Simulating Parton Showers in High-Energy Collisions

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Abstract

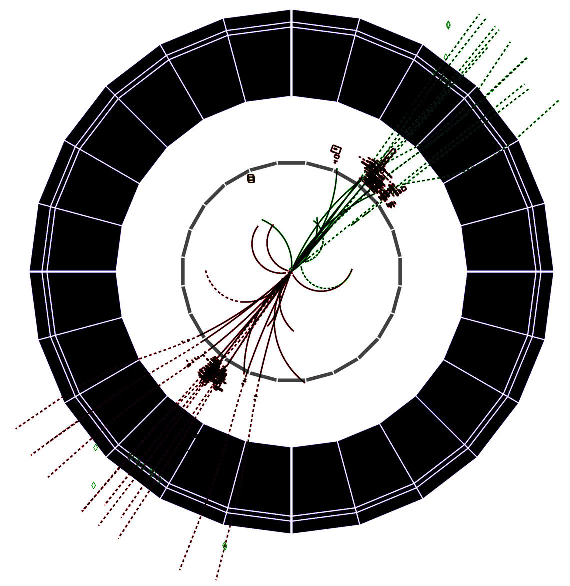

Figure 1. A cross-sectional view of a collider event at LEP.

The goal of high-energy particle colliders is to understand the nature of matter and energy at the fundamental level. In particular, the analysis of collisions can shed light on the theory of the strong force, that keeps nuclei bound together, described by **Quantum Chromodynamics (QCD)**. These collision 'events' can contain a multitude of particles, see e.g. fig. 1. Simulating such complex events entails many parts, and is essential for designing *and* comprehending experiments.

Particularly, the inclusion of **QCD radiation** is absolutely crucial in order to fully describe a collision, such as that of an electron and a positron annihilating into a quark and an antiquark ($e^+e^- \to q\bar{q}$). **Parton showers** can achieve this, incorporating the cascade of multiple QCD emissions. **We present the first-ever parton shower implemented in the `Julia` programming language**, designed to be fast and robust, and employ it to describe real experimental data.

Introduction

Electric charge quantifies the strength of electromagnetic (e/m) interactions between particles. These interactions can be envisioned as the exchange of the e/m force carriers between electrically-charged particles, the photons. In a similar way, 'color' charge describes QCD interactions between particles that 'feel' the strong force, with the force carriers being the gluons. Whereas electric charge can be positive or negative, color charge consists of three colors and three anti-colors.

QCD is substantially 'stronger' than e/m, resulting in significantly more radiation. The strong coupling constant, α_S, is used to quantify this strength, and due to it becoming small at large energies, QCD calculations can be performed through a perturbative series of the form: $\mathrm{LO} + \mathrm{NLO} + \ldots$, such that $\mathrm{LO} \gg \mathrm{NLO} \gg \ldots$. Here, the leading order (LO) calculation includes the lowest power of α_S, the next-to-leading order (NLO) includes the next higher power in α_S, etc.. Fully calculating these series out is extremely challenging beyond a certain fixed order in α_S. However, higher-order terms can involve enhancements that contribute significantly and cannot be ignored; **using parton showers allows us to include the enhanced pieces of the perturbation series, at *all* orders in α_S, into Monte Carlo simulations** [1].

The Parton Shower

The parton shower algorithm involves an 'evolution' variable, t, that controls the emissions of partons. The evolution between emissions uses the **Sudakov form factor** which gives the probability of an emission *not* occurring between two values of t. This probability is used along with a random number to determine the next evolution scale [2].

Solving for the Sudakov form factor can involve integrating functions that may not be integrable, which is why instead a known 'overestimate' is used. This overestimate gives a straightforward analytical solution that is easier to implement. Having a relatively 'simple' function allows us to more easily solve for the next evolution scale. Since this is not the actual function, we then have to correct by 'vetoing' (rejecting) according to the ratio of the overestimate and the actual.

These operations are computationally demanding, and therefore the choice of coding language becomes important. While most modern generators use `C++`, here we implement the first-ever parton shower in the `Julia` language [3]. We chose **`Julia` since it can be as fast as `C++` while being designed specifically for scientific computing.**

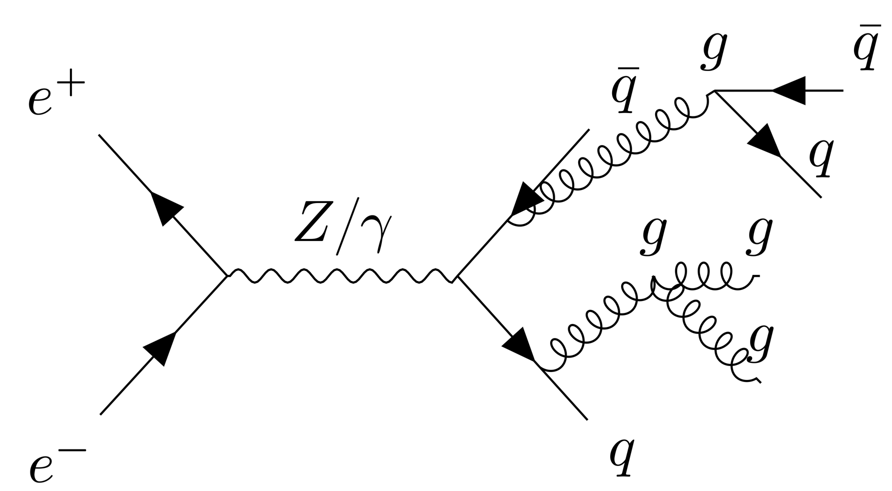

Figure 2. An example diagram of a parton shower in $e^+e^- \to q\bar{q}$.

Event Shape Variables at Colliders

In a collision, the resulting distribution of particles can help us understand the behavior of QCD radiation. To operationalize this, a variable known as the **thrust** defined as,

$$\mathbf{T} = \max_{\vec{n}_T} \left(\frac{\sum_i |\vec{p}_i \cdot \vec{n}_T|}{\sum_i |\vec{p}_i|} \right) ,$$

gives the overall 'shape' of the event, where the sum is over all particle momenta p_i, and $\vec{n}_T$ is the thrust vector. In the way this is defined, the limit $\mathbf{T} \to 1/2$ describes events which are 'spherical', and $\mathbf{T} \to 1$ represents 'pencil-like' events, where the two jets are anti-parallel. Another notable variable is the thrust major, $\mathbf{T}_{\mathbf{major}}$, defined in a similar way, but in a direction **perpendicular** to $\vec{n}_T$.

Results

In fig. 3, we compare our parton shower for $e^+e^- \to q\bar{q}$, to `HERWIG 7`, a widely-used event generator [4], showing good agreement.

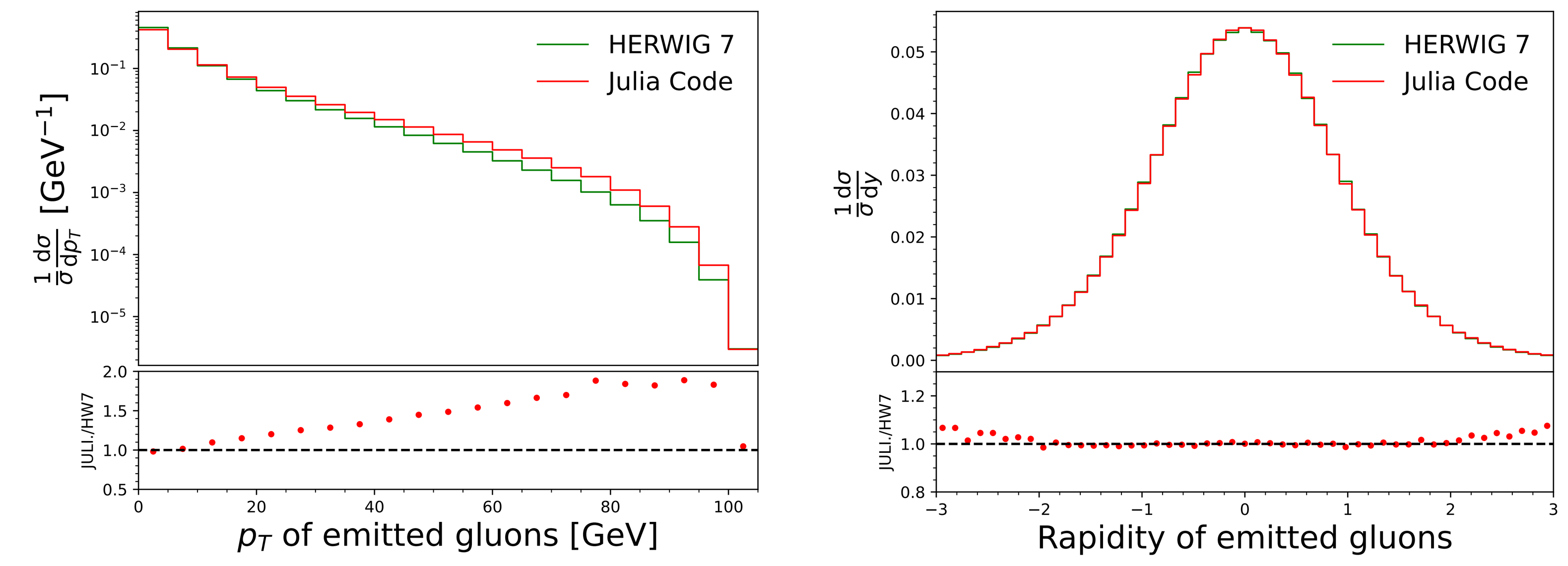

Figure 3. Transverse momentum (left) and rapidity (right) of emitted gluons.

Figure 4 shows a comparison to **experimental data collected at the CERN Large Electron-Positron (LEP)** [5]. The LO and NLO results clearly *cannot* describe the LEP thrust distributions. In contrast, the parton showers (`Julia` and `HERWIG 7`) provide good agreement.

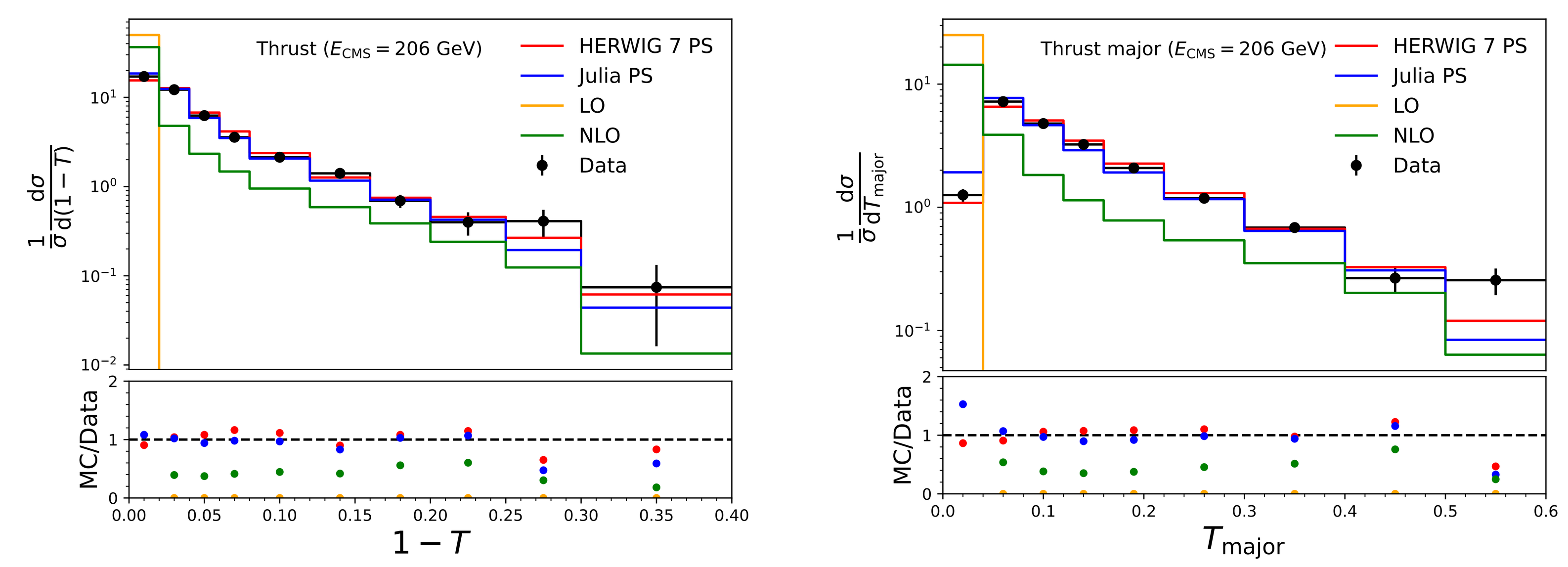

Figure 4. Thrust (left) and thrust major (right) comparisons for LEP data.

Conclusions & Outlook

We have presented the first-ever parton shower in the `Julia` programming language and employed it to compare to distributions in LEP data, showing significantly better agreement than with fixed-order calculations. This demonstrates the need for parton showers in the description of collider data. Our parton shower can be readily extended to include initial-state radiation, as well as the effects of polarization.

References

[1] K.R. Ellis, J.W. Stirling and B.R. Webber, *QCD and Collider Physics*, Cambridge University Press (1996).
[2] A. Papaefstathiou, *Pyresias: How to write a toy parton shower*, 2024.
[3] C. Garrett, *Julia parton shower*, 2025.
[4] G. Bewick et al., *Herwig 7.3 release note*, *Eur. Phys. J. C* **84** (2024) 1053 [2312.05175].
[5] ALEPH collaboration, *Studies of QCD at e+ e- centre-of-mass energies between 91 GeV and 209 GeV*, *Eur. Phys. J. C* **35** (2004) 457.

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