Multi-Scalar Production at Current & Future Colliders

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KENNESAW STATE UNIVERSITY



@ Brookhaven National Lab, June 30th, 2026

The Plan

01 Multi-Scalar Production: Motivation & Current State

02 Theory & Phenomenology: ***hhh*** *production & perturbative unitarity*

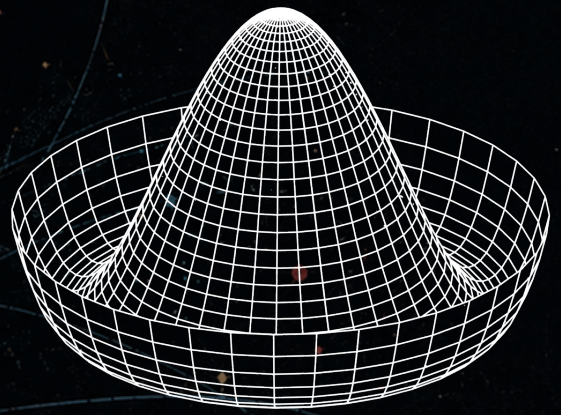
03 Future Prospects: Self-couplings & Extended Scalar Sectors

04 Hot Off the Press! Quadruple ***h*** & Charge pairing

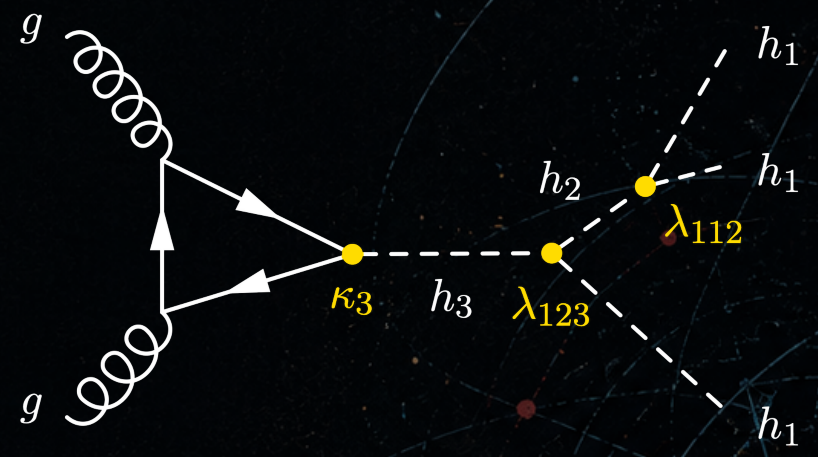
05 Conclusions

Main Reasons to study multi-scalar production:

I: The Higgs Potential



II: Extended Scalar Sectors



SECTION 01-I

I: The Higgs Potential

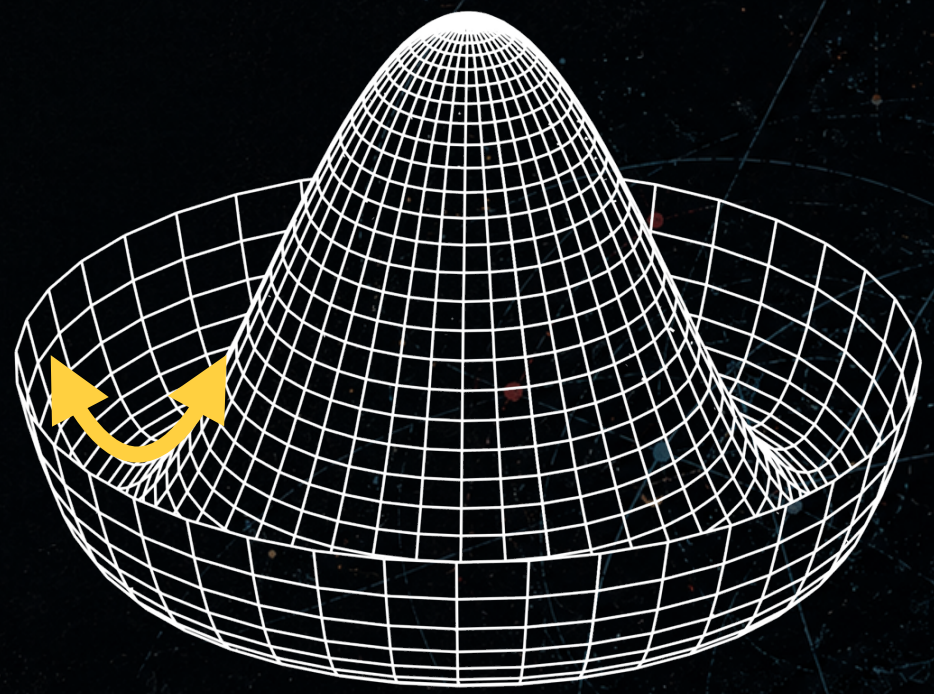
The Higgs Potential

Higgs doublet ϕ acquires **VEV**: $\phi \sim \langle \phi \rangle + h$

$$\Rightarrow \mathcal{V}(\langle \phi \rangle + h) = \frac{1}{2} m_h^2 h^2 + \frac{m_h^2}{2v} h^3 + \frac{m_h^2}{8v^2} h^4$$

Electroweak Symmetry Breaking

Life takes place about one of the degenerate minima:



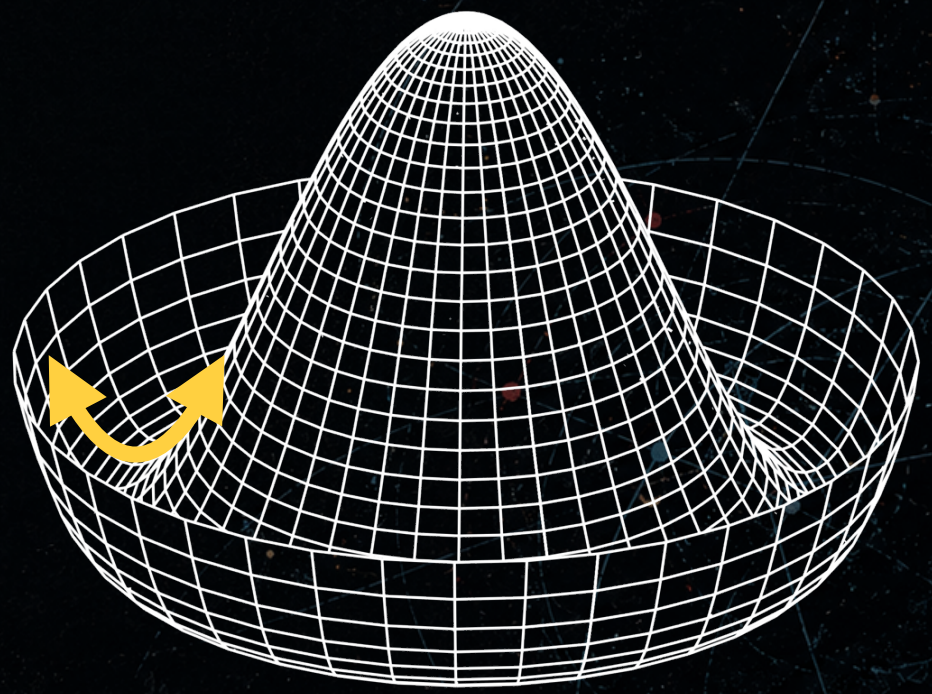
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Measuring The Higgs Boson's Potential: Mass

$$\mathscr{V}(\langle\phi\rangle + h) = \frac{1}{2}m_h^2 h^2 + \kappa_3 \frac{m_h^2}{2v} h^3 + \kappa_4 \frac{m_h^2}{8v^2} h^4$$

Verify SM predictions:

$$\kappa_{3,4} \stackrel{?}{=} 1$$

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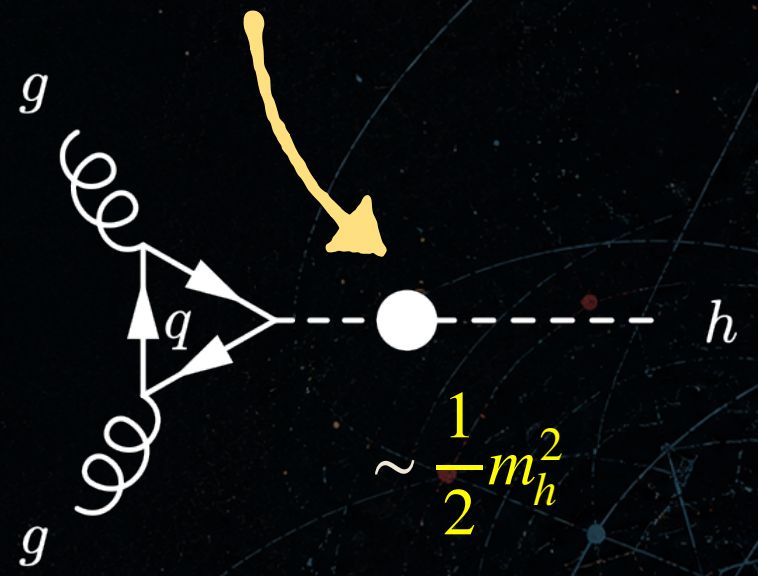
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Mass measurement:



Measuring The Higgs Boson's Potential: Mass

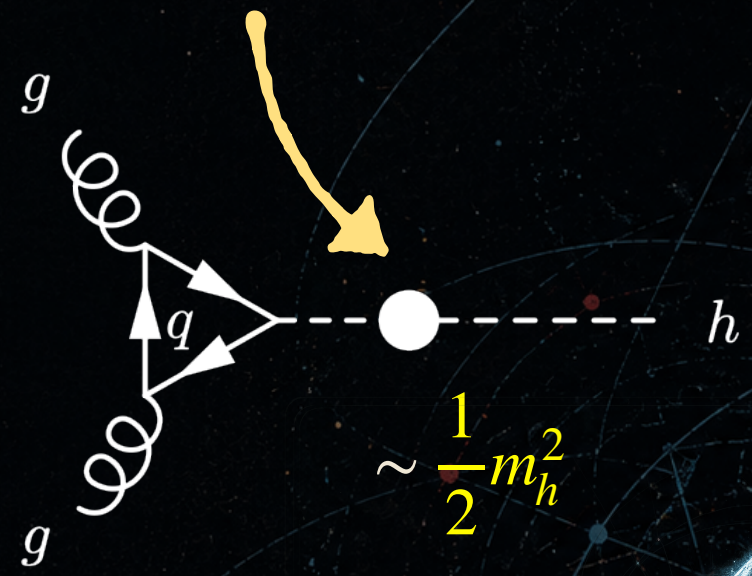
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Step 1: Single Higgs boson production:

Mass measurement:



$$\sim \frac{1}{2}m_h^2$$

$$m_h \simeq 125.1 \pm 0.1 \text{ GeV}$$

[CMS: PRD 111 (2025) 092014, ATLAS: Phys. Lett. B 847 (2023) 138315]

Measuring The Higgs Boson's Potential: Trilinear

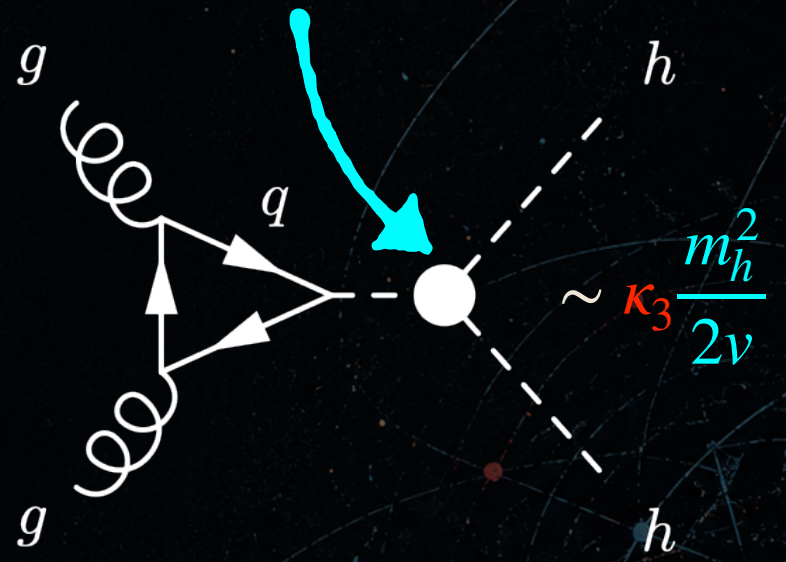
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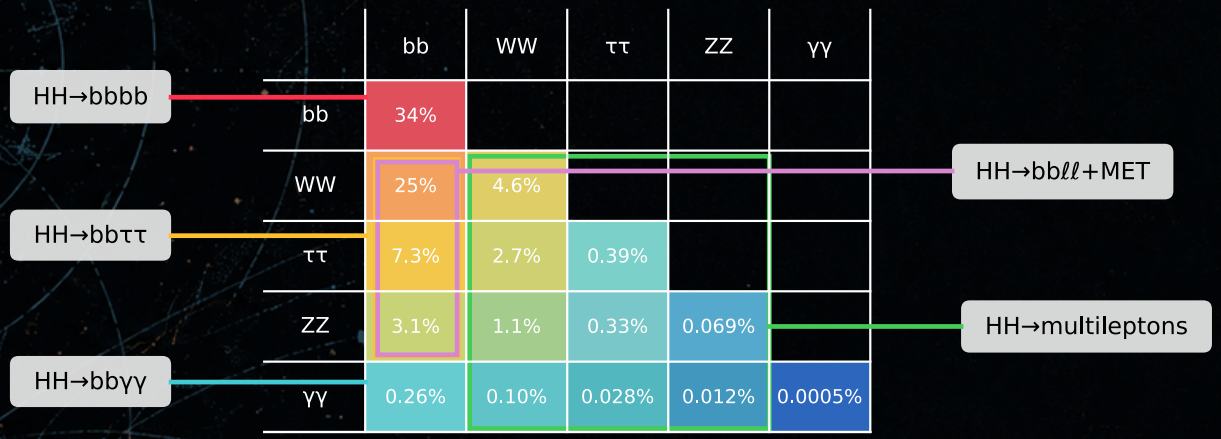
Trilinear coupling measurement:



Measuring The Higgs Boson's Potential: Trilinear

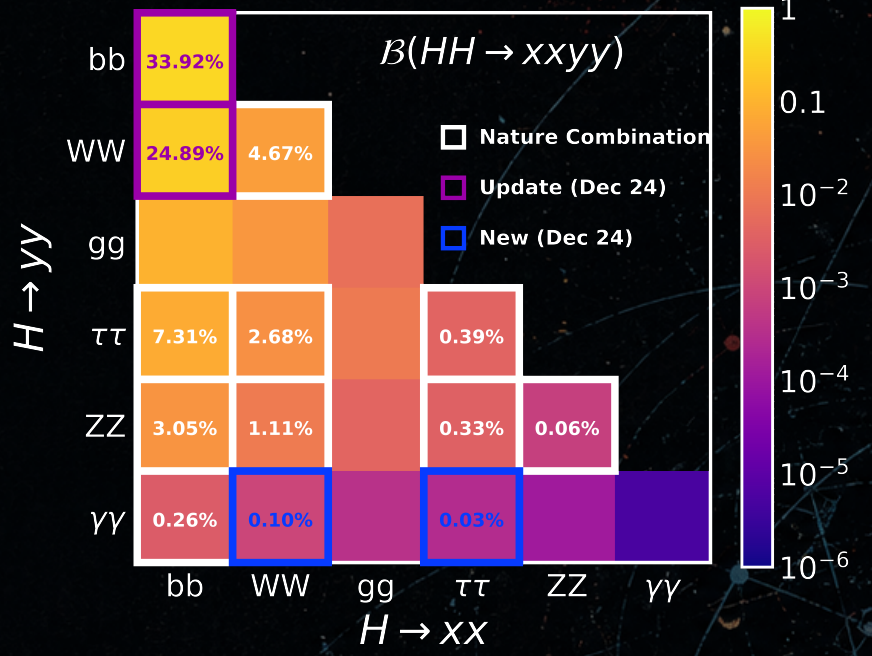
$pp \to hh \to (xx)(yy)$ final states @ LHC, Branching Ratios:

ATLAS:



[A. Sidley, HIGGS 2025]

CMS:

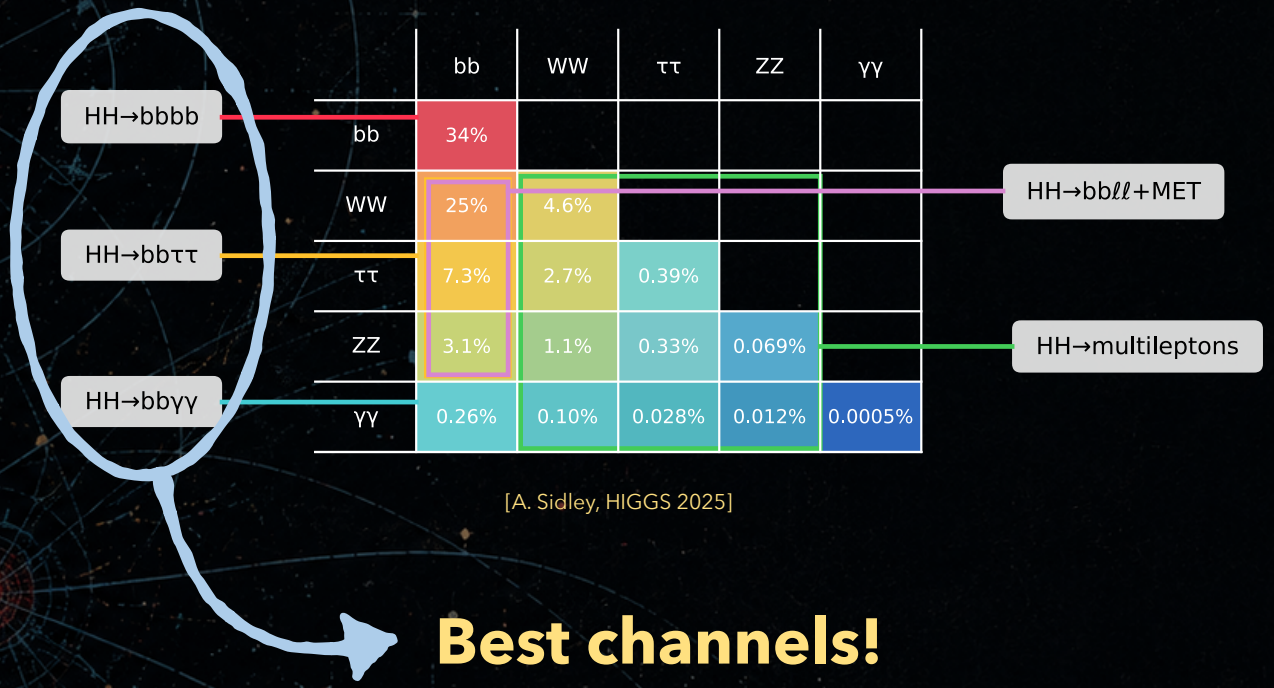


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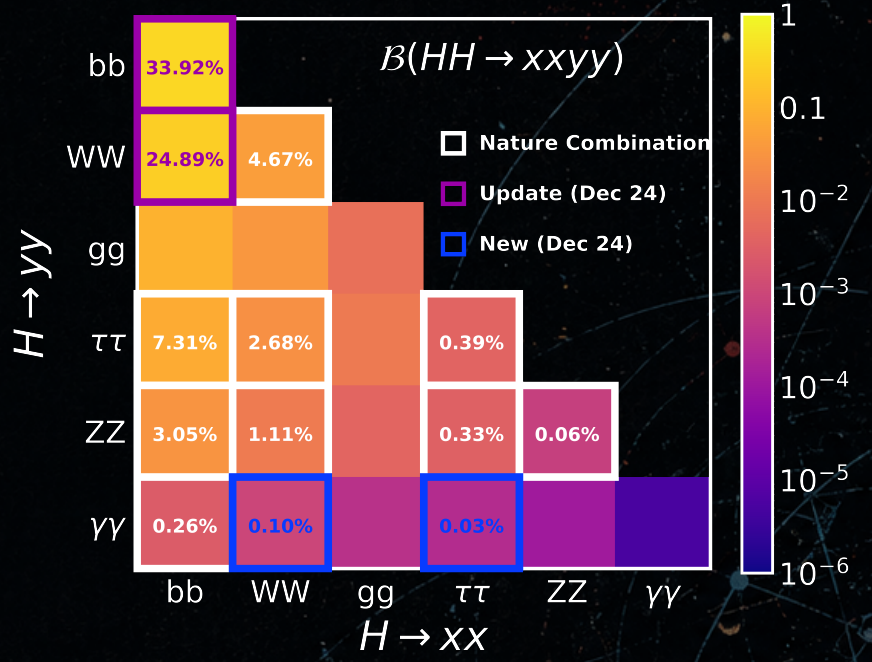
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Best channels!

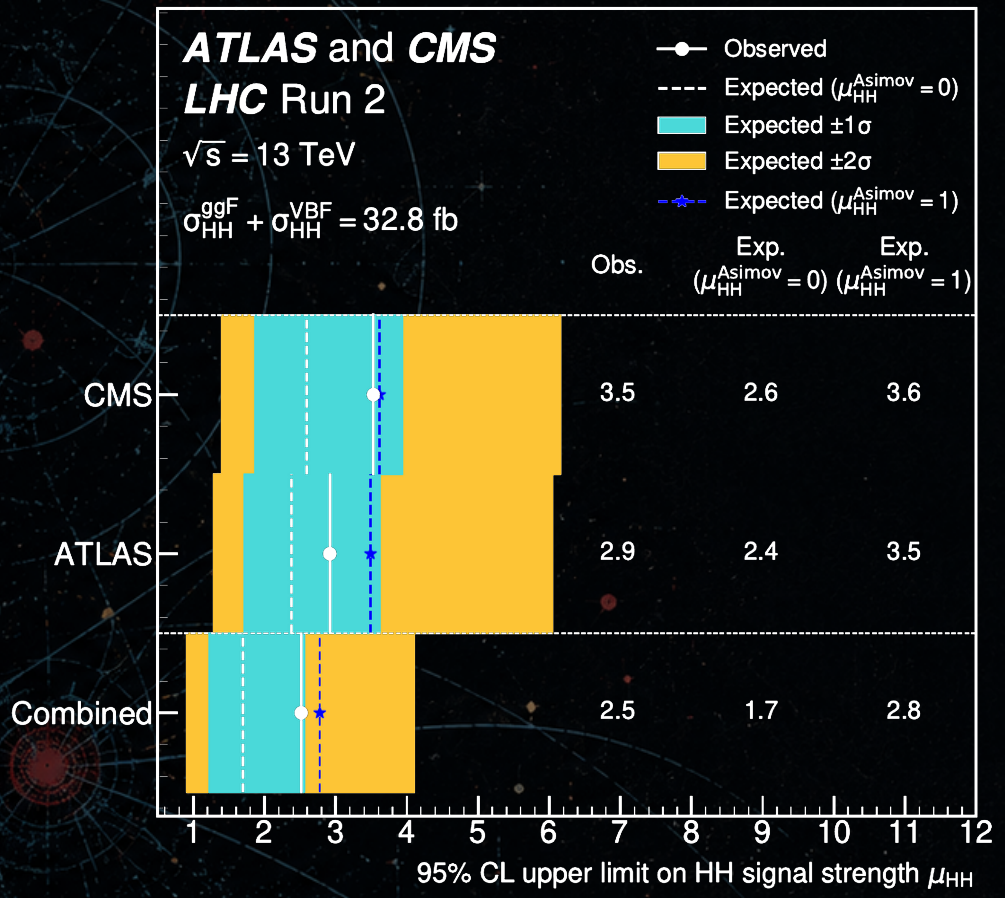
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Trilinear @ LHC (Today!)

ATLAS + CMS Combo (13 TeV): [arXiv:2602.23991]

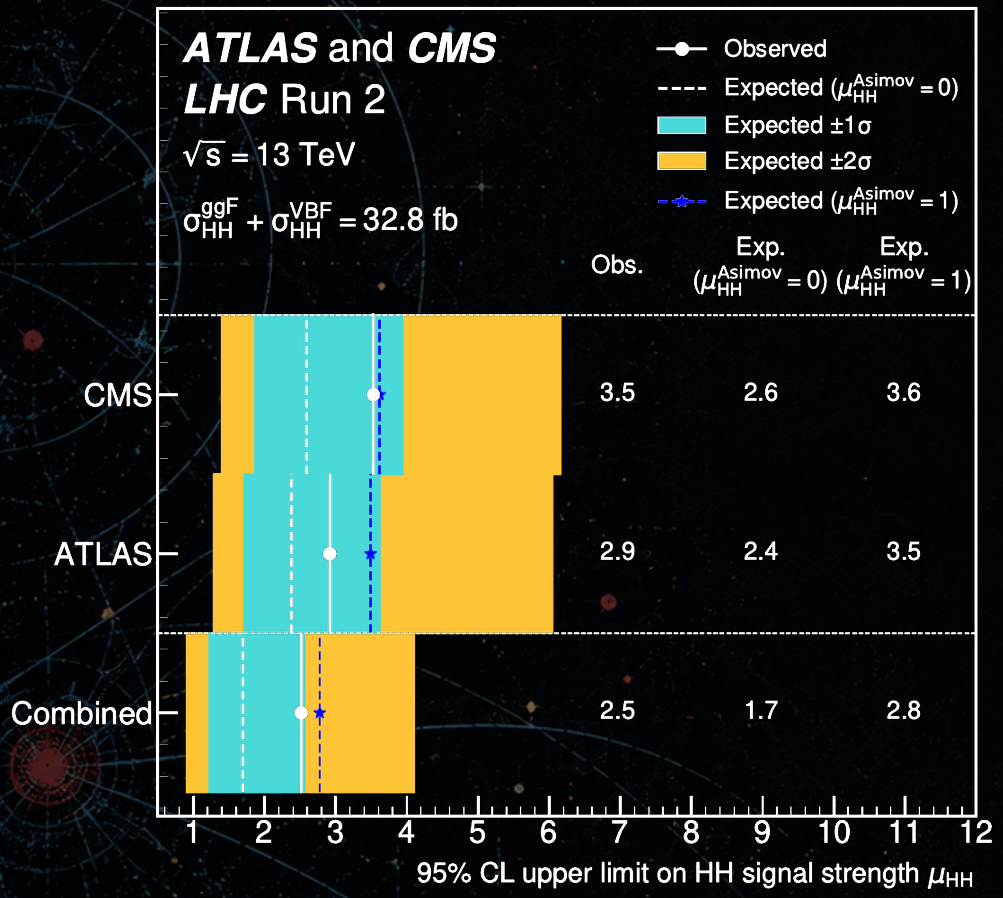


$\mu_{HH} = \sigma_{\text{obs}}/\sigma_{\text{SM}}$

	Channel	Observed signal strength μ_{HH}	Observed κ_3 95% CL interval
ATLAS	$bbbb$	< 5.4	$[-3.4, 11]$
	$bb\tau\tau$	< 5.8	$[-3.3, 9.1]$
	$bb\gamma\gamma$	< 4.1	$[-1.6, 7.0]$
	$bb\ell\ell + E_{\text{T}}^{\text{miss}}$	< 9.6	$[-6.5, 13]$
	Multilepton	< 17	$[-6.4, 12]$
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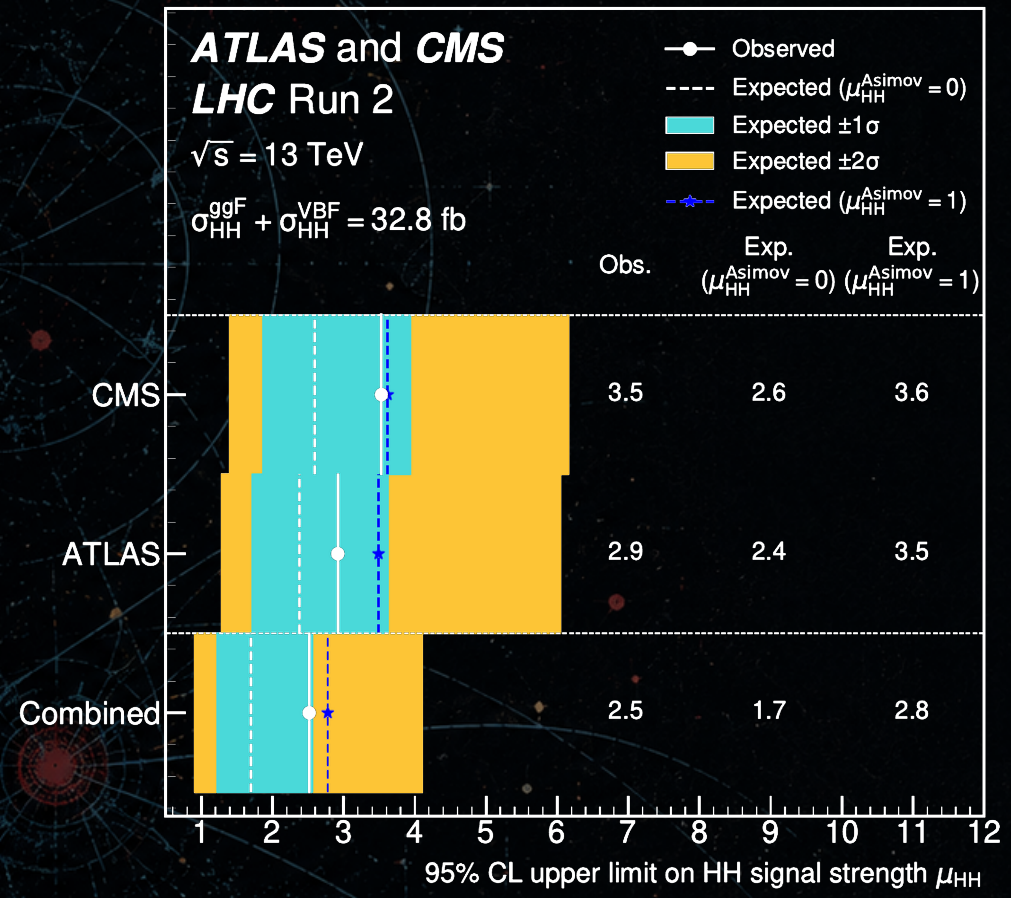


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$\Rightarrow -0.71 < \kappa_3 < 6.1$



Measuring The Higgs Boson's Potential: Quartic

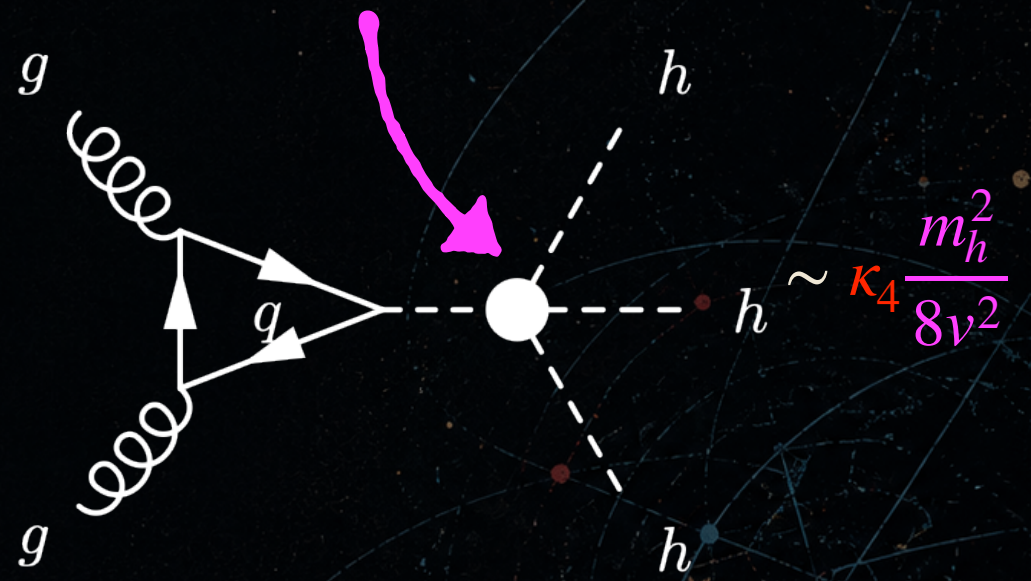
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Quartic coupling measurement:



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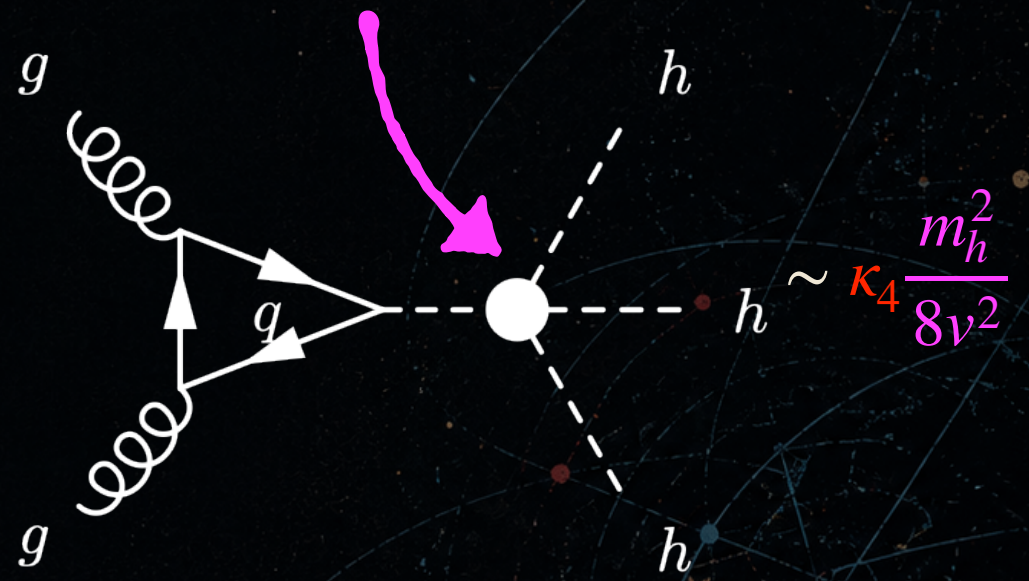
Tiny cross section @ LHC:

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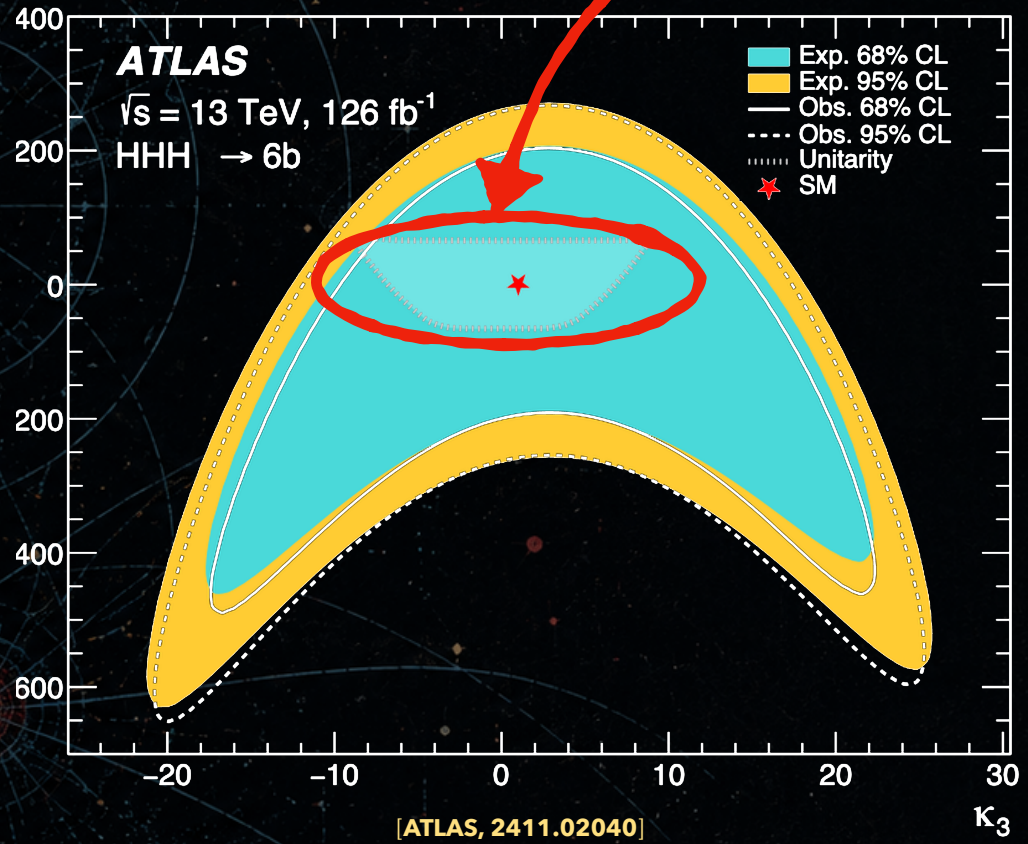
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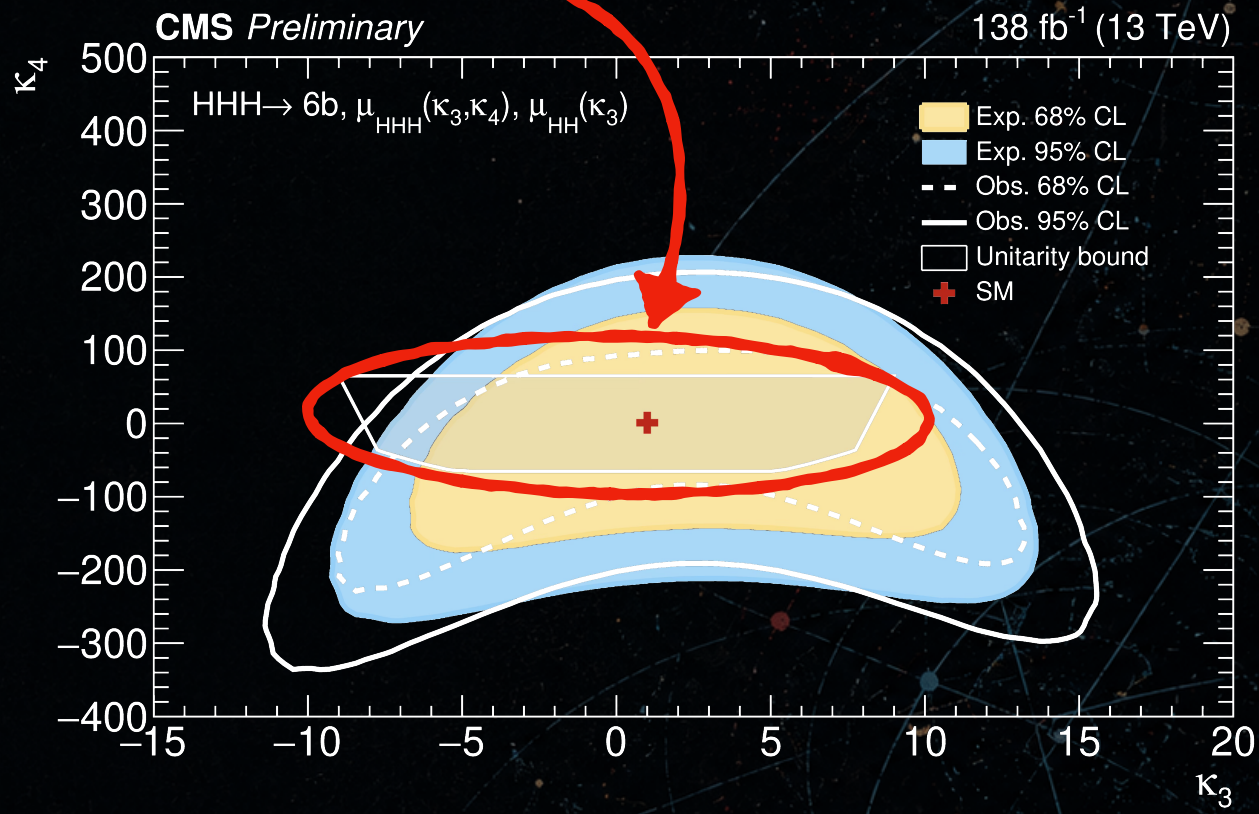
Quartic @ LHC (Present)

Final state: $pp \to (b\bar{b})(b\bar{b})(b\bar{b})$ ➡ **6 b-jets!**

perturbative unitarity*



[ATLAS, 2411.02040]



[CMS-PAS-HIG-24-012 & CMS-PAS-HIG-24-015]

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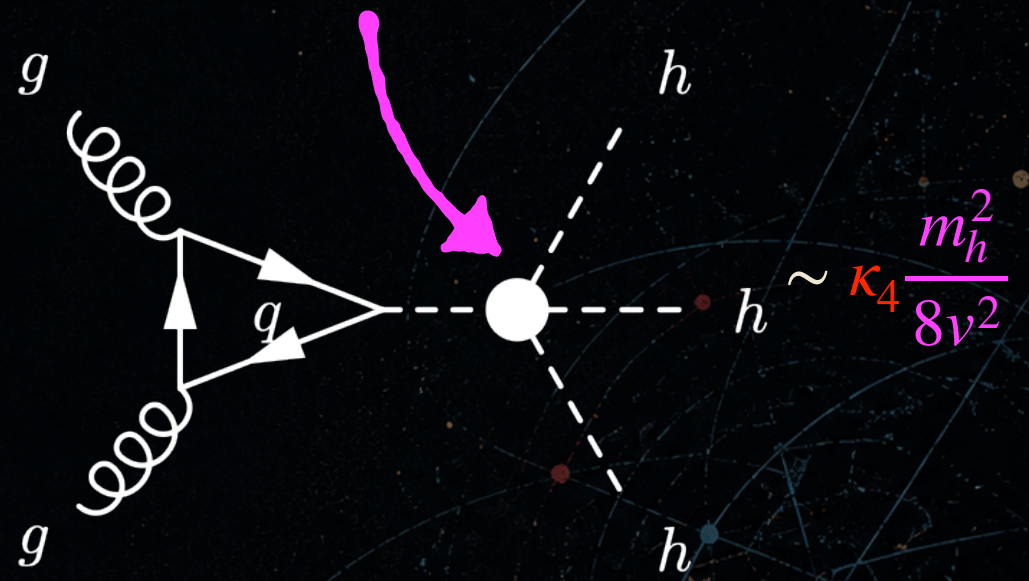
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$\kappa_4 \in [-533, 541]$ [CMS, 4b2γ]

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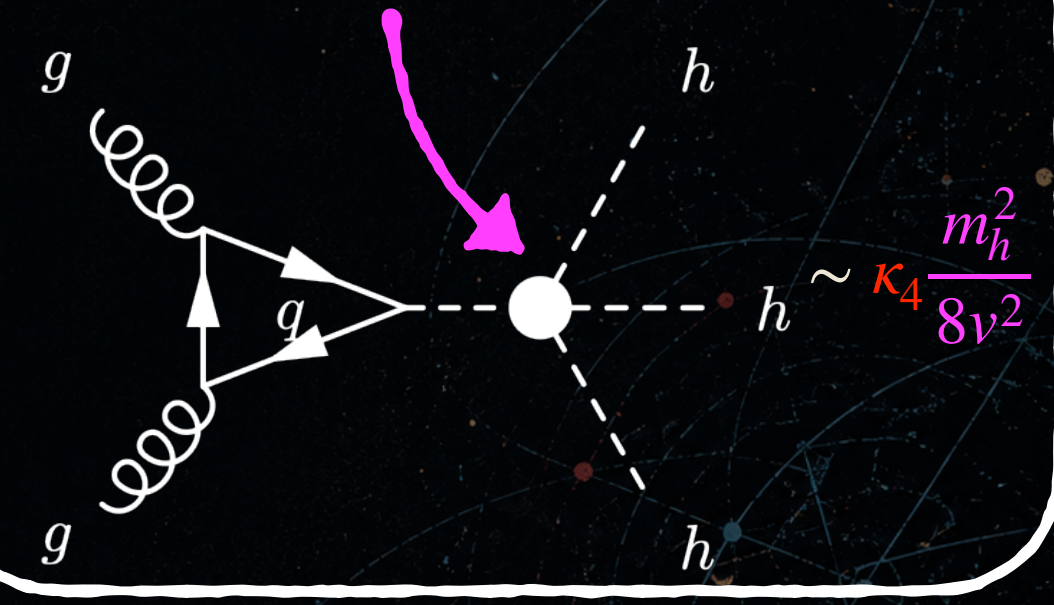
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SECTION 01-II

II: Extended Scalar Sectors

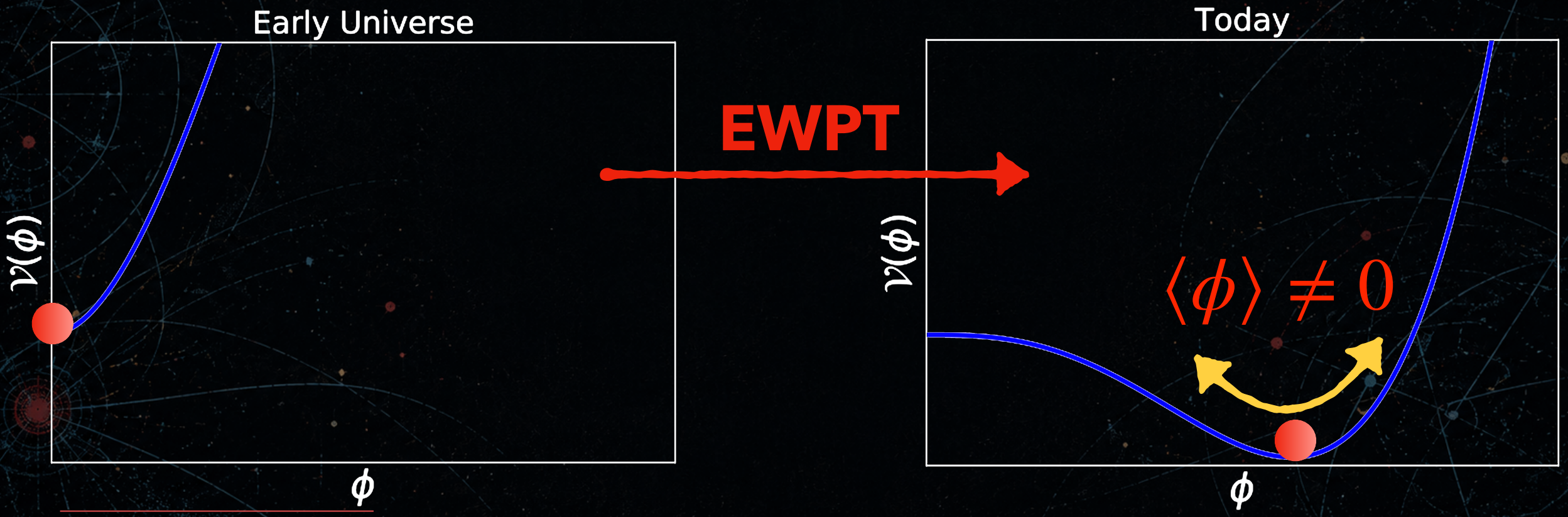
Extended Scalar Sectors:

Can catalyze a strong first-order **E**lectro**W**eak **P**hase **T**ransition, causing electroweak baryogenesis!



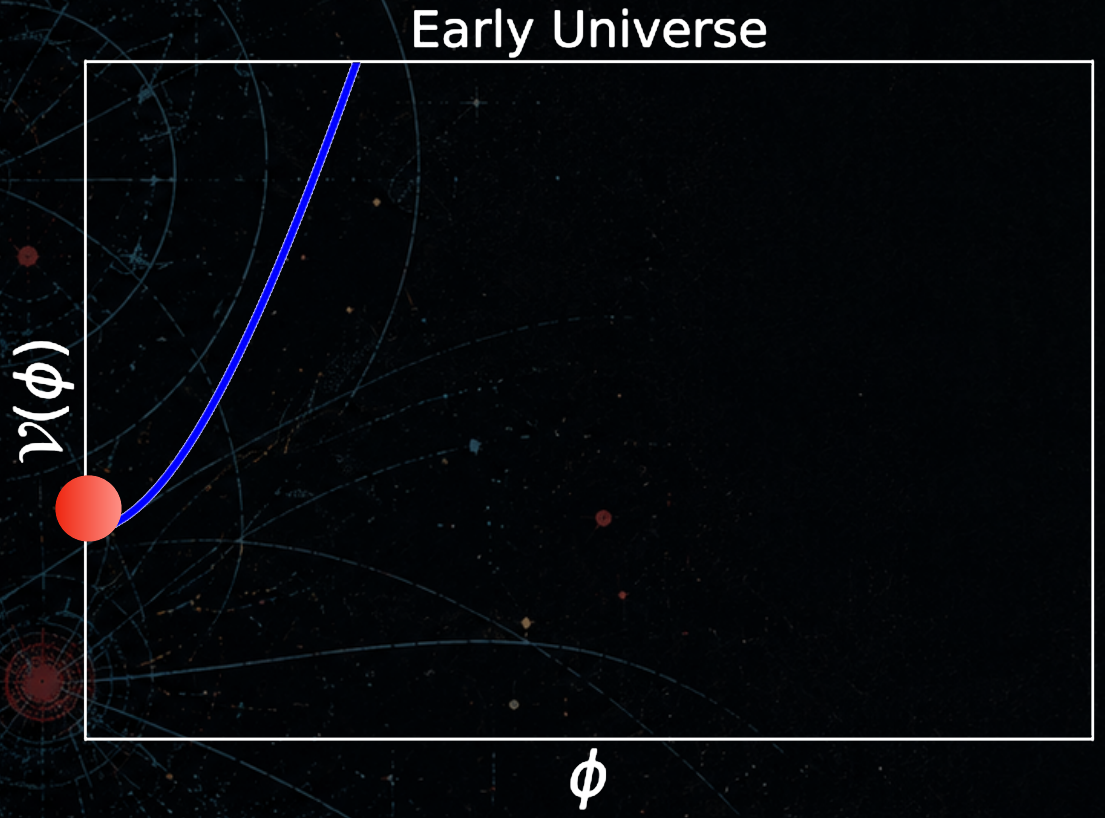
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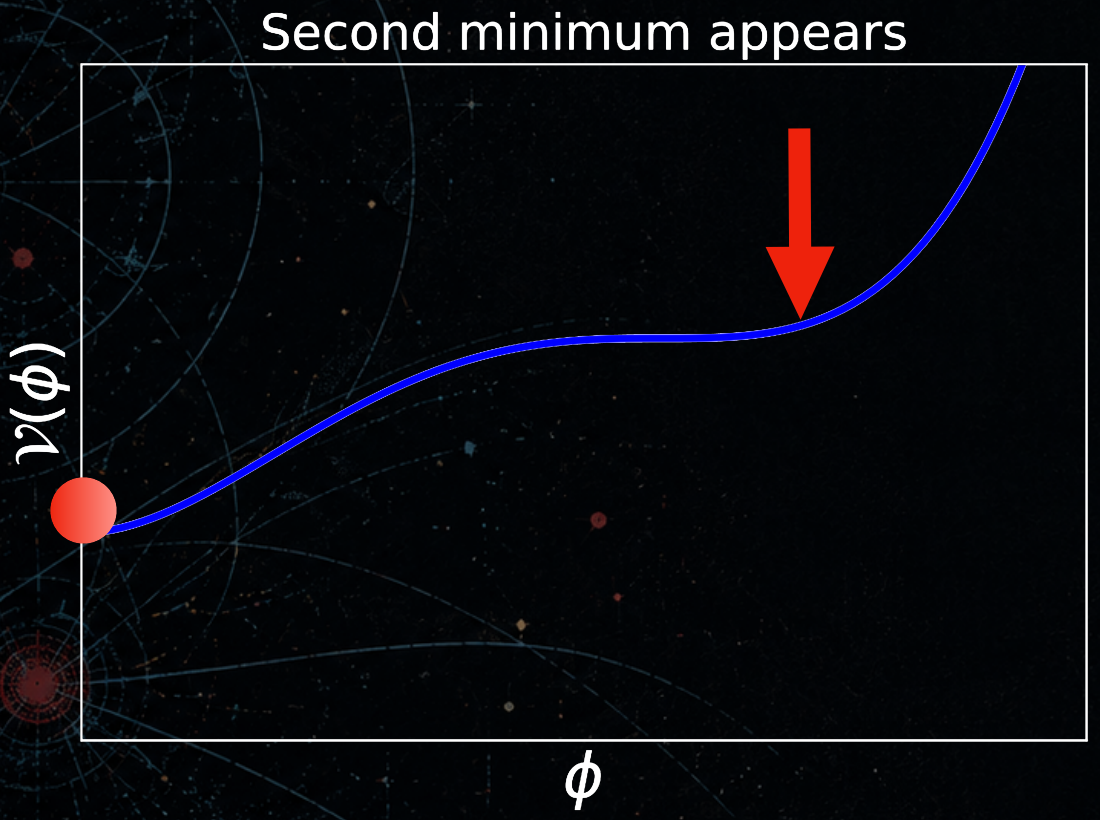


Higgs Potential evolution,

as the universe cools down.

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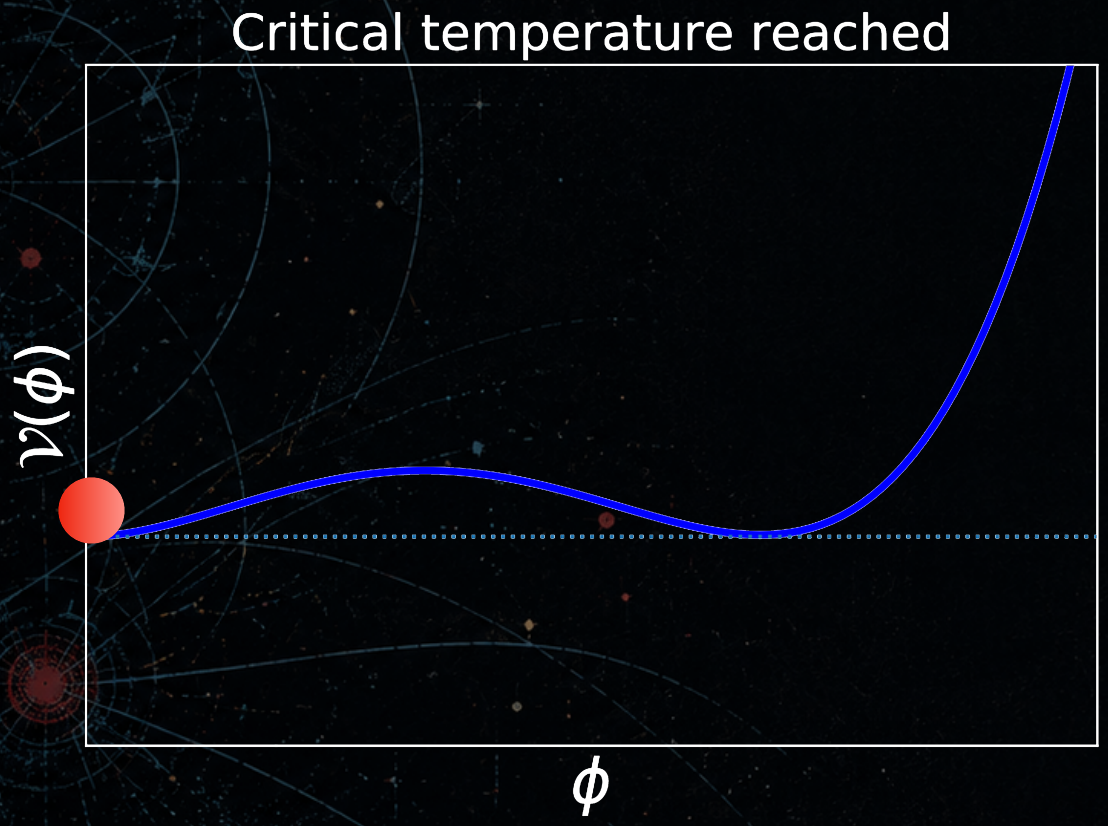


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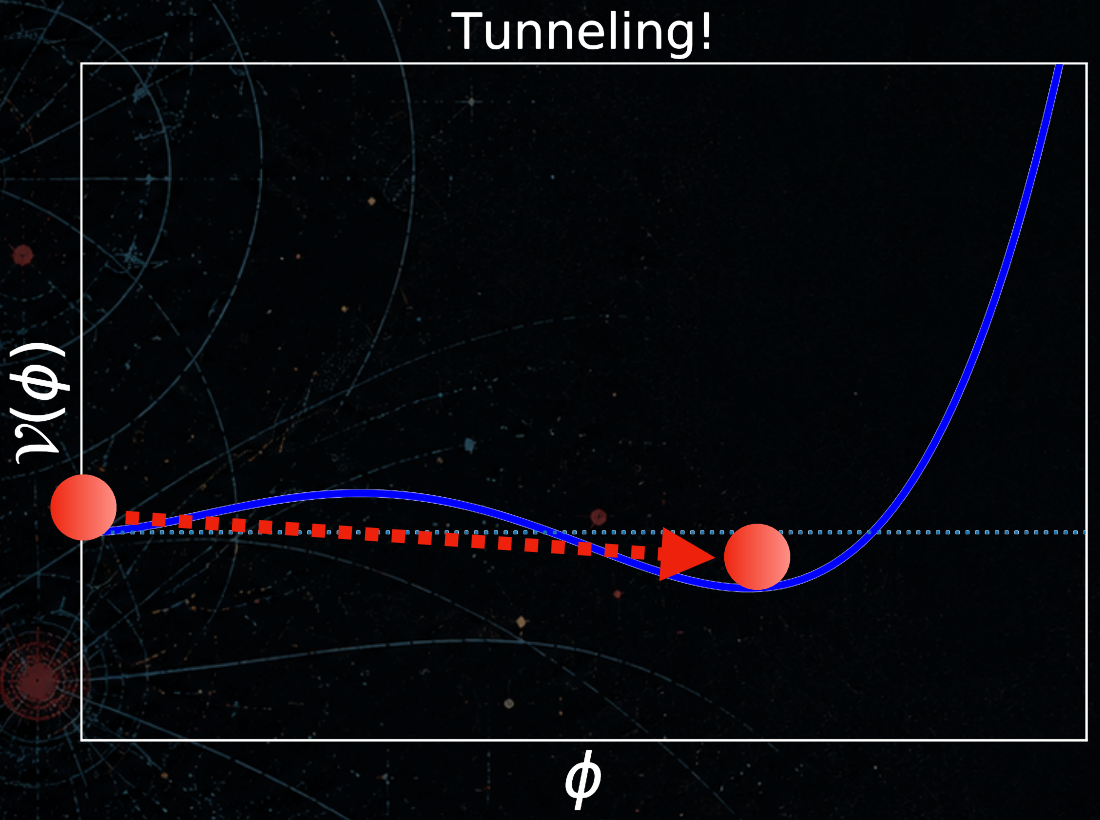


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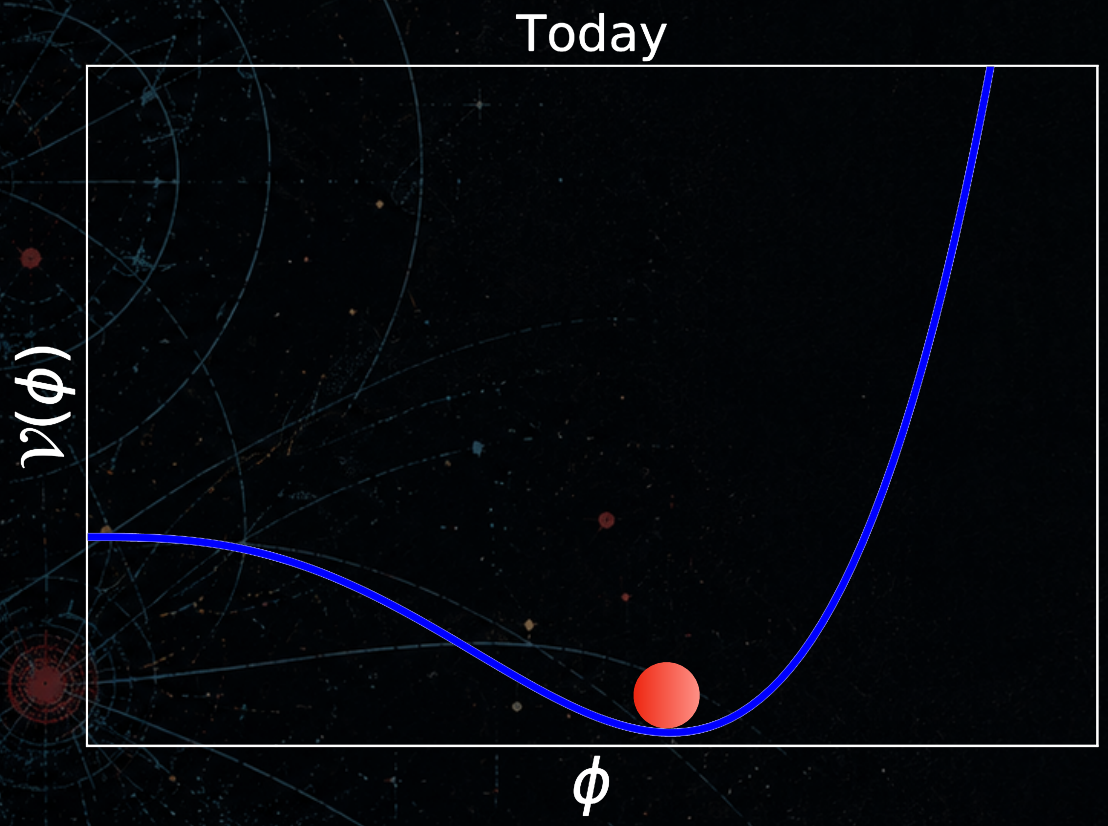


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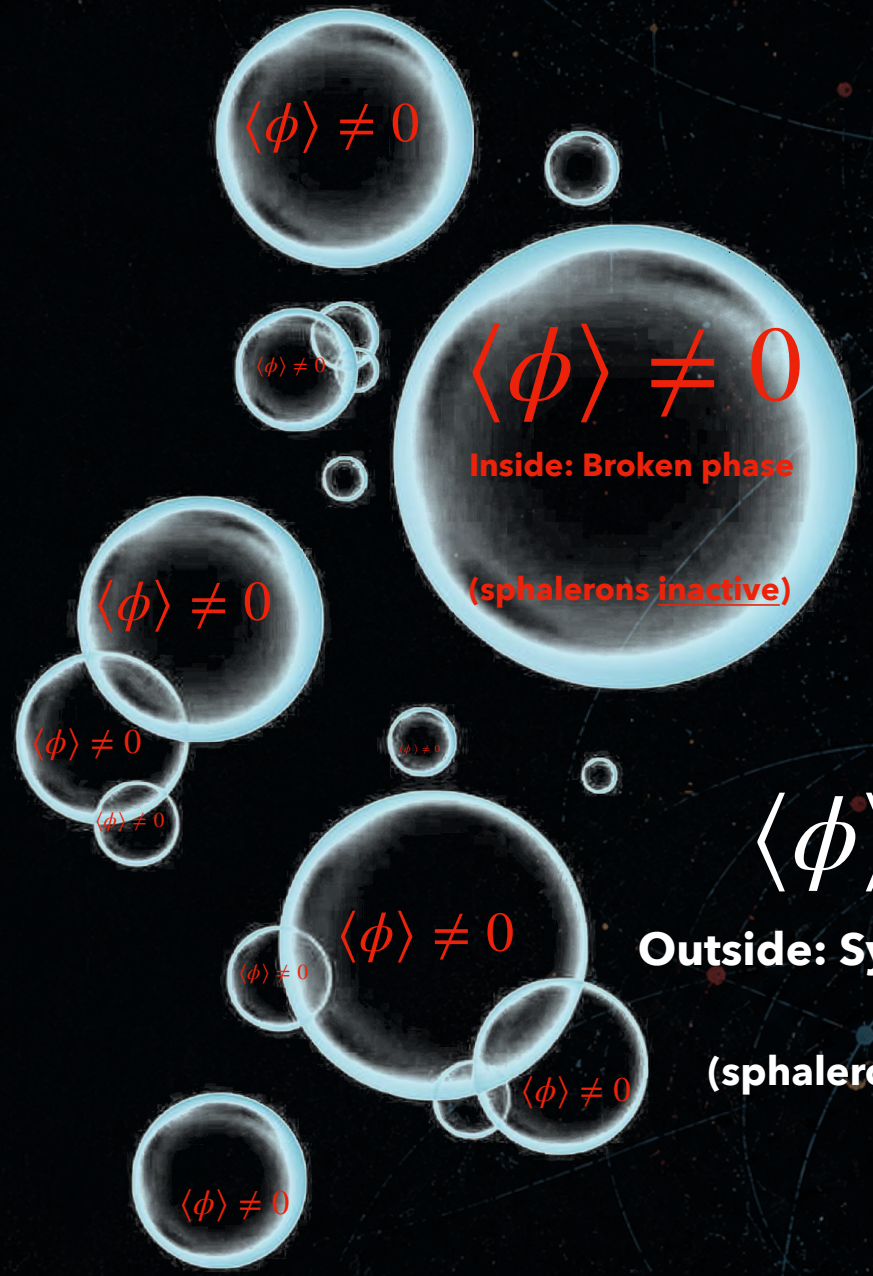


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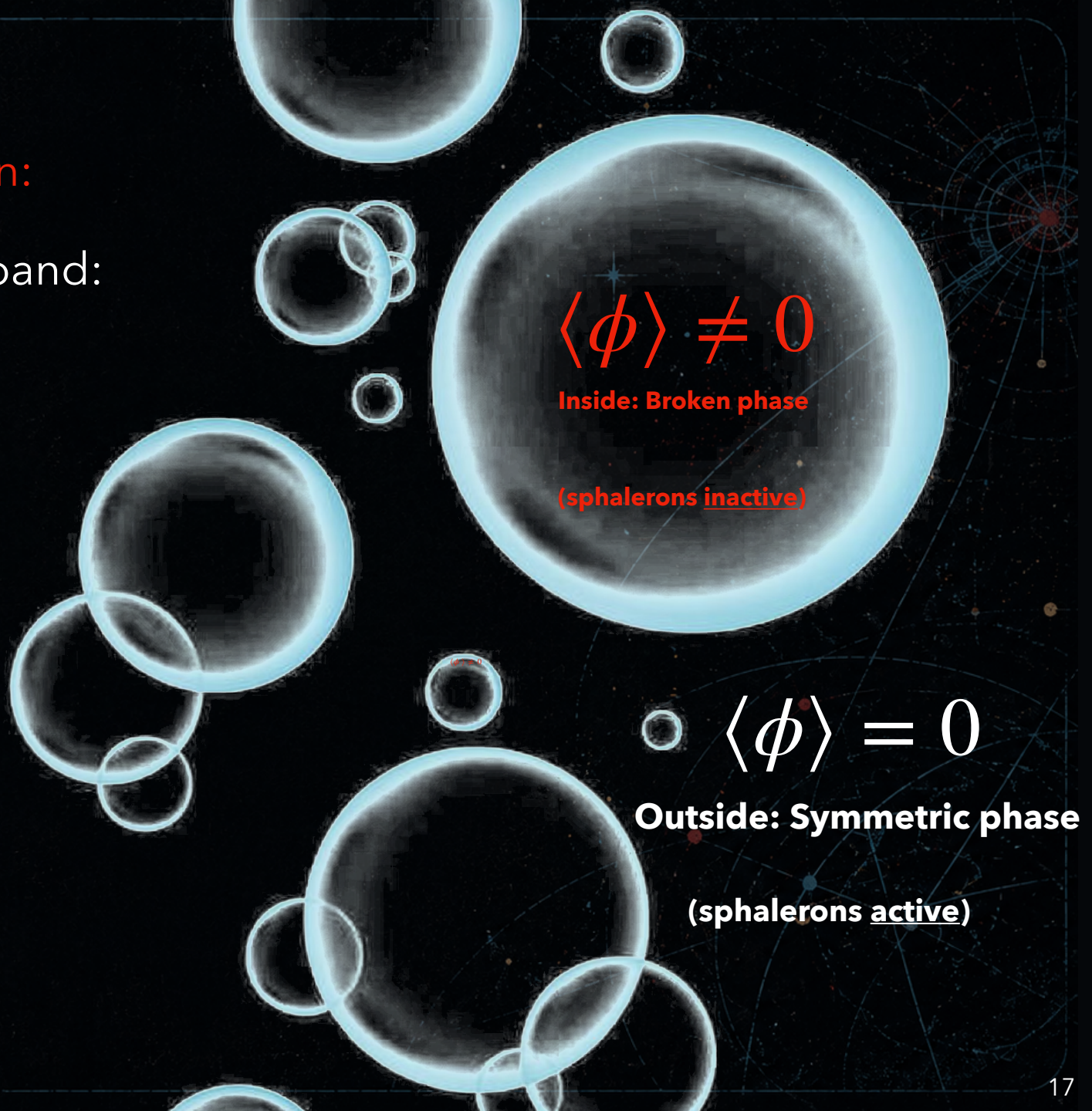
First-order **E**lectro**W**eak **P**hase **T**ransition:

Broken-phase Bubbles nucleate and expand:



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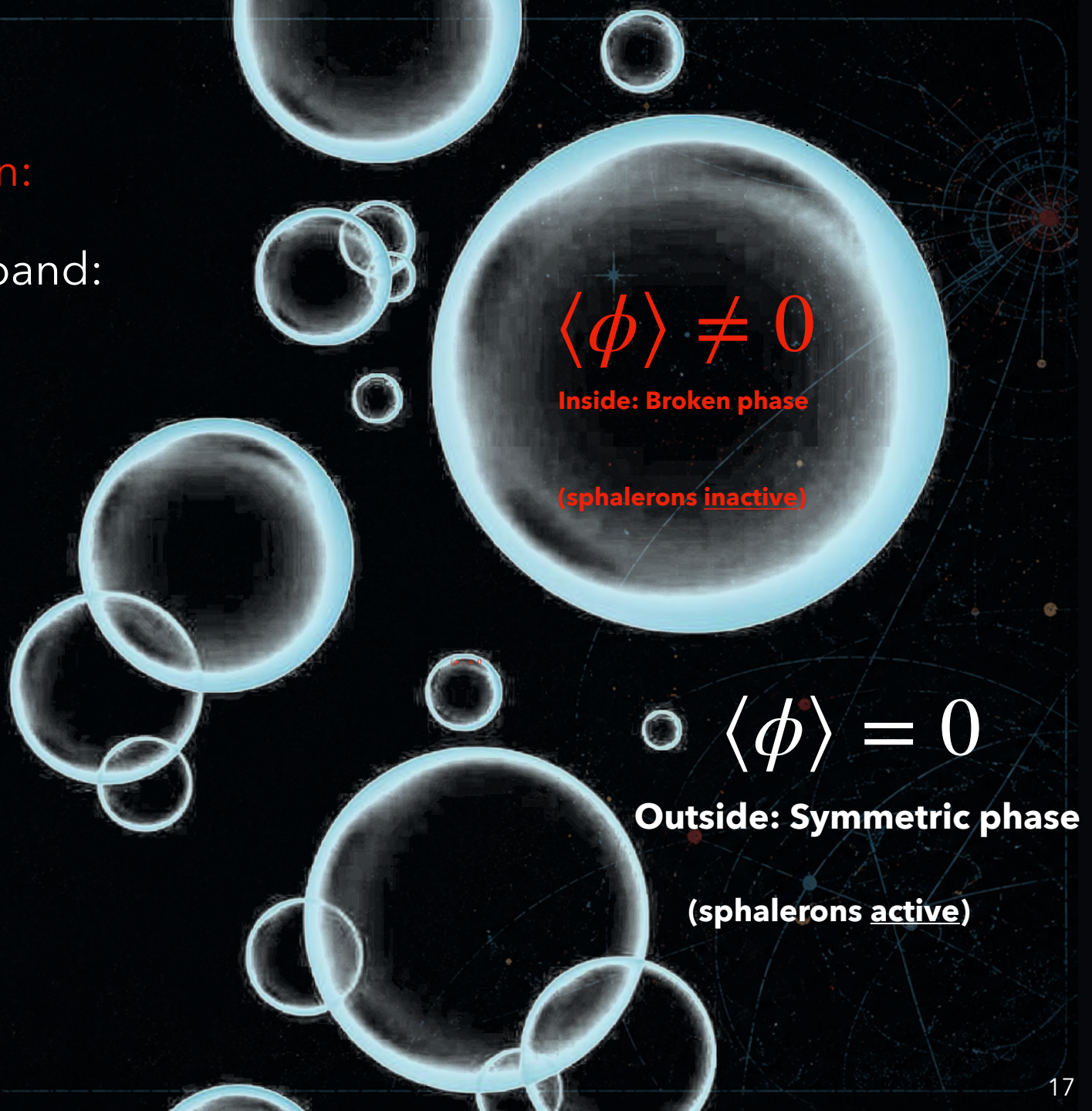
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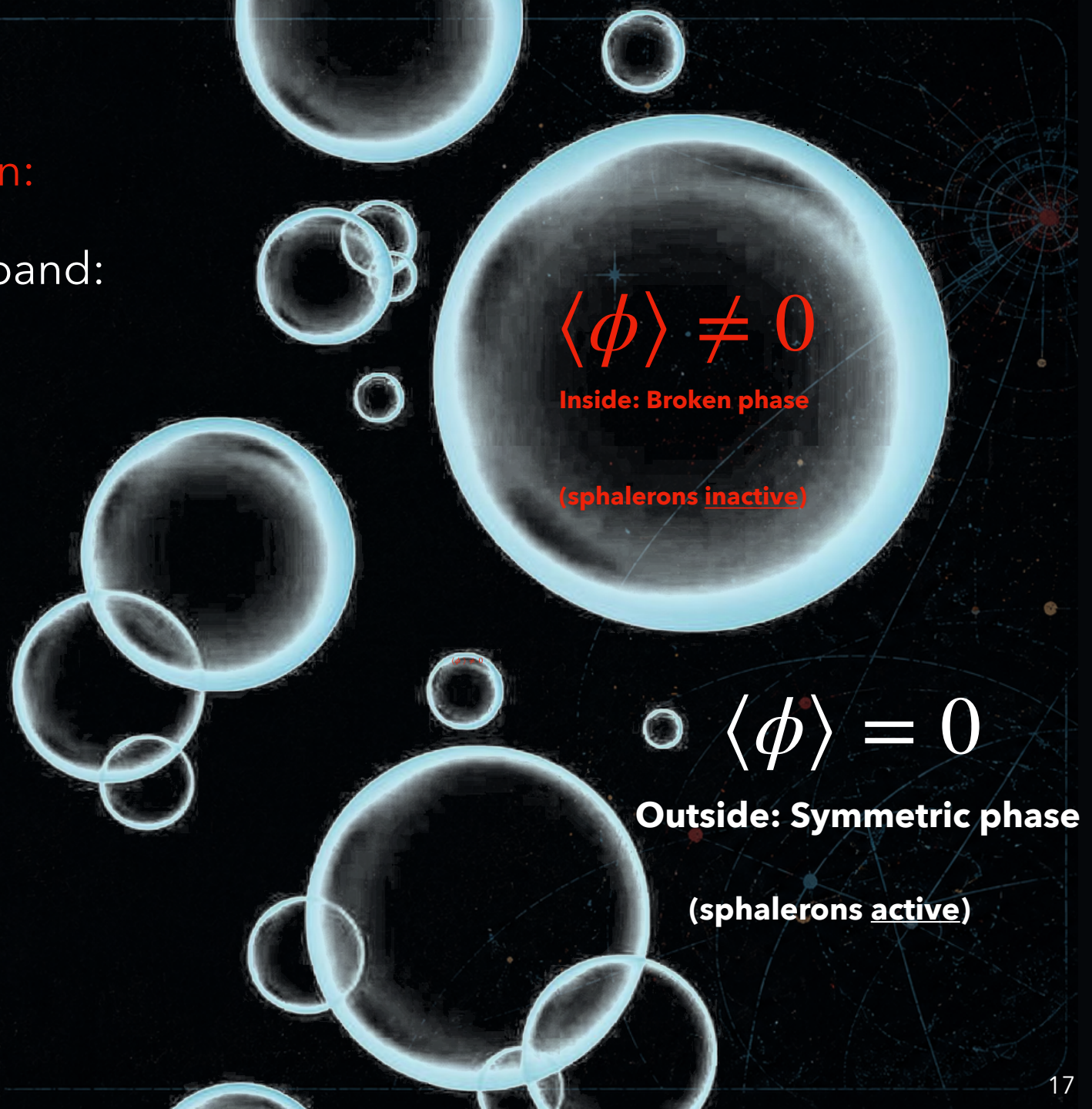


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  This is then "frozen" inside expanding bubbles (where **sphalerons** are **suppressed ➡ requires "Strong" EWPT**)

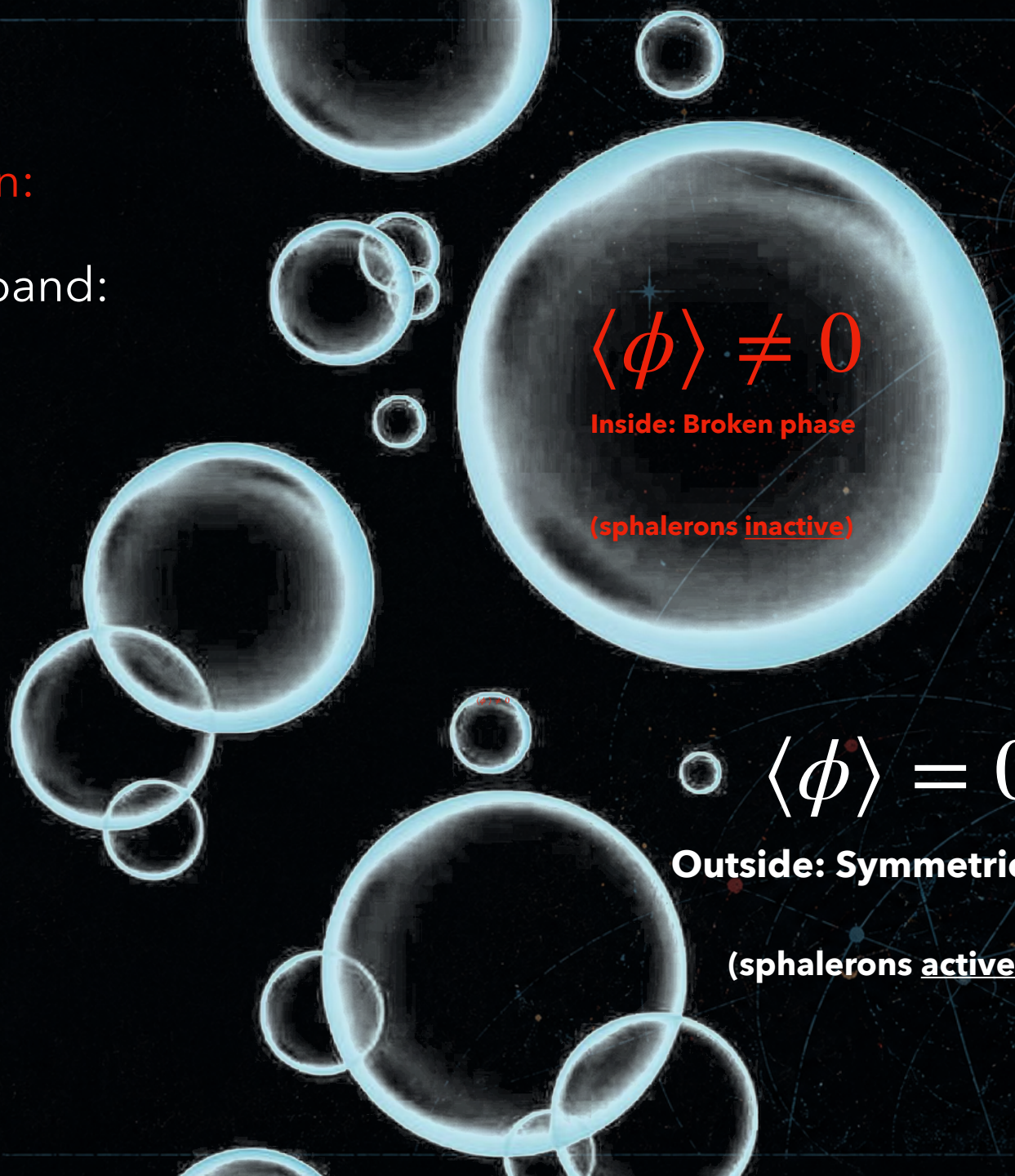


First-order **E**lectro**W**eak **P**hase **T**ransition:

- Broken-phase Bubbles nucleate and expand:

The problem? This ***does not*** occur in the SM! ➡ EWPT is "cross over"!

[Kajantie, Laine, Rummukainen, Shaposhnikov, hep-ph/9605288]



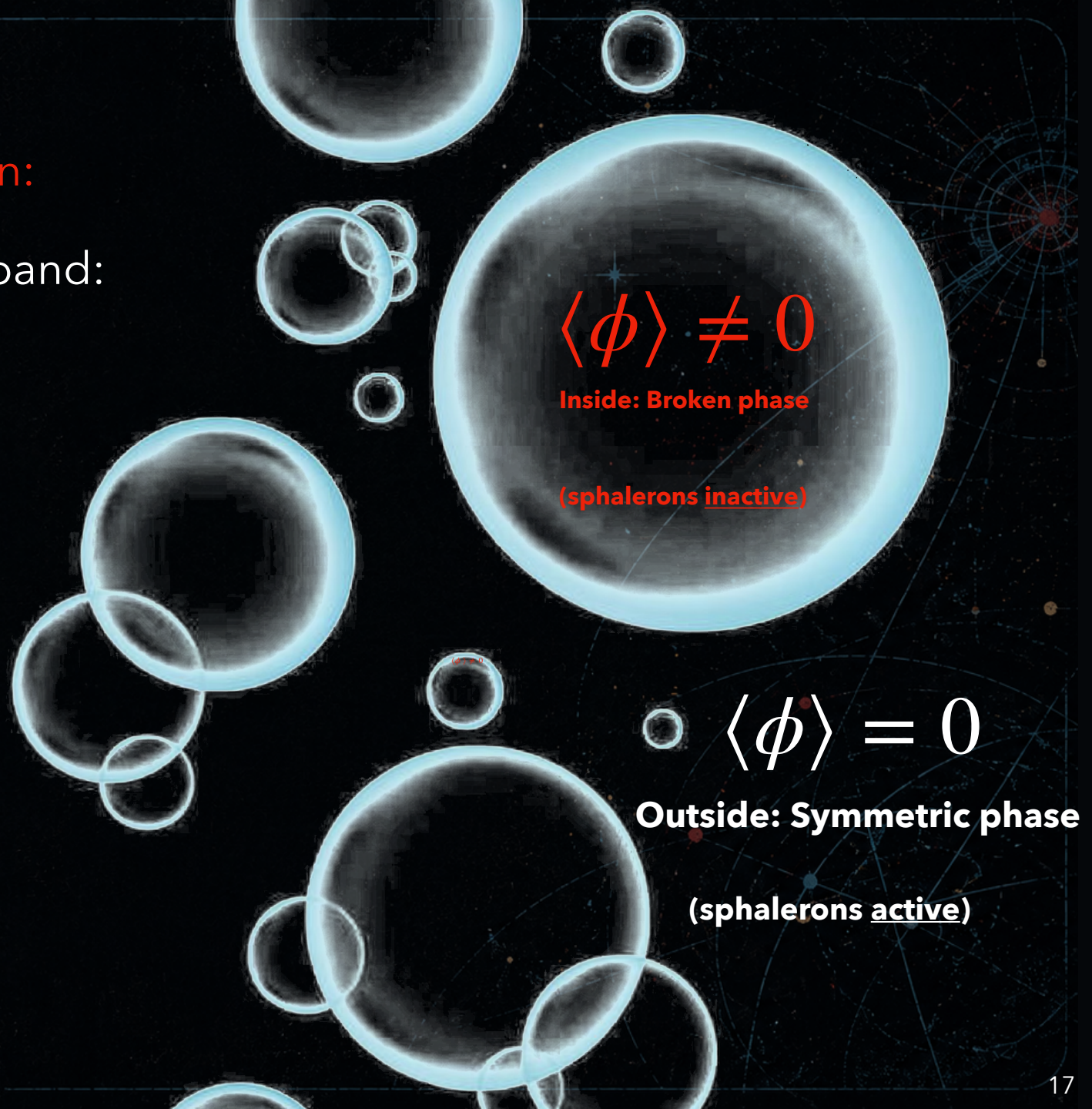
First-order ElectroWeak Phase Transition:

- Broken-phase Bubbles nucleate and expand:

The problem? This ***does not*** occur in the SM! ➡ EWPT is "cross over"!

[Kajantie, Laine, Rummukainen, Shaposhnikov, hep-ph/9605288]

Extended Scalar Sectors can modify the potential evolution, creating a barrier! ➡ **First order EWPT!**



Extended Scalar Sectors:

- They also may contain dark matter candidates!



Extended Scalar Sectors:

- They also may contain dark matter candidates!

A simple extension to SM: add **one** singlet scalar field:

$$\mathcal{V}(\phi, S) = \bullet\,|\phi|^2 + \blacksquare\,|\phi|^4$$

(Singlet = *No* SM charges)

$$+ \bullet\, S^2 + \blacktriangle\, S^3 + \blacksquare\, S^4$$



$$+ \blacktriangle\, |\phi|^2 S + \blacksquare\, |\phi|^2 S^2$$ ← "**Portal**" interactions.

$$+ S \times (\text{Hidden Sector}) + \cdots$$ ← **Dark Matter?**

Extended Scalar Sectors:

Another extension: add **two** singlet scalar fields + $\mathbb{Z}_2$ symmetries = TRSM:

$$\mathcal{V}(\phi, S, X) = \bullet\,|\phi|^2 + \blacksquare\,|\phi|^4 + \bullet\, S^2 + \blacksquare\, S^4 + \bullet\, X^2 + \blacksquare\, X^4 + \blacksquare\, S^2 X^2 + \blacksquare\, |\phi|^2 S^2 + \blacksquare\, |\phi|^2 X^2$$

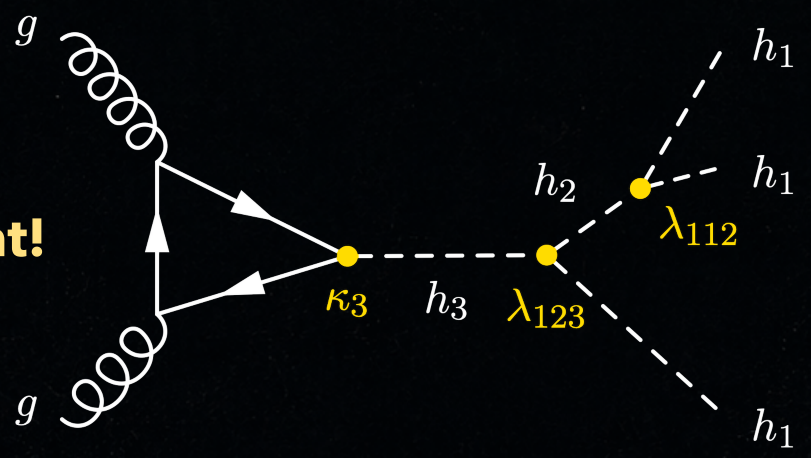
If ***no* vacuum expectation value** in X direction ➡ X = **Dark Matter?**

[Karkout, **AP**, Robens, Tetlalmatzi-Xolocotzi, in preparation]

Extended Scalar Sectors: Multi-scalar Pheno

SM + one singlet: Contains **one new heavy scalar** ➡ enhancements in ***hh*** production & gauge boson final states.

TRSM ➡ **two** new scalars, richer pheno, e.g.:



double-resonant enhancement!

[Robens, Stefaniak, Wittbrodt, 1908.08554,
AP, Robens, Tetlalmatzi-Xolocotzi, 2101.00037,
Karkout, **AP**, Postma, Tetlalmatzi-Xolocotzi, van de Vis, du Pree, 2404.12425
AP, Tetlalmatzi-Xolocotzi, 2501.14866]

Requires:

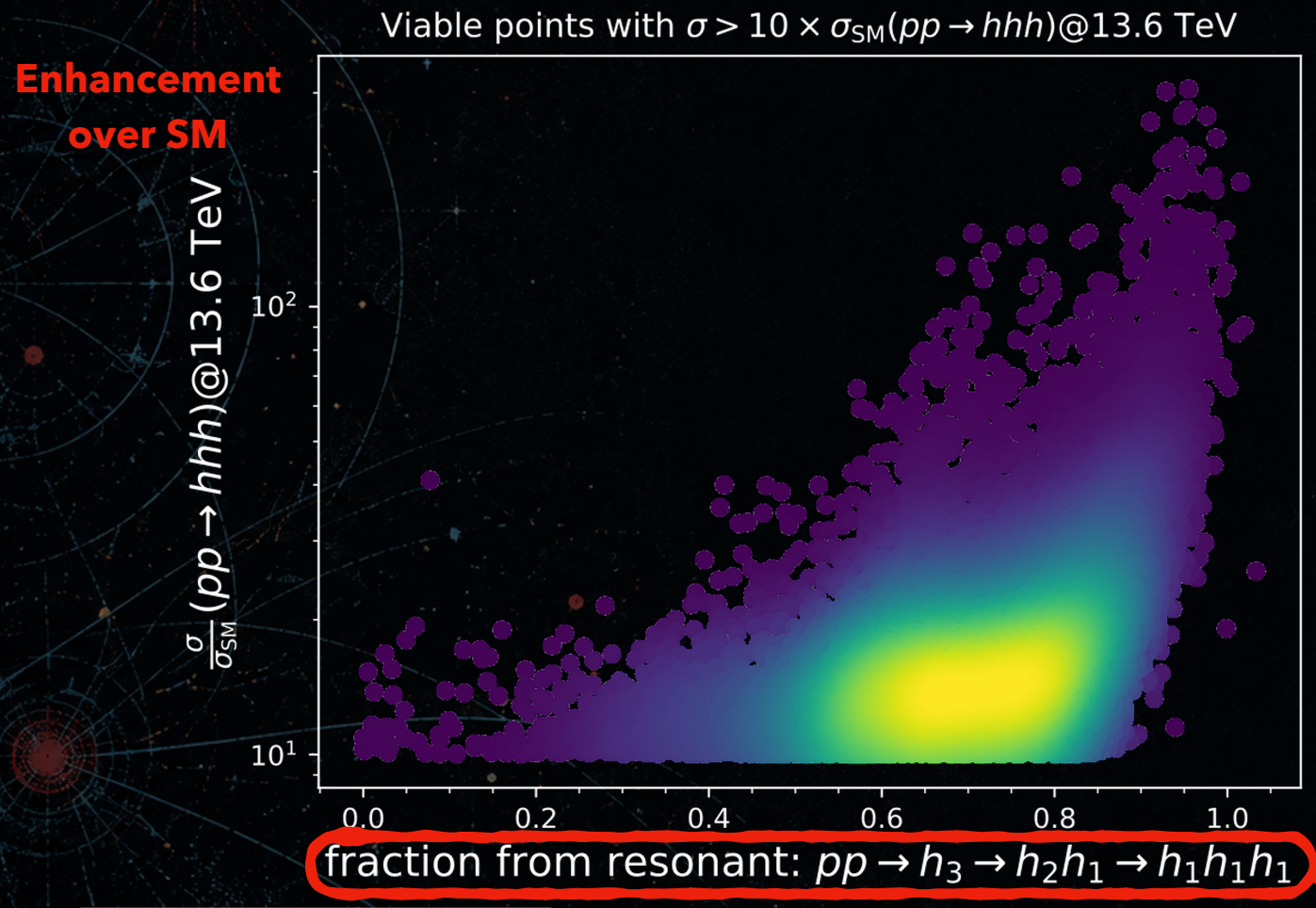
$m_3 > m_2 + m_1,\ m_2 > 2m_1$

and

VEVs for X & S nonzero!

Extended Scalar Sectors: Multi-scalar Pheno

- Enhancement of ***hhh*** ⇔ Large "fraction" of double-resonant process!

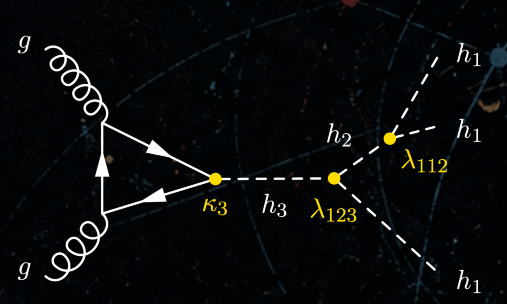


[Karkout, **AP**, Postma, Tetlalmatzi-Xolocotzi, van de Vis, du Pree, **2404.12425**]

TRSM parameter-space scan:

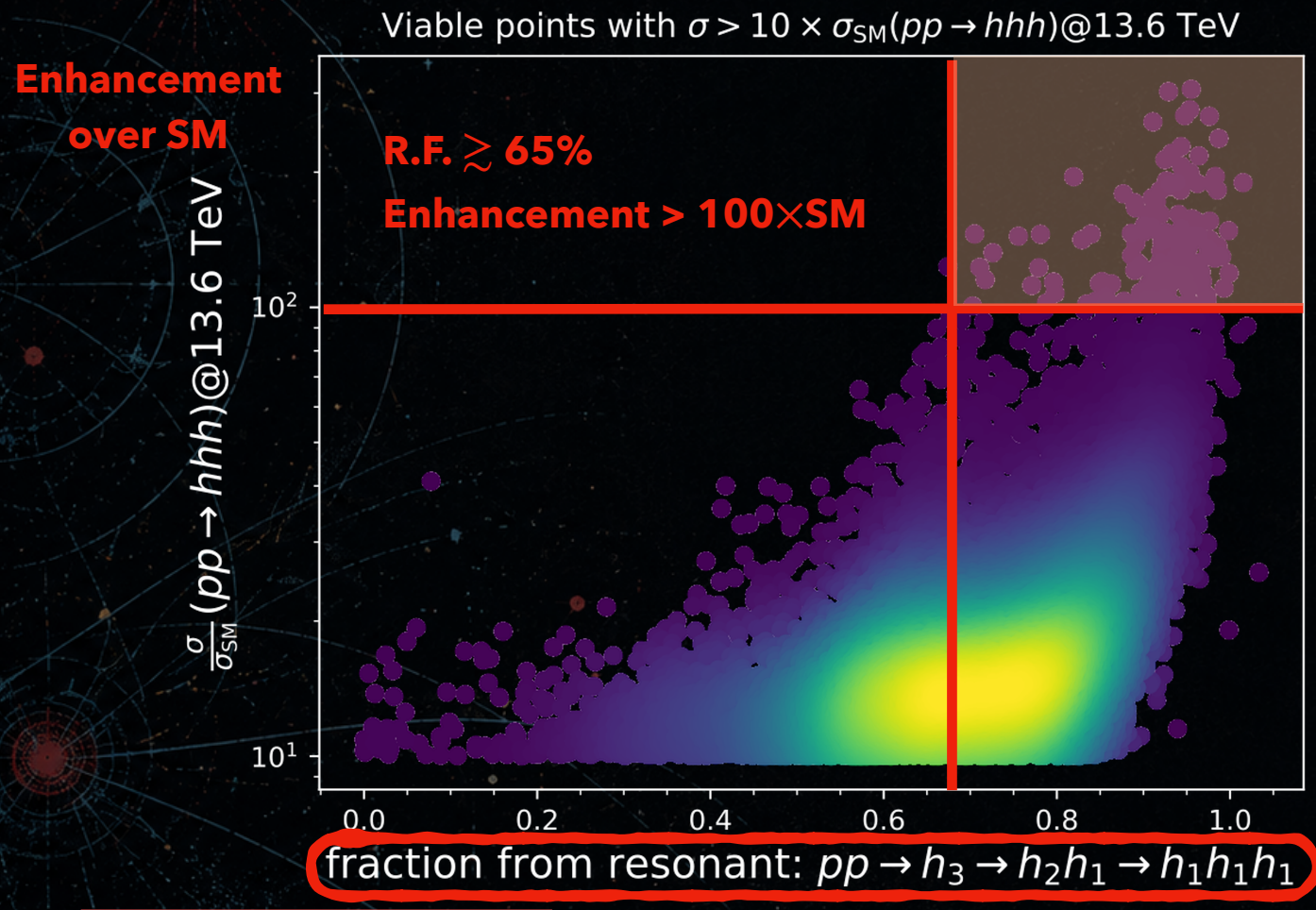
Including boundedness of potential + perturbativity + `HiggsTools` constraints.

Resonant Fraction (R.F.) =
How much of the total cross section comes from… ?



Extended Scalar Sectors: Multi-scalar Pheno

- Enhancement of ***hhh*** ⇔ Large "fraction" of double-resonant process!

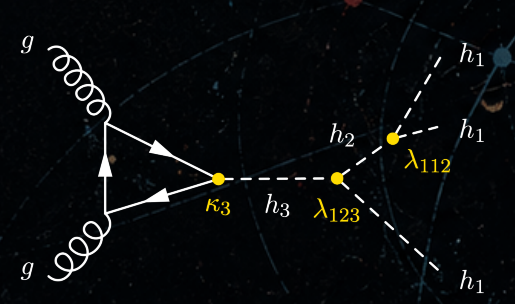


[Karkout, **AP**, Postma, Tetlalmatzi-Xolocotzi, van de Vis, du Pree, **2404.12425**]

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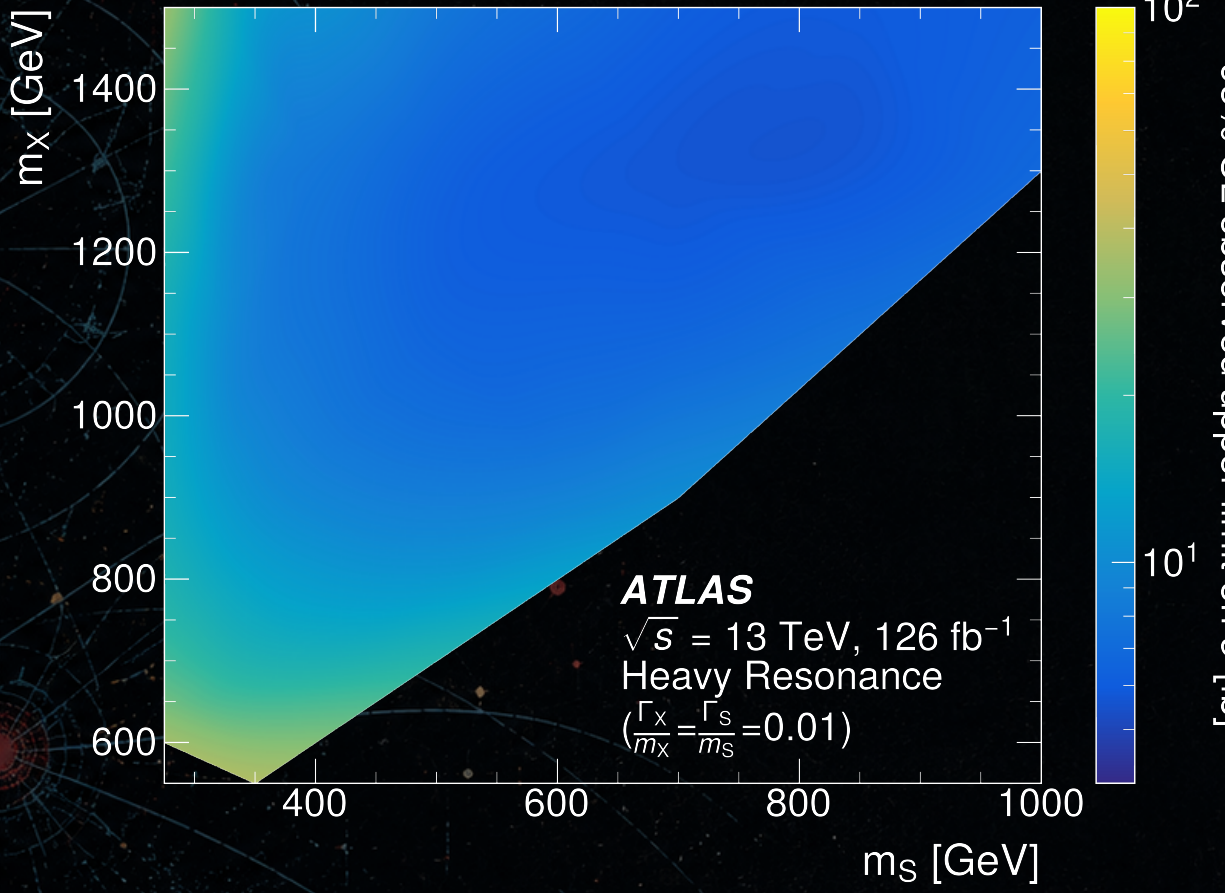
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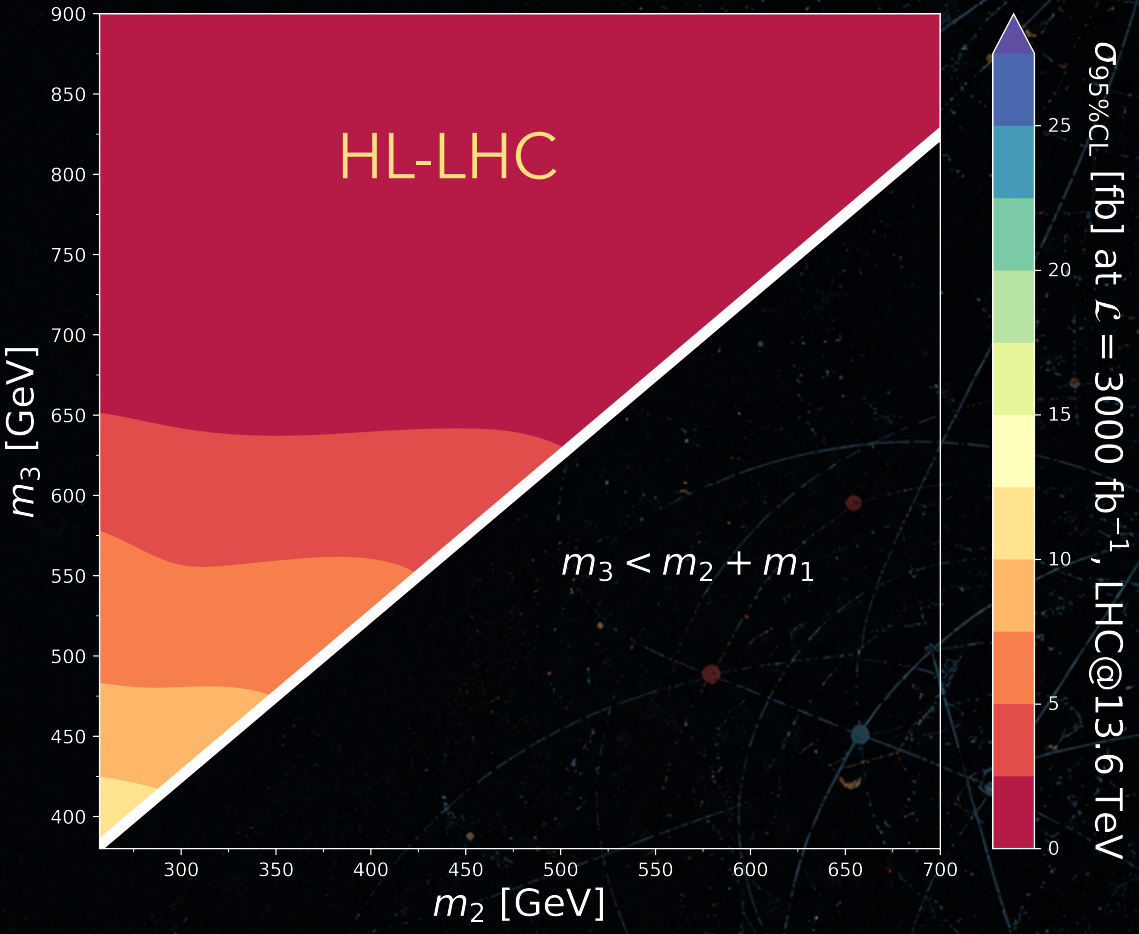
Extended Scalar Sectors: Double-Resonant *hhh*

Already being searched for in ATLAS $pp \to hhh \to 6b$ jets!

[ATLAS, 2411.02040]



[**AP**, Tetlalmatzi-Xolocotzi, 2501.14866]



SECTION 02

Theory & Phenomenology

A Brief History of Triple Higgs Boson Pheno

2005 → 2006 → 2012 → 2015+

[Plehn, Rauch, hep-ph/057321]

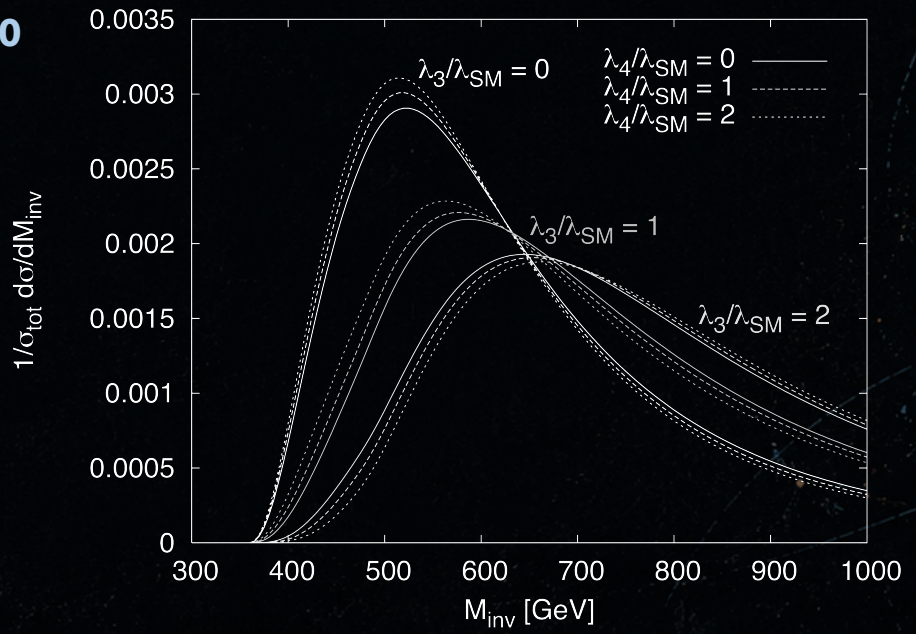
"VLHC" @ 200 TeV → "Little hope" & "Measuring the quartic requires good knowledge of trilinear"

[Binoth, Karg, Kauer, Rückl, hep-ph/0608057]

"Multi-Higgs rates sensitive to BSM, extreme values of quartic lead to $\mathcal{O}$(10 fb) @ LHC"

[Higgs Boson Discovered!]

Several studies!
See next slide!



(Modern) History of Triple Higgs Boson Pheno

"Interesting" final states + at least 100 events @ 100 TeV @ 30 ab^{-1}

First studies (mostly) at 100 TeV pp!

$hhh \to$ final state	BR (%)	σ (ab)	$N_{30\text{ab}^{-1}}$
$(b\bar{b})(b\bar{b})(b\bar{b})$	19.21	1110.338	33310
$(b\bar{b})(b\bar{b})(WW_{1\ell})$	7.204	416.41	12492
$(b\bar{b})(b\bar{b})(\tau\bar{\tau})$	6.312	364.853	10945
$(b\bar{b})(\tau\bar{\tau})(WW_{1\ell})$	1.578	91.22	2736
$(b\bar{b})(b\bar{b})(WW_{2\ell})$	0.976	56.417	1692
$(b\bar{b})(WW_{1\ell})(WW_{1\ell})$	0.901	52.055	1561
$(b\bar{b})(\tau\bar{\tau})(\tau\bar{\tau})$	0.691	39.963	1198
$(b\bar{b})(b\bar{b})(ZZ_{2\ell})$	0.331	19.131	573
$(b\bar{b})(WW_{2\ell})(WW_{1\ell})$	0.244	14.105	423
$(b\bar{b})(b\bar{b})(\gamma\gamma)$	0.228	13.162	394
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$(\tau\bar{\tau})(WW_{1\ell})(WW_{1\ell})$	0.099	5.702	171
$(\tau\bar{\tau})(\tau\bar{\tau})(WW_{1\ell})$	0.086	4.996	149
$(b\bar{b})(ZZ_{2\ell})(WW_{1\ell})$	0.083	4.783	143
$(b\bar{b})(\tau\bar{\tau})(ZZ_{2\ell})$	0.073	4.191	125

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[**AP**, Sakurai, **1508.06524**, Chen, Yan, Zhao, Zhao, Zhong, **1510.04013**, Fuks, Kim, Lee, **1510.07697, ...**]

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[Fuks, Kim, Lee, **1510.07697**, Fuks, Kim, Lee, **1704.04298**]

[Kilian, Sun, Yan, Zhao, Zhao, **1702.03554**]

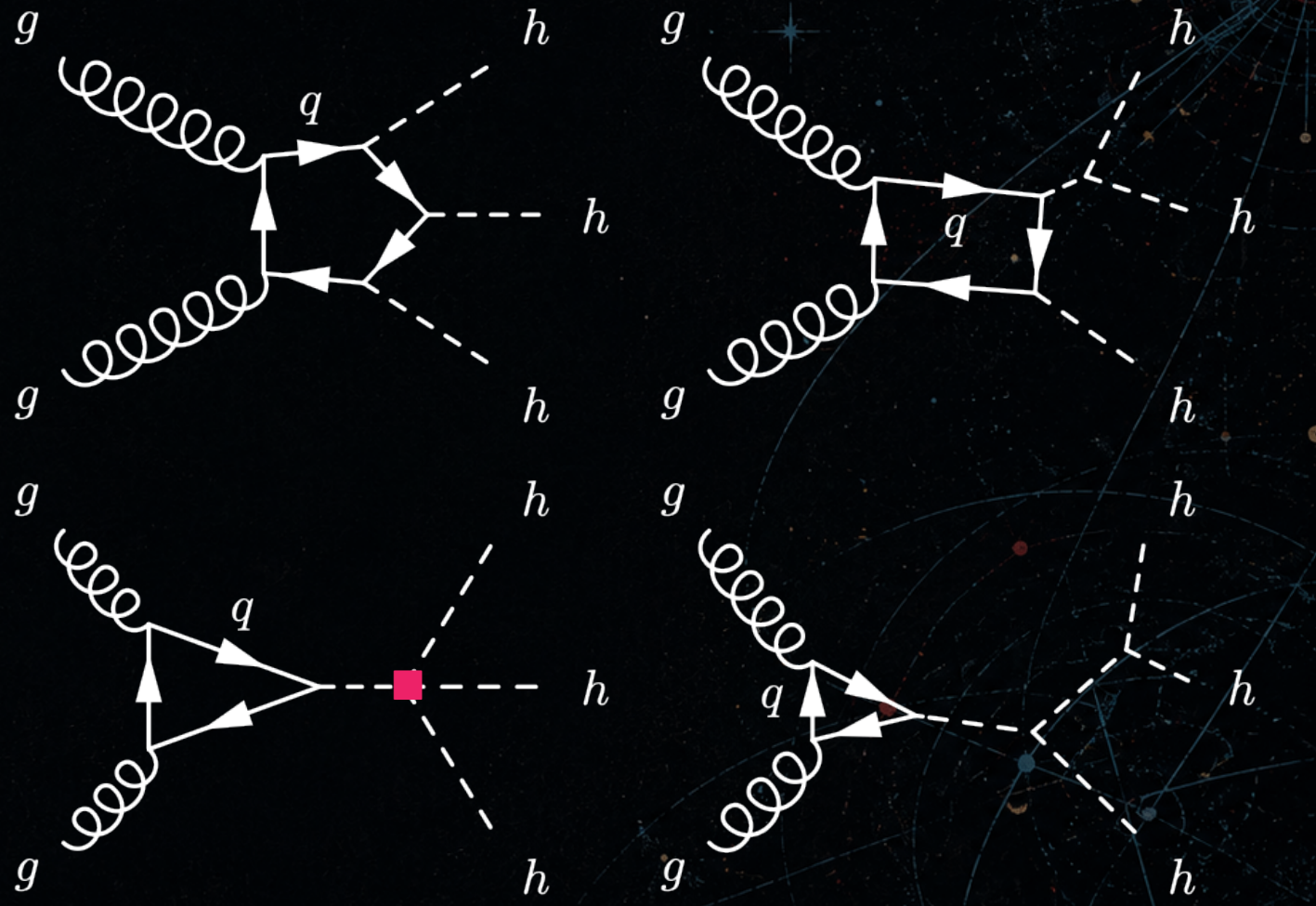
[Stylianou, Weiglein, **2312.04646**, Dong, Sun, Goo, Zhang, Li, Wang, **2504.04037**]

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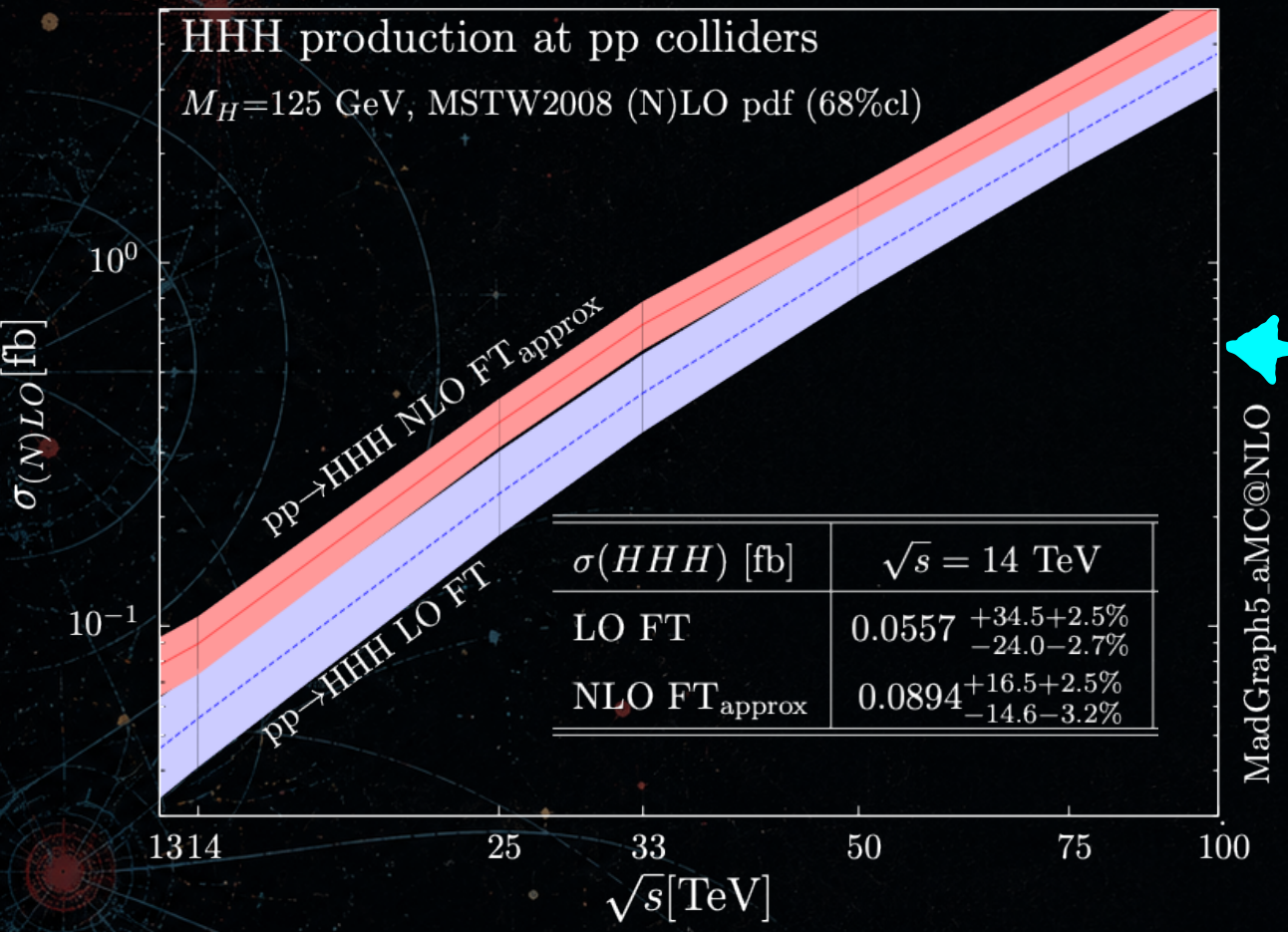
The $pp \to hhh$ Cross Section

At **LO** in SM: Loop-induced!

$\sigma^{\text{LO}}_{\text{SM}}(14\ \text{TeV}) \sim 0.05\ \text{fb}$

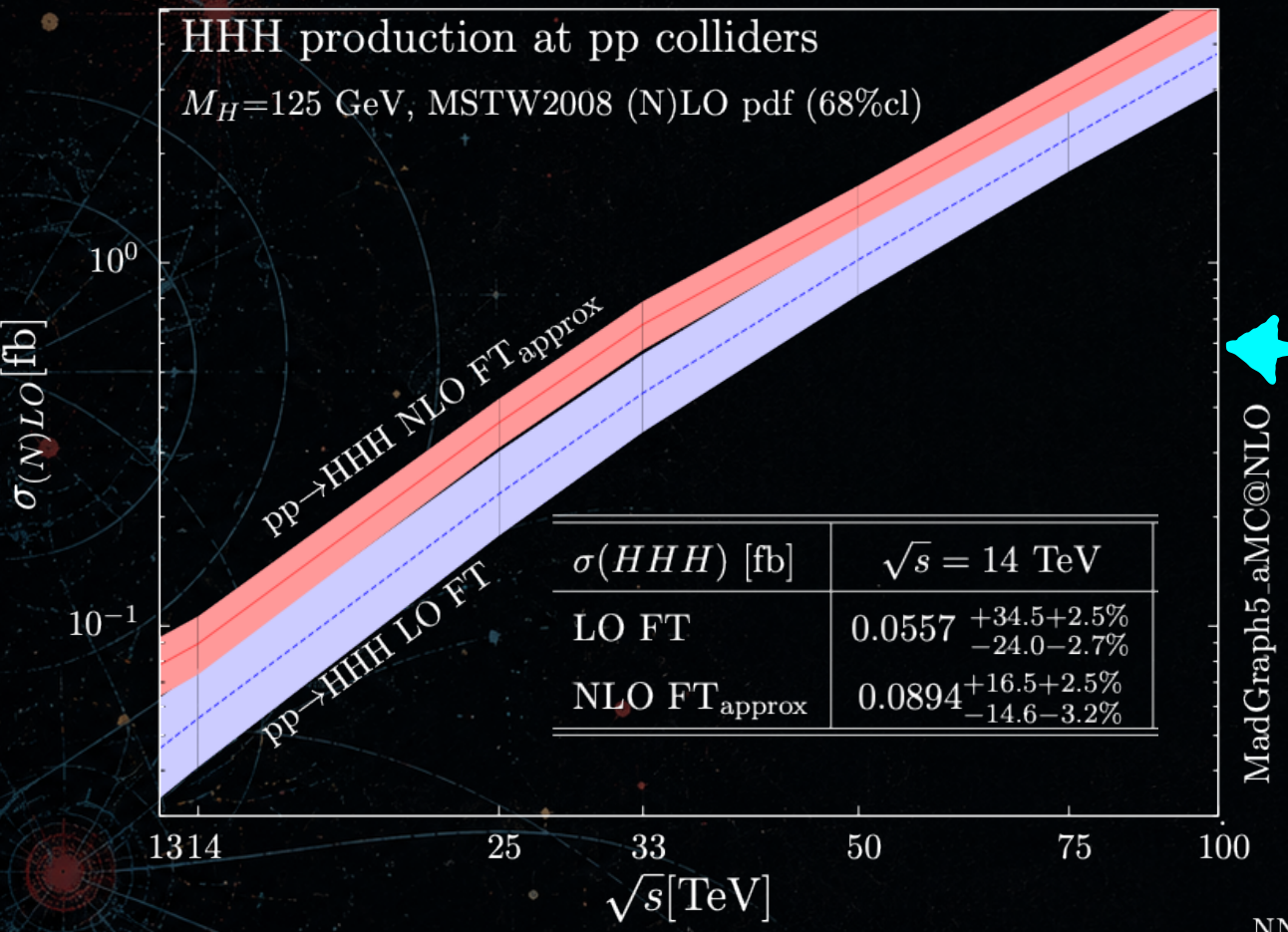


$pp \to hhh$ @ Higher Orders



NLO: [Maltoni, Vryonidou, Zaro, **1408.6542**] → $m_t \to \infty$ plus full real emission (= FT$_{approx}$).

$pp \to hhh$ @ Higher Orders



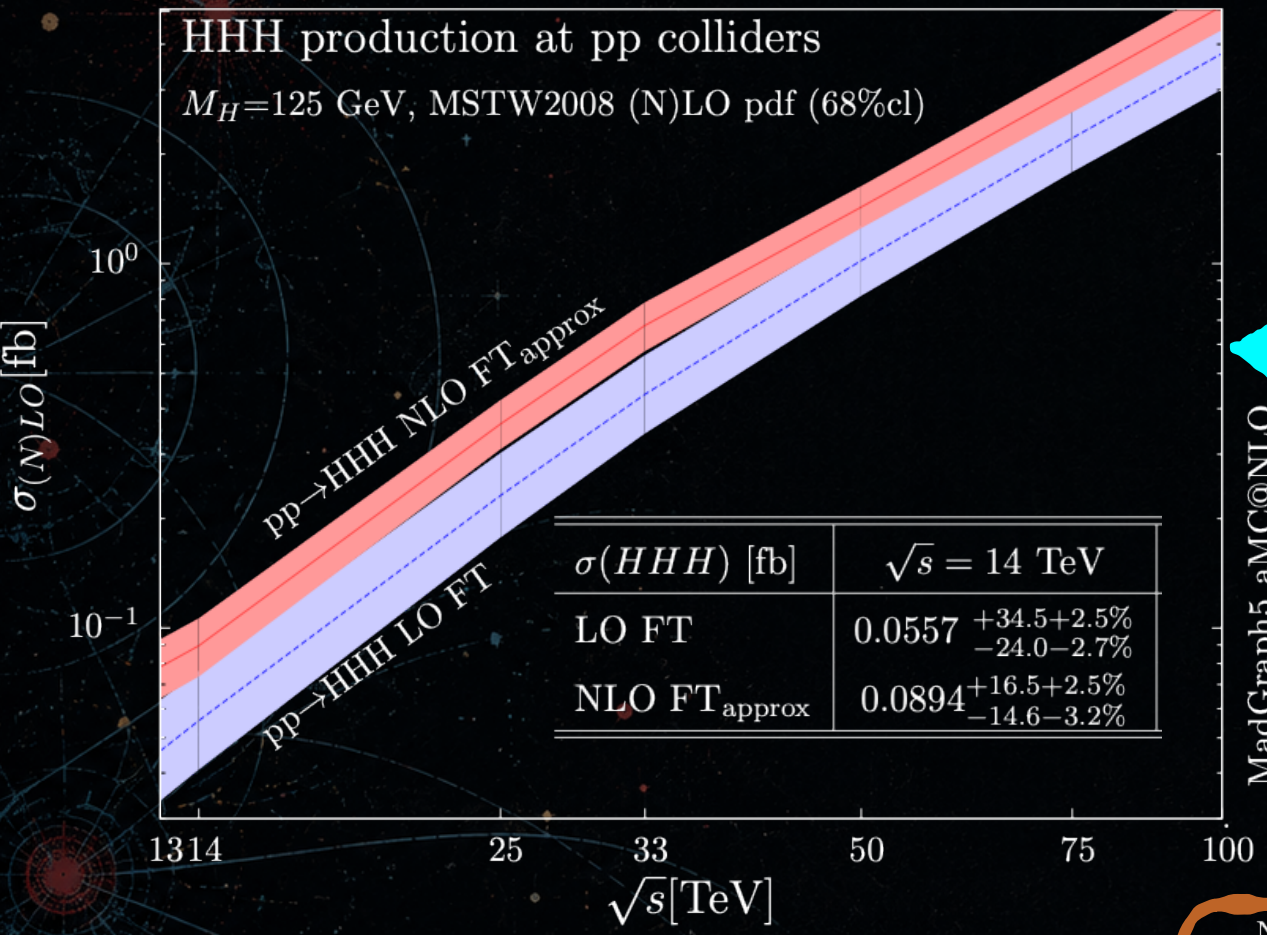
$\sigma(HHH)$ [fb]	$\sqrt{s} = 14$ TeV
LO FT	$0.0557\ ^{+34.5+2.5\%}_{-24.0-2.7\%}$
NLO FT$_{approx}$	$0.0894^{+16.5+2.5\%}_{-14.6-3.2\%}$

NLO: [Maltoni, Vryonidou, Zaro, **1408.6542**] → $m_t \to \infty$ plus full real emission (= FT$_{approx}$).

NNLO: [De Florian, Fabre, Mazzitelli, **1912.02760**] → Full NNLO in $m_t \to \infty$, "Born-Improved" ⇒ ~10% increase wrt. NLO.

$$\sigma^{\rm NNLO}_{\rm Best} = 0.103^{+5\%}_{-8\%} \pm 15\%\,{\rm fb}, \qquad K^{\rm NNLO}_{\rm Best} = 1.70, \qquad (14\ {\rm TeV})$$

$pp \to hhh$ @ Higher Orders



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Attack of the Anomalous Couplings

Anomalous couplings can enhance ***hhh***! e.g. *inspired* by an Effective Field Theory.

$$\mathscr{V}(h) \supset \underbrace{(1+c_3)}_{\kappa_3}\frac{m_h^2}{2v}h^3 + \underbrace{(1+d_4)}_{\kappa_4}\frac{m_h^2}{8v^2}h^4$$

⇒ ***hhh*** cross section as ratio to SM (highest-to-lowest coefficient):

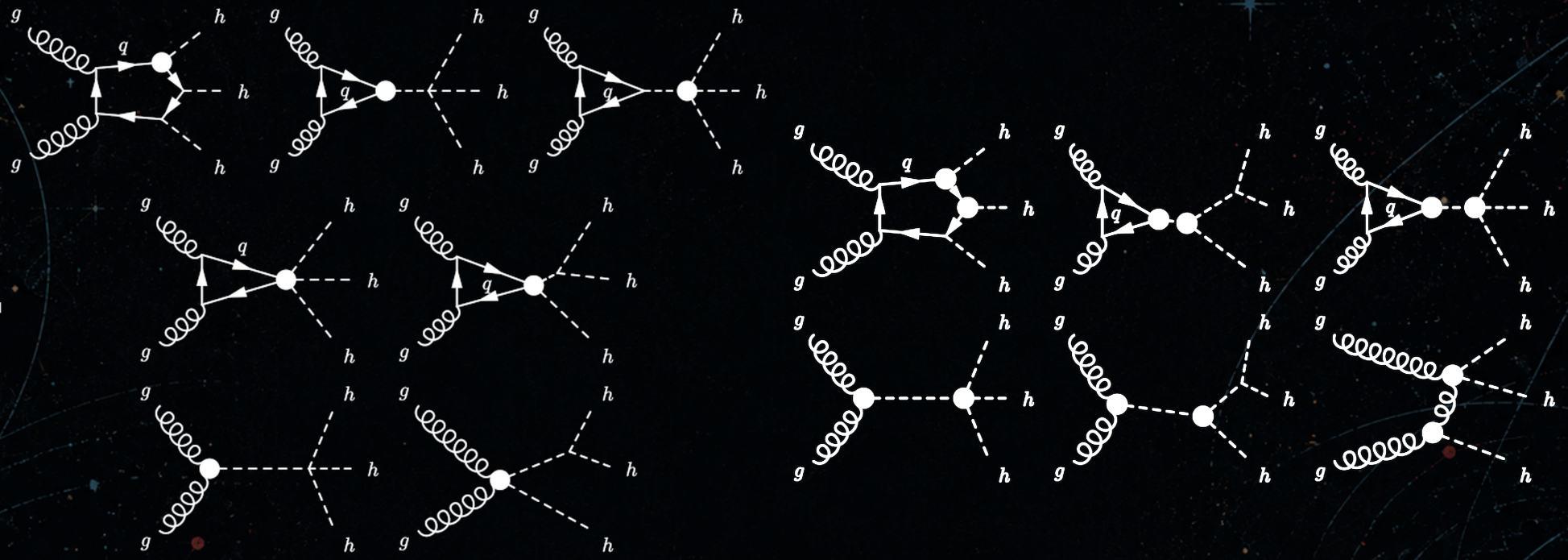
$$\sigma/\sigma_{\rm SM}(13.6\ \text{TeV}) - 1 \approx 0.922c_3^2 - 0.828c_3 - 0.277c_3^3 - 0.178c_3d_4 - 0.0993d_4 + 0.0510c_3^2d_4 - 0.0393c_3^4 + 0.0174d_4^2$$

⇒ Not very sensitive to the quartic!

Return of the Anomalous Couplings

Additional anomalous interactions can also be considered, e.g. $t\bar{t}h^2$, $t\bar{t}h^3$, ggh^2 …:

SM +



MadGraph5_aMC model at: https://gitlab.com/apapaefs/multihiggs_loop_sm

[**AP**, Tetlalmatzi-Xolocotzi, 2312.13562]

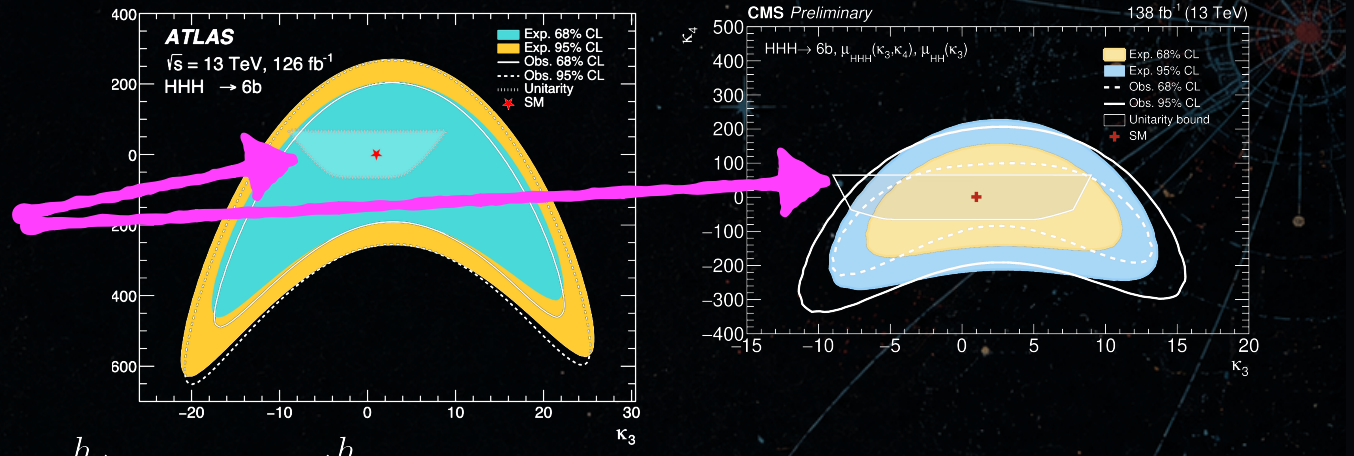
Includes description of interference effects.

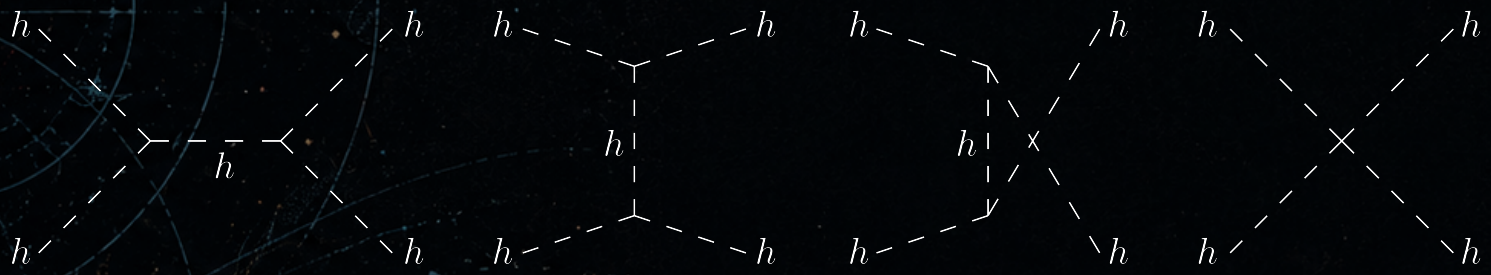
Perturbative Unitarity Constraints

Experiments show limits on (κ_3, κ_4) like:

Including **perturbative unitarity** constraints

From tree-level $hh \to hh$:





$$\mathscr{M} = -\lambda_3^2\left(\frac{1}{s-m_h^2}+\frac{1}{t-m_h^2}+\frac{1}{u-m_h^2}\right)-\lambda_4$$

Via matrix element $\mathscr{M} \sim \langle f\,|\,T\,|\,i\rangle \to T$, interacting part the ***S***-matrix: $S = 1 + iT$, requiring $S^\dagger S = 1$, i.e. unitarity.

<u>All theories must satisfy this!</u> If violated ➡ tree level is ***not*** a good approximation for $\mathscr{M}$!

Higher-order contributions to $\mathscr{M}$ can't be neglected ➡ perturbation theory has broken down!

[Jacob, Wick, Annals Phys. 7, 404 (1959), Di Luzio, Kamenik, Nardecchia, 1604.05746 Di Luzio, Gröber, Spannowsky, 1704.02311, Liu, Lyu, Zhu, 1803.04359, Chang, Luty, 1902.05556, Stylianou, Weiglein, 2312.04646,Fuks, AP, Tetlalmatzi-Xolocotzi, 2509.16364]

See [Logan, 2207.01064] for a **pedagogical** introduction.

More Perturbative Unitarity Constraints

- What about constraints from ***other*** scattering processes? e.g. longitudinal gauge boson scattering at high energy:

$$W_L W_L \to W_L W_L hh\,,\; W_L W_L \to W_L^4,\; W_L W_L \to h^4, \ldots$$

For these, ∃ breakdown energy scale Λ, e.g.: [Mahmud, Tobioka, 2406.03522, 2501.16439]

$$\left|\delta_3 - \frac{1}{5}\delta_4\right| < \left(\frac{12.3\ \text{TeV}}{\Lambda}\right)^2, \qquad |\delta_3| < \left(\frac{18.9\ \text{TeV}}{\Lambda}\right)^2, \qquad \left|\delta_3 - \frac{1}{3}\delta_4\right| < \left(\frac{20.9\ \text{TeV}}{\Lambda}\right)^2, \ldots$$

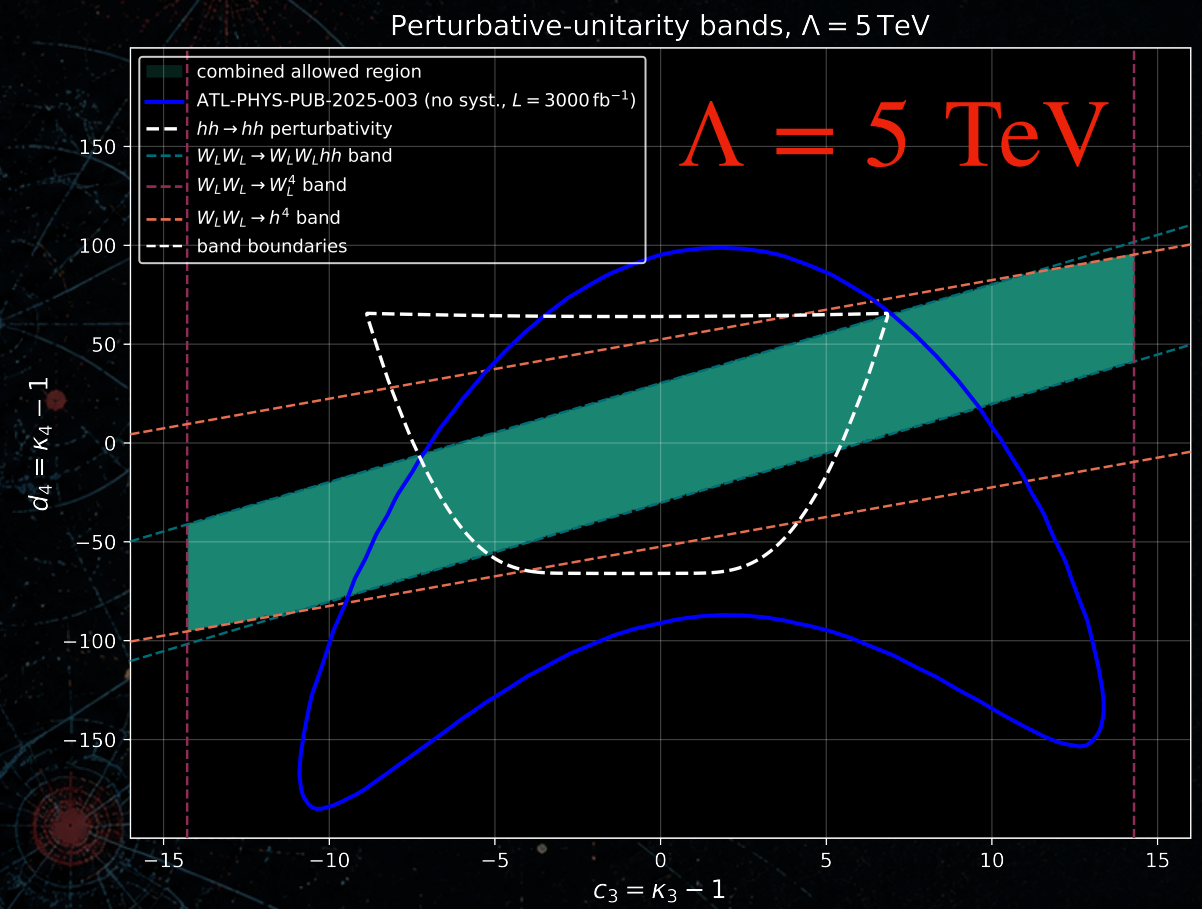
[**quintic** interaction set to zero]

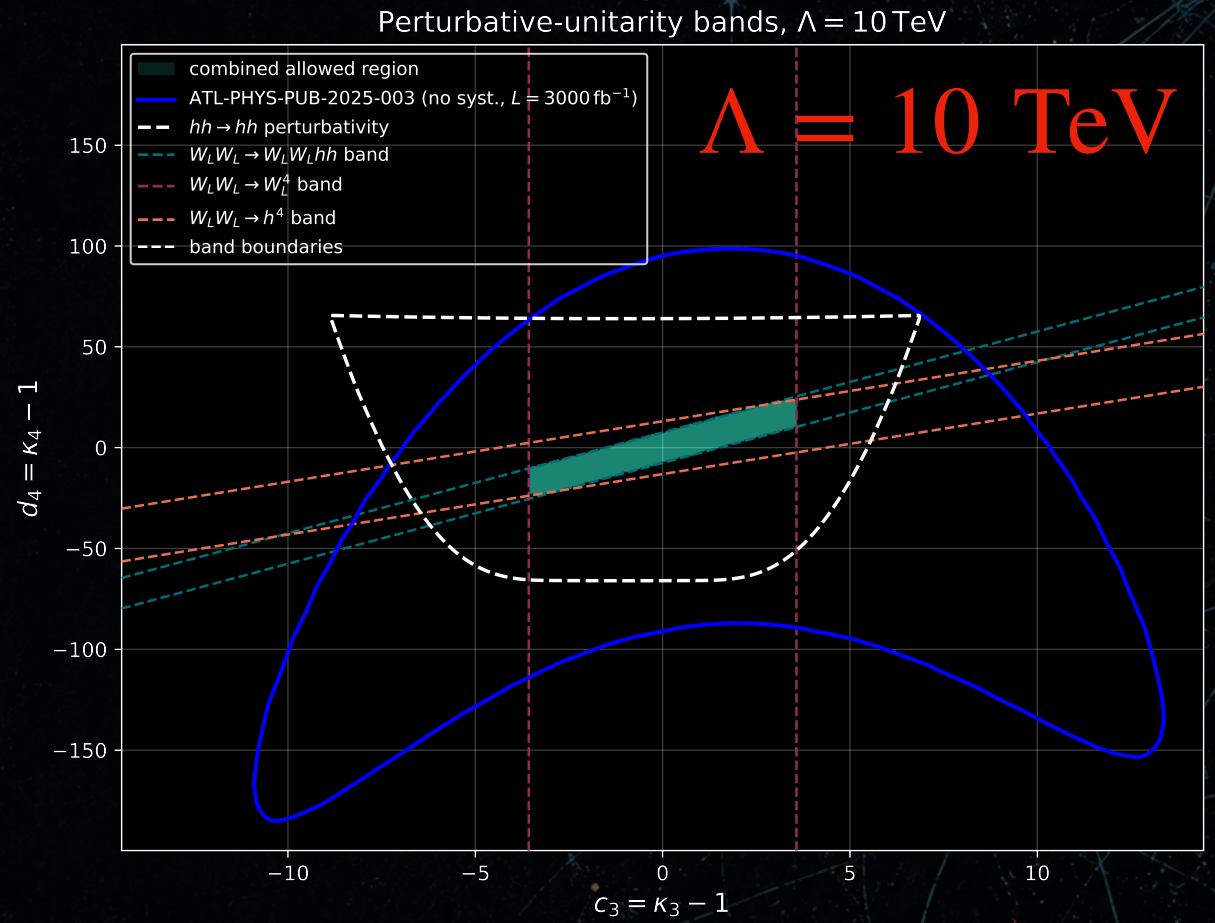
[In order of most constraining processes.]

[*Note*: The $\Lambda \to \infty$ line is a **leading-power** null direction: sub-leading terms become important.]

More Perturbative Unitarity Constraints

Using: $W_L W_L \to W_L W_L hh\,,\; W_L W_L \to W_L^4\,,\; W_L W_L \to h^4$ [Mahmud, Tobioka, 2406.03522, 2501.16439]





Statements on perturbative unitarity should be associated with a breakdown scale Λ!

[*Note*: The $\Lambda \to \infty$ line is a **leading-power** null direction: sub-leading terms become important.]

Perturbative Unitarity Constraints @ NLO?

Calculate perturbative unitarity at **one loop**, in Higgs Effective Field Theory:

Already available for: $W_L W_L \to W_L W_L$, $W_L W_L \to hh$, $hh \to hh$. [Asáin, Espriu, Mescia, 2109.02673l, 2301.13030]

More ***model dependent*** than tree-level:

e.g. @ one loop $hh \to hh$: κ_3, κ_4 ***not*** sufficient: e.g. need derivative operator $\frac{\gamma}{v^4}(\partial_\mu h\, \partial^\mu h)^2$!

Also:

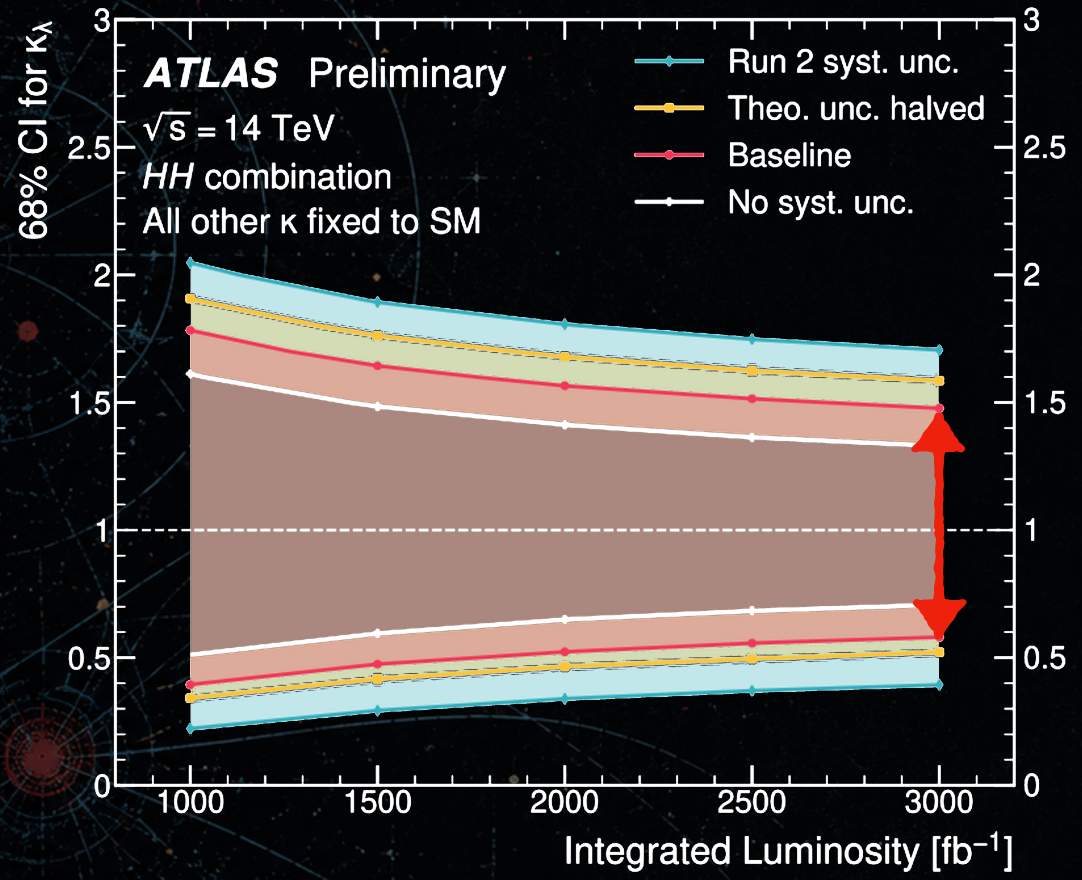
- **Massless** formulas for $hh \to hh$ are insufficient (no κ_3, κ_4 dependence).
- They set $g' = 0$ (neglect e/m) & assume exact custodial symmetry.
- Calculation is in bosonic HEFT (no top quarks).

SECTION 03

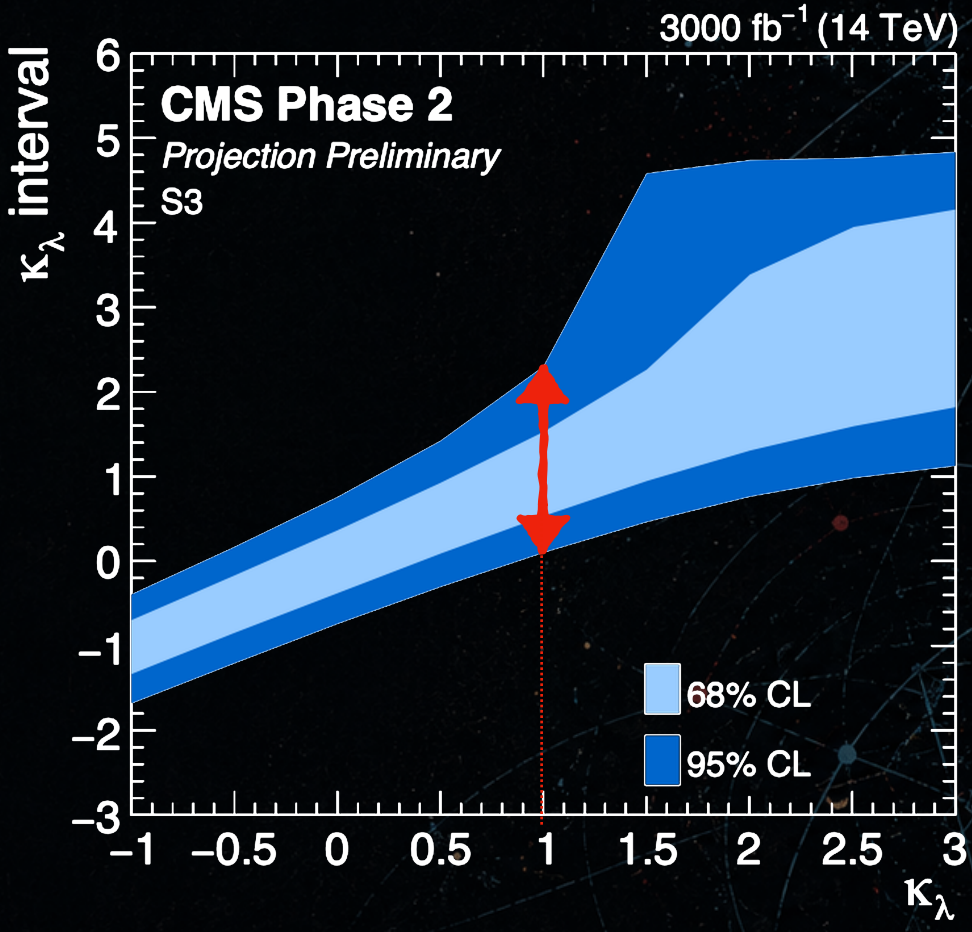
Future Prospects

Trilinear @ HL-LHC

ATLAS: $\kappa_3 = 1^{+0.48}_{-0.42}$ [ATL-PHYS-PUB-2025-006]



CMS: $\kappa_3 = 1^{+0.50}_{-0.47}$ [CMS NOTE -2025/006]



Trilinear @ Other Colliders

FCC-ee @ 240 GeV/365 GeV:

⇒ not enough for *hhZ* ➡ use loop diagrams:



$\delta(\kappa_3) \sim 28\,\%$

$\delta(\kappa_3) \sim 18\,\%$ (HL-LHC combo)

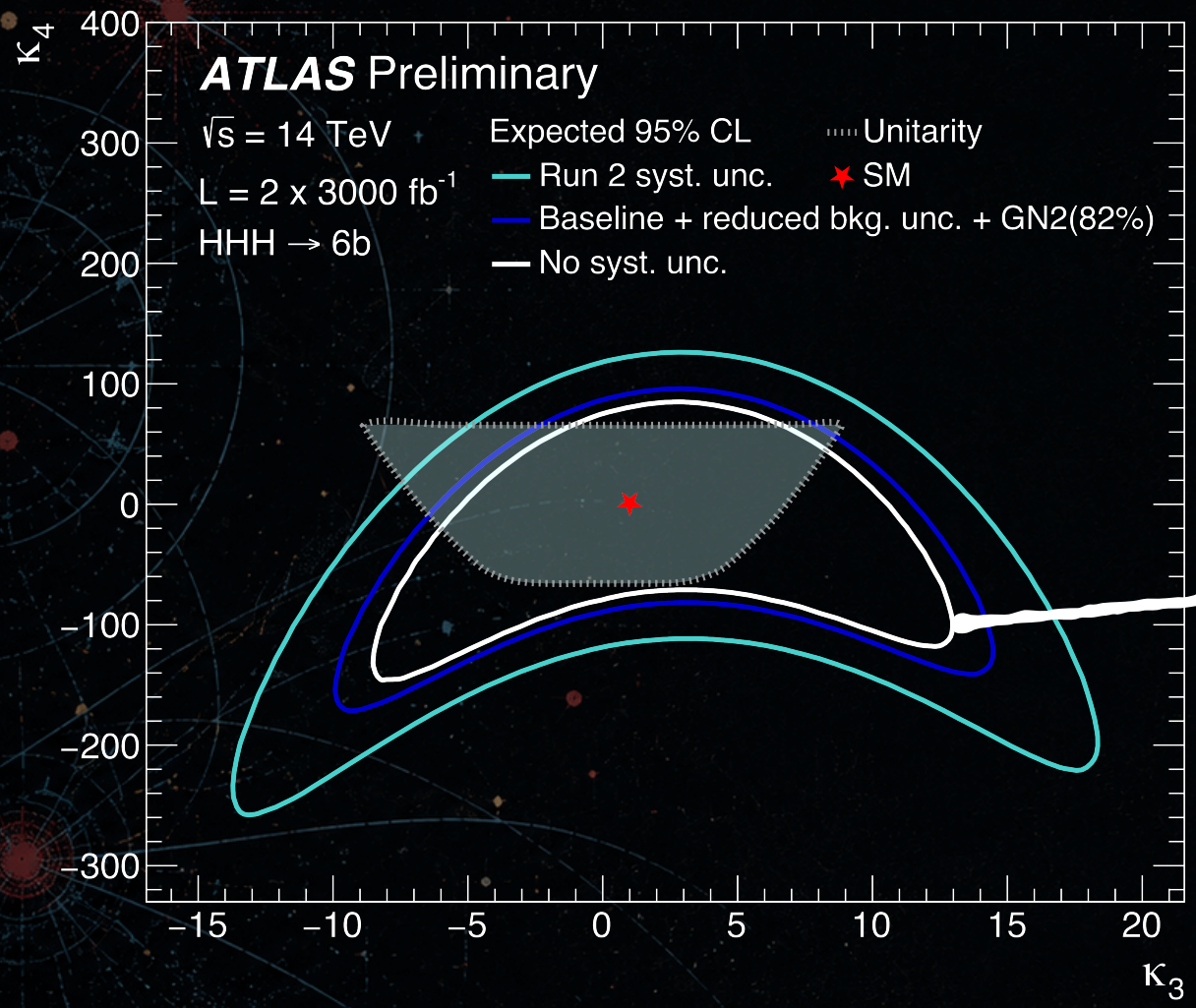
FCC-hh (compared to others):



$\delta(\kappa_3) = 3.2\% - 5.4\,\%$ @84 TeV

$\delta(\kappa_3) = 2.8\% - 4.8\,\%$ @100 TeV

Quartic @ HL-LHC

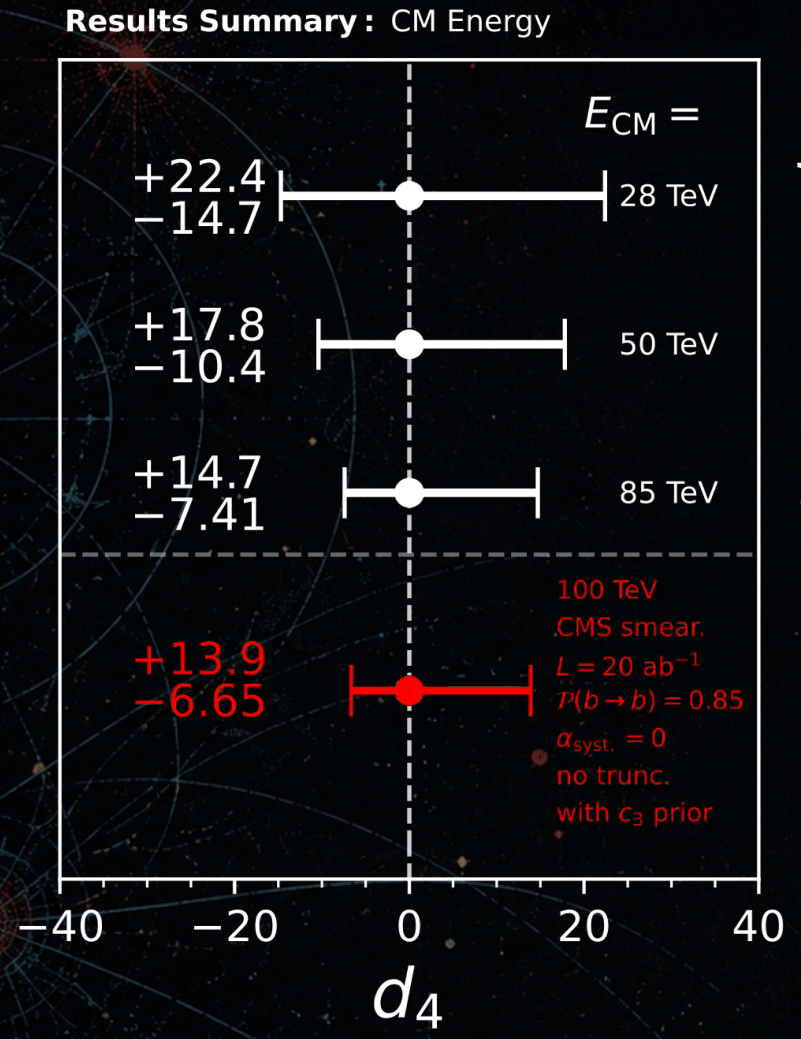


ATLAS HL-LHC extrapolation of **6 b-jet** final state (**6000 fb^{-1}**) with reduced *or* no systematics. [ATL-PHYS-PUB-2025-003]

Best-case scenario:
$\kappa_4 \in [-65, 76]$

[**Shaded region:** $hh \to hh$ **perturbative unitarity**]

Quartic @ Hadron Colliders @ Higher Energies



Analysis of **6 b-jet** final state [Fuks, **AP**, Tetalmatzi-Xolocotzi, **2509.16364**]

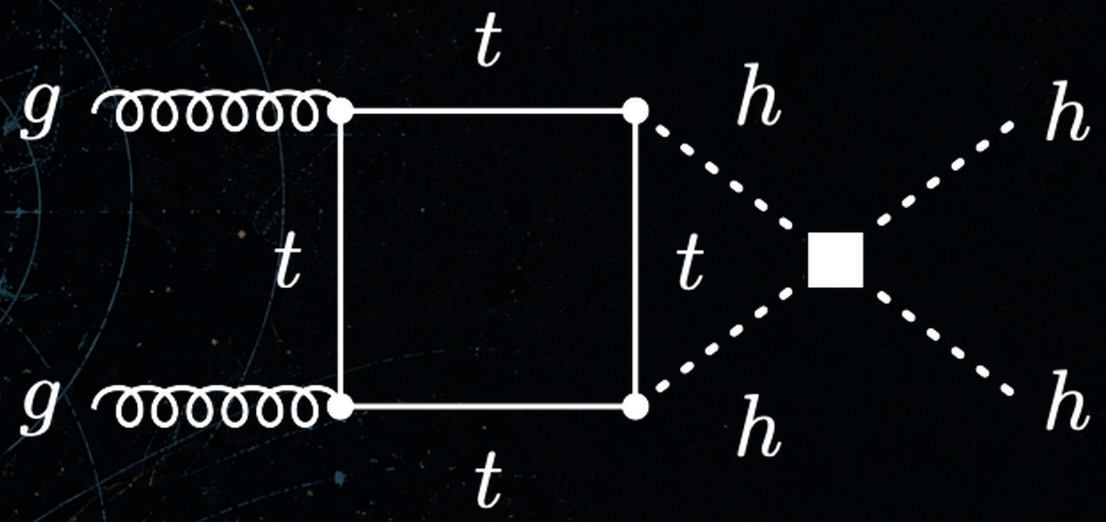
pp collider **energy** variation
(85 → 50 → 28 TeV)

hhh 100 TeV/20 ab^{-1} +
combination with future trilinear constraint,
assuming: $\delta c_3 = 5\,\%$

Note: $1 + d_4 = \kappa_4$

Quartic Indirect Constraints through *hh*

- Through electroweak corrections:

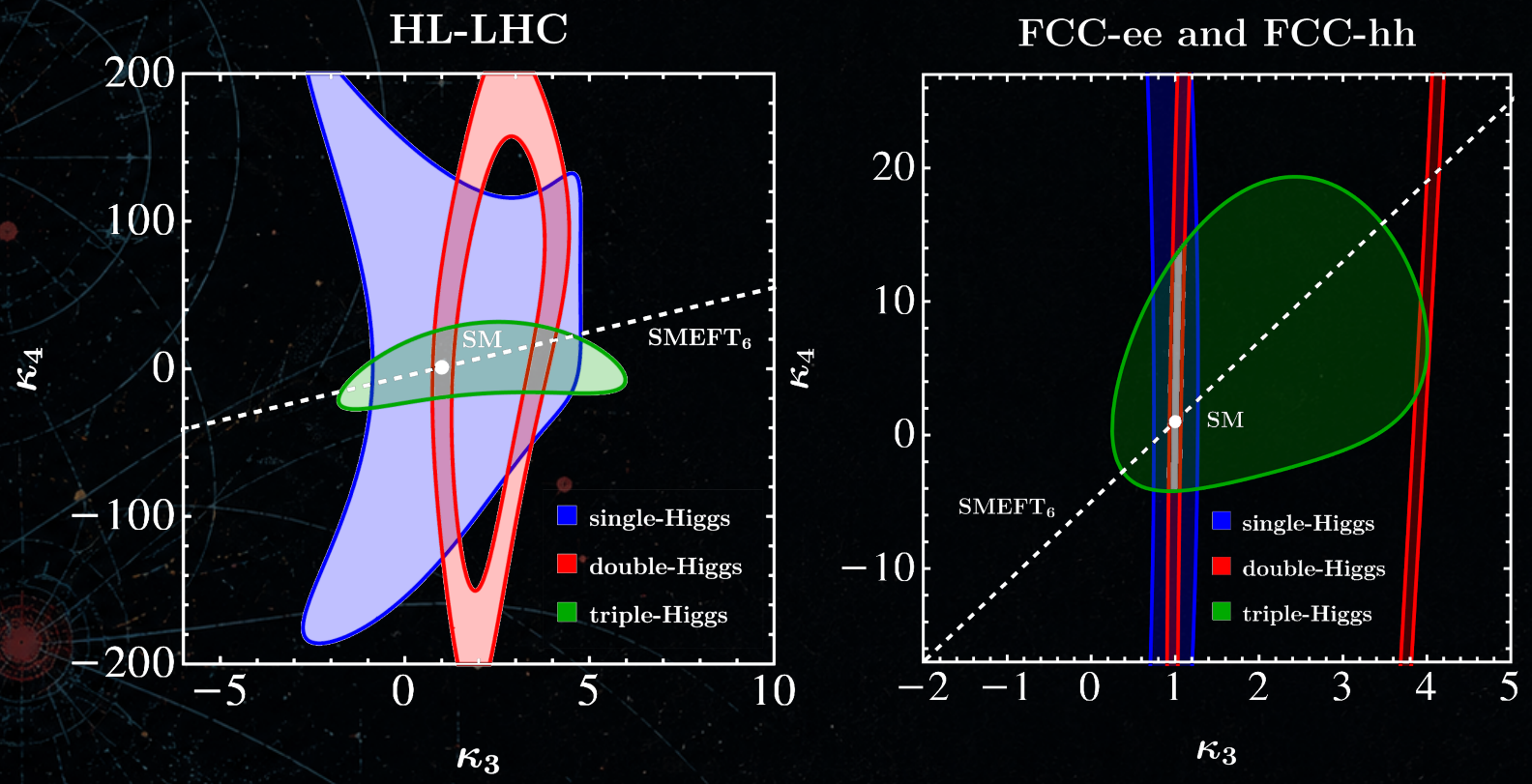


[Bizoń, Haisch, Rottoli, **1810.04665**, Li Si, Wang, Zhang, Zhao, **2407.14716**, Heinrich, Jones, Kerner, Stone, Vesten, **2407.04653**]

- But: **Full** EW corrections to ***hh*** are **only** available for the SM!

Quartic Indirect Constraints: Single h

- Through **corrections to the Higgs wave-function renormalization** from κ_3, κ_4 and κ_5 up to **two-loops**. [Haisch, Sankar, Zanderighi, **2505.20463**]



HL-LHC: $\kappa_4 \in [-21, 28]$

FCC-ee+hh: $\kappa_4 \in [-7, 15]$

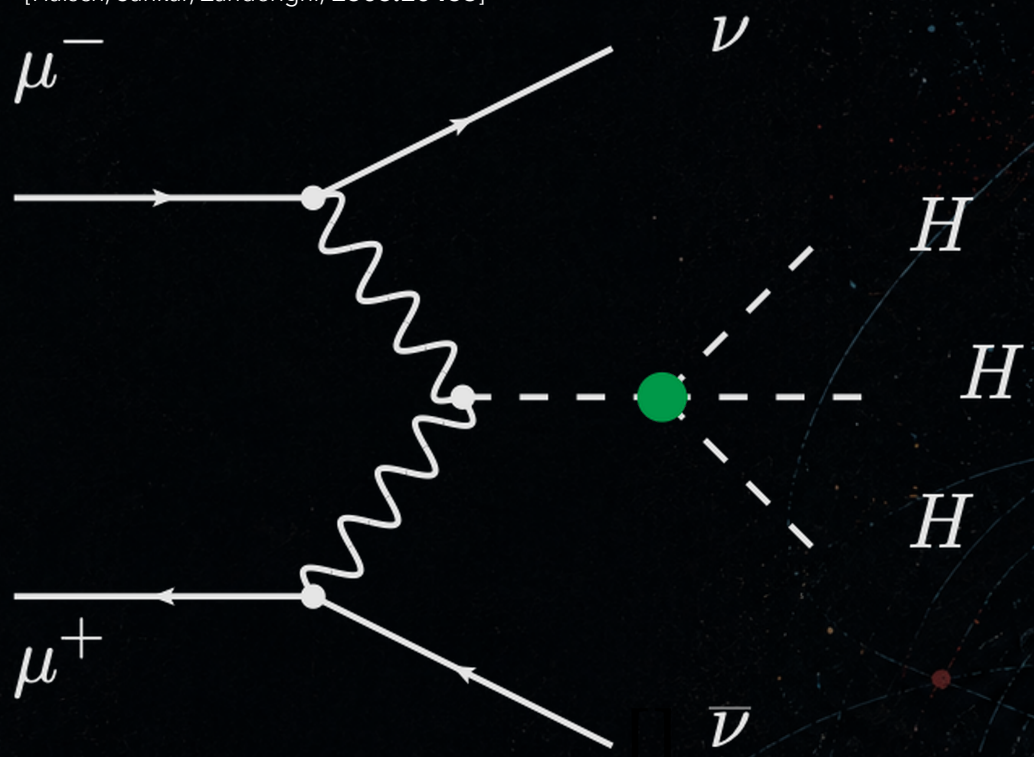
But somewhat more model-dependent than direct probes!

Quartic @ Lepton Colliders

FCC-ee: **insufficient** energy for ***Zhh or Zhhh*** production & constraints for k_4 very weak in ***Zh***.

[Haisch, Sankar, Zanderighi, **2505.20463**]

Muon Collider, through VBF:



e.g. for $\sqrt{s} = 14$ TeV, 33 ab^{-1}: $\kappa_4 \sim [0.65, 1.55]$

[Chiesa, Maltoni, Mantani, Mele, Piccinini, Zhao, **2003.13628**]

(Rough) Summary of Self-Coupling Prospects

Collider:	HL-LHC	FCC-ee	FCC-hh	Muon Coll.
k_3	$\sim 50\,\%$	$\sim 20-30\,\%$	$\sim 5\%$	$5-30\,\%$
k_4	$\mathcal{O}(10-100)$ [ATL-PHYS-PUB-2025-003, ATL-PHYS-PUB-2025-018 CMS-HIG-25-002]	???	$\mathcal{O}(1-10)$ [Fuks, **AP**, Tetlalmatzi-Xolocotzi, **2509.16364**]	$50-100\,\%$ [Chiesa, Maltoni, Mantani, Mele, Piccinini, Zhao, **2003.13628**]

SECTION 04

Hot Off the Press!

[**AP**, Tetlalmatzi-Xolocotzi, in progress]

Quadruple Higgs Boson Production (!?)



Quartic (d_4) constraints are likely to remain weak at HL-LHC.

Quadruple Higgs boson production includes more diagrams involving d_4 at LO.

At **14 TeV**, **LO SM**: $\sigma(hhh)/\sigma(4h) \sim 350$ (➡ $\mathcal{O}(10^{-4}\ \text{fb})$).

Also: Rate for ***4h*** can be large in **extended scalar sectors** [see e.g.: Cheng, Chiang, Low, **2202.02954**, Lane, Lewis, Sullivan, **2403.18003**, Li, Liu, Wang, **2604.14284**, Roy, Wagner, **2605.29345, ...**]. e.g. ***4h > 3h*** or ***4h > hh***!

QCD 8 b-jet backgrounds are also likely to be much **smaller**! → $\mathcal{O}(1\ \text{fb})$ with $p_{T,b} > 20\ \text{GeV}$!

Is it worth investigating?

[**AP**, Tetlalmatzi-Xolocotzi, in progress]

Quadruple Higgs Boson Production (!?)



Cross-section fits (14 TeV):

$$\sigma(gg \to hhh)/\sigma_{\rm SM}(14\ \text{TeV}) - 1 \approx 0.881c_3^2 - 0.872c_3 - 0.261c_3^3 - 0.168c_3d_4 - 0.0934d_4 + 0.0490c_3^2d_4 + 0.0397c_3^4 + 0.0170d_4^2$$

$$\sigma(gg \to hhhh)/\sigma_{\rm SM}(14\ \text{TeV}) - 1 \approx 0.549c_3^2 + 0.0992c_3^4 + 0.0799c_3^3 + 0.0327c_3 - 0.0184c_3^5 + 0.00329c_3^6 - 0.360d_4 - 0.107c_3d_4 - 0.104c_3^2d_4 - 0.0368c_3^3d_4 + 0.0130c_3^4d_4 + 0.0412d_4^2 + 0.00294c_3d_4^2 + 0.0141c_3^2d_4^2$$

For $d_4 \sim 70$, enhancement is ~ 200 (=100-200 events @ 3000 fb^{-1})

[**AP**, Tetlalmatzi-Xolocotzi, in progress]

Quadruple Higgs Boson Production (!?)



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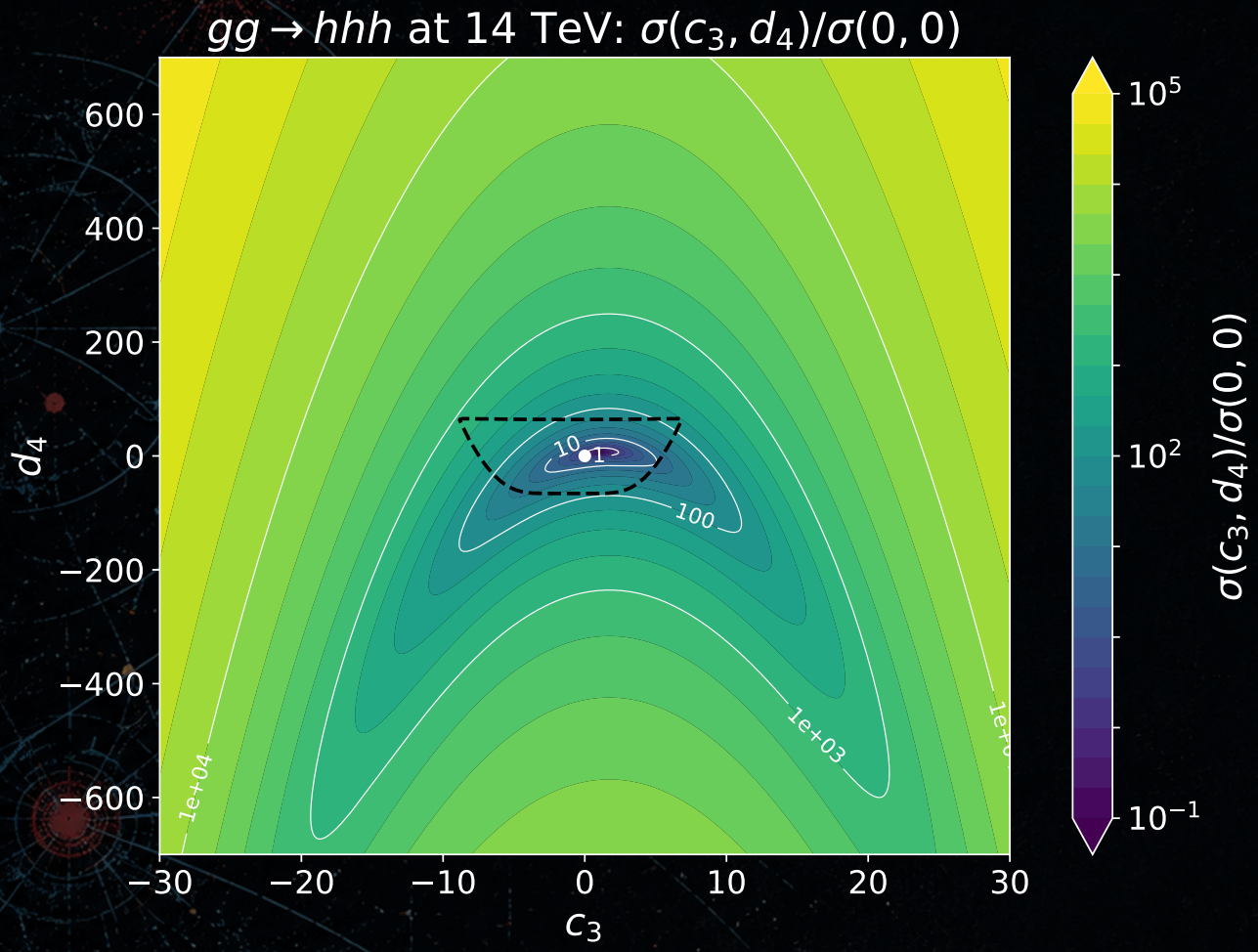
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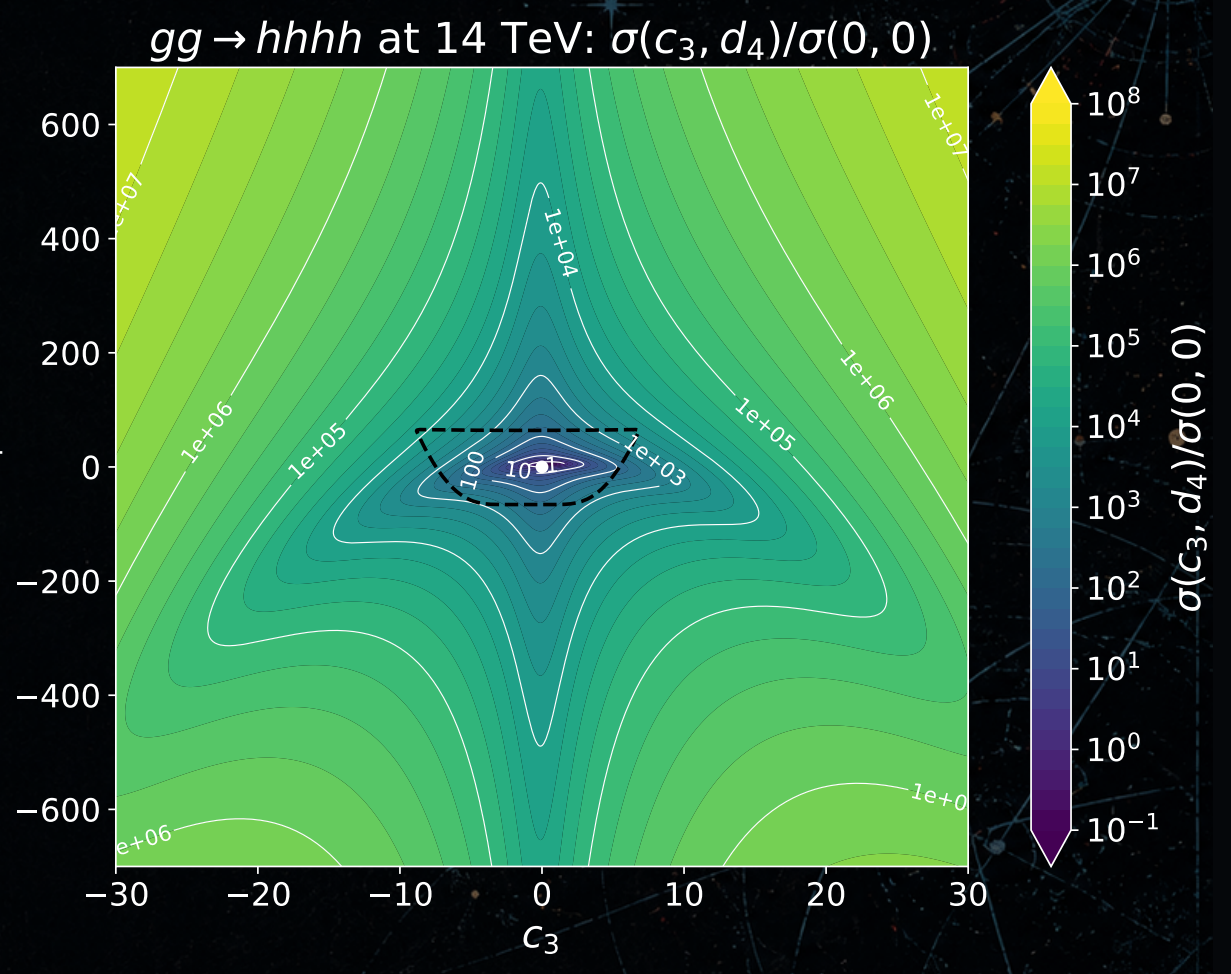
[**AP**, Tetlalmatzi-Xolocotzi, in progress]

PRELIMINARY

Quadruple Higgs Boson Production (!?)

Cross section contours [Black dashed curve = $hh \to hh$ perturbative unitarity]:





[**AP**, Tetlalmatzi-Xolocotzi, in progress]

Quadruple Higgs Boson Production (!?)



Main background: $gg \to 8b$ jets ➡ ***Extremely*** challenging to generate events ➡ Sherpa LHE + Herwig 7.

Simple **smearing** for jets applied (see e.g. **[Fuks, AP, Tetalmatzi-Xolocotzi, 2509.16364]**).

Pair 8 b-jets into four Higgs candidates and minimize:

$$\chi^2_8(P) = \sum_{k=1}^{4} \left(m_{h,k}(P) - m_{h,k}^{\text{target}} \right)^2 \quad \text{with:} \quad (m_{h,1}^{\text{target}}, m_{h,2}^{\text{target}}, m_{h,3}^{\text{target}}, m_{h,4}^{\text{target}}) = (120,115,110,105)\ \text{GeV}\,.$$

XGBoost analysis on a complete set of observables.

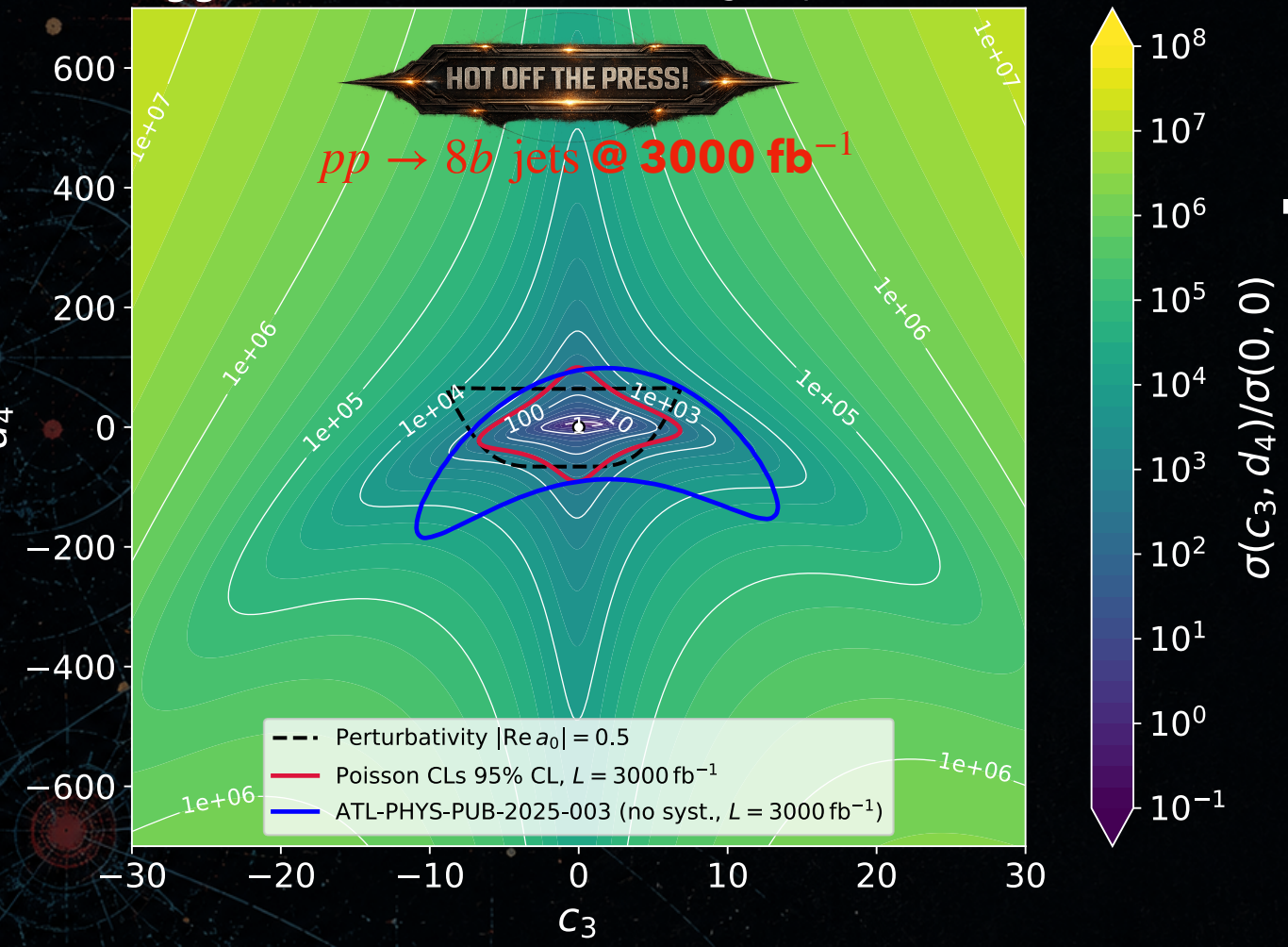
Poisson limited, with *very* low numbers of events! ➡ $B \sim \mathcal{O}(0.1)$ expected events for $gg \to 8b$ jets**!**

[**AP**, Tetlalmatzi-Xolocotzi, in progress]

Quadruple Higgs Boson Production (!?)

PRELIMINARY

$gg \to hhhh$ at 14 TeV: $\sigma(c_3, d_4)/\sigma(0,0)$



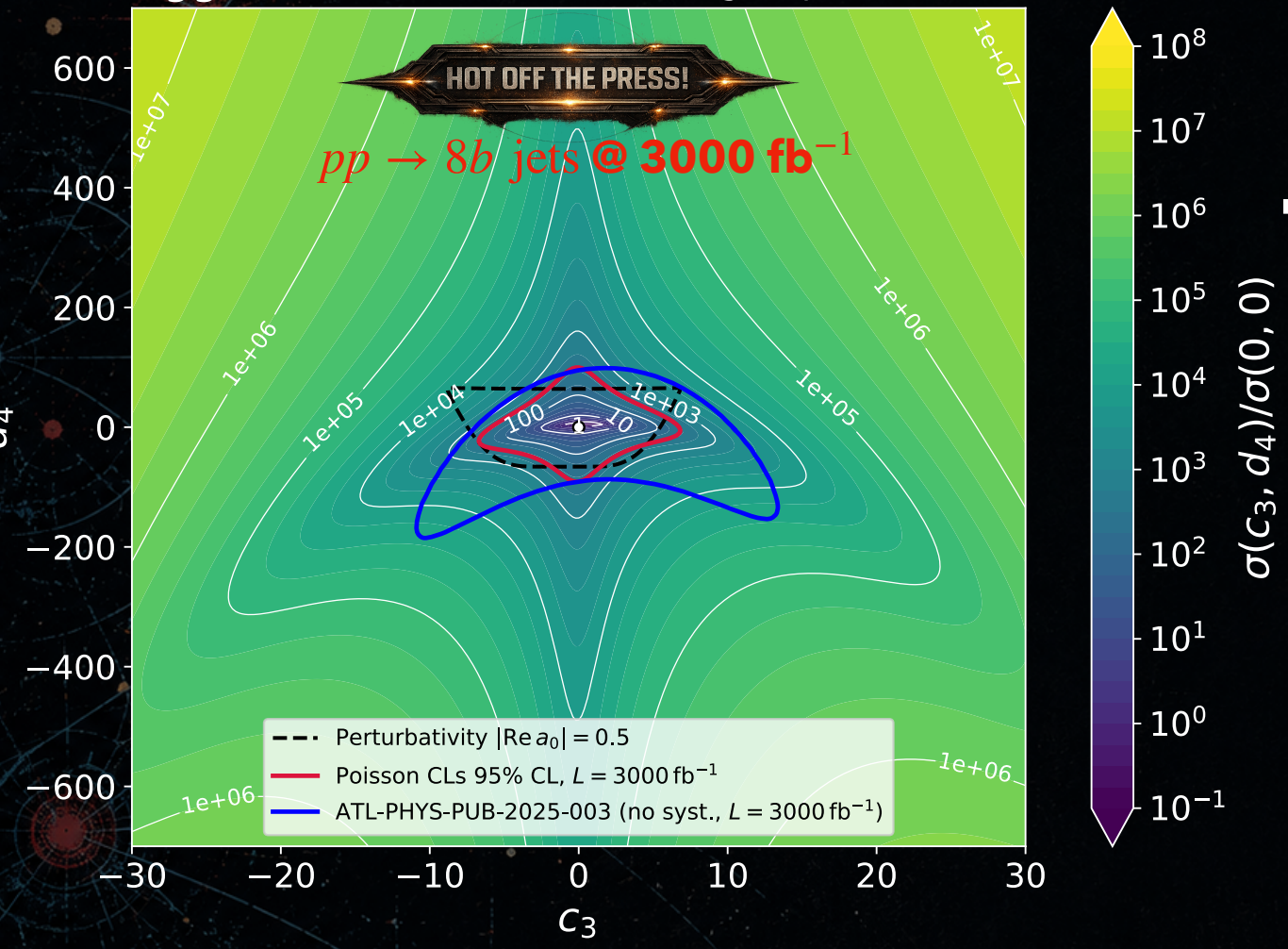
➡ **Potentially competitive with** $hhh \to 6b$ jets**!**

[**AP**, Tetlalmatzi-Xolocotzi, in progress]

Quadruple Higgs Boson Production (!?)

PRELIMINARY

$gg \to hhhh$ at 14 TeV: $\sigma(c_3, d_4)/\sigma(0,0)$



➡ **Potentially competitive with** $hhh \to 6b$ jets**!**

WARNING: ONLY QCD 8b background included so far!

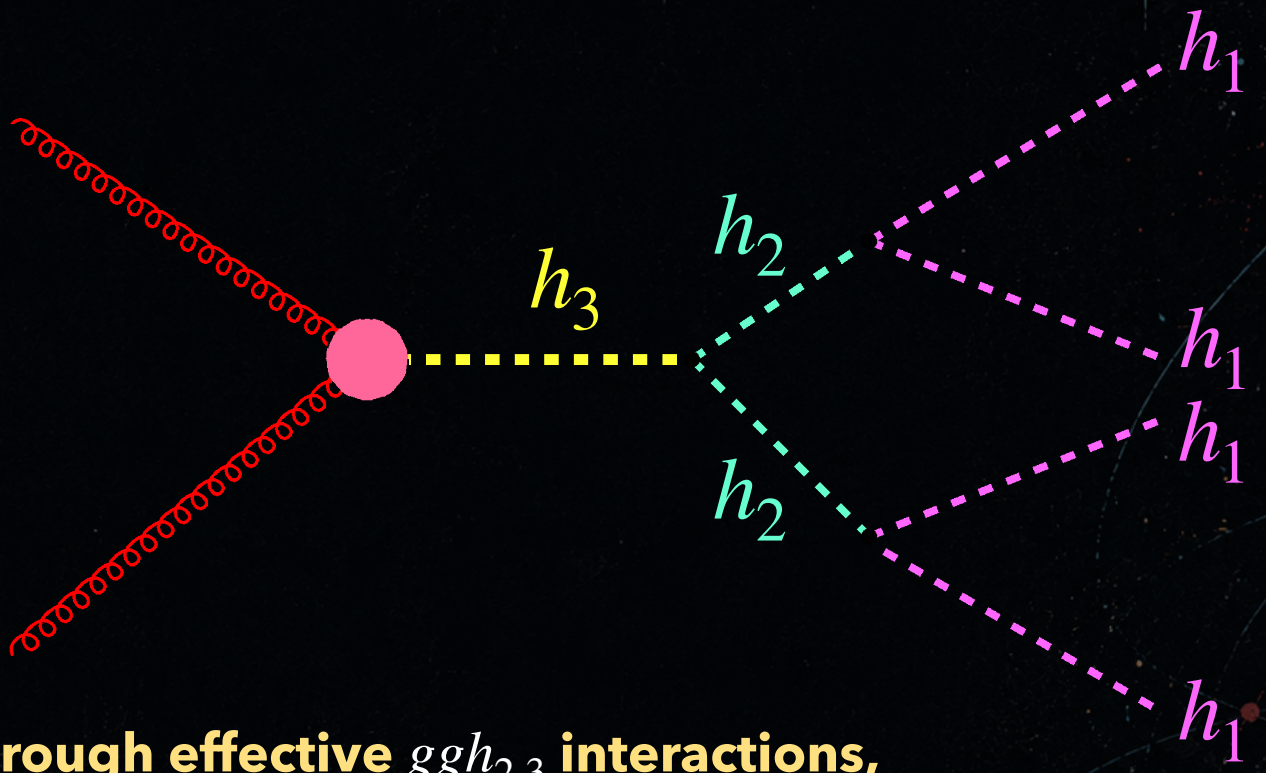
Open questions:

- **What about reducible backgrounds, e.g.** $6b + c\bar{c}/jj$**?**
- **What about** $hhh + b\bar{b}$ **or** $hh + b\bar{b}b\bar{b}$ **"backgrounds"?**
- **Systematics will be investigated!**
- **…**

STAY TUNED!

Quadruple Higgs Boson Production in Extended Scalar Sectors

- Rate for ***4h*** can be large in **extended scalar sectors:**



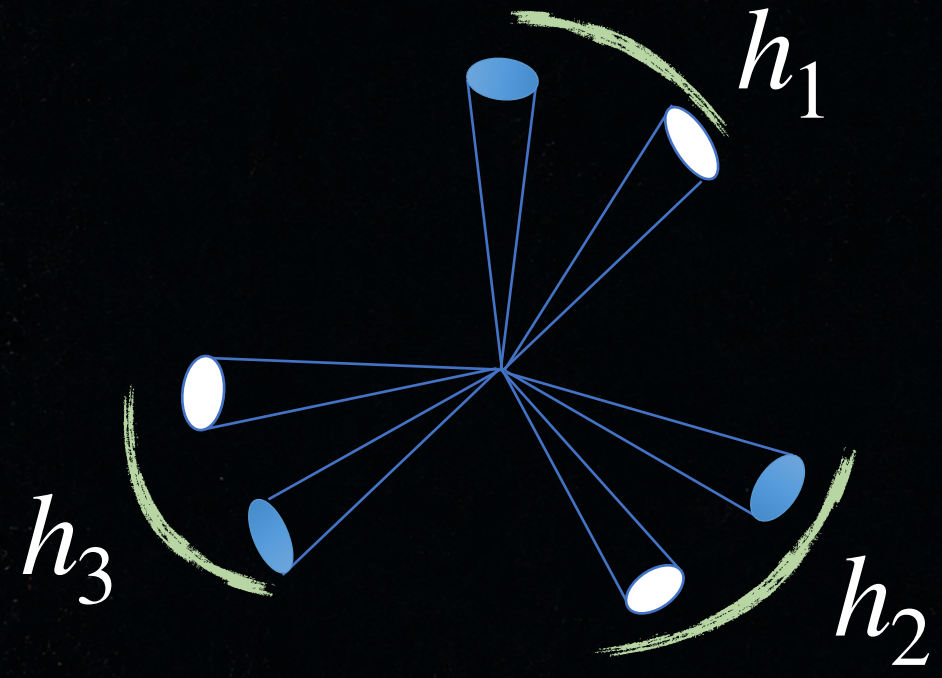
[Roy, Wagner, 2605.29345]: **Through effective $ggh_{2,3}$ interactions,**

& "**alignment limit**": suppresses gauge boson decays + maintains large scalar couplings.

Charging Up Multi-Higgs Boson Production

PRELIMINARY

6 b-jets: if charge is unknown → **15** combinations of 6 b-jets into pairs.



$hh \to 4b$ jets: **3** combos

$hhhh \to 8b$ jets: **105 (!)**

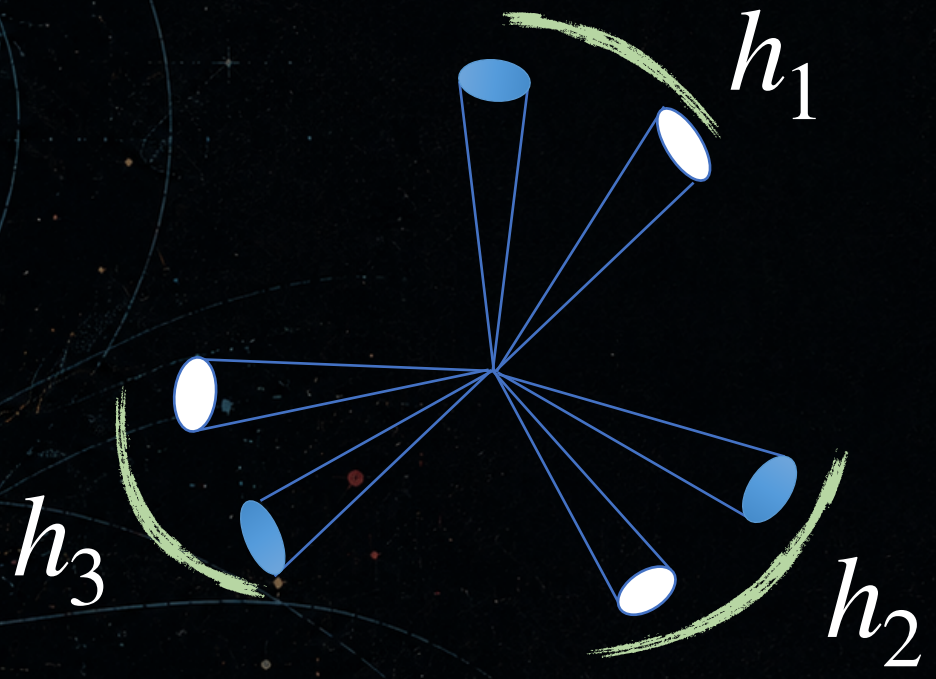
Jet-charge tagging reduces the amount of combinations: e.g. CMS DP-2025/07 → **70**% accuracy on b-jets (= **30%** mis-id).

[see e.g. talk by M. Stamenkovic @ HIGGS 2025]

Charging Up Multi-Higgs Boson Production



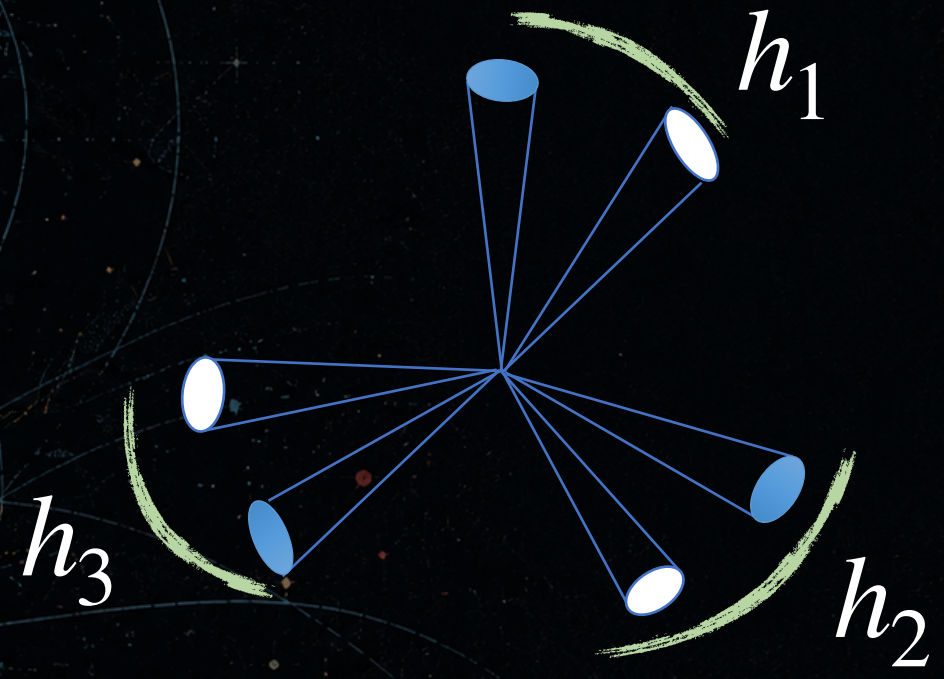
Preliminary analysis with **perfect** charge identification: ~ **factor of two better rejection of QCD backgrounds in** $hhh \to 6b$ jets **!**



Charging Up Multi-Higgs Boson Production



Preliminary analysis with **perfect** charge identification: ~ **factor of two better rejection of QCD backgrounds in** $hhh \to 6b$ jets **!**





SECTION 05

Conclusions

Conclusions & Outlook

Multi-Higgs boson production → **Directly** probe self-couplings & extended scalar sectors.

Extremely rare & challenging pheno.

Quadruple production tiny but *potentially* complementary for κ_4 (& extended scalar sectors).

B-jet charge tagging could improve significance in multi-Higgs boson production.

Future colliders *could* provide **few %** for κ_3 and $\lesssim \mathcal{O}(1)$ for κ_4 constraints **by the end of the century**!

Conclusions & Outlook

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Thanks!

Questions?