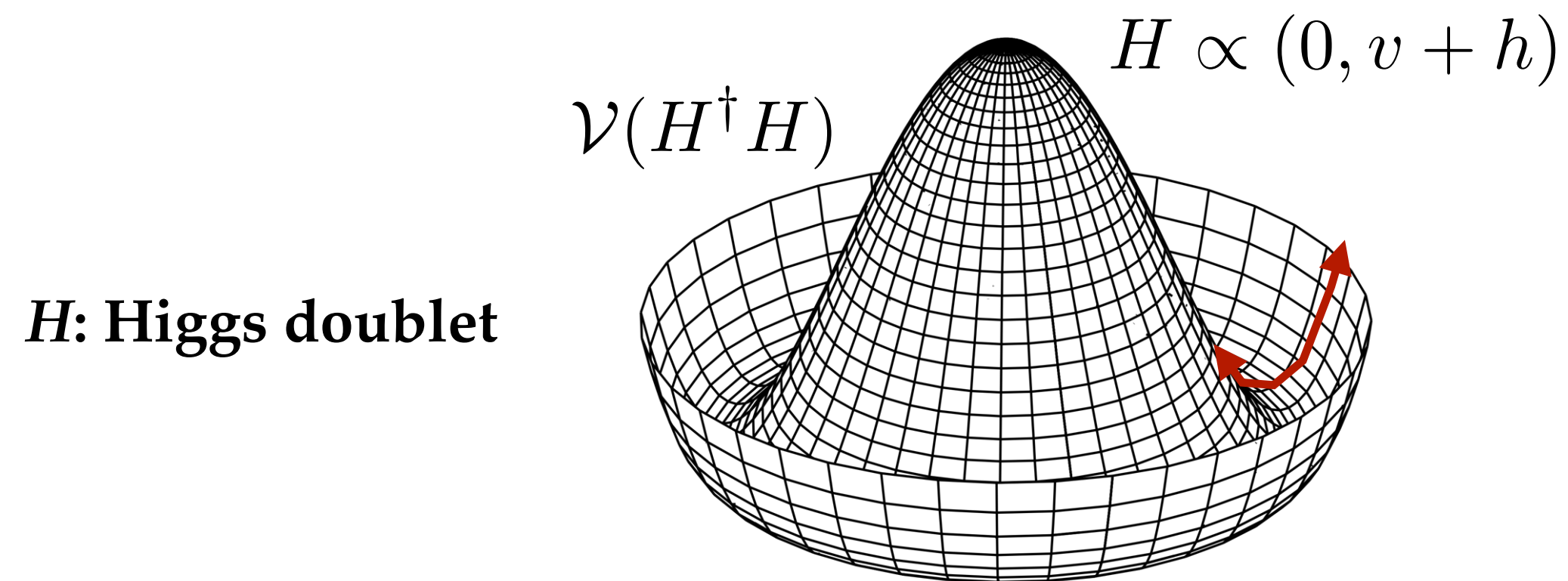
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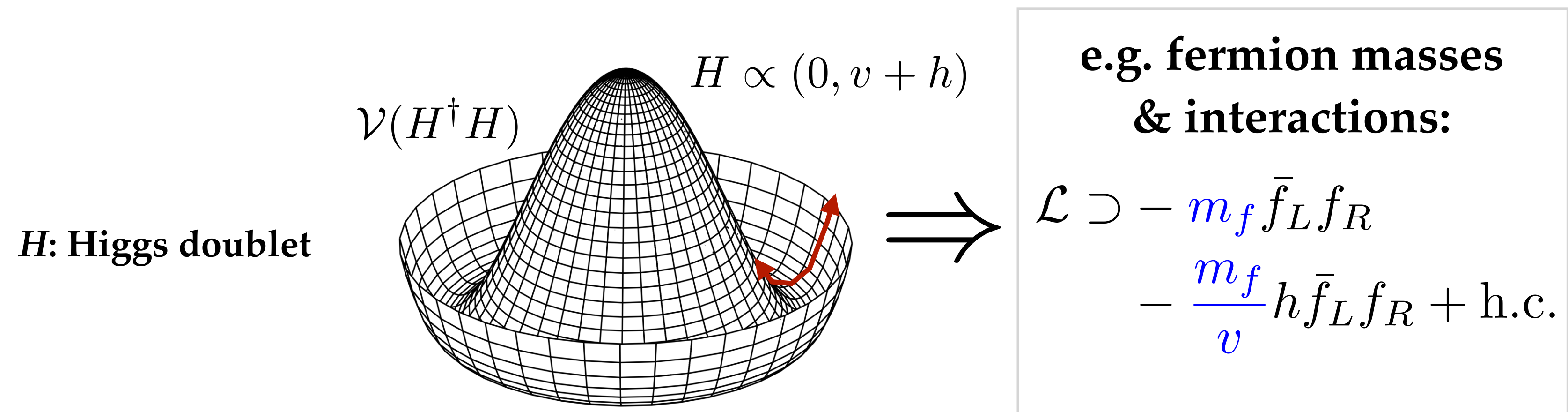
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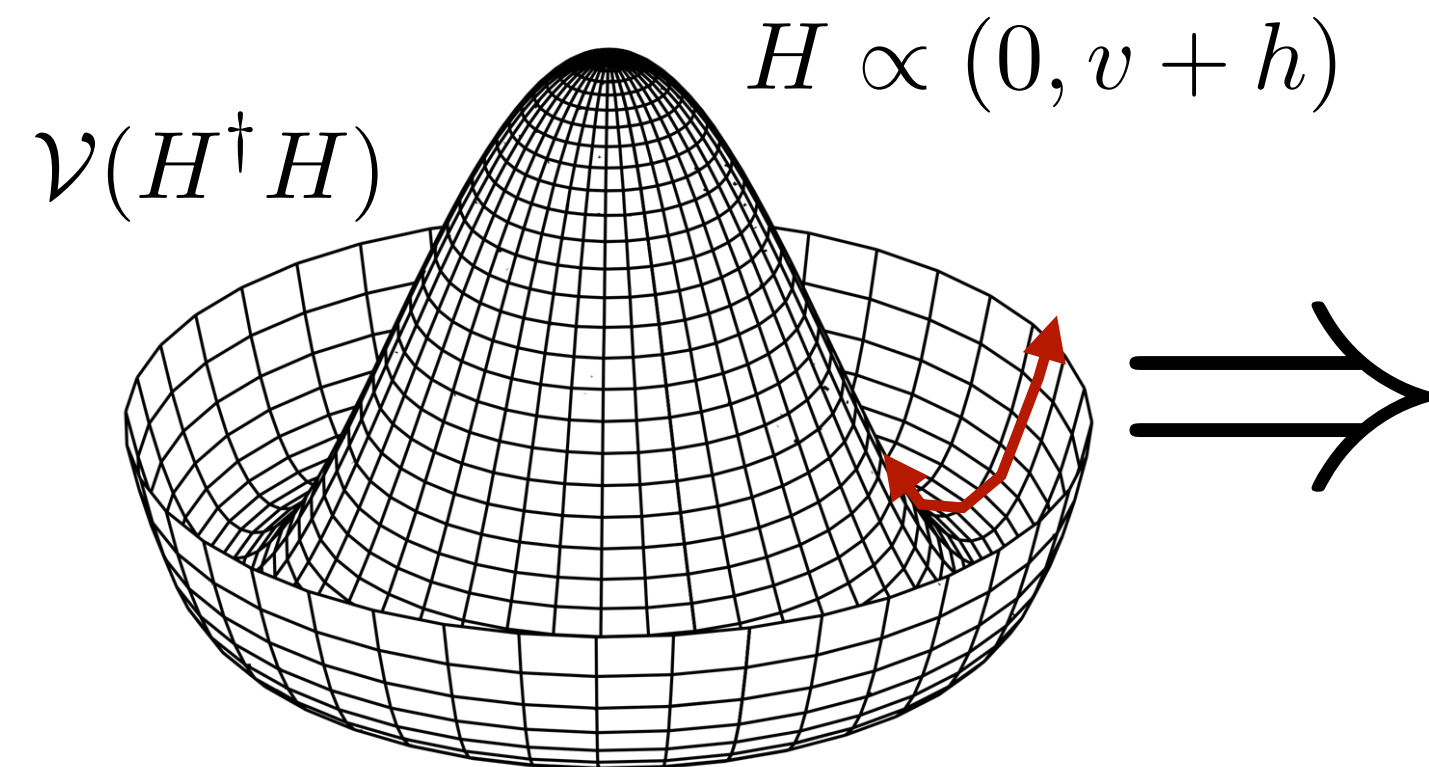


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e.g. gauge boson masses
& interactions:

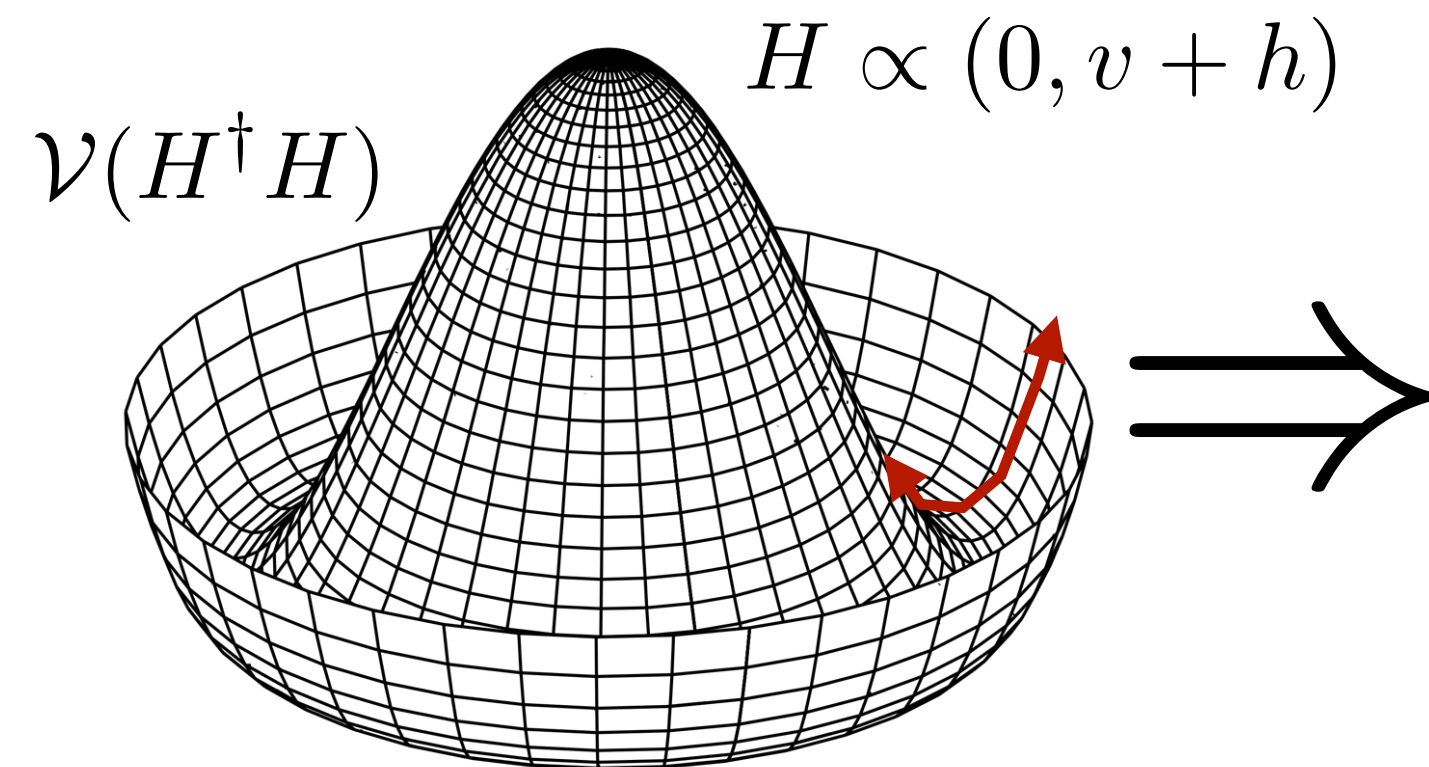
$$\mathcal{L} \supset [m_W^2 W^{\mu+} W_{\mu}^- + \frac{1}{2} m_Z^2 Z^{\mu} Z_{\mu}] \times \left(1 + \frac{h}{v}\right)^2$$

- an important characteristic of the Higgs boson is the way it couples to itself:

The Importance of the Higgs sector

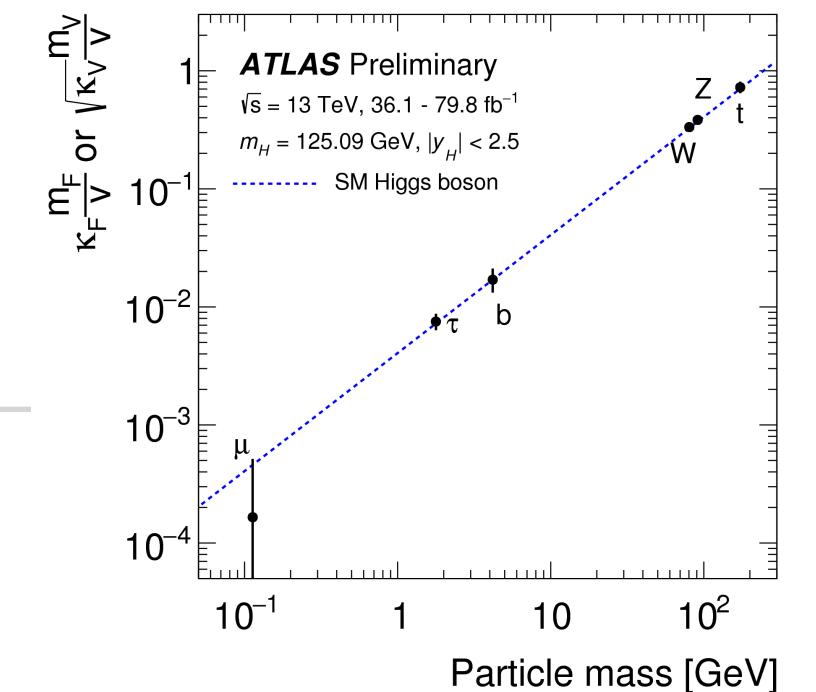
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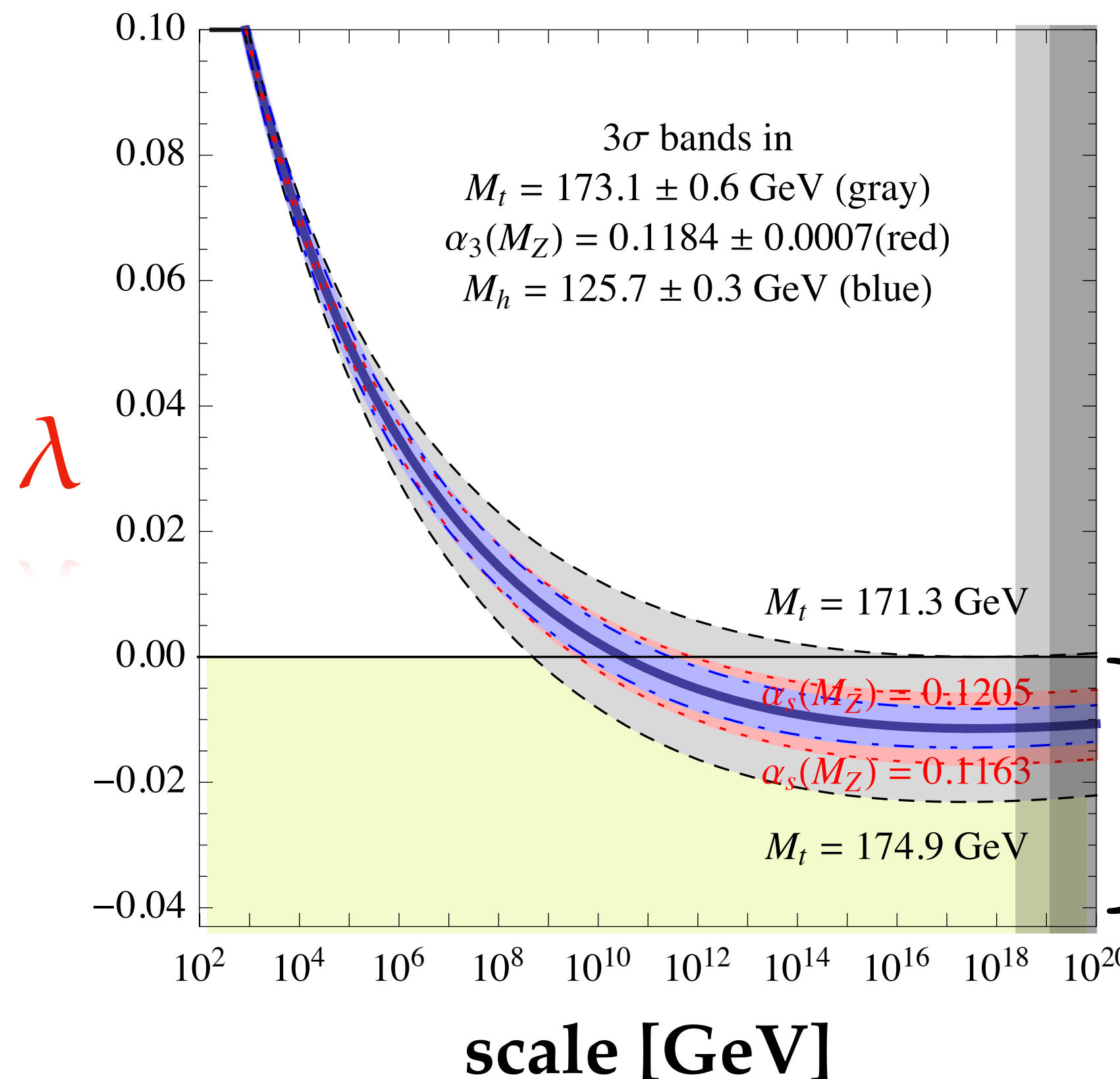
- an important characteristic of the Higgs boson is the way it couples to itself:

Vacuum Stability

- SM potential for the Higgs doublet:

$$\mathcal{V}(H^\dagger H) = -m^2(H^\dagger H) + \lambda(H^\dagger H)^2$$

- renormalisation group evolution of the coupling λ :

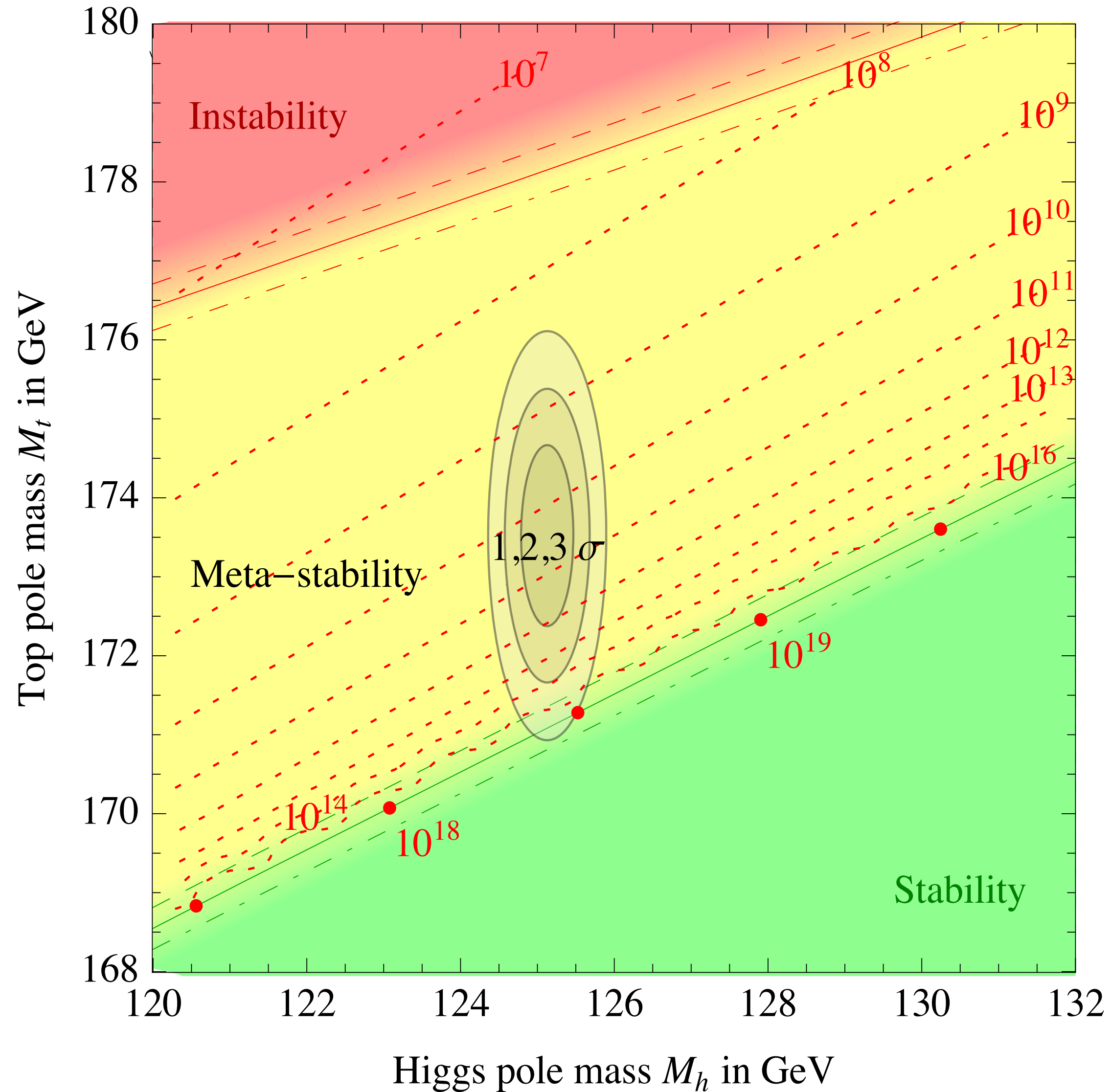


[Cabibbo, Maiani, Parisi, Petronzio, 1979, Hung, 1979, ..., Degrassi, Di Vita, Elias-Miró, Giudice, Isidori, Strumia, 1205.6497, Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia, 1307.3536 ..., Espinosa, 1512.01222]

**potentially
 unstable or
 meta-stable
 vacuum!**

vacuum stability

[Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia, 1307.3536, Espinosa, 1512.01222]



in deriving this: **assumed Standard Model.**

➡ a hint for a non-standard Higgs sector?

➡ further investigation necessary.

Sphaleron/Instanton Processes

Instantons and Baryon-# Violation

- toy model:

(1+1)-dimensions, Abelian gauge field A^μ , complex scalar Φ , Dirac fermion of unit charge Ψ .

- Euclidean space action:

$$S = \int d^2x \left[\frac{1}{4} F_{\mu\nu}^2 + |(\partial_\mu - ieA_\mu)\Phi|^2 + V(\Phi) + i\bar{\Psi}(\partial_\mu - ieA_\mu)\gamma^\mu\Psi \right]$$

“Higgs potential”: $V(\Phi) = \lambda(\Phi^*\Phi - v^2)^2 \implies$ “EWSB” $\implies M_A, M_h$

Instantons and Baryon-# Violation

- consider the current:

$$K_\mu = \frac{e}{2\pi} \epsilon_{\mu\nu} A_\nu$$

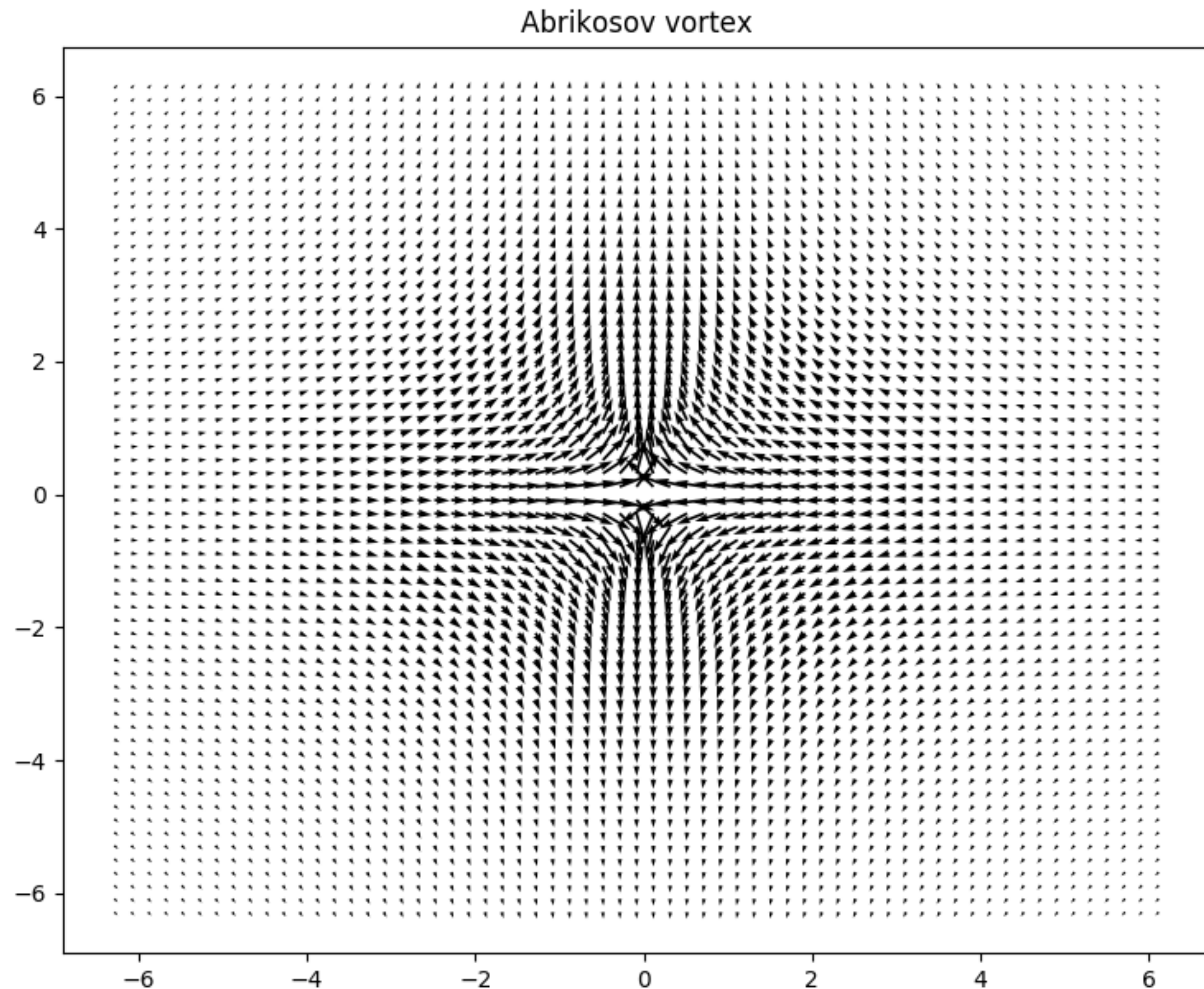
- corresponds to “charge density”:

$$N_{\text{CS}} = \int dx K_0 = \frac{e}{2\pi} \int dx A_1$$

- known as the “winding” or “Chern-Simons” number.

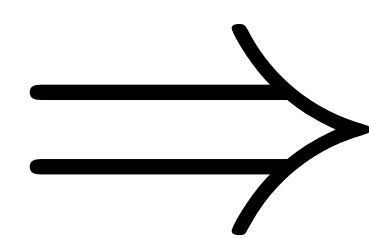
Instantons and Baryon-# Violation

- a classical solution to equations of motion is the “Abrikosov vortex”:



$$A_r = 0, \quad A_\theta = \frac{1}{er} f(r),$$

$$f(0) = 0, \quad 1 - f(r) \sim e^{-M_A r}$$



changes the Chern-Simons number by one unit:

$$\Delta N_{\text{CS}} = \int d^2x \partial_\mu K^\mu = \frac{e}{4\pi} \int d^2x \epsilon_{\mu\nu} F^{\mu\nu} = 1$$

Instantons and Baryon-# Violation

- “instanton” transition necessarily accompanied by change of chirality of fermions by two units:

$$j_{\mu}^5 = \bar{\Psi} \gamma_{\mu} \gamma_5 \Psi$$

$$\frac{1}{2} \partial_{\mu} j^{5\mu} = \frac{e}{4\pi} \epsilon_{\mu\nu} F^{\mu\nu} \quad \text{anomalous divergence of the axial-vector current.}$$

$$\Delta N_{\text{CS}} = \int d^2x \partial_{\mu} K^{\mu} = \frac{e}{4\pi} \int d^2x \epsilon_{\mu\nu} F^{\mu\nu} = 1$$

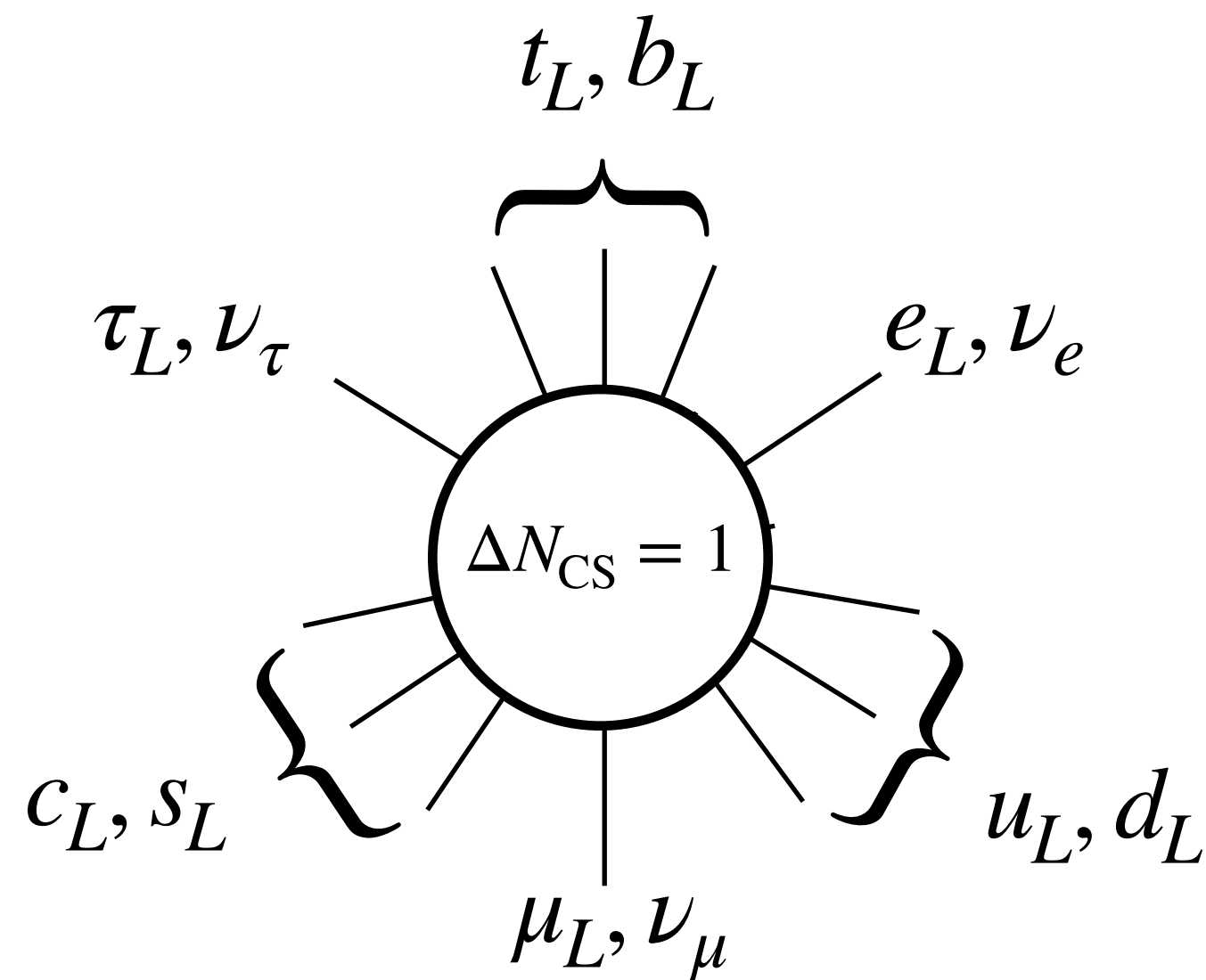
$$\Rightarrow \Delta Q_5 = \Delta \int dx j_0^5 = 2$$

EW Sphalerons at colliders?

- Rate and observability of sphaleron processes at colliders debated.

e.g. [Bezrukov, Levkov, Rebbi, Rybakov, Tinyakov, hep-ph/0304180] VS. [Tye, Wong, 1505.0360, 1710.07223].

- Ponder: **Sphaleron-induced interactions at hadron colliders:**

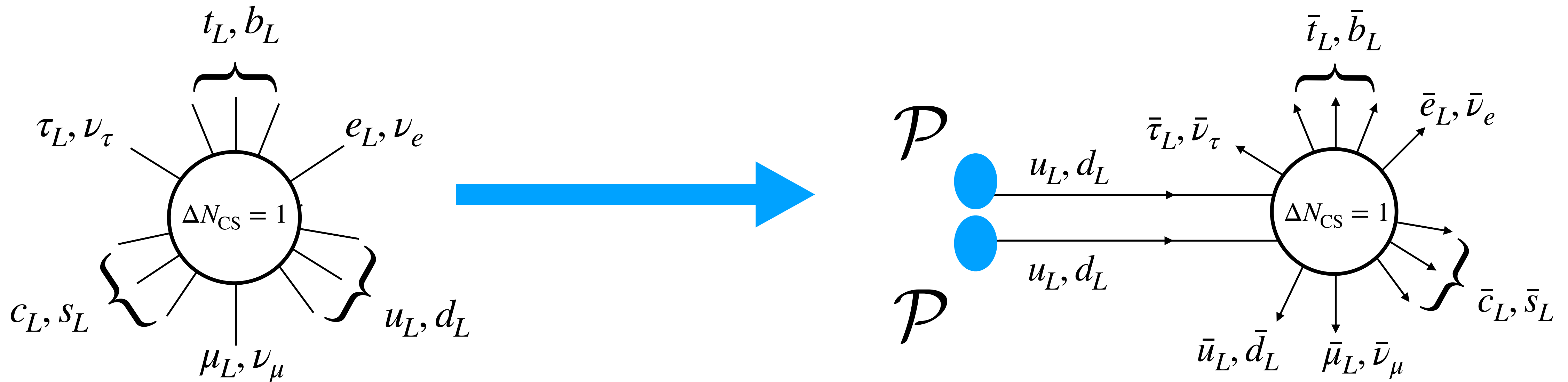


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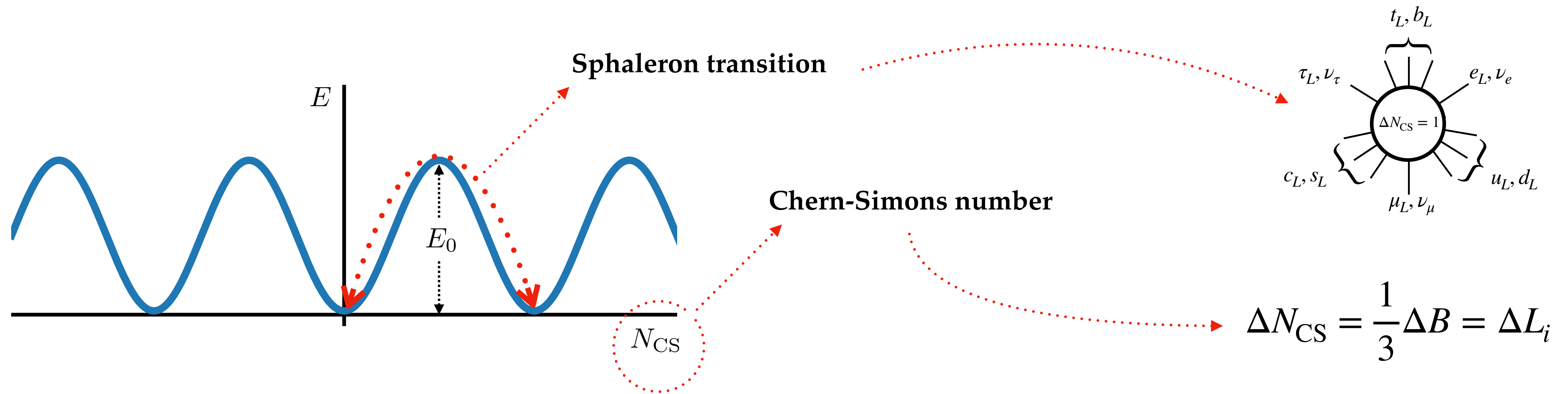
- Ponder: **Sphaleron-induced interactions at hadron colliders:**



What is the Sphaleron?



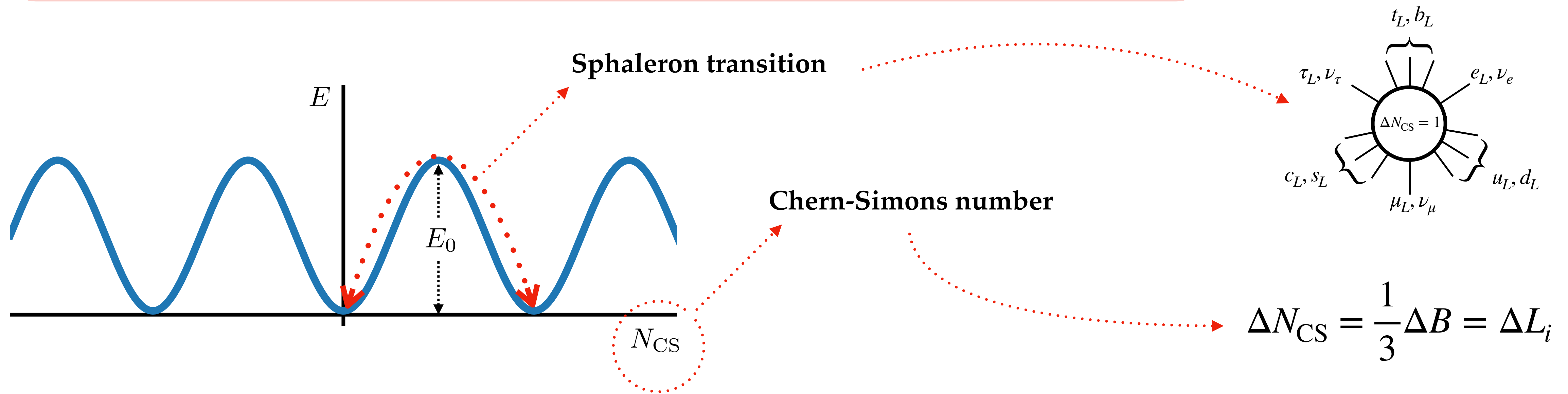
- $E_0 \sim \mathcal{O}(10)$ TeV, separates degenerate Electro-Weak vacua.



What is the Sphaleron?



- $E_0 \sim \mathcal{O}(10)$ TeV, separates degenerate Electro-Weak vacua.



The Sphaleron energy depends crucially on the Higgs sector!

A Note on Sphaleron Suppression

- **Suppression of sphaleron rate inside bubble**

⇒ Baryon Asymmetry “swept in” broken phase and “frozen in”.

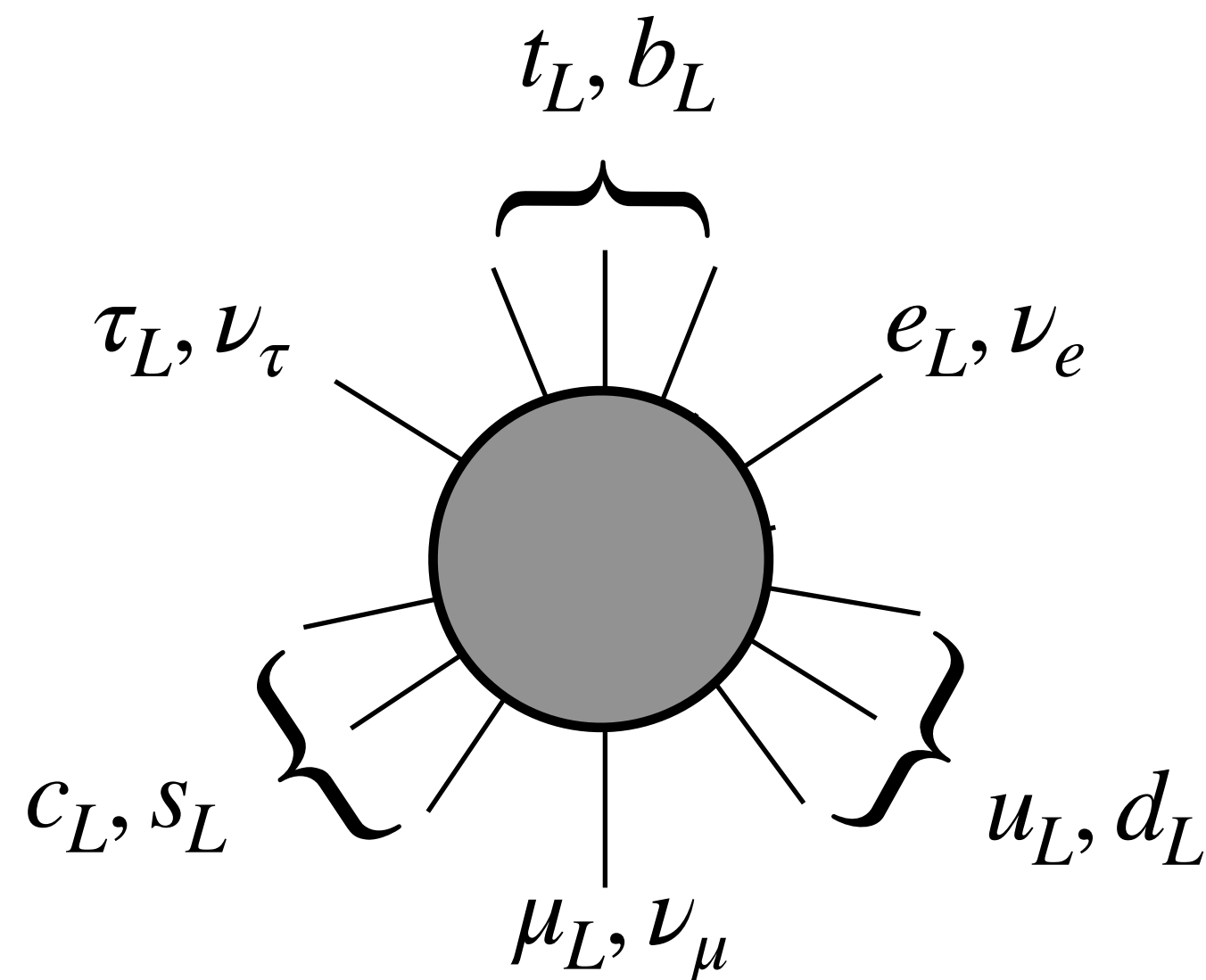
- **Rate** $\sim \exp[-\langle \phi(T_C) \rangle / T_C \times \dots]$,

[T_C : the critical temperature.]

- ⇒ Require: $\langle \phi(T_C) \rangle / T_C \geq 1 \Rightarrow$ a “**Strong**” First-Order EWPT (**SFO-EWPT**).

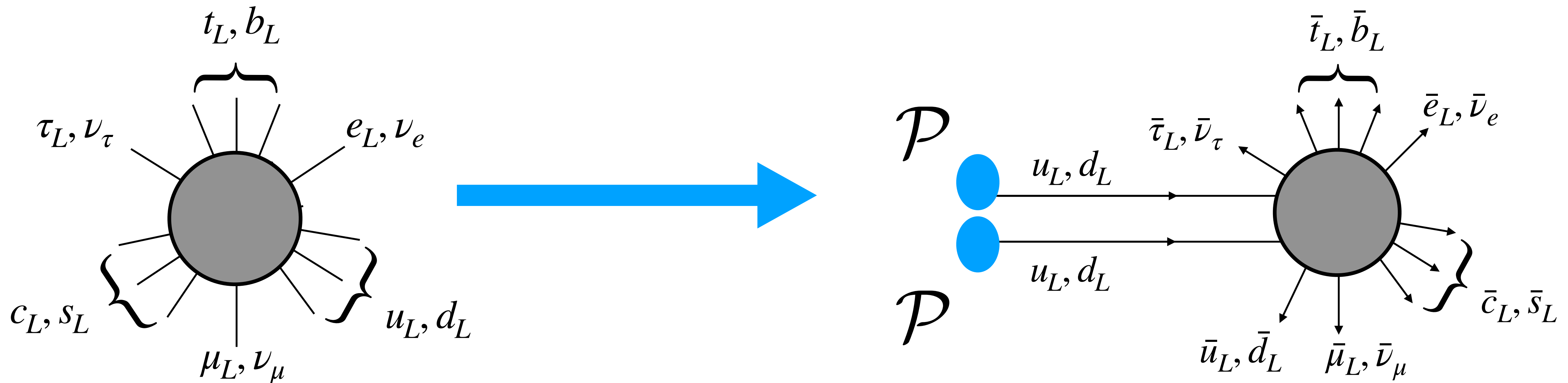
Sphaleron Suppression

- Inside the bubble: ~~Sphaleron~~
- Suppression requires “**Strong**” First-Order EWPT (**SFO-EWPT**).
- **Despite suppression:** Sphalerons @ colliders?

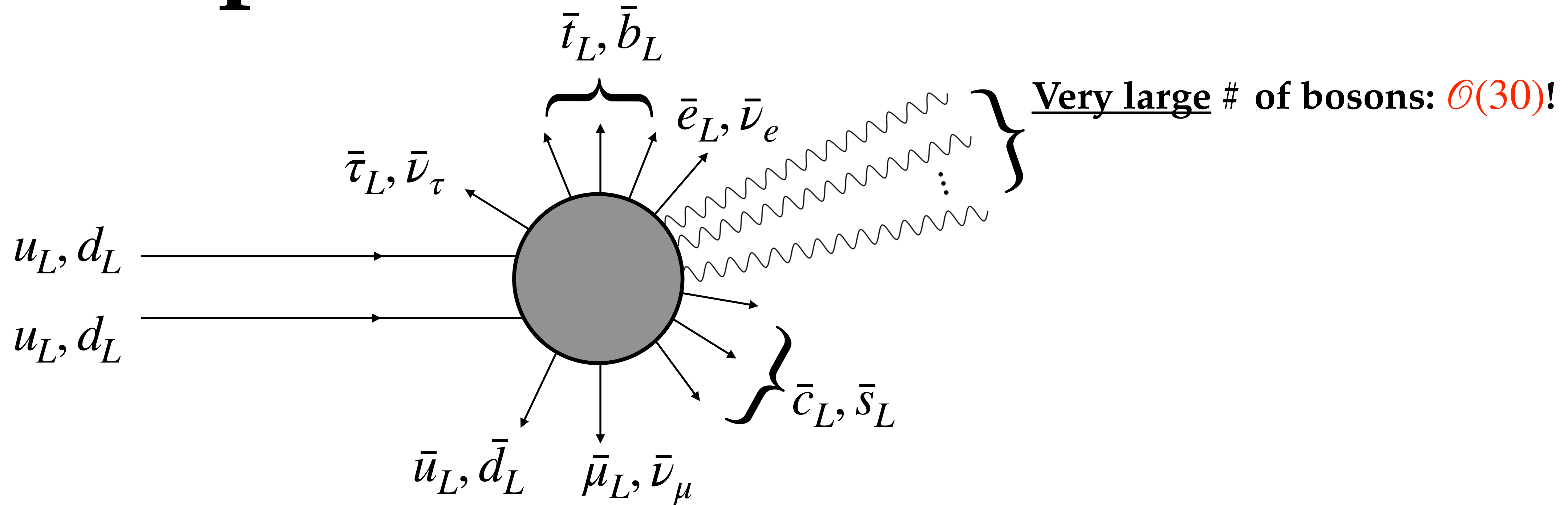


Sphaleron Suppression

- Inside the bubble: ~~Sphaleron~~
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- **Despite suppression: Sphalerons @ colliders?**



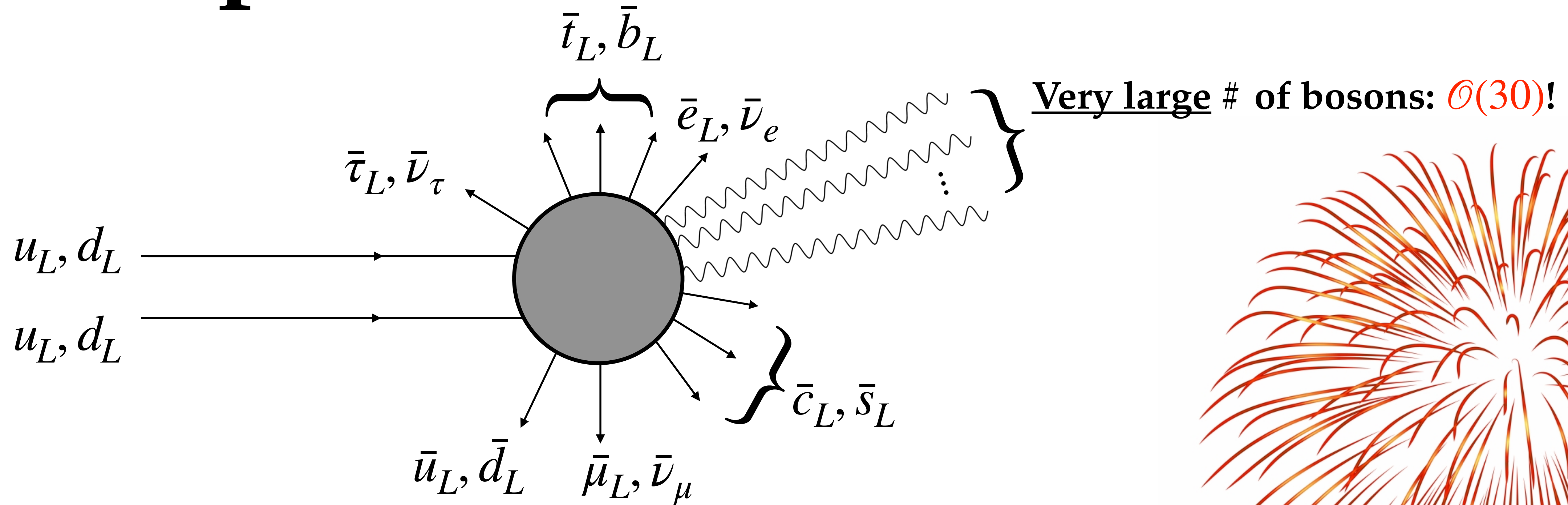
EW Sphalerons at Colliders?



- Possible enhancement if large number of bosons,

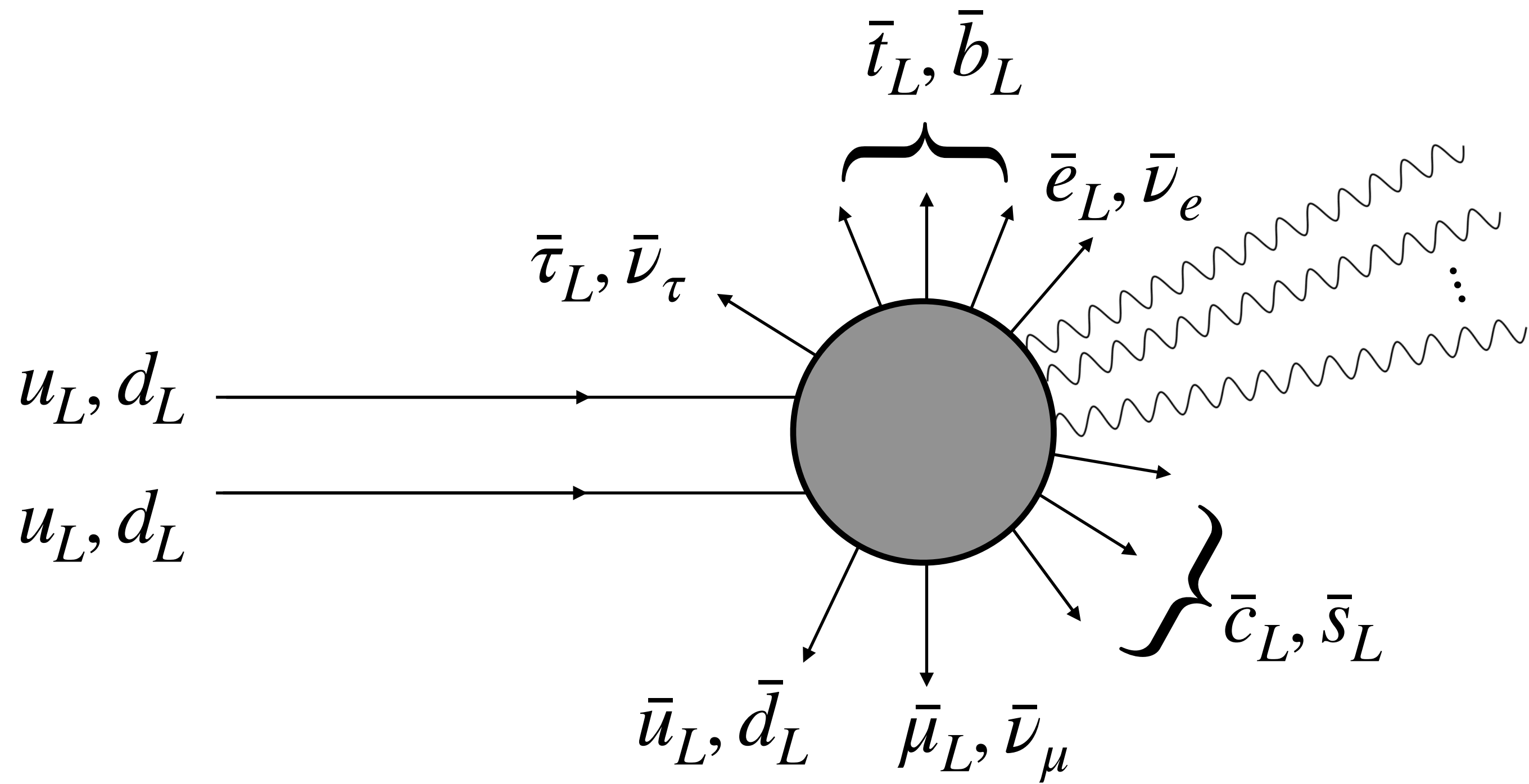
⇒ Events would **spectacularly light up detectors** at experiments!

EW Sphalerons at Colliders?



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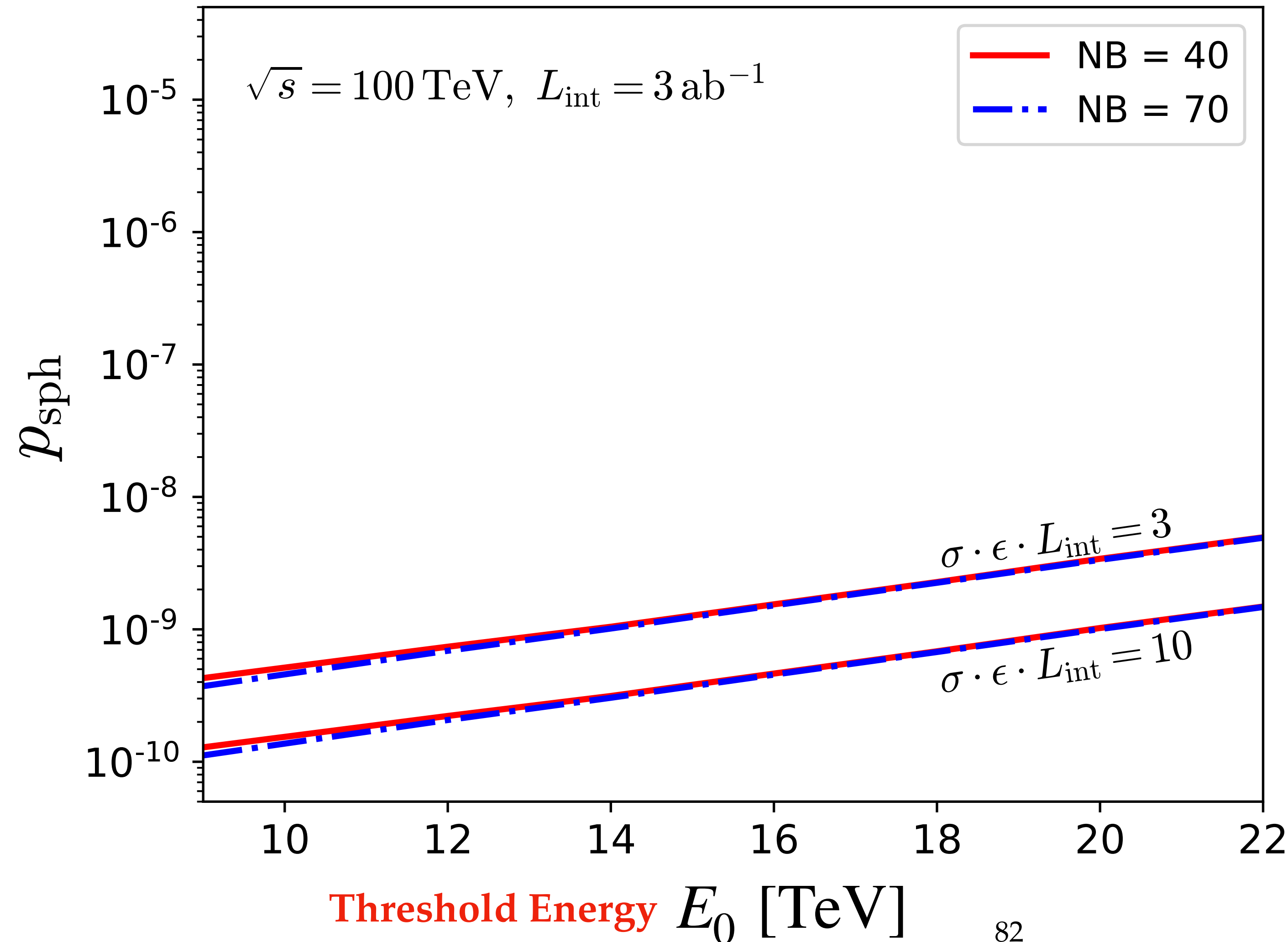
⇒ Events would **spectacularly light up detectors** at experiments!



Sphalerons at the FCC

- Parametrise parton-parton cross section by \mathcal{P}_{sph} :

$$\hat{\sigma}(E) = \frac{\mathcal{P}_{\text{sph}}}{m_W^2} \Theta(E - E_0)$$



→ Event Generator within HERWIG 7.

[[AP](#), Sakurai, Plätzer, [arXiv:1910.4761](#)]

EW Sphalerons at Colliders?

[[AP](#), Sakurai, Plätzer, [arXiv:1910.4761](#)]

⋮

EW Sphalerons at Colliders?

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Homework:

- (i) What can we learn about the Higgs sector and EWBG?
- (ii) New theoretical features in Sphaleron MC.
- (iii) Model discrimination, e.g. VS micro-black holes.
- (iv) Collaboration with experimentalists for measurements.



hhh: Final states

Assume: K-factor = 2.

[Maltoni, Vryonidou, Zaro, 1408.6542]

$hhh \rightarrow$ final state	BR (%)	N_{20ab}^{-1}	
$(b\bar{b})(b\bar{b})(b\bar{b})$	19.21	22207	
$(b\bar{b})(b\bar{b})(WW_{1\ell})$	7.20	8328	
$(b\bar{b})(b\bar{b})(\tau\bar{\tau})$	6.31	7297	\rightarrow Fuks, Kim, Lee, 1510.07697, Fuks, Kim, Lee, 1704.04298.
$(b\bar{b})(\tau\bar{\tau})(WW_{1\ell})$	1.58	1824	
$(b\bar{b})(b\bar{b})(WW_{2\ell})$	0.98	1128	
$(b\bar{b})(WW_{1\ell})(WW_{1\ell})$	0.90	1041	\rightarrow Kilian, Sun, Yan, Zhao, Zhao, 1702.03554.
$(b\bar{b})(\tau\bar{\tau})(\tau\bar{\tau})$	0.69	799	
$(b\bar{b})(b\bar{b})(\gamma\gamma)$	0.23	263	\rightarrow <u>AP</u> , Sakurai, 1508.06524, Chen, Yan, Zhao, Zhao, Zhong, 1510.04013, Fuks, Kim, Lee, 1510.07697.

[AP, Sakurai, 1508.06524]

Singlet model details

$$m_h^2 \equiv \frac{d^2V}{dh^2} = 2\lambda v_0^2$$

$$m_s^2 \equiv \frac{d^2V}{ds^2} = b_3 x_0 + 2b_4 x_0^2 - \frac{a_1 v_0^2}{4x_0}$$

$$m_{hs}^2 \equiv \frac{d^2V}{dhds} = (a_1 + 2a_2 x_0) \frac{v_0}{2}$$

$$h_1 = h \cos \theta + s \sin \theta$$

$$h_2 = -h \sin \theta + s \cos \theta$$

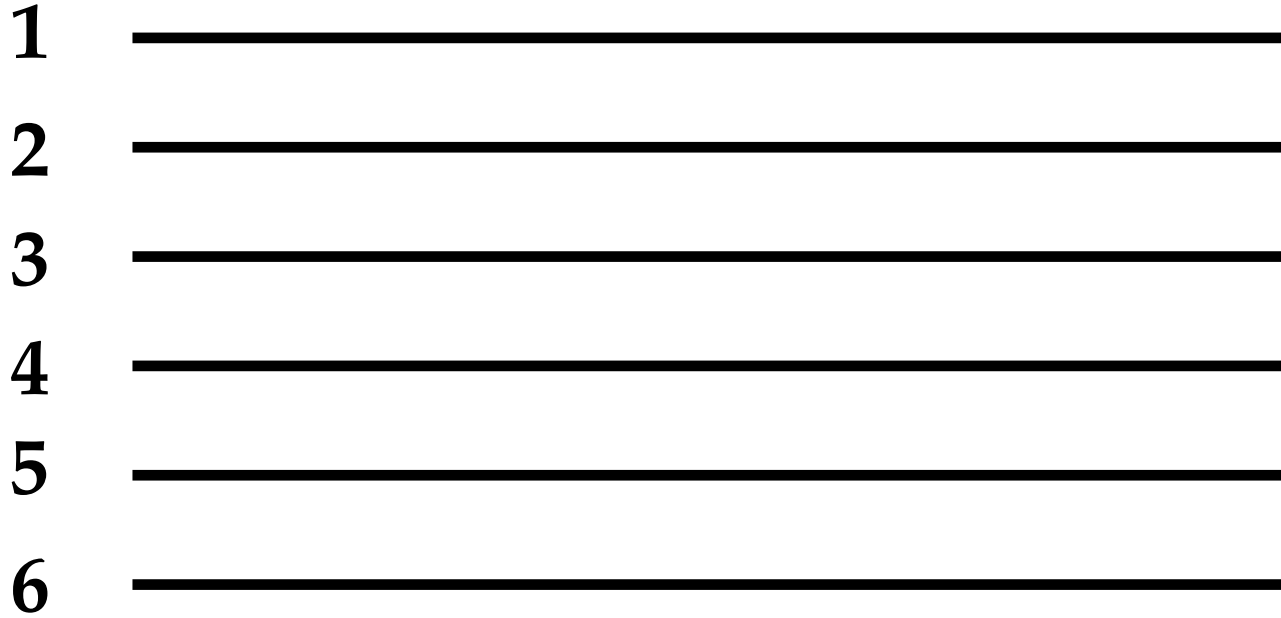
$$m_{2,1}^2 = \frac{m_h^2 + m_s^2 \pm |m_h^2 - m_s^2| \sqrt{1 + \left(\frac{m_{hs}^2}{m_h^2 - m_s^2}\right)^2}}{2},$$

$$\sin 2\theta = \frac{(a_1 + 2a_2 x_0) v_0}{m_1^2 - m_2^2}$$

The 6b final state, analysis [[AP](#), Gilberto Tetlalmatzi-Xolocotzi, Marco Zaro, arXiv:1909.09166]

- What can we learn about the anomalous couplings via **hhh** at 13.6 TeV?
- Begin by using the 6 **b-jet final state!**

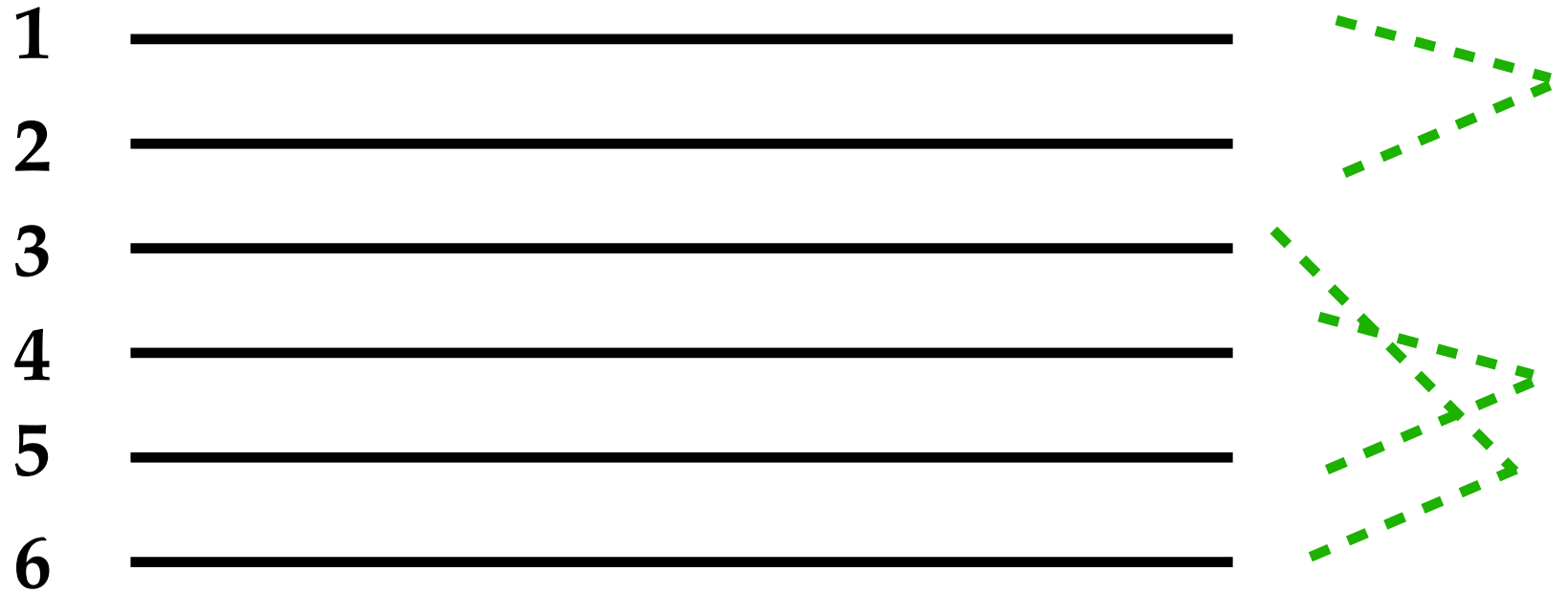
1. Require 6 tagged b-jets.



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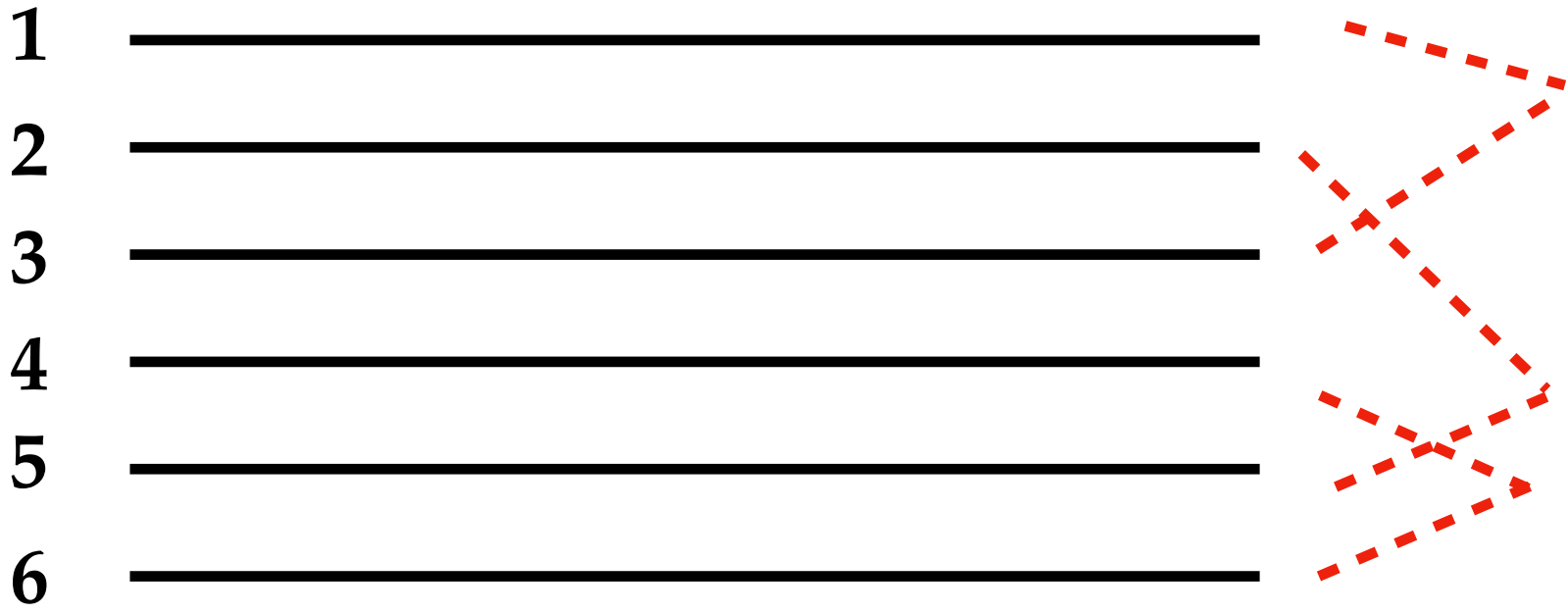
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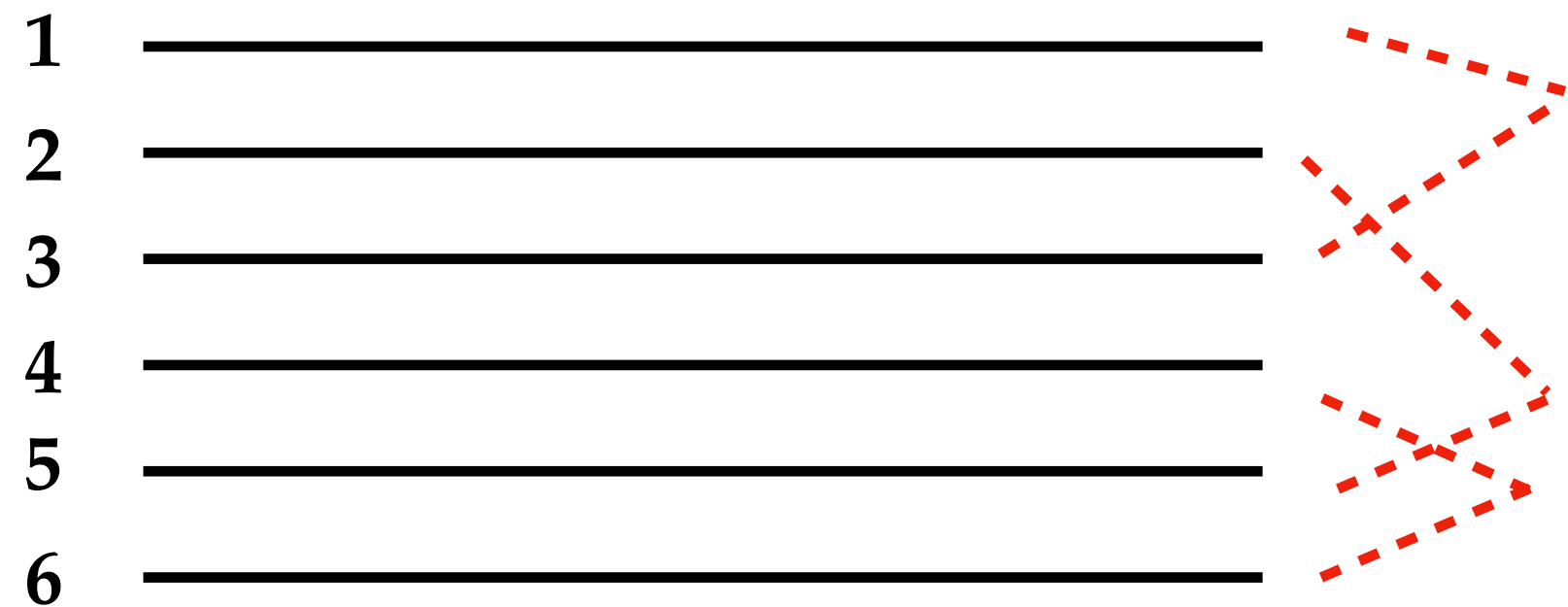


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3. For each pairing construct:

$$\chi^2 = \sum_{qr \in \text{pairings } I} (M_{qr} - m_h^2)^2$$

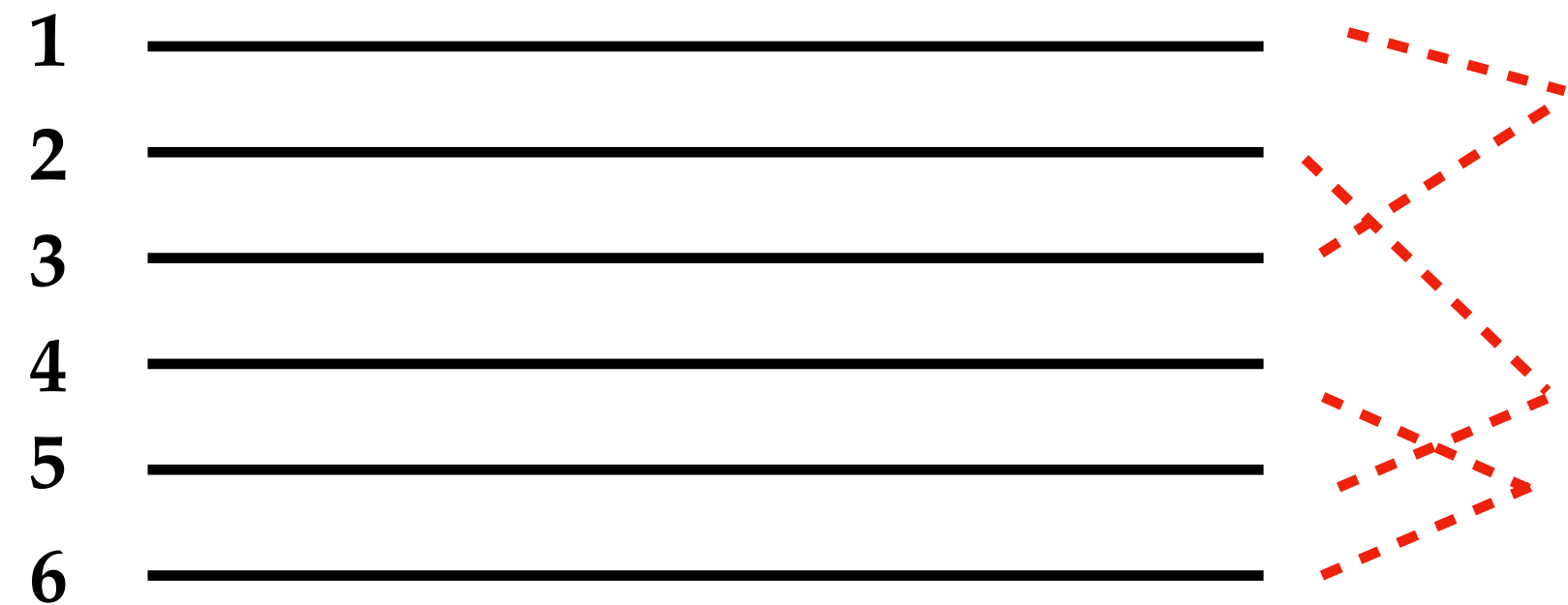
≡ sum of squared differences from Higgs mass (~125 GeV)

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⇒ 4. Pairing that gives minimum χ^2 determines “reconstructed Higgs boson”.

$$\chi_{\min}^2$$

The 6b final state, analysis

$h_r^i \rightarrow$ Higgs boson candidates

observable	cut
$p_{T,b}$	$> 45 \text{ GeV}$
$ \eta_b $	< 3.2
$\Delta R_{b,b}$	> 0.3
$p_T(h_r^i)$	$> [170, 120, 0] \text{ GeV}, i = 1, 2, 3$
χ_{\min}^2	$< 17 \text{ GeV}$
$\Delta m_{\min, \text{mid}, \text{max}}$	$< 8, 8, 11 \text{ GeV}$ ← the three terms in χ_{\min}^2.
$\Delta R(h_r^i, h_r^j)$	$< [3.5, 3.5, 3.5], (i, j) = [(1, 2), (1, 3), (2, 3)]$
$\Delta R_{bb}(h_r^i)$	$< [3.5, 3.5, 3.5], i = 1, 2, 3$

signal/backgrounds after analysis

Process	σ_{GEN} (pb)	$\sigma_{\text{NLO}} \times \text{BR}$ (pb)	$\mathcal{E}_{\text{analysis}}$	$N_{20 \text{ ab}^{-1}}^{\text{cuts}}$
hhh (SM)	2.88×10^{-3}	1.06×10^{-3}	0.0131	278
QCD $(b\bar{b})(b\bar{b})(b\bar{b})$	26.15	52.30	2.6×10^{-5}	27116
$q\bar{q} \rightarrow hZZ \rightarrow h(b\bar{b})(b\bar{b})$	8.77×10^{-4}	4.99×10^{-4}	1.8×10^{-4}	~ 2
$q\bar{q} \rightarrow ZZZ \rightarrow (b\bar{b})(b\bar{b})$	7.95×10^{-4}	7.95×10^{-4}	1.2×10^{-5}	< 1
ggF $hZZ \rightarrow h(b\bar{b})(b\bar{b})$	1.08×10^{-4}	1.23×10^{-4}	$\mathcal{O}(10^{-3})$	~ 2
ggF $ZZZ \rightarrow (b\bar{b})(b\bar{b})$	1.36×10^{-5}	2.73×10^{-5}	2×10^{-5}	$\ll 1$
$h(b\bar{b})(b\bar{b})$	1.46×10^{-2}	1.66×10^{-2}	5.4×10^{-4}	179
$hh(b\bar{b})$	1.40×10^{-4}	9.11×10^{-5}	2.8×10^{-4}	~ 1
$hhZ \rightarrow hh(b\bar{b})$	4.99×10^{-3}	1.61×10^{-3}	7.2×10^{-4}	23
$hZ(b\bar{b}) \rightarrow h(b\bar{b})(b\bar{b})$	9.08×10^{-3}	1.03×10^{-2}	1.4×10^{-4}	29
$ZZ(b\bar{b}) \rightarrow (b\bar{b})(b\bar{b})(b\bar{b})$	2.87×10^{-2}	5.74×10^{-2}	1×10^{-5}	11
$Z(b\bar{b})(b\bar{b}) \rightarrow (b\bar{b})(b\bar{b})(b\bar{b})$	0.93	1.87	3×10^{-5}	1121
Σ backgrounds				2.8×10^4

Reducible backgrounds

process	σ_{GEN} (pb)	$\sigma_{\text{GEN}} \times \mathcal{P}(6 b - \text{jets})$ (pb)
$(b\bar{b})(b\bar{b})(c\bar{c})$	76.8	0.768
$(b\bar{b})(c\bar{c})(c\bar{c})$	75.6	0.00756
$(c\bar{c})(c\bar{c})(c\bar{c})$	22.5	22.5×10^{-5}
$(b\bar{b})(b\bar{b})(jj)$	1.32×10^4	1.32
$(b\bar{b})(jj)(jj)$	9.79×10^5	0.00979
$(jj)(jj)(jj)$	1.37×10^6	1.37×10^{-6}

c.f. $\sigma_{\text{GEN}}(6b) = 26.15$ pb

↑
applied:

$$\mathcal{P}_{c \rightarrow b} = 0.1$$

$$\mathcal{P}_{j \rightarrow b} = 0.01$$

⇒ Assuming perfect b-tagging + identical analysis efficiency to QCD 6b:

→ **~10% contribution from reducible backgrounds.**

for $\text{P}(b\text{-tagging}) = 0.8$:

→ **~30% contribution.**

Scalar singlet model self-couplings

triple:

$$\lambda_{111} = \lambda v_0 c_\theta^3 + \frac{1}{4}(a_1 + 2a_2 x_0) c_\theta^2 s_\theta ,$$

$$+ \frac{1}{2} a_2 v_0 s_\theta^2 c_\theta + \left(\frac{b_3}{3} + b_4 x_0 \right) s_\theta^3 ,$$

$$\lambda_{112} = v_0 (a_2 - 3\lambda) c_\theta^2 s_\theta - \frac{1}{2} a_2 v_0 s_\theta^3$$

$$+ \frac{1}{2} (-a_1 - 2a_2 x_0 + 2b_3 + 6b_4 x_0) c_\theta s_\theta^2 + \frac{1}{4} (a_1 + 2a_2 x_0) c_\theta^3$$

$$\lambda_{122} = v_0 (3\lambda - a_2) s_\theta^2 c_\theta + \frac{1}{2} a_2 v_0 c_\theta^3$$

$$+ (b_3 + 3b_4 x_0 - \frac{1}{2} a_1 - a_2 x_0) s_\theta c_\theta^2 + \frac{1}{4} (a_1 + 2a_2 x_0) s_\theta^3 ,$$

$$\lambda_{222} = \frac{1}{12} [4(b_3 + 3b_4 x_0) c_\theta^3 - 6a_2 v_0 c_\theta^2 s_\theta$$

$$+ 3(a_1 + 2a_2 x_0) c_\theta s_\theta^2 - 12\lambda v_0 s_\theta^3] ,$$

quartic:

$$\lambda_{1111} = \frac{1}{4} (\lambda c_\theta^4 + a_2 c_\theta^2 s_\theta^2 + b_4 s_\theta^4) ,$$

$$\lambda_{1112} = -\frac{1}{2} [-b_4 + \lambda + (-a_2 + b_4 + \lambda)(2c_\theta^2 - 1)] c_\theta s_\theta ,$$

$$\lambda_{1122} = \frac{1}{16} \{ a_2 + 3(b_4 + \lambda)$$

$$+ 3(a_2 - b_4 - \lambda)[(c_\theta^2 - s_\theta^2)^2 - (s_\theta c_\theta)^2] \} ,$$

$$\lambda_{1222} = \frac{1}{4} [b_4 - \lambda + (-a_2 + b_4 + \lambda)(c_\theta^2 - s_\theta^2)] s_\theta c_\theta ,$$

$$\lambda_{2222} = \frac{1}{4} (b_4 c_\theta^4 + a_2 c_\theta^2 s_\theta^2 + \lambda s_\theta^4) .$$

TRSM hhh \rightarrow 6b analysis details

Introduce two observables: $\chi^{2,(4)} = \sum_{qr \in I} \left(M_{qr} - M_1 \right)^2$

$$\chi^{2,(6)} = \sum_{qr \in J} \left(M_{qr} - M_1 \right)^2$$

\rightarrow constructed from different pairings of 4 and 6 b-tagged jets, M_{qr} is the invariant mass of the pairing qr .

TRSM $hhh \rightarrow 6b$ analysis details

Label	(M_2, M_3) [GeV]	$< P_{T,b}$ [GeV]	$\chi^{2,(4)} <$ [GeV ²]	$\chi^{2,(6)} <$ [GeV ²]	$m_{4b}^{\text{inv}} <$ [GeV]	$m_{6b}^{\text{inv}} <$ [GeV]
A	(255, 504)	34.0	10	20	-	525
B	(263, 455)	34.0	10	20	450	470
C	(287, 502)	34.0	10	50	454	525
D	(290, 454)	27.25	25	20	369	475
E	(320, 503)	27.25	10	20	403	525
F	(264, 504)	34.0	10	40	454	525
G	(280, 455)	26.5	25	20	335	475
H	(300, 475)	26.5	15	20	352	500
I	(310, 500)	26.5	15	20	386	525
J	(280, 500)	34.0	10	40	454	525

Table 3. The optimised selection cuts for each of the benchmark points within **BP3** shown in table 2. The cuts not shown above are common for all points, as follows: $|\eta|_b < 2.35$, $\Delta m_{\text{min, med, max}} < [15, 14, 20]$ GeV, $p_T(h_1^i) > [50, 50, 0]$ GeV, $\Delta R(h_1^i, h_1^j) < 3.5$ and $\Delta R_{bb}(h_1) < 3.5$. For some of the points a m_{4b}^{inv} cut is not given, as this was found to not have an impact when combined with the m_{6b}^{inv} cut.

TRSM hhh \rightarrow 6b analysis details (Signal vs Bkg)

Label	(M_2, M_3) [GeV]	$\varepsilon_{\text{Sig.}}$	$S _{300\text{fb}^{-1}}$	$\varepsilon_{\text{Bkg.}}$	$B _{300\text{fb}^{-1}}$	sig $_{300\text{fb}^{-1}}$ (syst.)	sig $_{3000\text{fb}^{-1}}$ (syst.)
A	(255, 504)	0.025	14.12	8.50×10^{-4}	19.16	2.92 (2.63)	9.23 (5.07)
B	(263, 455)	0.019	17.03	3.60×10^{-5}	8.12	4.78 (4.50)	15.10 (10.14)
C	(287, 502)	0.030	20.71	9.13×10^{-5}	20.60	4.01 (3.56)	12.68 (6.67)
D	(290, 454)	0.044	37.32	1.96×10^{-4}	44.19	5.02 (4.03)	15.86 (6.25)
E	(320, 503)	0.051	31.74	2.73×10^{-4}	61.55	3.76 (2.87)	11.88 (4.18)
F	(264, 504)	0.028	18.18	9.13×10^{-5}	20.60	3.56 (3.18)	11.27 (5.98)
G	(280, 455)	0.044	38.70	1.96×10^{-4}	44.19	5.18 (4.16)	16.39 (6.45)
H	(300, 475)	0.054	41.27	2.95×10^{-4}	66.46	4.64 (3.47)	14.68 (4.94)
I	(310, 500)	0.063	41.43	3.97×10^{-4}	89.59	4.09 (2.88)	12.94 (3.87)
J	(280, 500)	0.029	20.67	9.14×10^{-5}	20.60	4.00 (3.56)	12.65 (6.66)

Table 4. The resulting selection efficiencies, $\varepsilon_{\text{Sig.}}$ and $\varepsilon_{\text{Bkg.}}$, number of events, S and B for the signal and background, respectively, and statistical significances for the sets of cuts presented in table 3. A b -tagging efficiency of 0.7 has been assumed. The number of signal and background events are provided at an integrated luminosity of 300 fb^{-1} . Results for 3000 fb^{-1} are obtained via simple extrapolation. The significance is given at both values of the integrated luminosity excluding (including) systematic errors in the background according to Eq. (5.1) (or Eq. (5.2) with $\sigma_b = 0.1 \times B$).

TRSM BP3 Definition

Parameter	Value
M_1	125.09 GeV
M_2	[125, 500] GeV
M_3	[255, 650] GeV
θ_{hS}	-0.129
θ_{hX}	0.226
θ_{SX}	-0.899
v_S	140 GeV
v_X	100 GeV
κ_1	0.966
κ_2	0.094
κ_3	0.239

TRSM BP3 Benchmark Point Info

Label	(M_2, M_3)	Γ_2 [GeV]	Γ_3 [GeV]	$\text{BR}_{2 \rightarrow 11}$ [GeV]	$\text{BR}_{3 \rightarrow 11}$	$\text{BR}_{3 \rightarrow 12}$
A	(255, 504)	0.086	11	0.55	0.16	0.49
B	(263, 455)	0.12	7.6	0.64	0.17	0.47
C	(287, 502)	0.21	11	0.70	0.16	0.47
D	(290, 454)	0.22	7.0	0.70	0.19	0.42
E	(320, 503)	0.32	10	0.71	0.18	0.45
F	(264, 504)	0.13	11	0.64	0.16	0.48
G	(280, 455)	0.18	7.4	0.69	0.18	0.44
H	(300, 475)	0.25	8.4	0.70	0.18	0.43
I	(310, 500)	0.29	10	0.71	0.17	0.45
J	(280, 500)	0.18	10.6	0.69	0.16	0.47

Table 5. The total widths and new scalar branching ratios for the parameter points considered in the analysis. For the SM-like h_1 , we have $M_1 = 125$ GeV and $\Gamma_1 = 3.8$ MeV for all points considered. The other input parameters are specified in table 1. The on-shell channel $h_3 \rightarrow h_2 h_2$ is kinematically forbidden for all points considered here.

Monte Carlo Implementation of Anomalous Couplings

- Get the **MG5_aMC model** at: https://gitlab.com/apapaefs/multihiggs_loop_sm.
- [A patch to MG5_aMC to enable **Loop × Tree** is included].
- Can generate events either at:

- **SM² + interference of [SM × One-Insertion diagrams]**, i.e.:

$$|\mathcal{M}|^2 = |\mathcal{M}_{\text{SM}}|^2 + 2\text{Re}\{\mathcal{M}_{\text{SM}}^* \mathcal{M}_{1\text{-ins.}}\} \propto 1 + c_i$$

or

- **SM² + interference of [SM × One or Two insertion diagrams] + [One Insertion]²**, i.e.:

$$|\mathcal{M}|^2 = |\mathcal{M}_{\text{SM}}|^2 + 2\text{Re}\{\mathcal{M}_{\text{SM}}^* \mathcal{M}_{1\text{-ins.}}\} + 2\text{Re}\{\mathcal{M}_{\text{SM}}^* \mathcal{M}_{2\text{-ins.}}\} + |\mathcal{M}_{1\text{-ins.}}|^2 \\ \propto 1 + c_i + c_j c_k + c_\ell^2$$

