Modeling Particle Interactions Using Monte Carlo Simulations

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Abstract

In this work, we discuss the general principles of the Monte Carlo method, applying it to perform numerical integration. We then discuss its application for the simulation of events at high-energy particle colliders, through the construction of Monte Carlo event generators, one of which we construct for electro-positron annihilation into muon-anti-muon pairs.

Introduction

When you think of Monte Carlo, most people would think of the city in Monaco that is famous for its numerous casinos, but in scientific computing, Monte Carlo is more synonymous with randomness, like that which is found in gambling. Monte Carlo simulations (MCs) use randomness and probability to generate data for many diverse types of processes, facilitating their theoretical treatment.

A Simple Monte Carlo Simulation

A simple example of an MC simulation is used to calculate the irrational number π by randomly selecting N points in a square with a circle of radius r inscribed in it. By counting the number of points that fall within the circle, one can obtain an approximation for π. The more points that are used in this simple "simulation", the closer the estimate is to π (fig. 1).

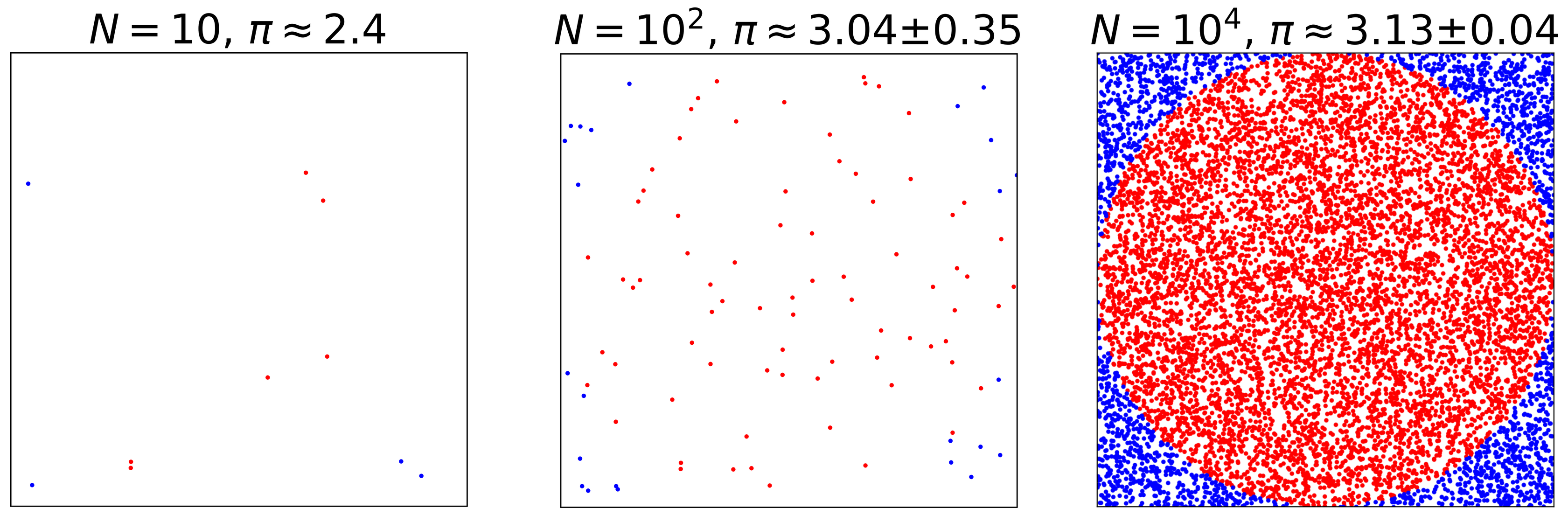

Figure 1: A simple Monte Carlo Simulation for calculating π.

Monte Carlo Integration

The Monte Carlo method also allows for a simple and powerful way to estimate integrals. In particular, if we pick N points randomly and uniformly in a region $[a, b]$, the average value of a function $f(x)$ in that region provides an approximation for the integral (fig. 2). Along with this, it is also easy to obtain the uncertainty in the calculation. An advantage of Monte Carlo integration over other methods is that the error scales as $1/\sqrt{N}$, irrespective of dimensionality of the problem, rendering it quintessential for integrals in many dimensions, such as those encountered in particle physics.

$$I = \int_a^b f(x)\,\mathrm{d}x \approx \frac{b-a}{N} \sum_{i=1}^{i=N} f(x_i)$$

$$\sigma_I^2 \approx \frac{\sigma_f^2}{N}$$

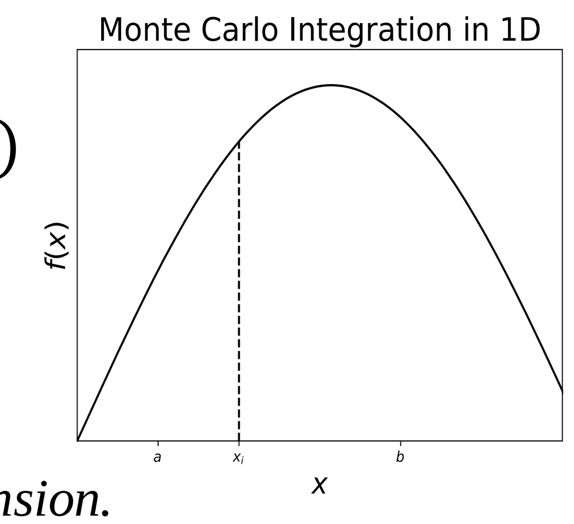

Figure 2: Monte Carlo Integration in one dimension.

Monte Carlos in Particle Physics

Due to its efficiency in higher dimensions, Monte Carlo integration is ideal for calculating cross sections for particle interactions at high energy colliders. The cross section can be roughly thought to represent the "frequency" at which a certain event is likely to occur at a particle collider, such as the Large Electron-Positron Collider (LEP) at CERN, Geneva, Switzerland, the predecessor of the Large Hadron Collider, operating between 1989 and 2000. In this work, we have considered the process where an electron and positron annihilate into a muon-anti-muon at LEP: $e^-e^+ \longrightarrow \mu^-\mu^+$ (fig. 3).

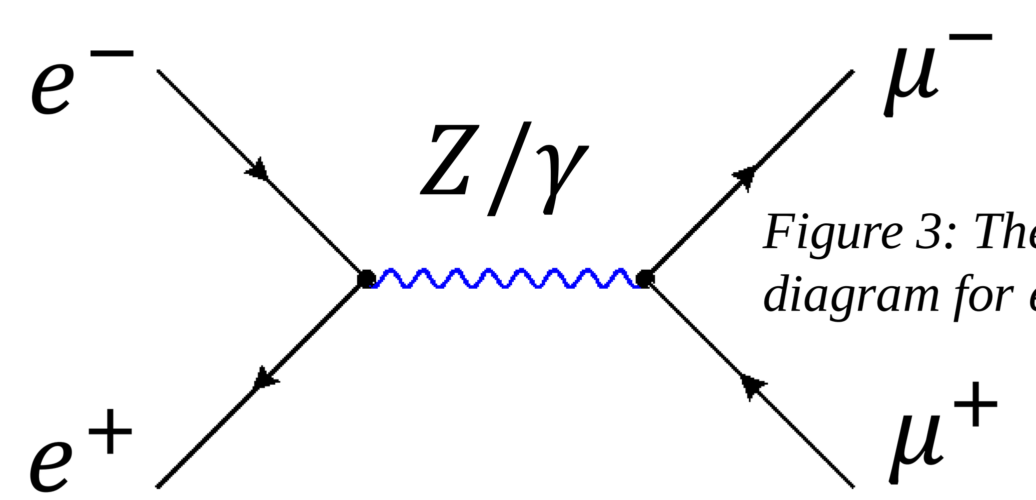

Figure 3: The leading-order Feynman diagram for $e^-e^+ \longrightarrow \mu^-\mu^+$.

Simulating Electron-Positron Annihilation

The differential cross section for electron-positron annihilation into muon-anti-muon pairs is given by:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\alpha^2}{4\hat{s}}\left[A_0(1+\cos^2\theta) + A_1\cos\theta\right]$$

where $\mathrm{d}\Omega$ is the solid angle element, θ is the scattering angle between the incoming e^- and outgoing μ^-, $\sqrt{\hat{s}} = 90$ GeV is the center-of-mass energy, and $\alpha \simeq 1/132.5$ is the QED coupling. $A_{0,1}$ are functions that depend on the contributing "virtual" particles, in this case the photon (γ) and the Z boson. To simulate the process, we employ the "hit-or-miss" method [1], where we "throw" random values of $\cos\theta$, and accept or reject according to the differential cross section.

Results

We show the $\cos\theta$ distribution in fig. 4, where we also show the graph *without* the Z boson (left). Another interesting observable is the forward-backward asymmetry, $A_{FB} = \frac{F-B}{F+B}$, which gives us a measure of the difference of the number of electrons moving in the forward direction, F, versus those in the backward direction, B. We find: $A_{FB} = 0.2490 \pm 0.009$.

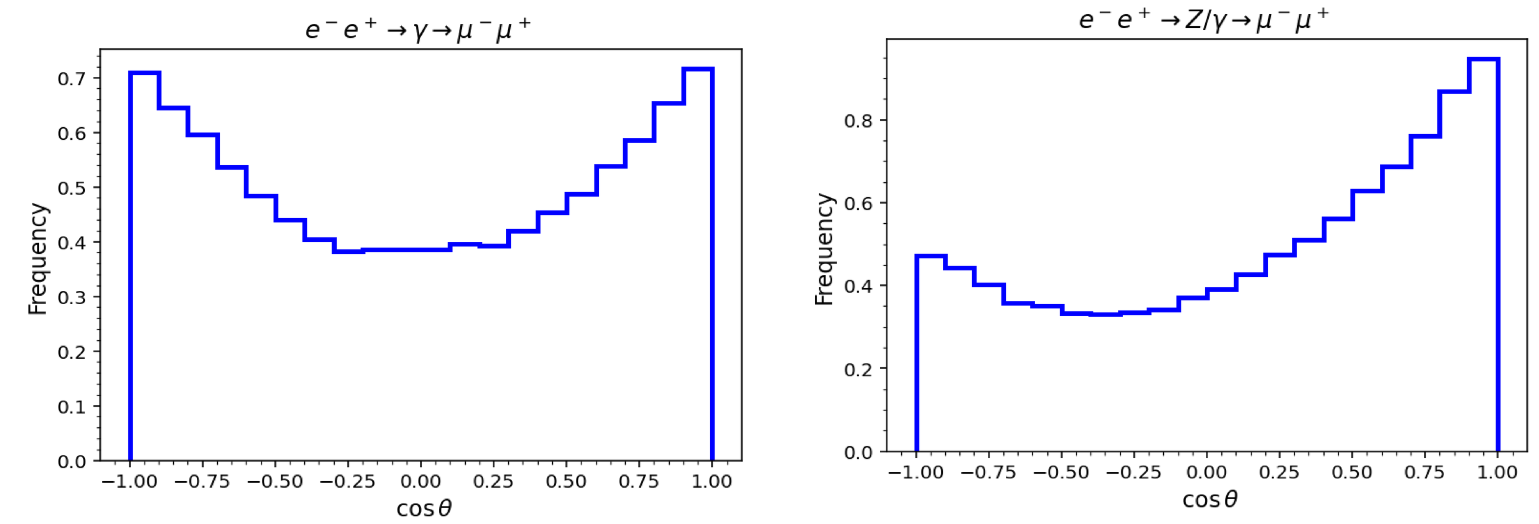

Figure 4: Results through the MC simulation of $e^-e^+ \longrightarrow \mu^-\mu^+$.

References

[1] A. Papaefstathiou, arXiv:1412.4677, Eur.Phys.J.Plus 135 (2020) 6, 497.

This work was made possible by the support of The National Science Foundation, through award No. PHY 2210161.