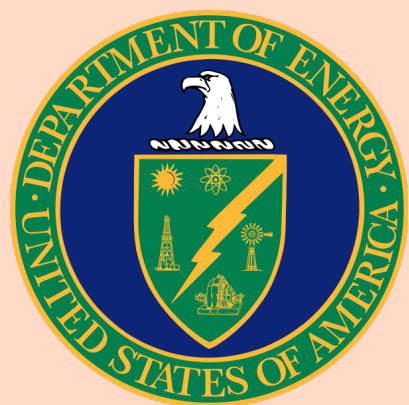


TRIPLE HIGGS BOSON

PRODUCTION

at the LHC and Beyond



Andreas Papaefstathiou
Kennesaw State University, GA, USA
@ HIGGS 2025 [Oct. 27-31, 2025]



KENNESAW STATE
UNIVERSITY

PART I: RAIDERS OF THE HIGGS POTENTIAL



Motivation: Measuring the Higgs Potential

🎩 Higgs field ϕ , “sits” in potential (SM): $\mathcal{V}(\phi) = \text{●} |\phi|^2 + \text{■} |\phi|^4$.

🎩 **Electroweak Symmetry Breaking:**

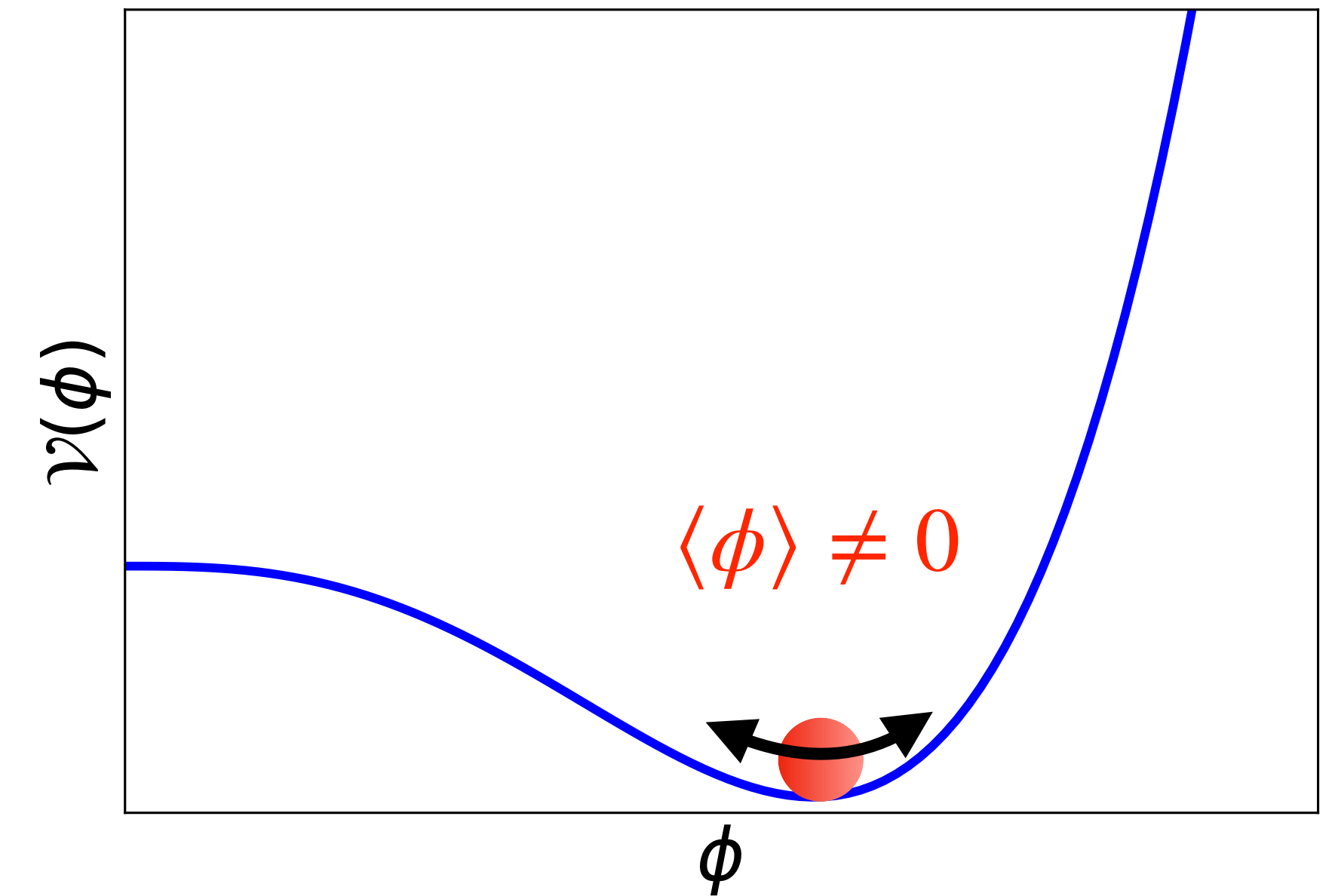
$$\mathcal{V}(\langle \phi \rangle + h) = \text{●} h^2 + \text{▲} h^3 + \text{■} h^4$$

where: h = the Higgs boson.

🎩 $\{\text{●}, \text{▲}, \text{■}\} \rightarrow$ the Higgs boson self-interactions!

🎩 SM: predicted via the Higgs boson mass & VEV:

$$\text{●}_{\text{SM}} = \frac{1}{2} m_h^2 ; \quad \underbrace{\text{▲}_{\text{SM}}}_{\lambda_3^{\text{SM}}} = \frac{m_h^2}{2v} ; \quad \underbrace{\text{■}_{\text{SM}}}_{\lambda_4^{\text{SM}}} = \frac{m_h^2}{8v^2}$$

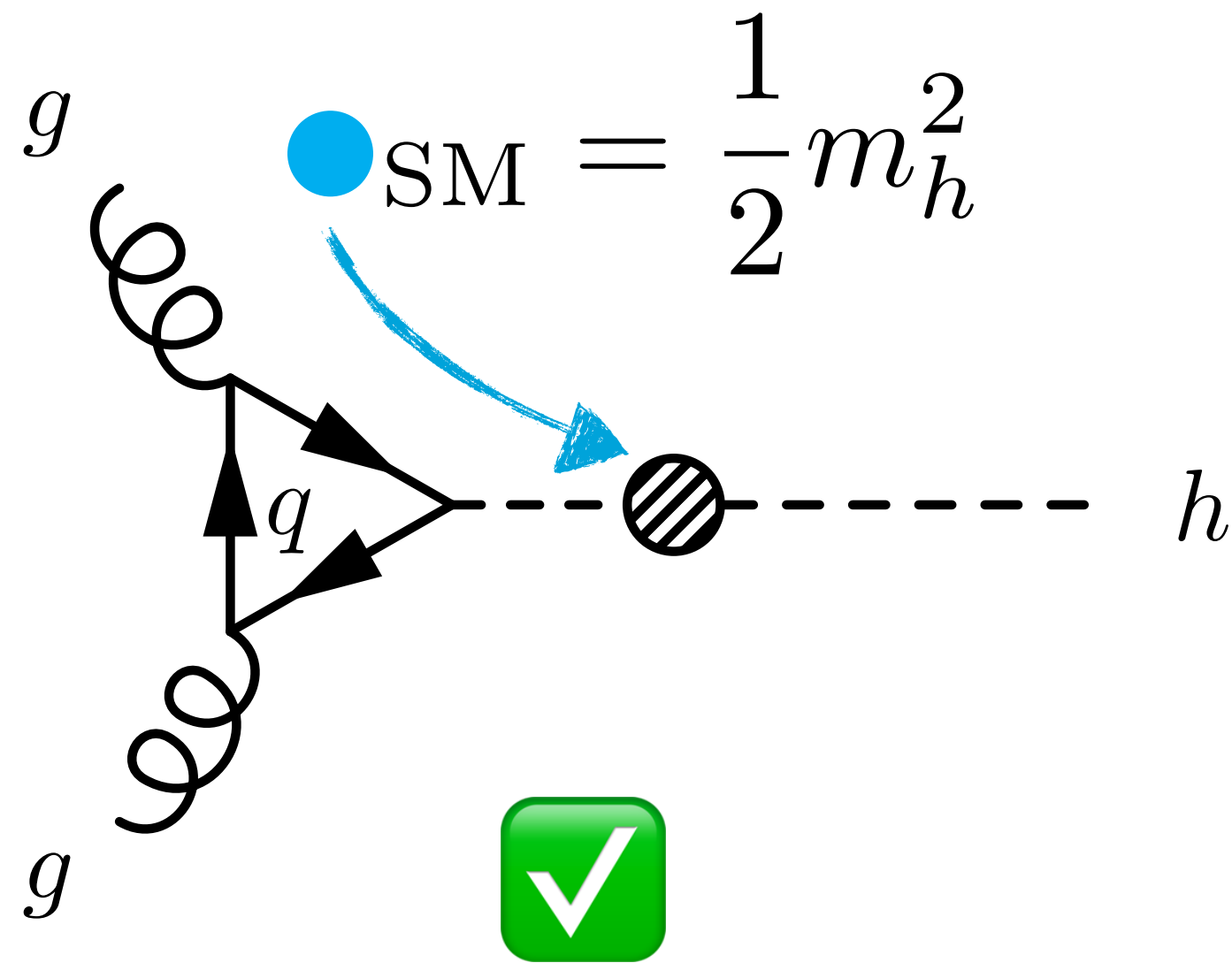


Motivation: Measuring the Higgs Self-Couplings

👒 **Verify** that $\{\bullet, \blacktriangle, \blacksquare\} \approx \{\bullet, \blacktriangle, \blacksquare\}_{\text{SM}}$ at hadron colliders, by measuring:

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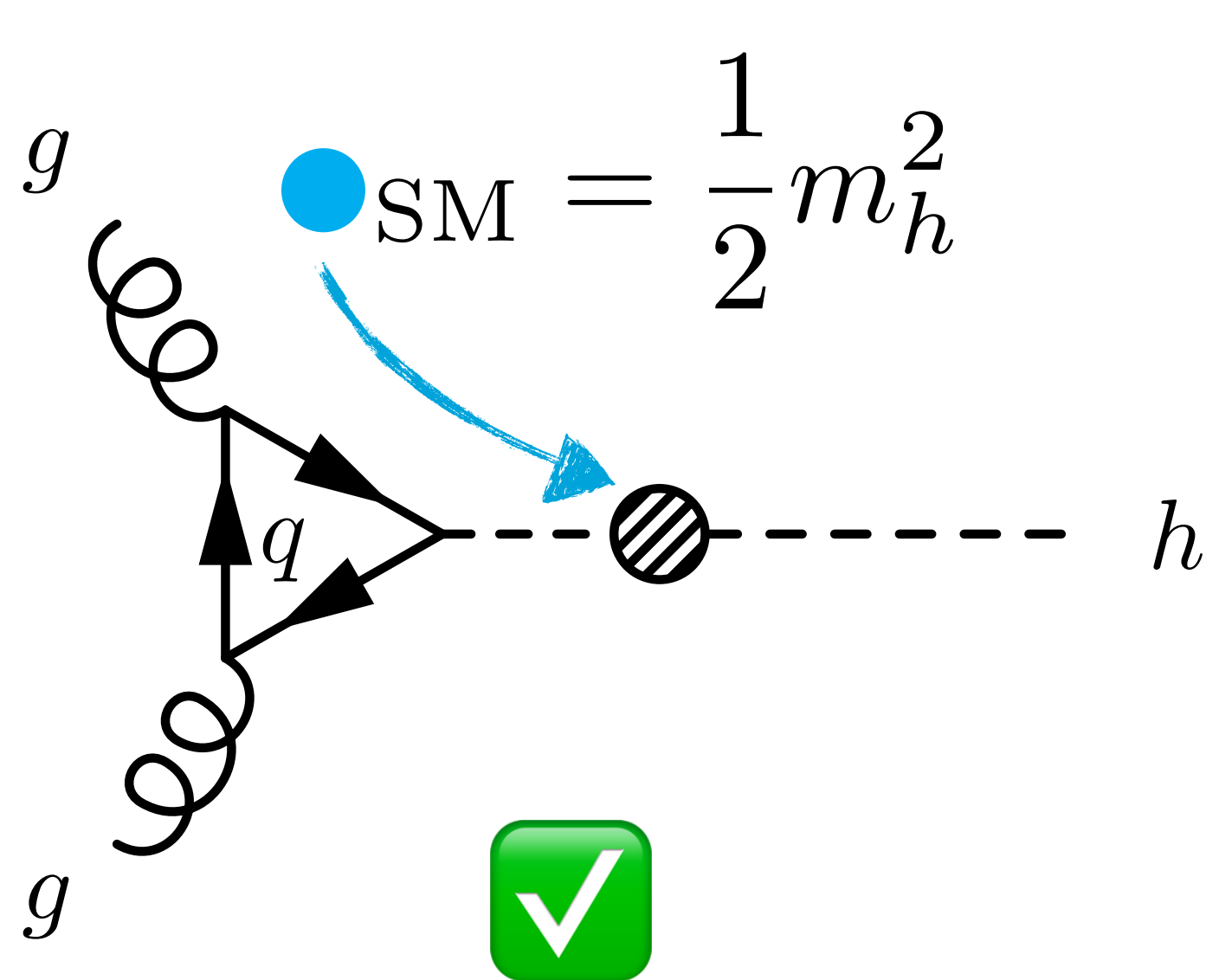
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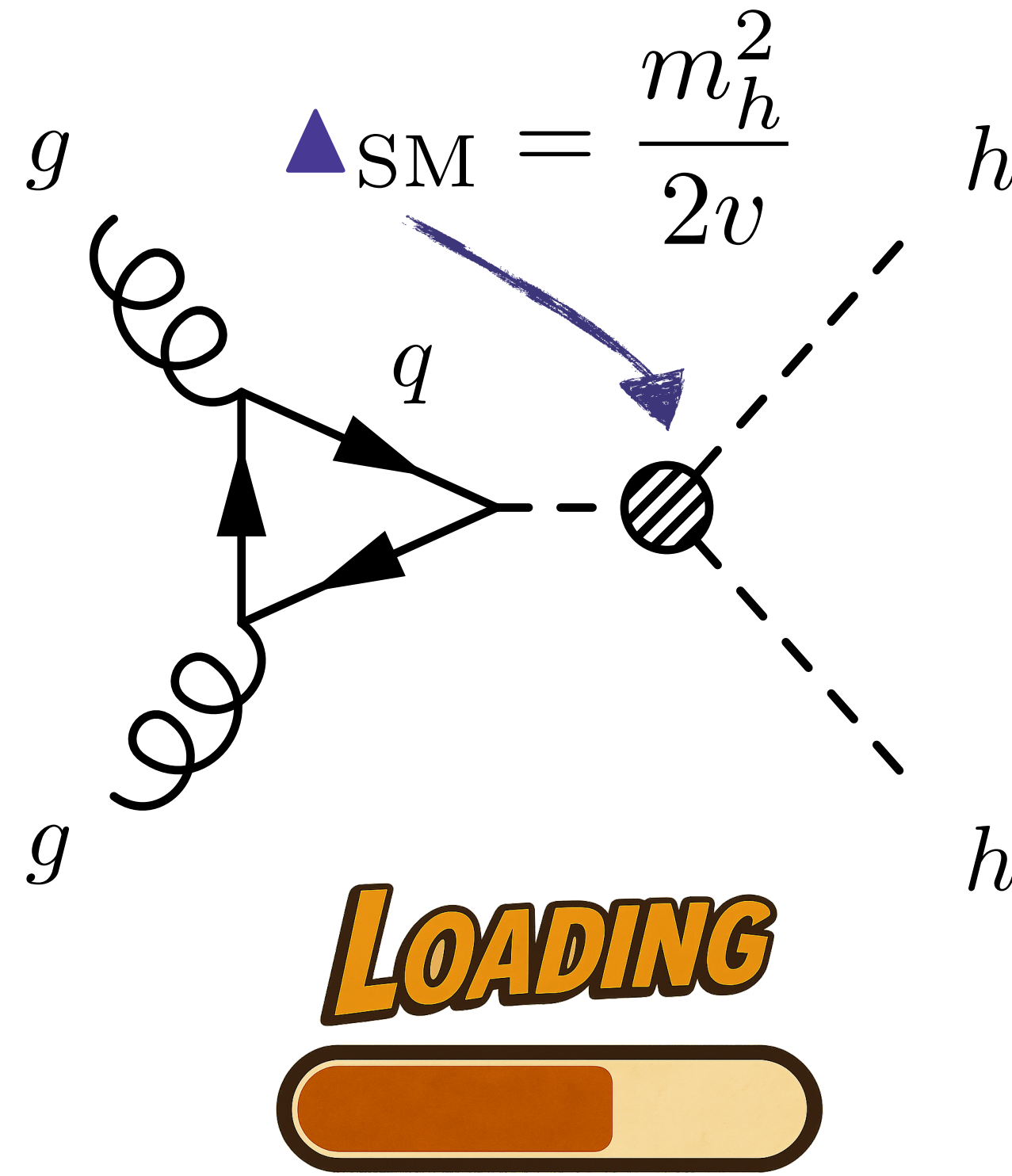
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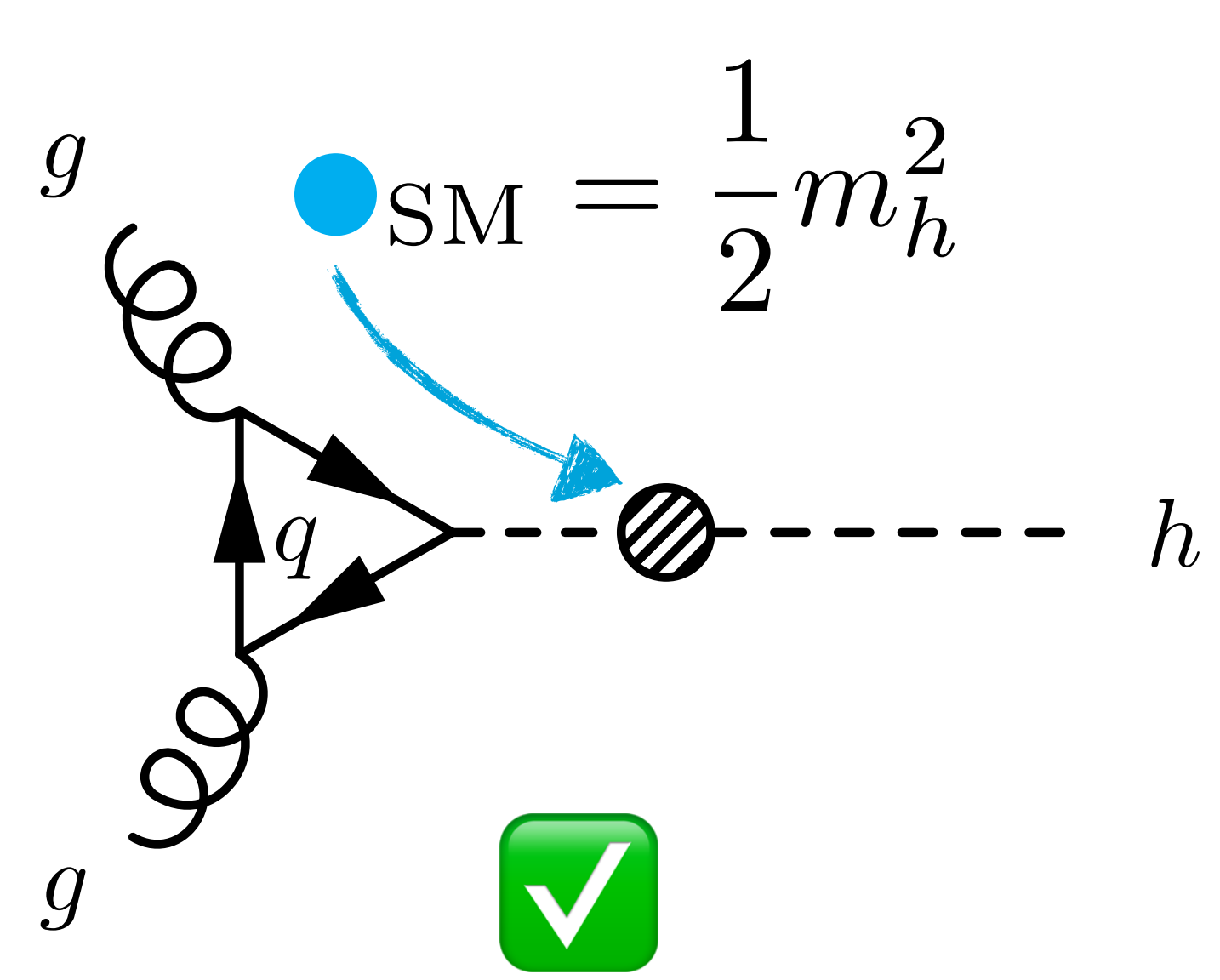


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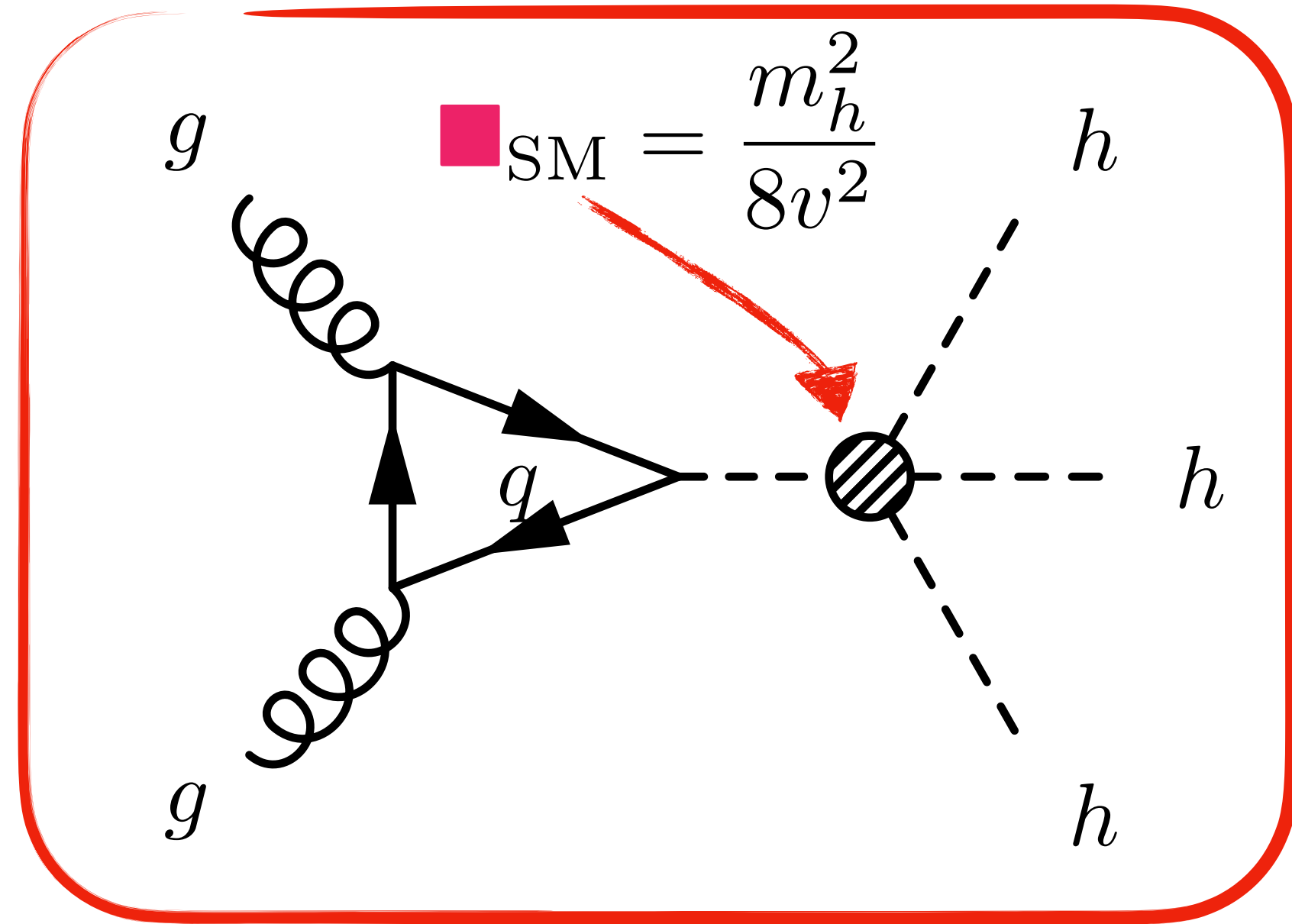
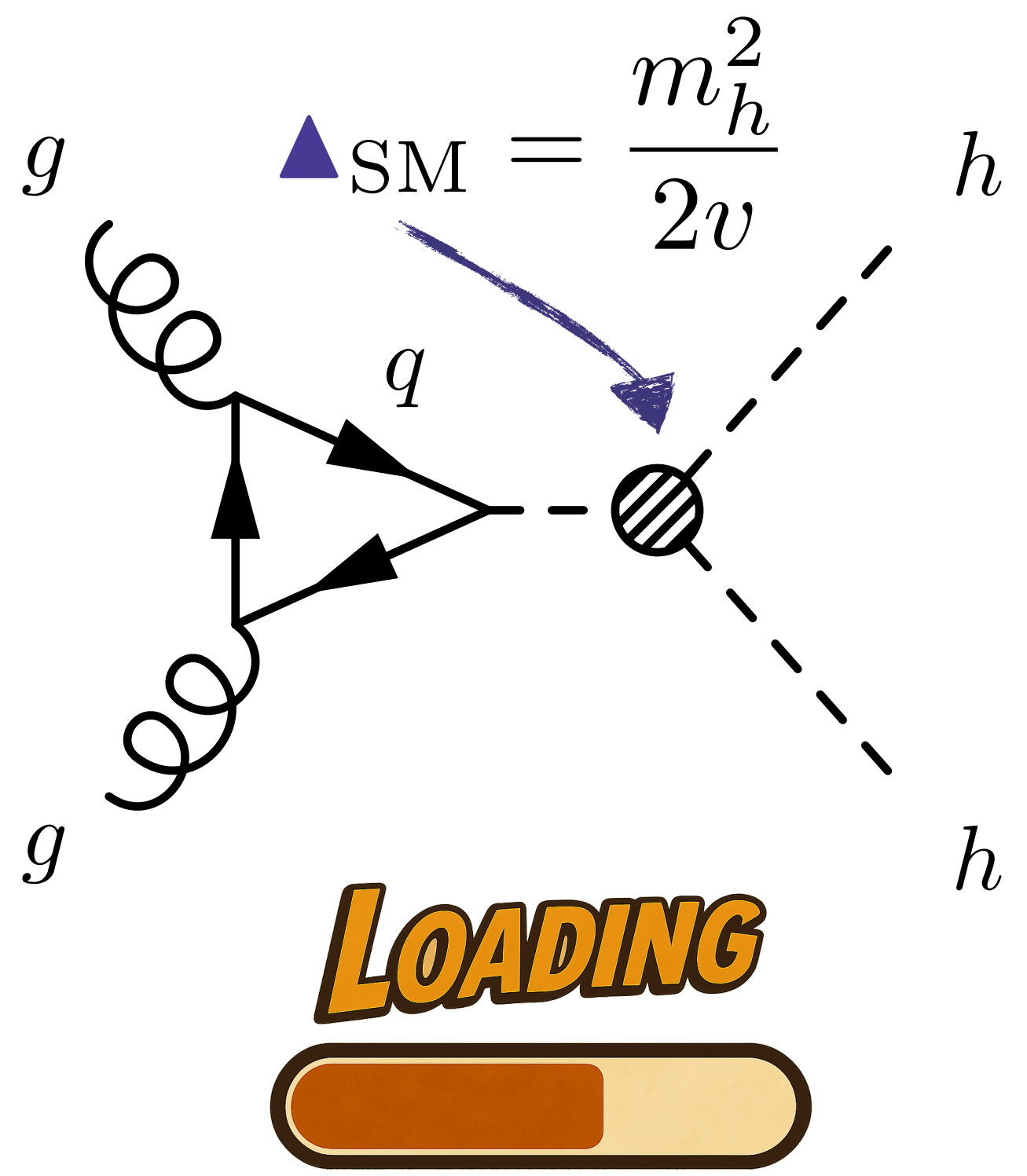


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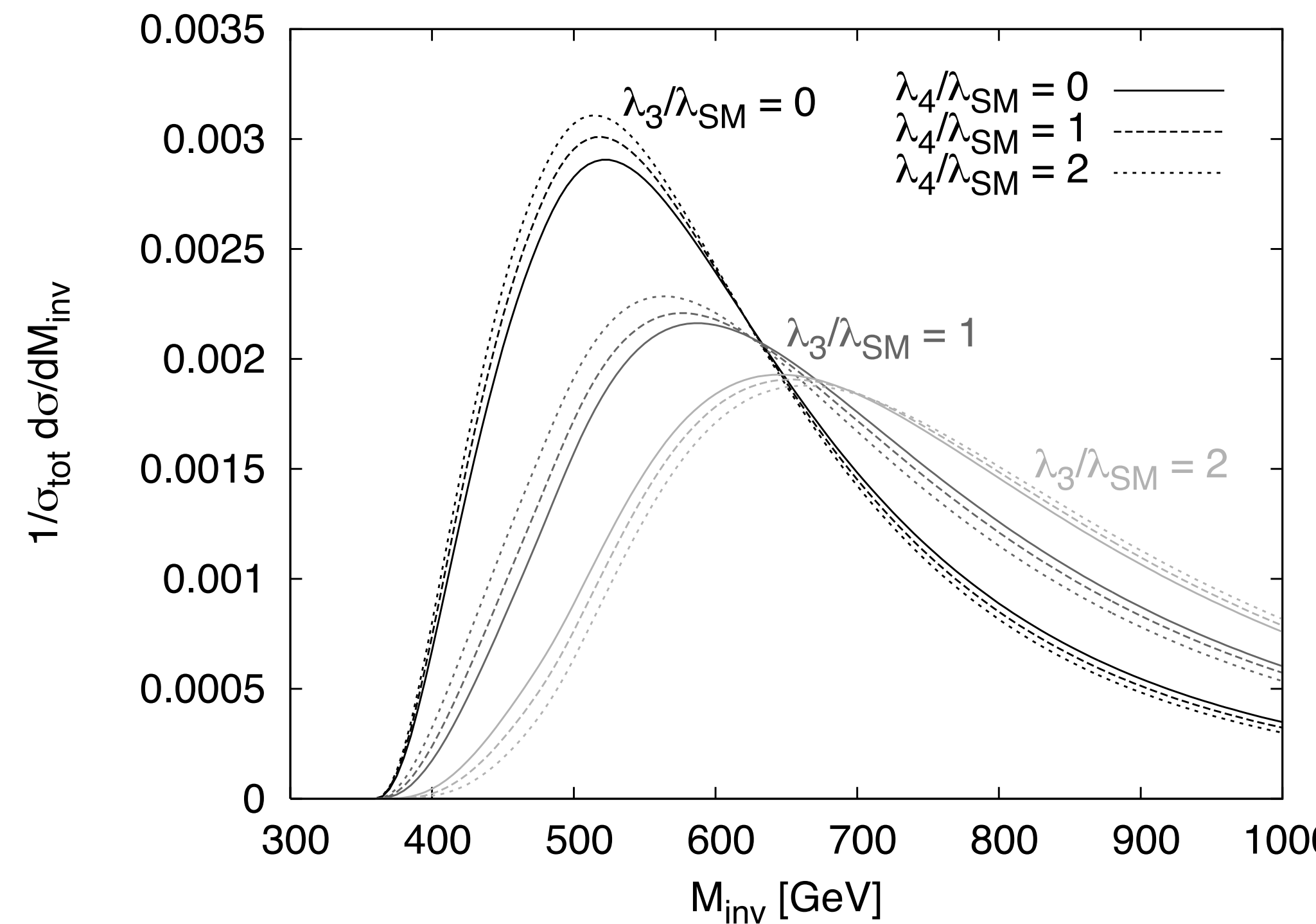
Tiny cross section @ LHC:
SM $\sigma(hhh @ 14 \text{ TeV}) \sim 0.1 \text{ fb!}$



Triple Higgs Boson Production: First Studies

👒 [Plehn, Rauch, hep-ph/0507321]: “VLHC” @ 200 TeV → “*Little Hope! & Measuring quartic requires good knowledge of trilinear*”.

👒 [Binoth, Karg, Kauer, Rückl, hep-ph/0608057] →
“*multi- h rates sensitive to BSM, extreme values of quartic lead to $\mathcal{O}(10 \text{ fb})$ cross sections at LHC*”.



Note: In these studies: no knowledge of h mass!

Post-Higgs Boson Discovery Studies

🎩 Following Higgs boson discovery $\rightarrow$ (most) pheno studies @ pp@100 TeV:

“Interesting” final states with at least 100 events @ 100 TeV @ 30 ab⁻¹

$hh \rightarrow$ final state	BR (%)	σ (ab)	$N_{30\text{ab}^{-1}}$
$(b\bar{b})(b\bar{b})(b\bar{b})$	19.21	1110.338	33310
$(b\bar{b})(b\bar{b})(WW_{1\ell})$	7.204	416.41	12492
$(b\bar{b})(b\bar{b})(\tau\bar{\tau})$	6.312	364.853	10945
$(b\bar{b})(\tau\bar{\tau})(WW_{1\ell})$	1.578	91.22	2736
$(b\bar{b})(b\bar{b})(WW_{2\ell})$	0.976	56.417	1692
$(b\bar{b})(WW_{1\ell})(WW_{1\ell})$	0.901	52.055	1561
$(b\bar{b})(\tau\bar{\tau})(\tau\bar{\tau})$	0.691	39.963	1198
$(b\bar{b})(b\bar{b})(ZZ_{2\ell})$	0.331	19.131	573
$(b\bar{b})(WW_{2\ell})(WW_{1\ell})$	0.244	14.105	423
$(b\bar{b})(b\bar{b})(\gamma\gamma)$	0.228	13.162	394
$(b\bar{b})(\tau\bar{\tau})(WW_{2\ell})$	0.214	12.359	370
$(\tau\bar{\tau})(WW_{1\ell})(WW_{1\ell})$	0.099	5.702	171
$(\tau\bar{\tau})(\tau\bar{\tau})(WW_{1\ell})$	0.086	4.996	149
$(b\bar{b})(ZZ_{2\ell})(WW_{1\ell})$	0.083	4.783	143
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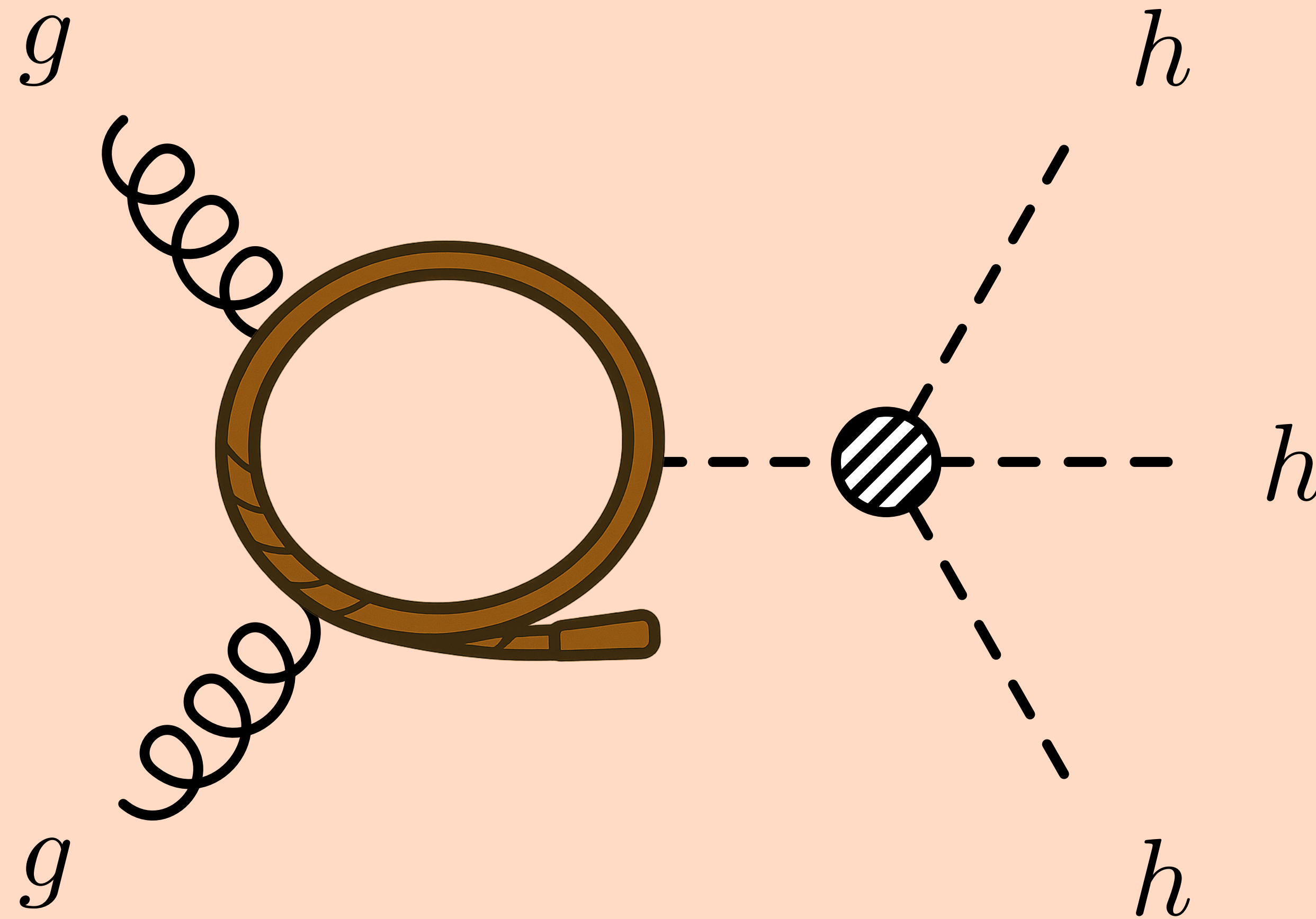
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PART II: THE NON-RESONANT CROSS SECTION

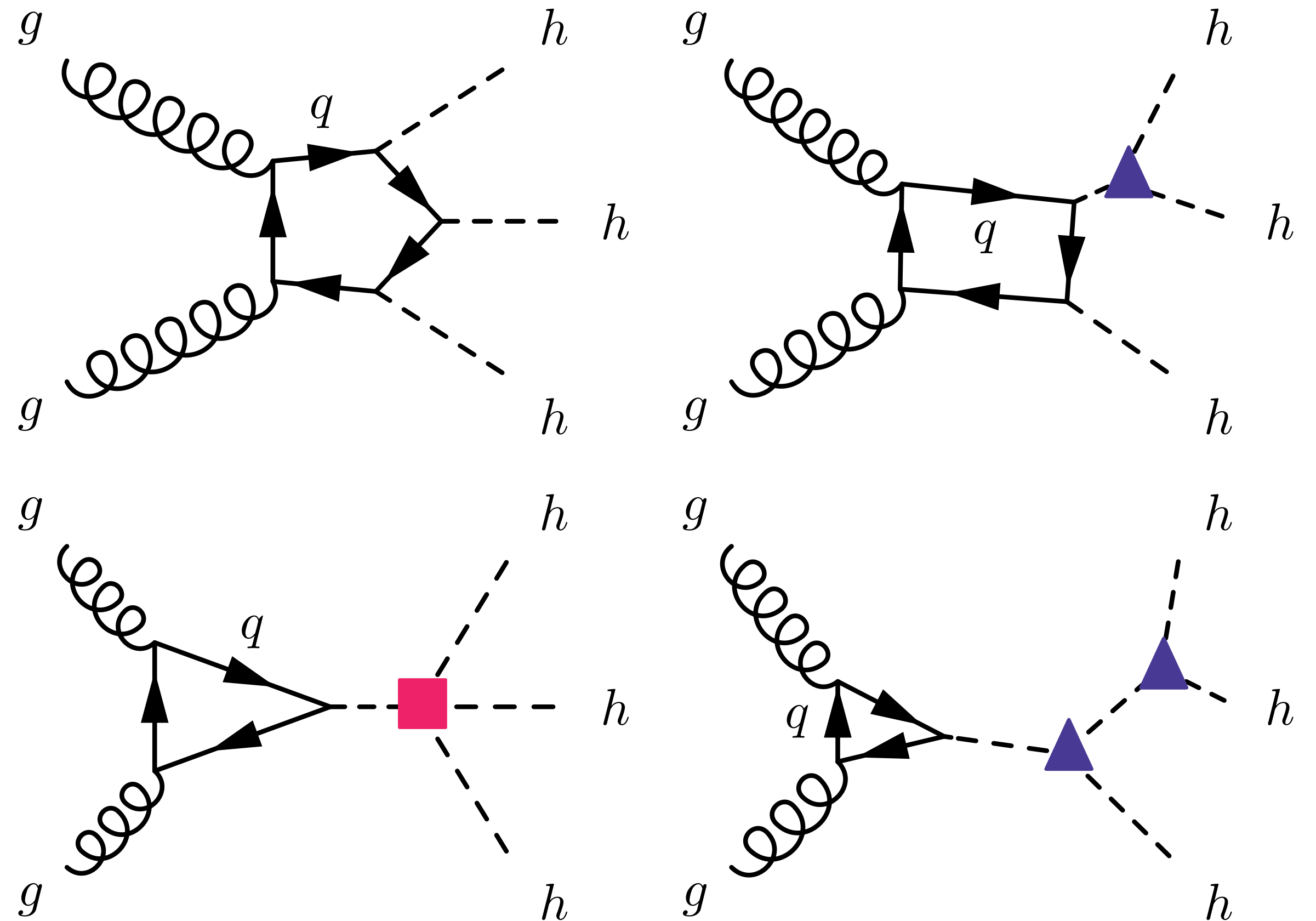


The $pp \rightarrow hhh$ Cross Section

👒 [Note: VBF hhh $\sigma \sim 10^{-4}$ fb@13 TeV $\Rightarrow$ Gluon fusion dominant by far, see: Belyaev, Schaefers, Thomas, **1101.10157**].

👒 At **LO** in SM: Loop-induced!

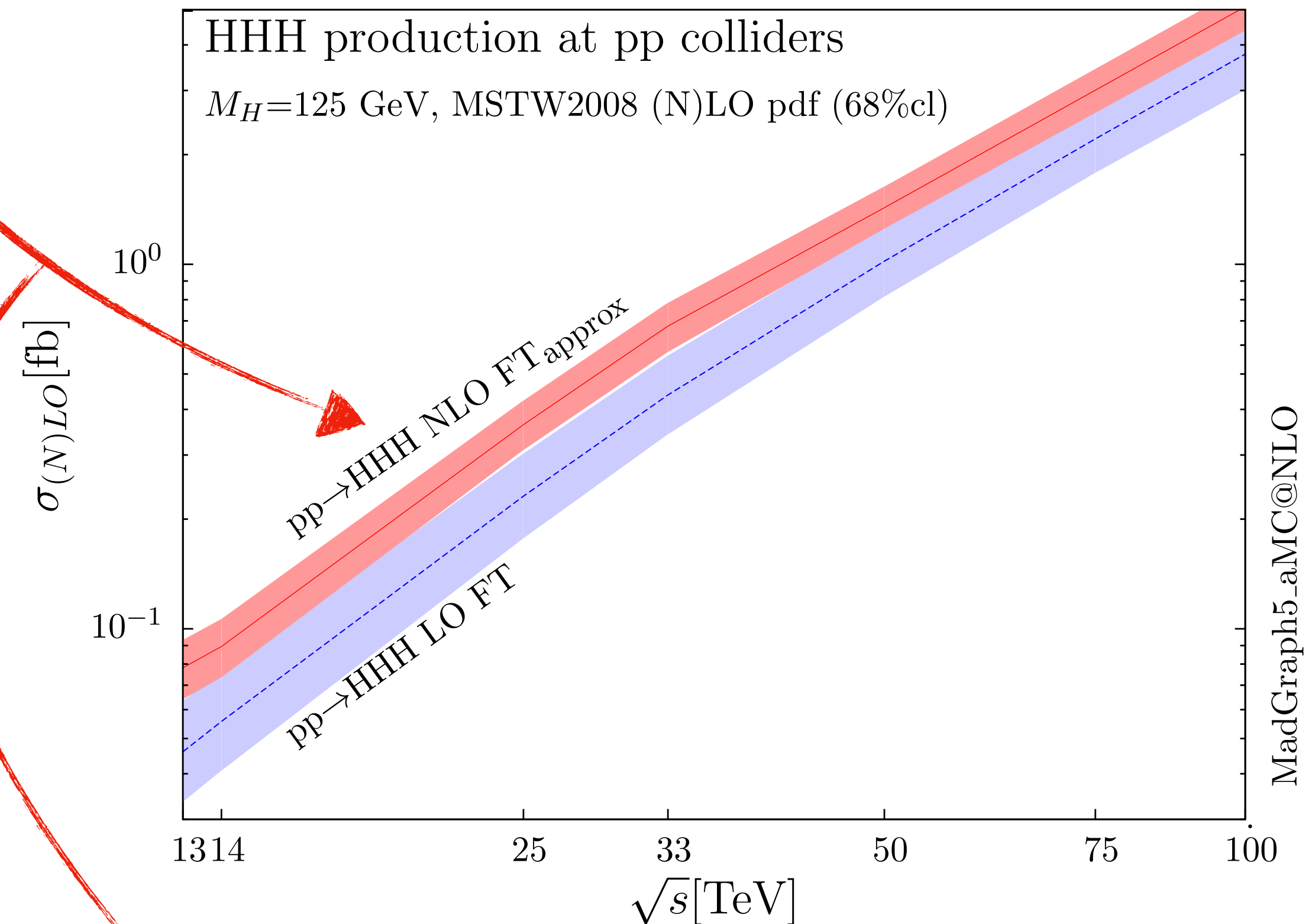
$$\sigma_{\text{SM}}^{\text{LO}}(14 \text{ TeV}) \sim 0.05 \text{ fb}$$



$pp \rightarrow hhh$ @ Higher Orders

🎩 **NLO**: [Maltoni, Vryonidou, Zaro, 1408.6542] $\rightarrow m_t \rightarrow \infty$ plus full real emission (= $\text{FT}_{\text{approx}}$).

🎩 **NNLO**: [De Florian, Fabre, Mazzitelli, 1912.02760] $\rightarrow$ Full NNLO in $m_t \rightarrow \infty$, “Born-Improved” $\Rightarrow \sim 10\%$ increase wrt. NLO.



$$\begin{aligned} \sigma_{\text{Best}}^{\text{NNLO}} &= 0.103^{+5\%}_{-8\%} \pm 15\% \text{ fb}, & K_{\text{Best}}^{\text{NNLO}} &= 1.70, & (14 \text{ TeV}) \\ \sigma_{\text{Best}}^{\text{NNLO}} &= 0.501^{+5\%}_{-7\%} \pm 15\% \text{ fb}, & K_{\text{Best}}^{\text{NNLO}} &= 1.70, & (27 \text{ TeV}) \\ \sigma_{\text{Best}}^{\text{NNLO}} &= 5.56^{+5\%}_{-6\%} \pm 20\% \text{ fb}, & K_{\text{Best}}^{\text{NNLO}} &= 1.43, & (100 \text{ TeV}) \end{aligned}$$

$\sigma(HHH)$ [fb]	$\sqrt{s} = 14 \text{ TeV}$	$\sqrt{s} = 33 \text{ TeV}$	$\sqrt{s} = 100 \text{ TeV}$
LO FT	$0.0557^{+34.5+2.5\%}_{-24.0-2.7\%}$	$0.438^{+26.8+1.5\%}_{+20.0-2.0\%}$	$3.78^{+24.1+0.9\%}_{-18.7-1.7\%}$
NLO $\text{FT}_{\text{approx}}$	$0.0894^{+16.5+2.5\%}_{-14.6-3.2\%}$	$0.677^{+14.5+1.4\%}_{-13.4-1.7\%}$	$5.09^{+13.5+1.0\%}_{-12.7-1.3\%}$

Attack of the Anomalous Couplings

👒 Anomalous couplings can enhance *hhh*! e.g. inspired by an Effective Field Theory.

$$\mathcal{L} \supset -\lambda_{\text{SM}} v \underbrace{(1+c_3)}_{K_3} h^3 - \frac{\lambda_{\text{SM}}}{4} \underbrace{(1+d_4)}_{K_4} h^4$$

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⇒ Cross section modifications (ratio to SM):

$$\begin{aligned} \sigma/\sigma_{\text{SM}}(13.6 \text{ TeV}) - 1 \approx & 0.88c_3^2 - 0.82c_3 - 0.32c_3^3 \\ & - 0.17c_3d_4 - 0.09d_4 + 0.05c_3^2d_4 \\ & - 0.02c_3^4 + 0.02d_4^2 \end{aligned}$$

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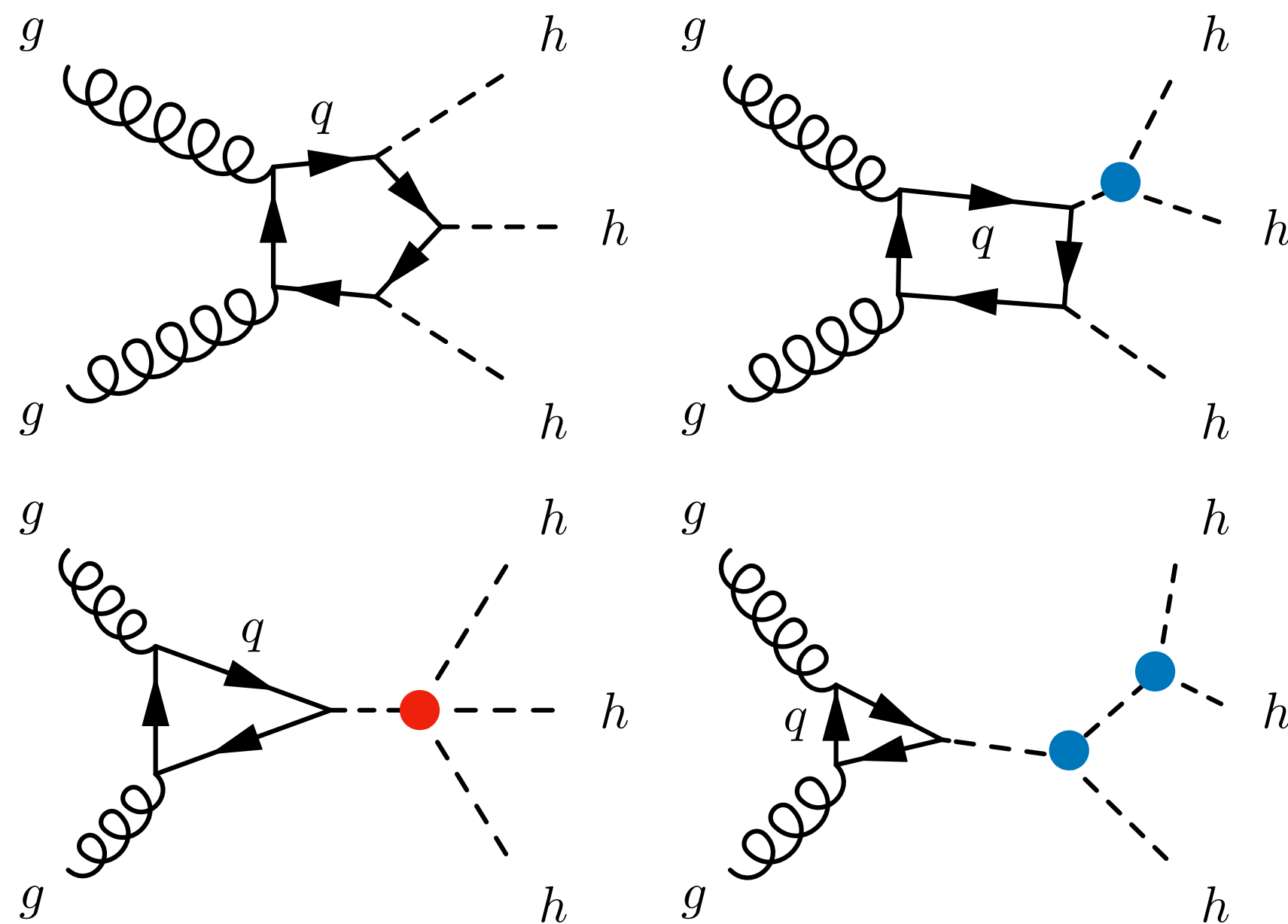
Q: Do all of these terms make sense in an effective field theory?

Matrix Element Truncation

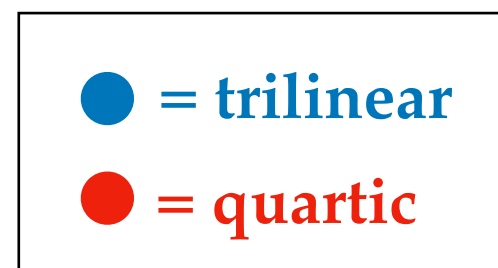
🎩 **Q:** Which anomalous contributions to keep @ the **matrix element-squared level**?

🎩 Effective Field Theory considerations: **not settled yet!** [$\rightarrow$ see, e.g. discussions of 2201.04974, 2304.01968].

🎩 Consider linear, quadratic or cubic truncations?



$$|\mathcal{M}|^2 \sim 1 + A c_3 + B d_4 \quad (\text{linear})$$

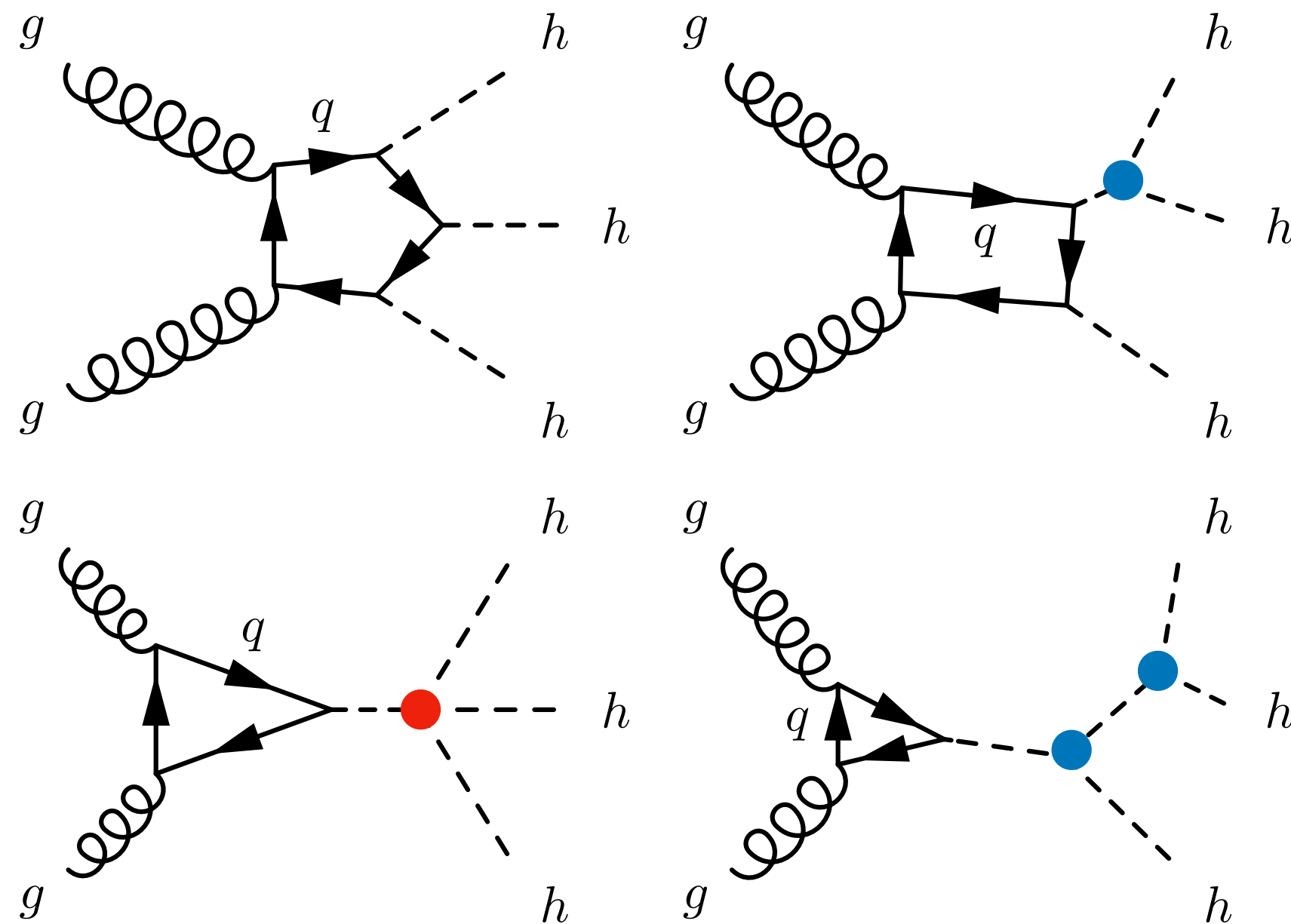


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$$+ C d_4^2 + D c_3 d_4 + E c_3^2 \quad (\text{quadratic})$$

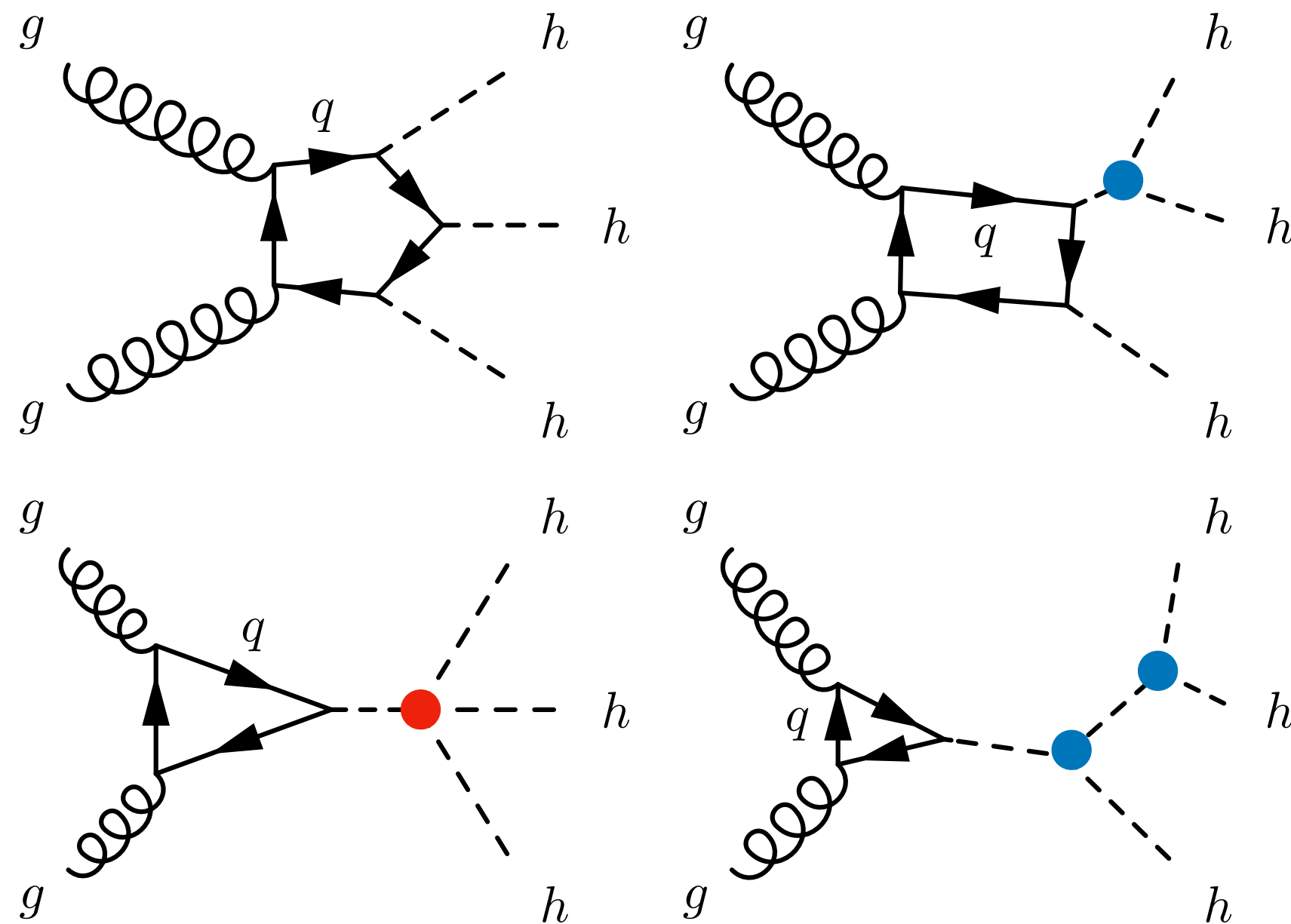
● = trilinear
● = quartic

Matrix Element Truncation

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🎩 Effective Field Theory considerations: **not settled yet!** [→ see, e.g. discussions of 2201.04974, 2304.01968].

🎩 Consider linear, quadratic or cubic truncations?



● = trilinear
● = quartic

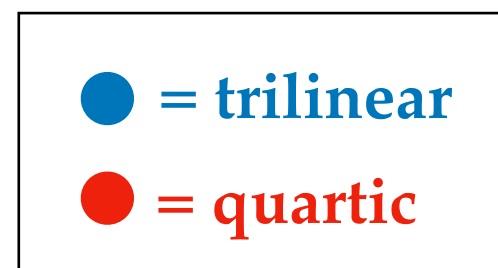
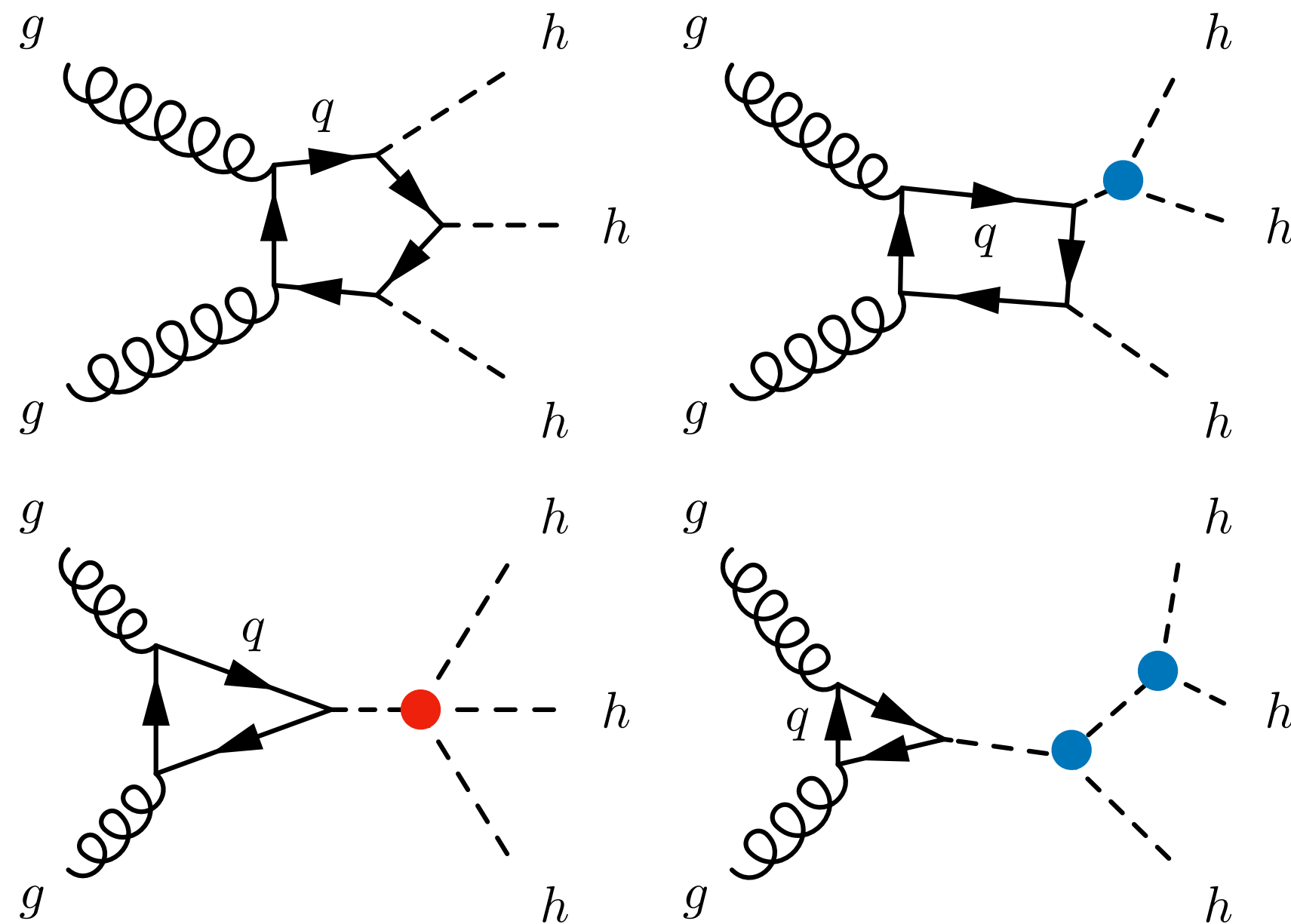
$$\begin{aligned}
 |\mathcal{M}|^2 \sim & 1 + A c_3 + B d_4 && \text{(linear)} \\
 & + C d_4^2 + D c_3 d_4 + E c_3^2 && \text{(quadratic)} \\
 & + F c_3^2 d_4 + G c_3^3 && \text{(cubic)}
 \end{aligned}$$

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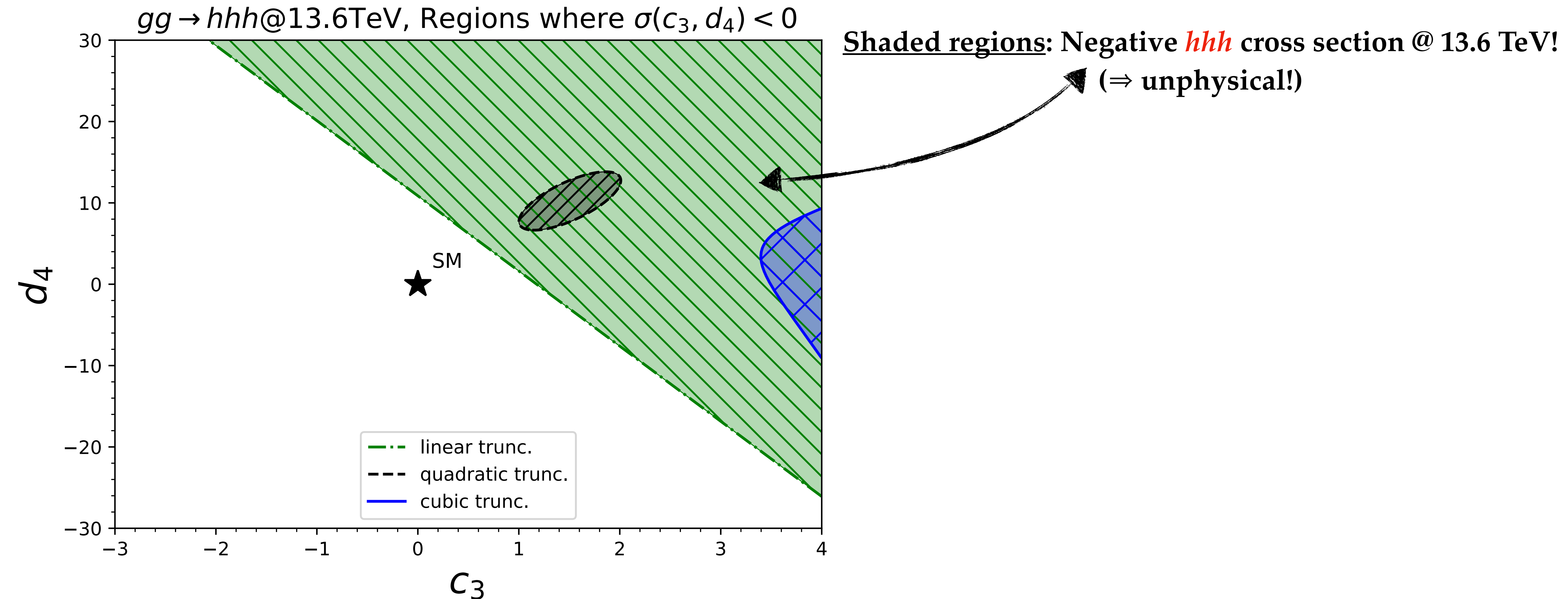
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 & + c_3^4 && \text{(no trunc.)}
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Matrix Element Truncation

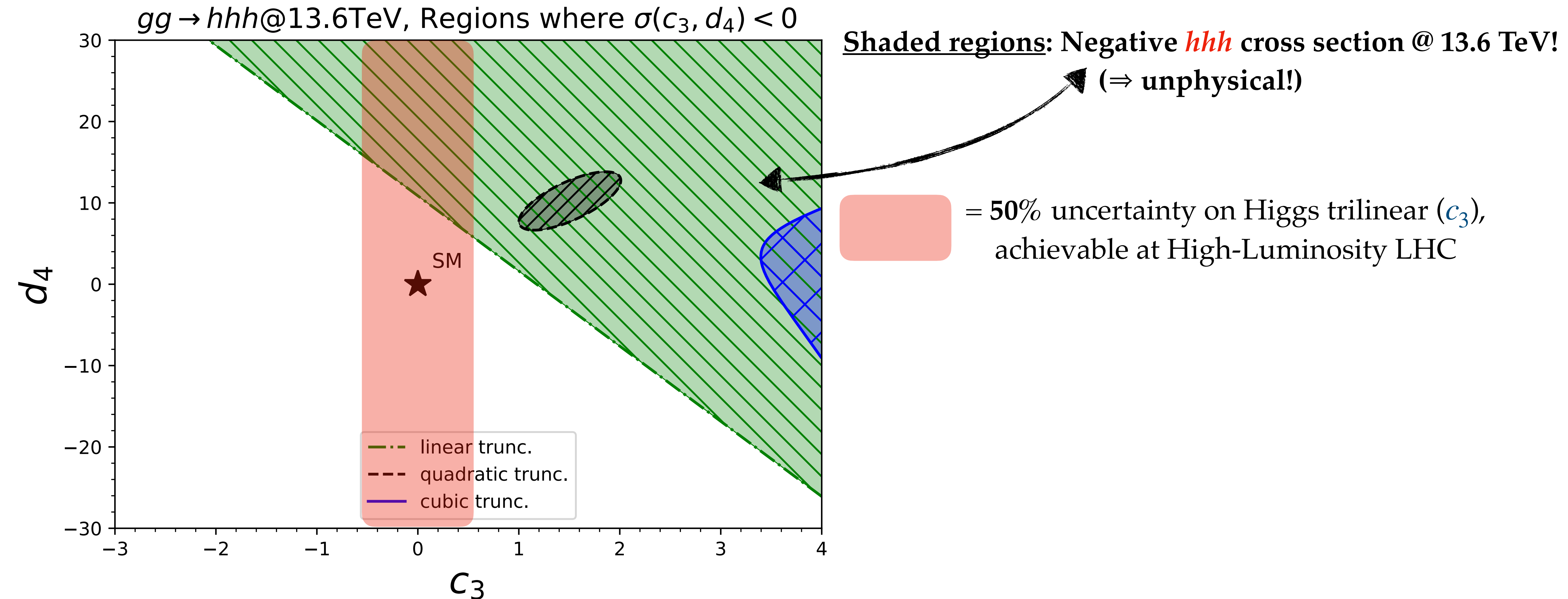
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[Fuks, AP, Tetlalmatzi-Xolocotzi, 2509.16364]

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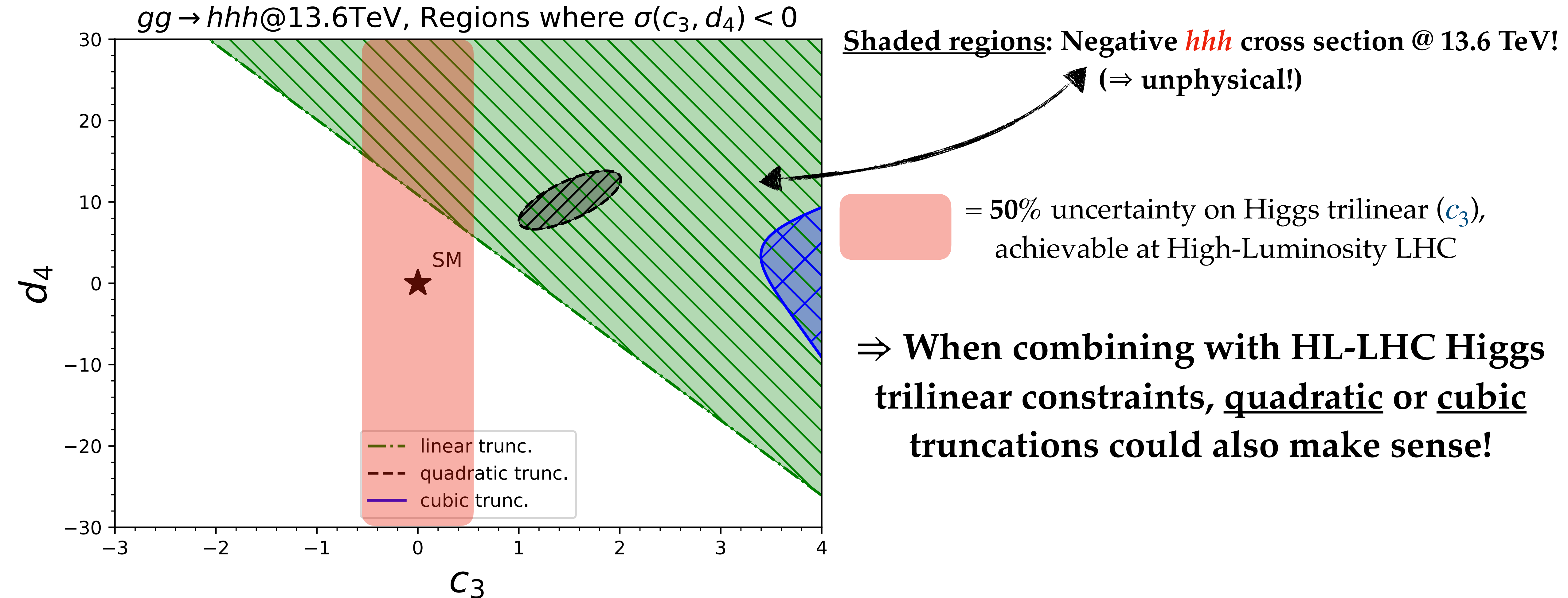
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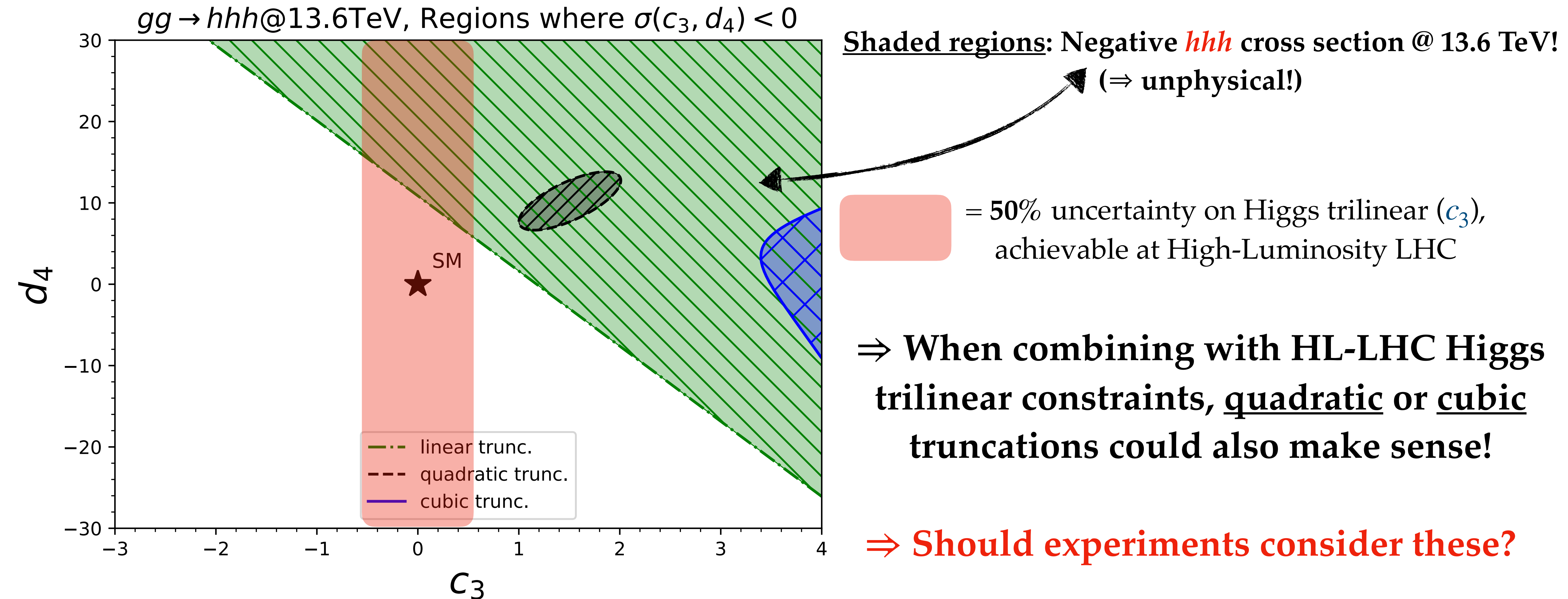
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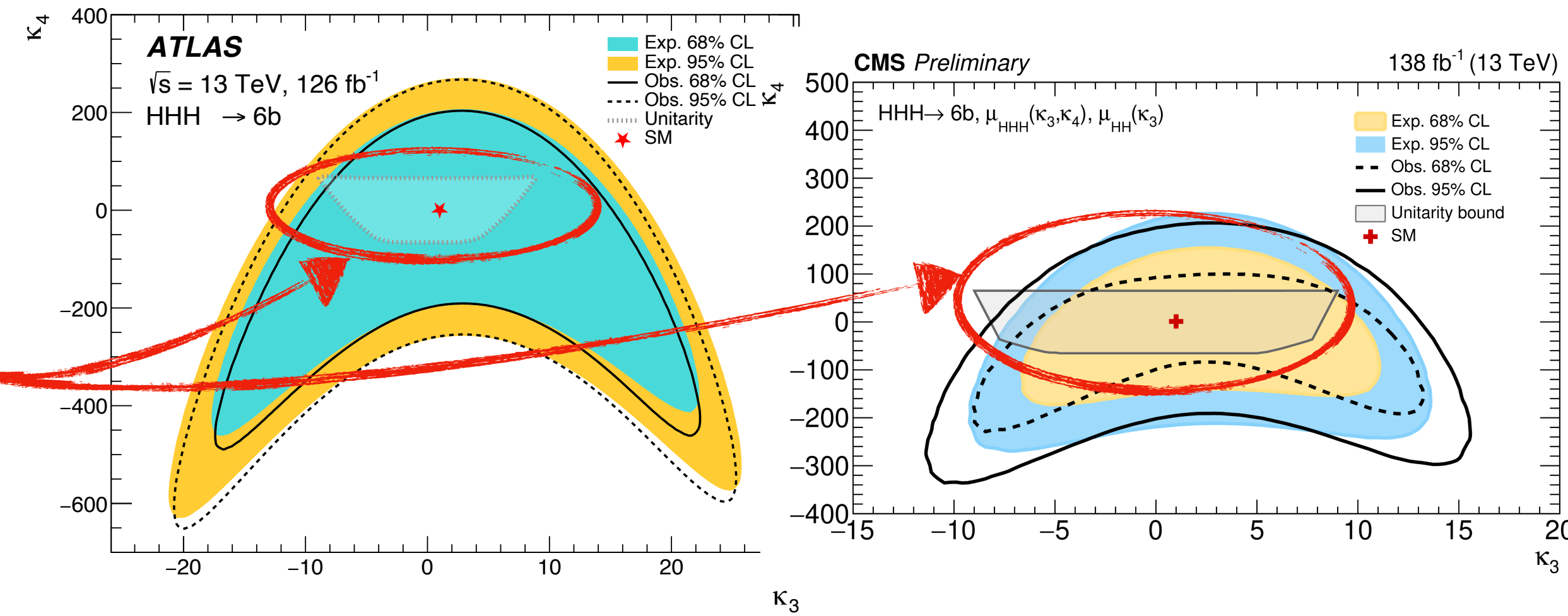
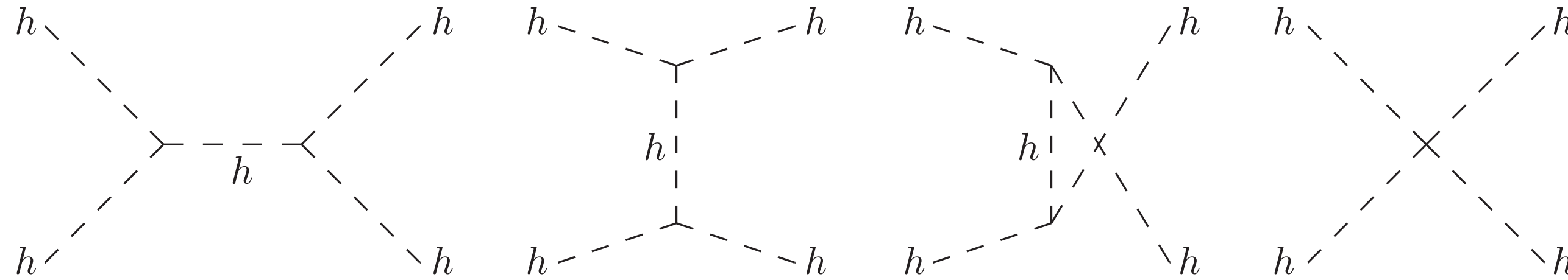
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[Fuks, AP, Tetlalmatzi-Xolocotzi, 2509.16364]

Perturbative Unitarity (in 5 easy steps)

- Experiments show limits on (κ_3, κ_4) like these:
- including constraint from **perturbative unitarity**,
- 1. Consider $hh \rightarrow hh$ scattering:



$$\mathcal{M} = -\lambda_3^2 \left(\frac{1}{s - m_h^2} + \frac{1}{t - m_h^2} + \frac{1}{u - m_h^2} \right) - \lambda_4$$

- via the matrix element $\langle f | T | i \rangle \rightarrow T$ the interacting part of the S-matrix: $S = 1 + iT$, requiring $S^\dagger S = 1$, i.e. **unitarity**.

- 2. Project into partial waves $a_{fi}^J \propto \int d(\cos \theta) \overbrace{d_{\mu_i \mu_f}^J(\theta)}^{\text{Wigner-}d \text{ function}} \langle f | T | i \rangle$ of total angular momentum J ,

- 3. consider same-helicity state and $J=0$ (higher typically smaller amplitudes) $\rightarrow d_{00}^0 = 1$.

Perturbative Unitarity (in 5 easy steps)

👒 4. If real amplitude, then require: $\left| \text{Re}(a_{ii}^J) \right| \leq \frac{1}{2}$, at all energies.

(in reality: issues arise before saturating this $\Rightarrow$ **bound is conservative**).

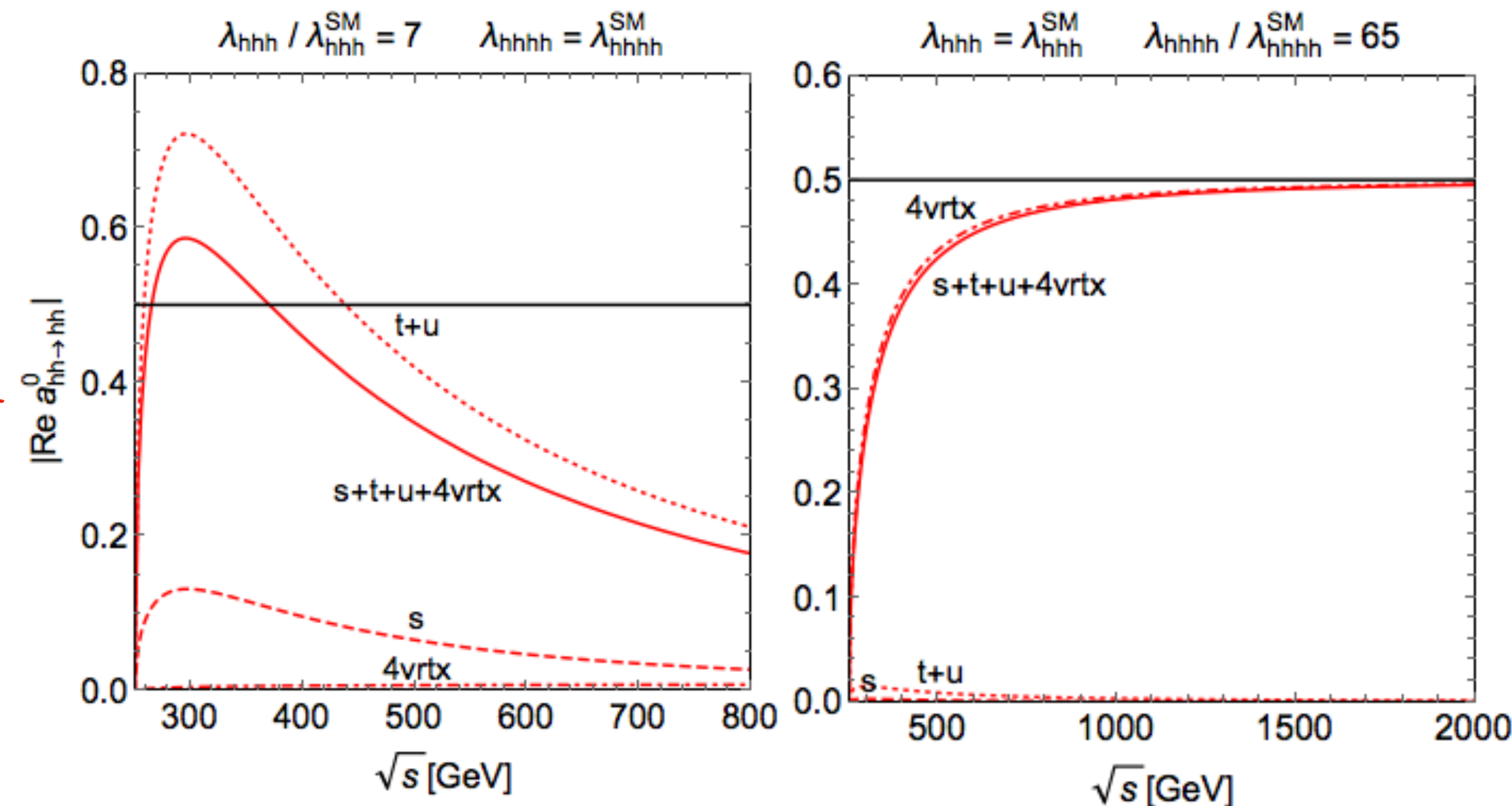
👒 5. Apply to $hh \rightarrow hh$: one finds that κ_3 bound is **maximized at low energy**, κ_4 at **high energy**:

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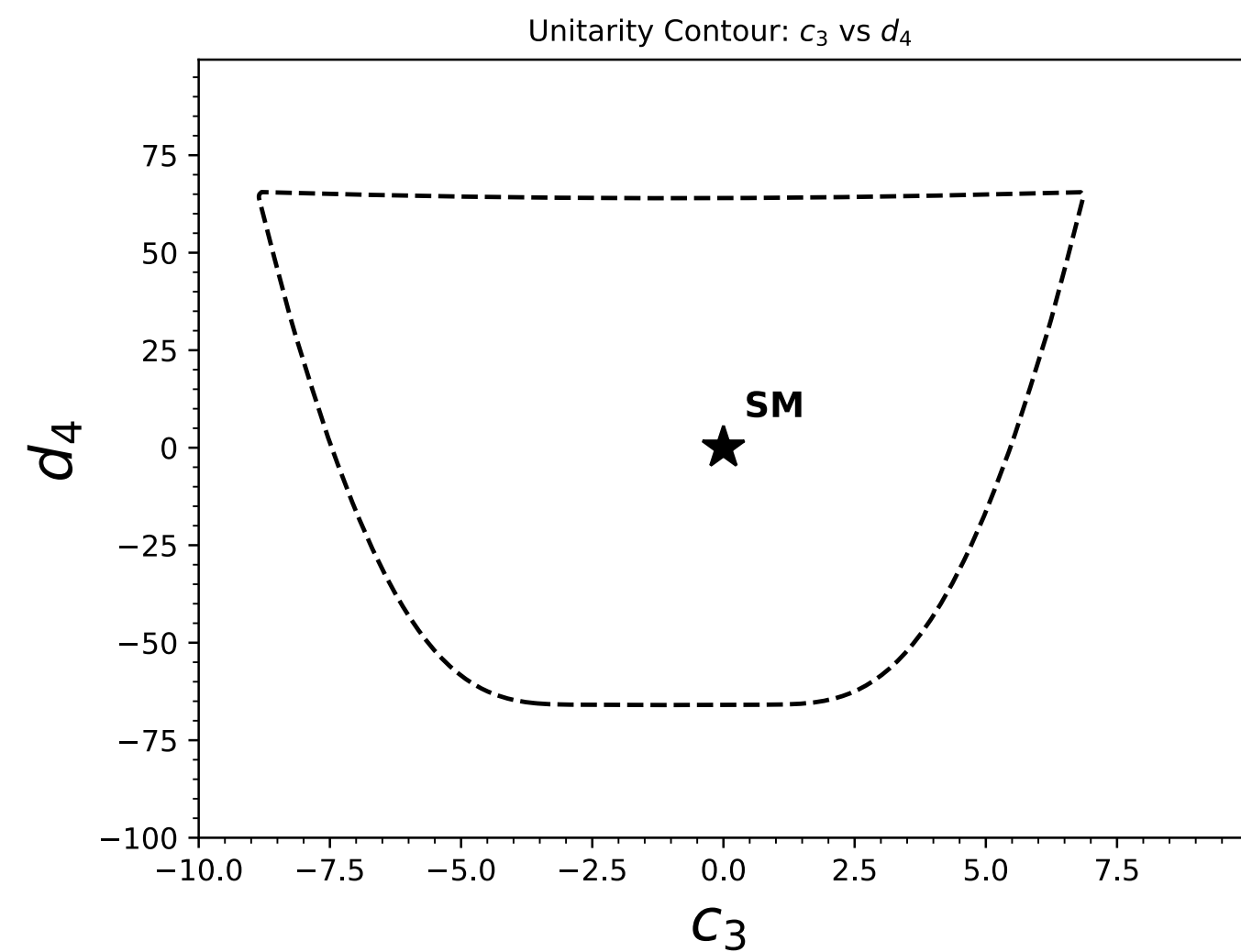
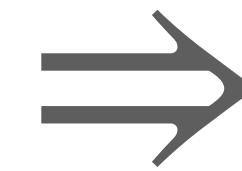
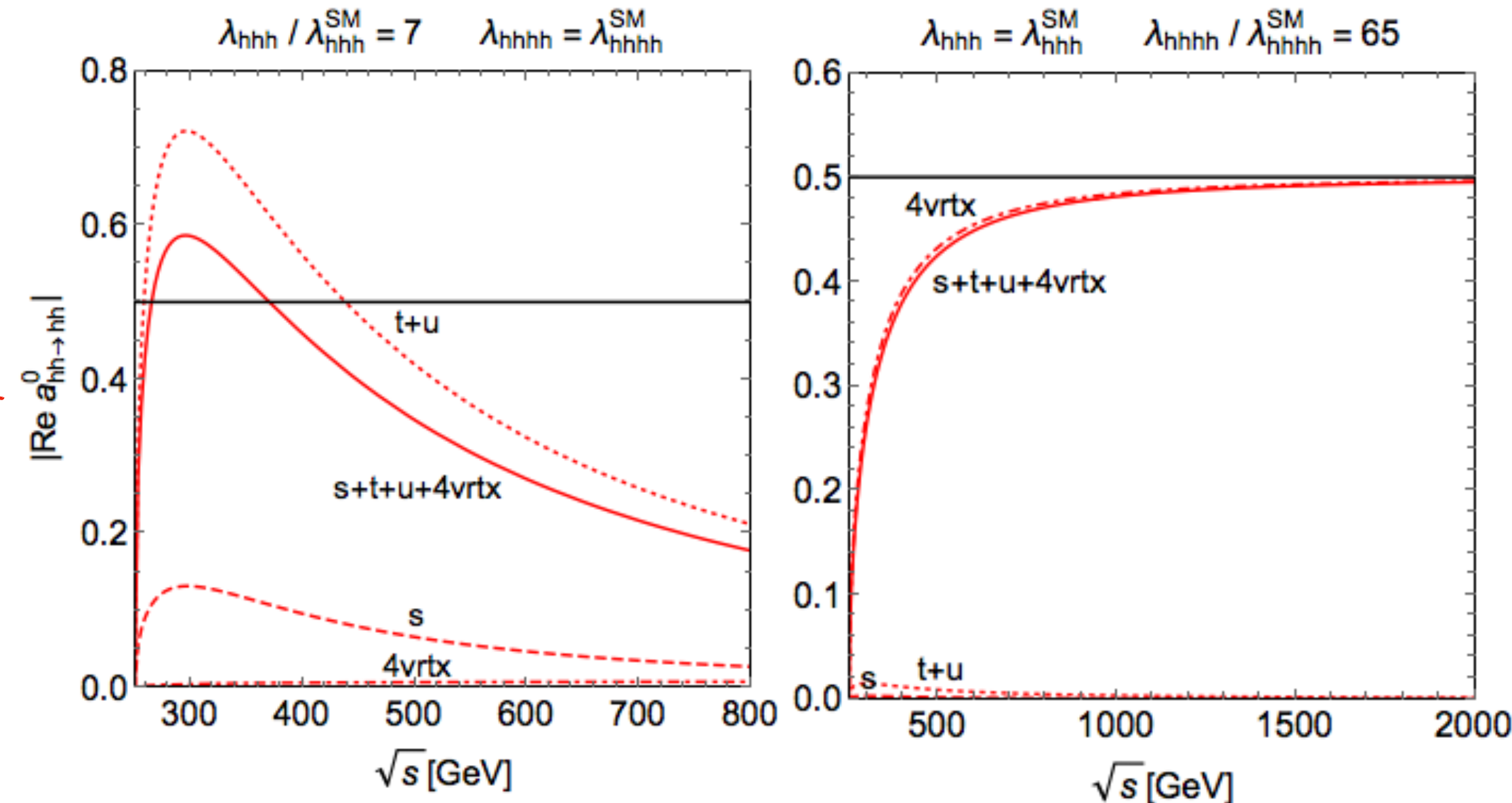
[Fuks, AP, Tetlalmatzi-Xolocotzi, 2509.16364]

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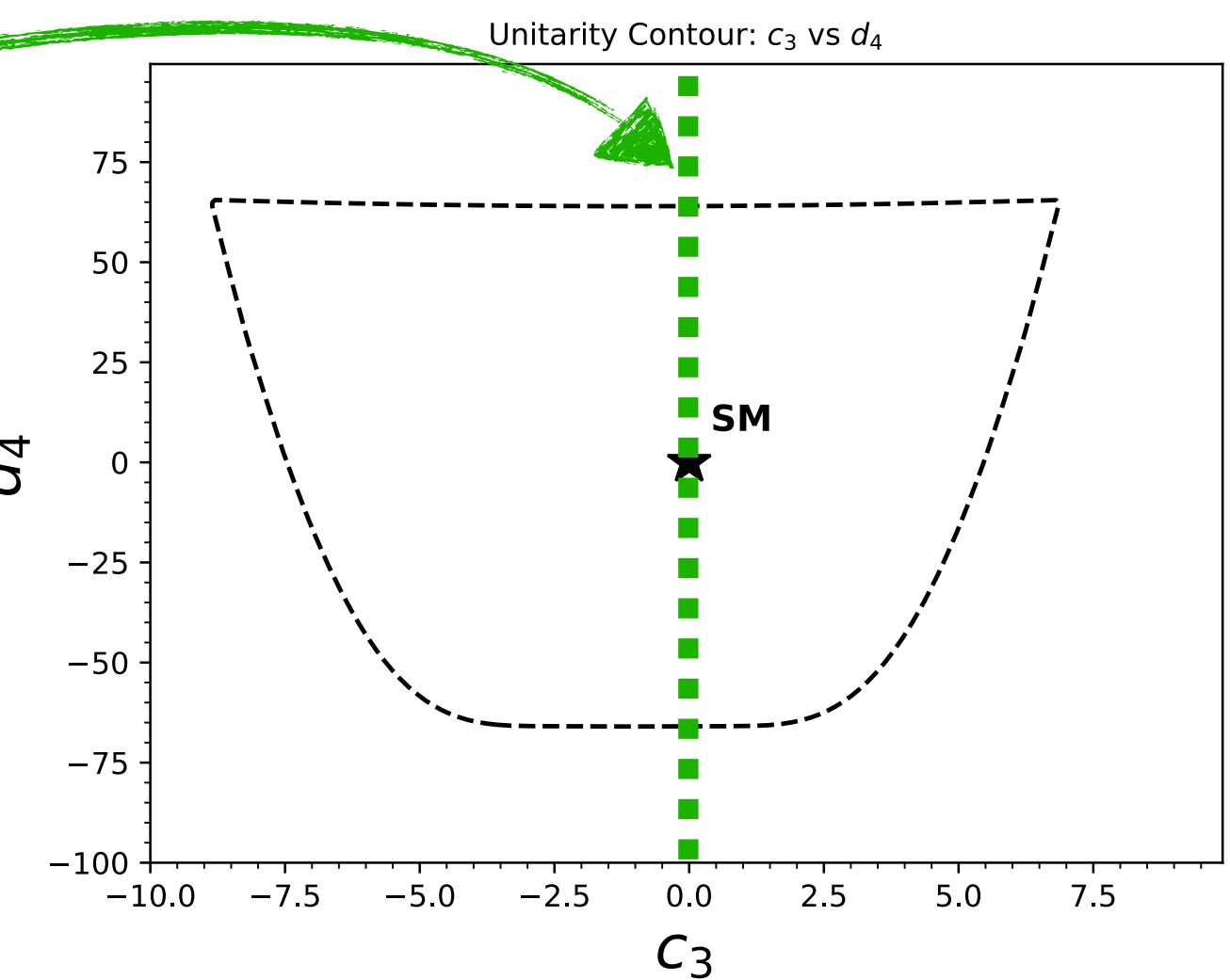
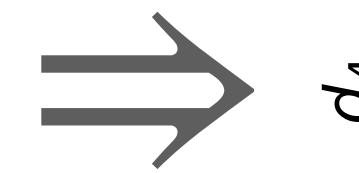
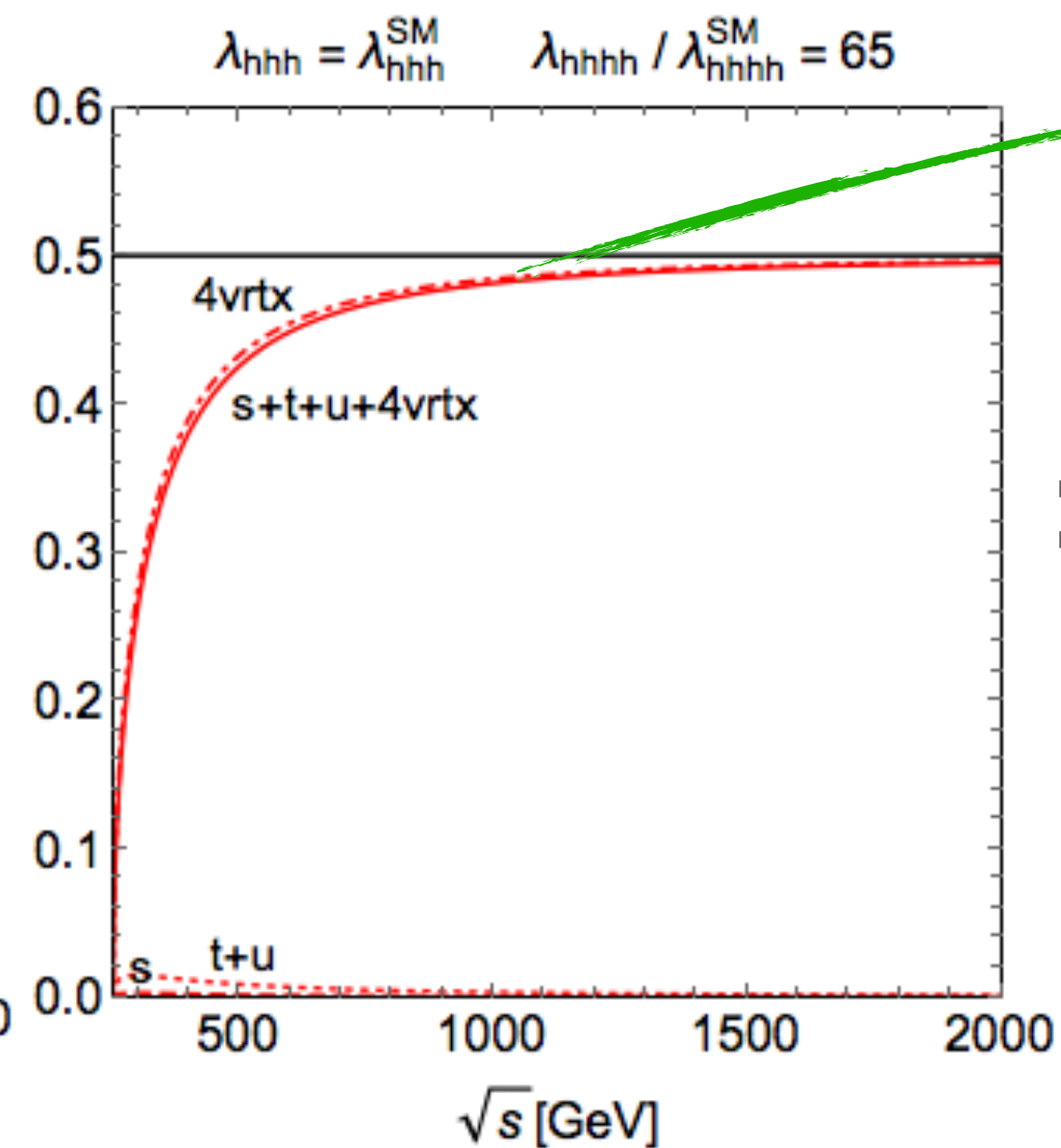
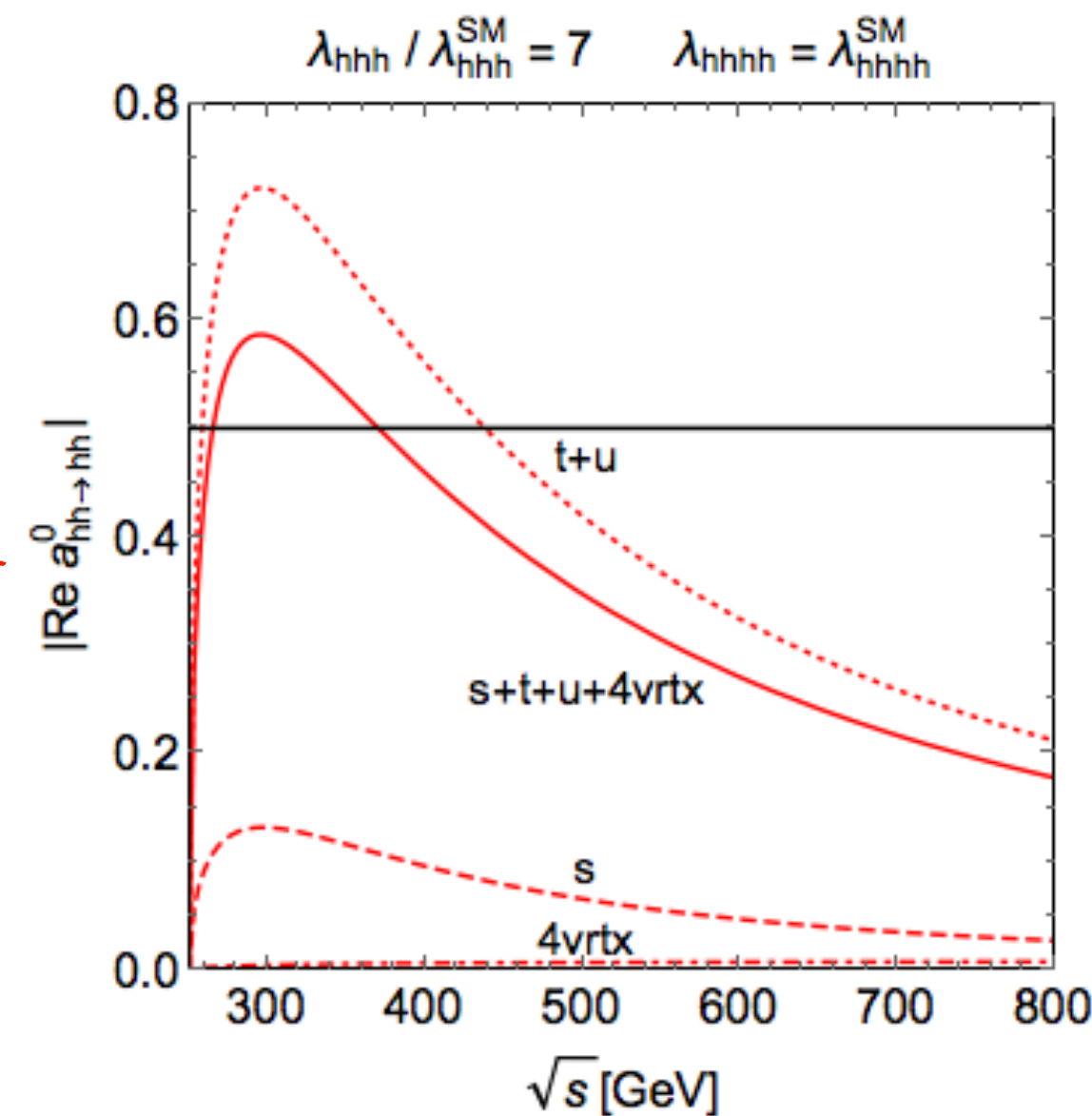
[Jacob, Wick, Annals Phys. 7, 404 (1959),
Di Luzio, Kamenik, Nardecchia, 1604.05746
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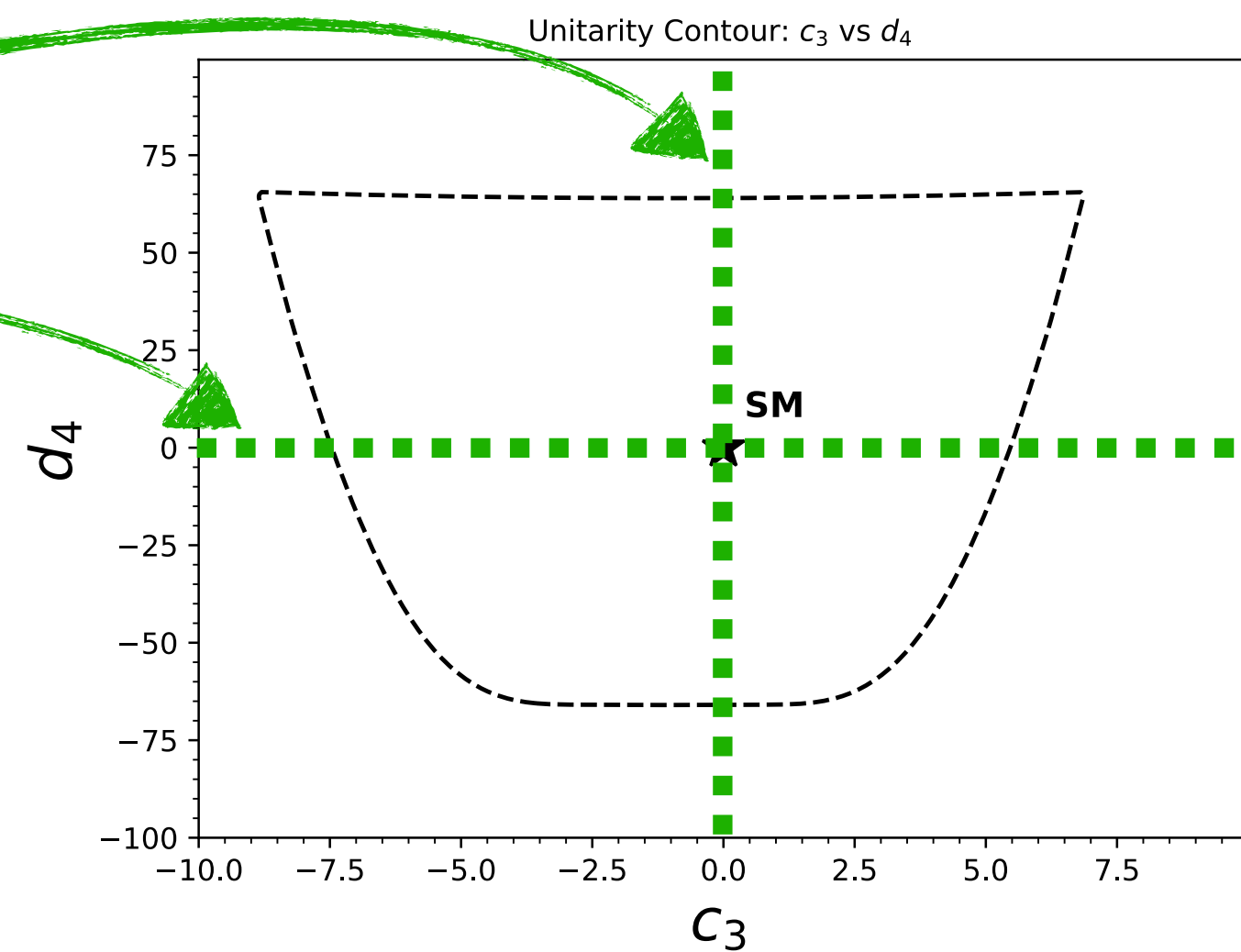
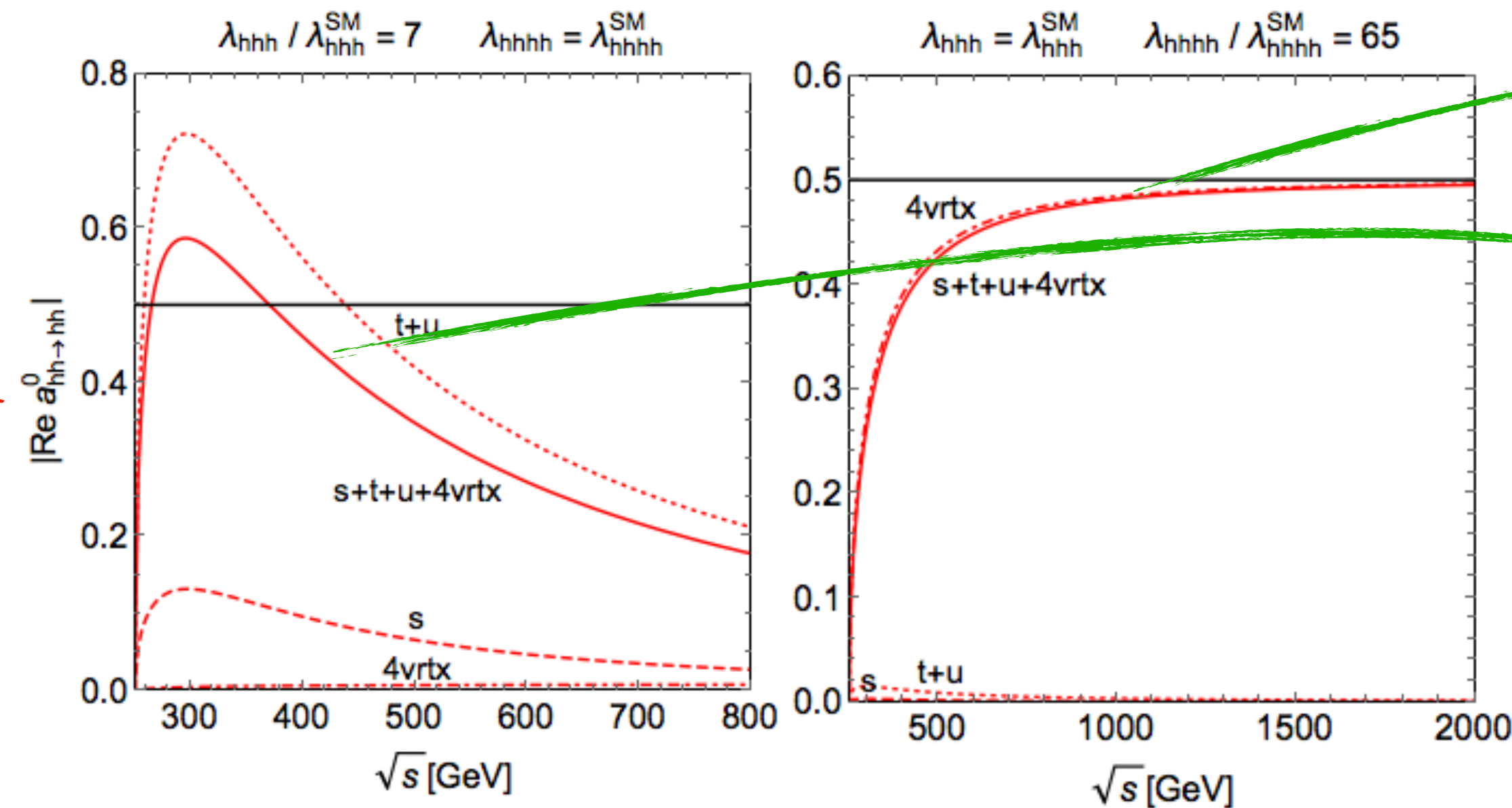
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Return of the Anomalous Couplings

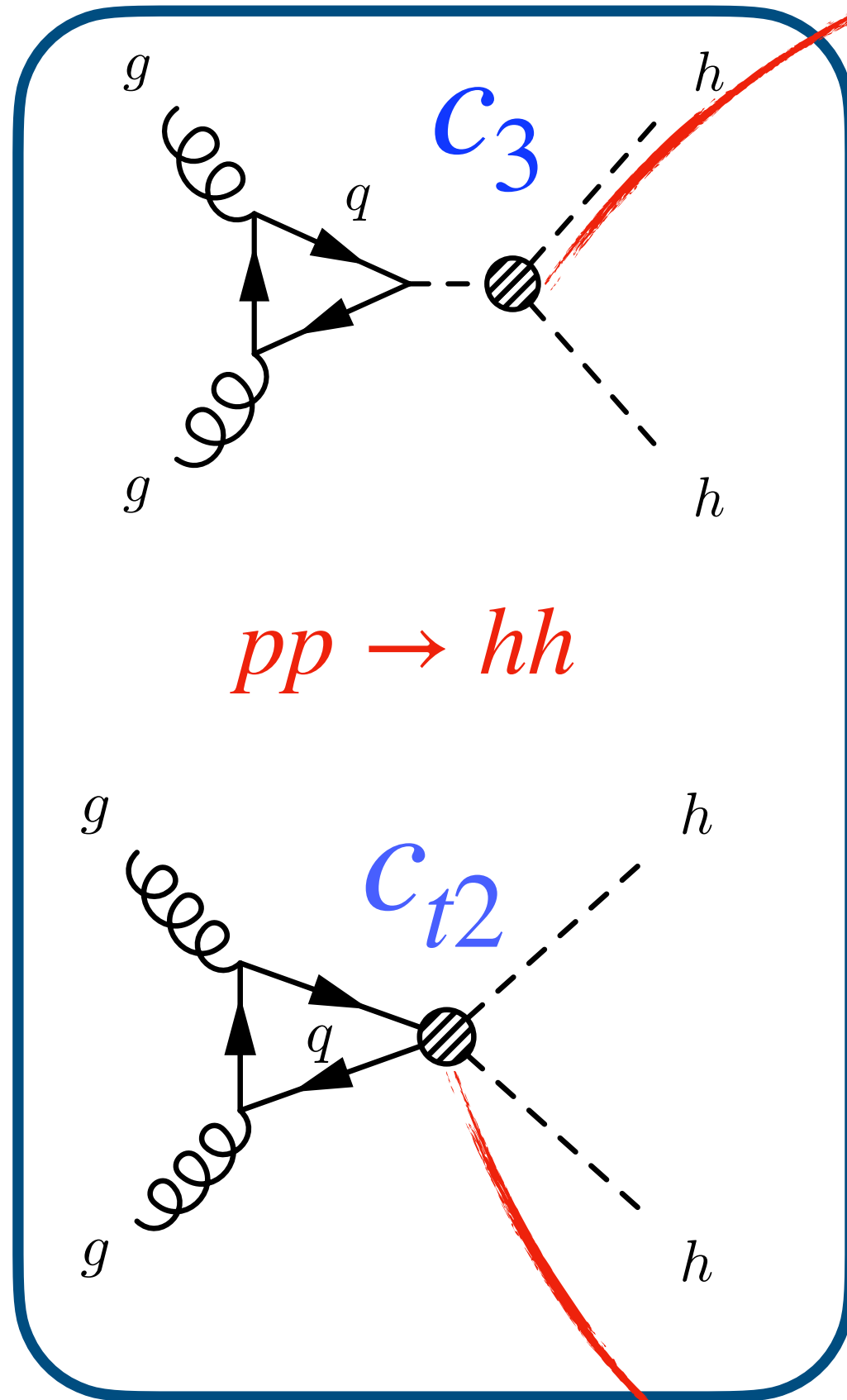
👒 Anomalous couplings can enhance ***hhh***! e.g. $\kappa_3 \neq 1, \kappa_4 \neq 1$, but also others!

$$\begin{aligned} \mathcal{L}_{\text{PhenoExp}} \supset & -\lambda_{\text{SM}} v (1+c_3) h^3 - \frac{\lambda_{\text{SM}}}{4} (1+d_4) h^4 \\ & + \frac{\alpha_s}{12\pi} \left(c_{g1} \frac{h}{v} - c_{g2} \frac{h^2}{2v^2} \right) G_{\mu\nu}^a G_a^{\mu\nu} \\ & - \left[\frac{m_t}{v} (1+c_{t1}) \bar{t}_L t_R h + \frac{m_b}{v} (1+c_{b1}) \bar{b}_L b_R h + \text{h.c.} \right] \\ & - \left[\frac{m_t}{v^2} c_{t2} \bar{t}_L t_R h^2 + \frac{m_b}{v^2} c_{b2} \bar{b}_L b_R h^2 + \text{h.c.} \right] \\ & - \left[\frac{m_t}{v^3} \left(\frac{c_{t3}}{2} \right) \bar{t}_L t_R h^3 + \frac{m_b}{v^3} \left(\frac{c_{b3}}{2} \right) \bar{b}_L b_R h^3 + \text{h.c.} \right] \end{aligned}$$

[AP, Tetlalmatzi-Xolocotzi, 2312.13562]

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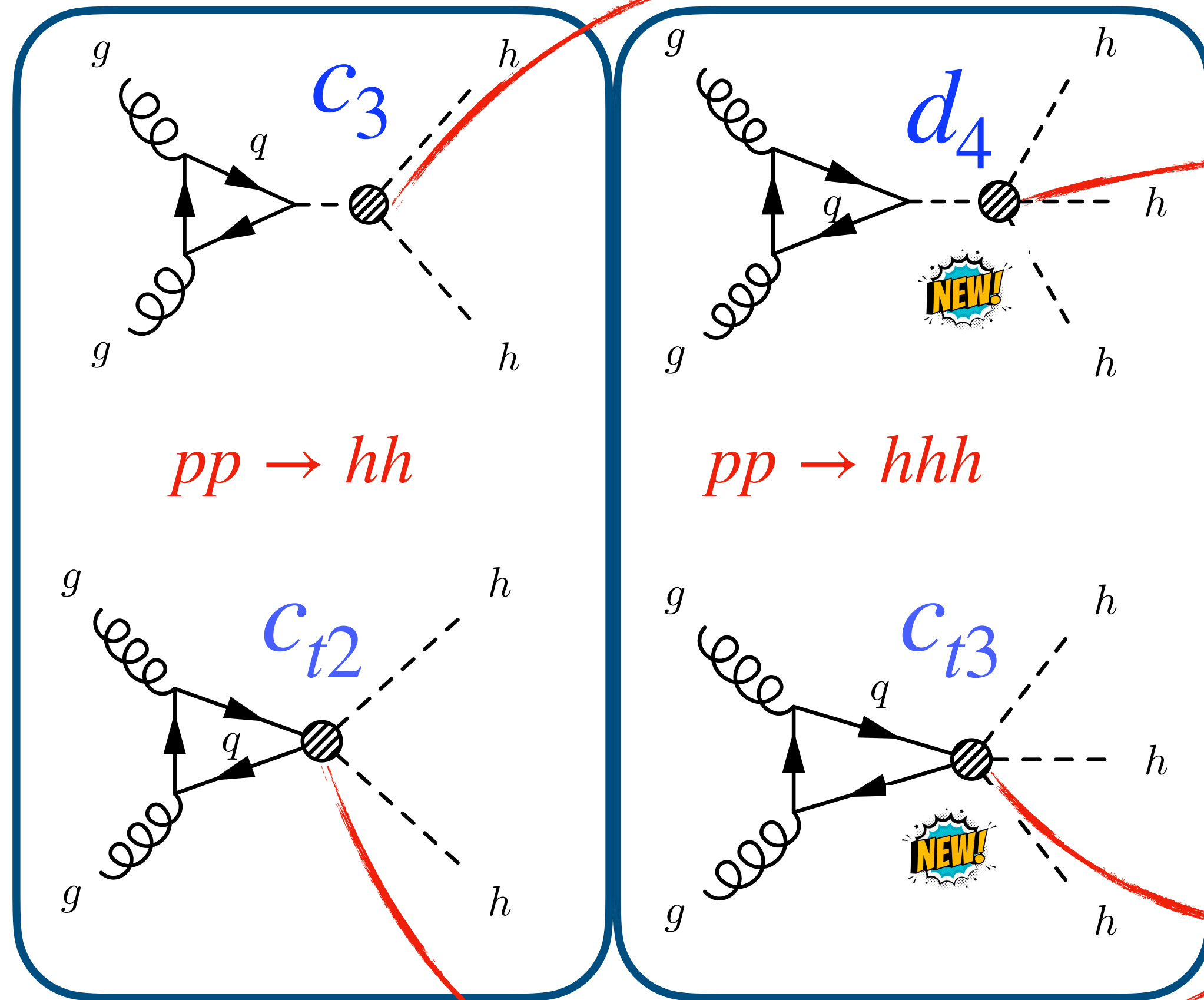
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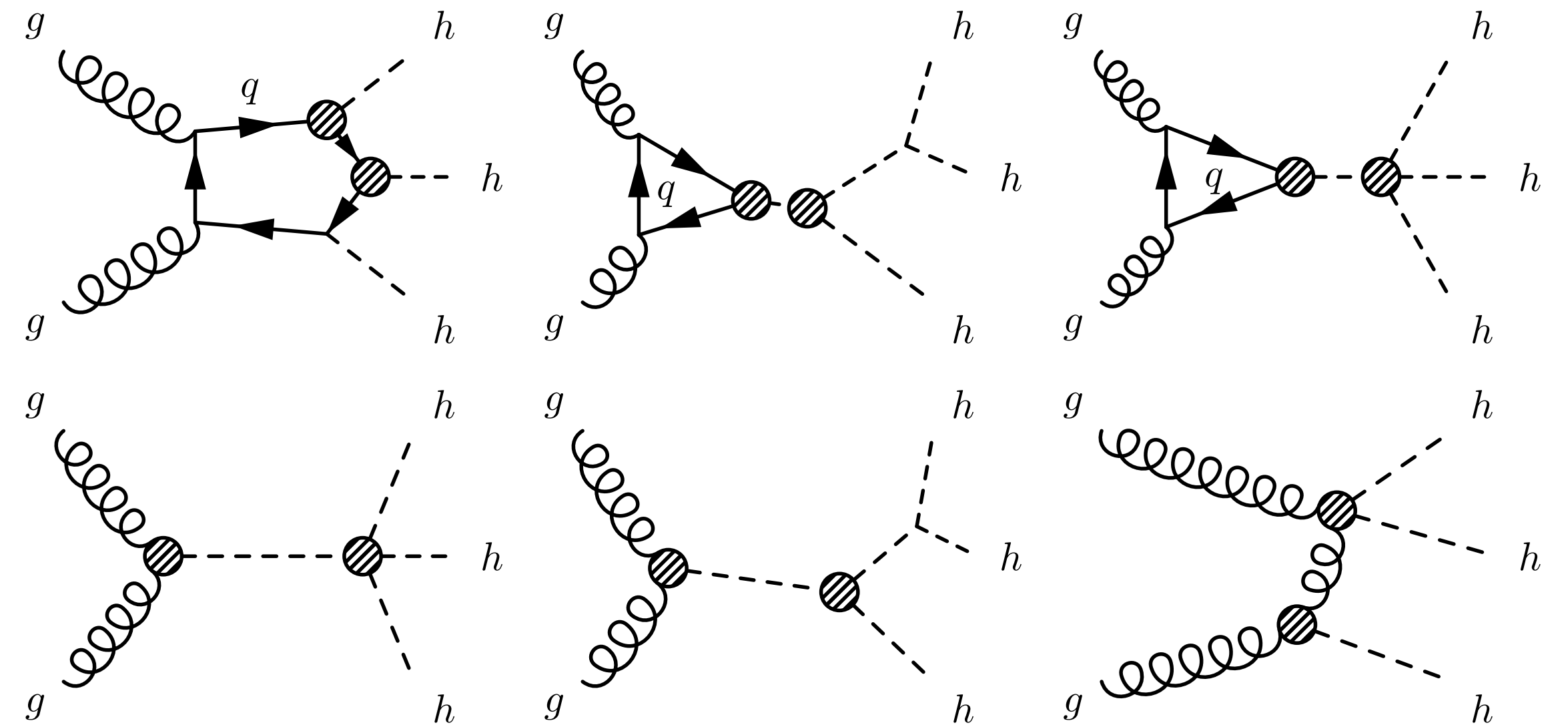
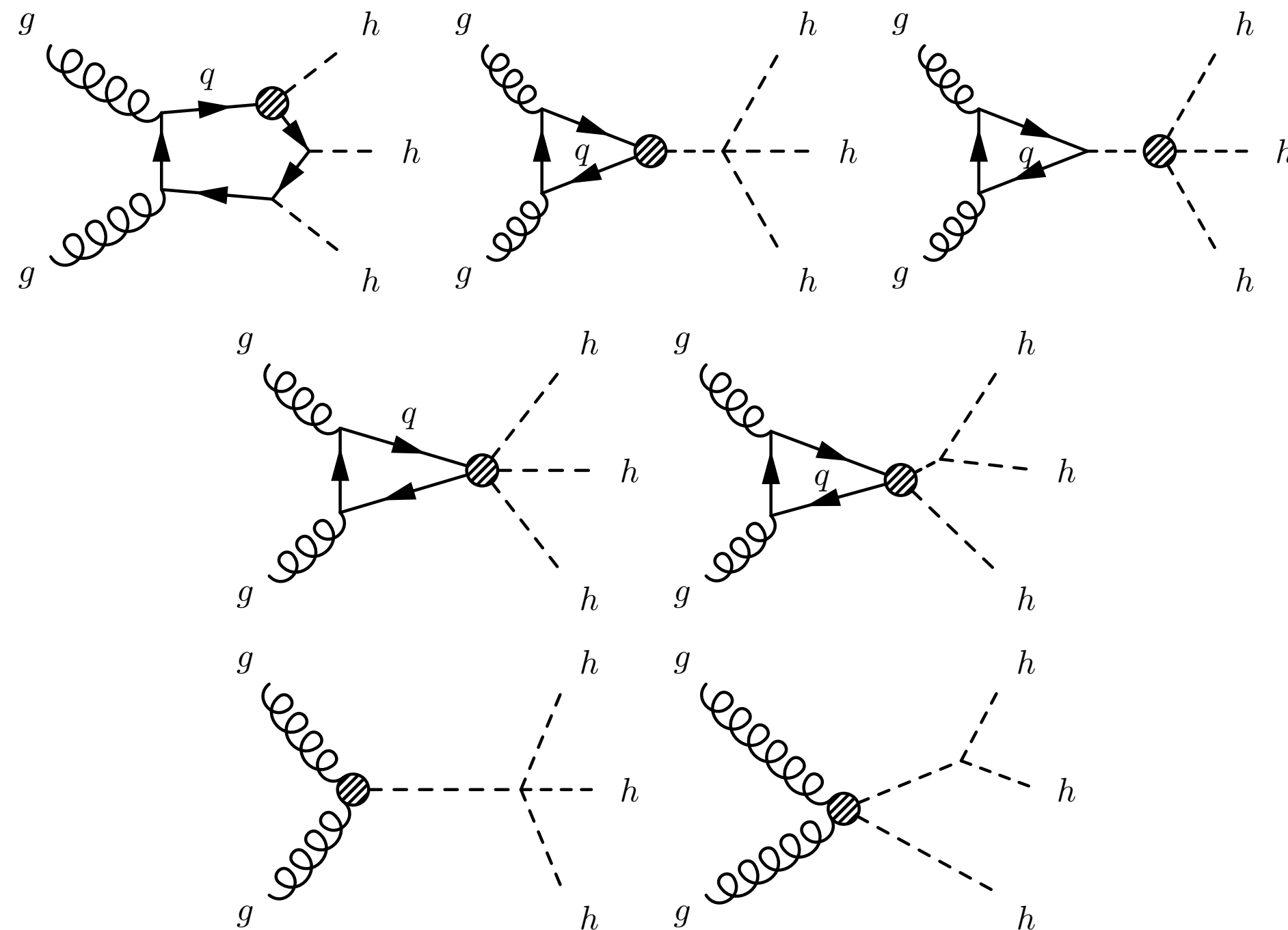
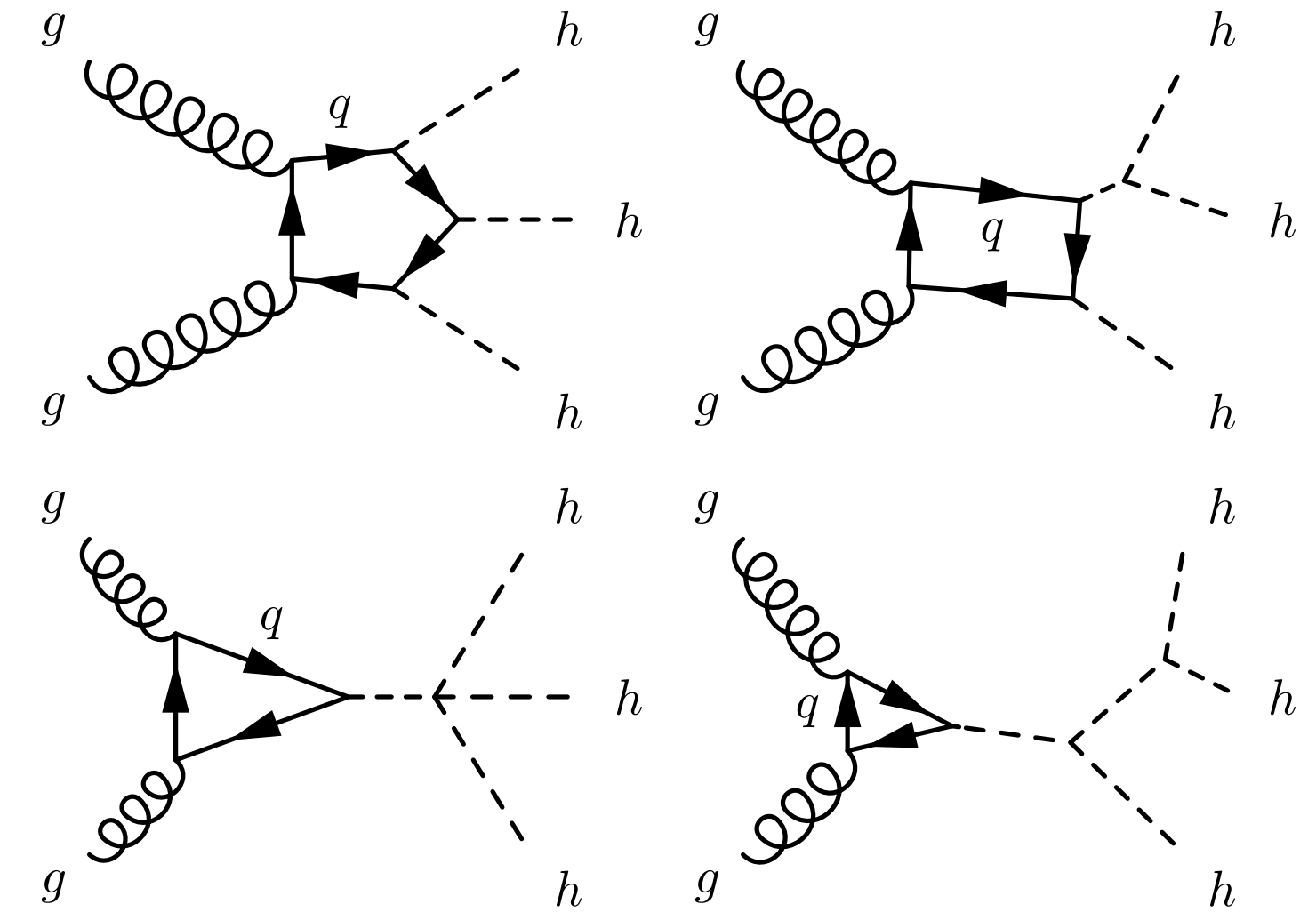
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Return of the Anomalous Couplings

MadGraph5_aMC model at:
https://gitlab.com/apapaefs/multihiggs_loop_sm

*Includes correct description of interference effects!



PART III: RESONANT PRODUCTION



Extended Scalar Sectors: Why?

- 🎩 **Dark Matter** (the scalar themselves, or portals to hidden sectors),
- 🎩 First-order EW phase transitions ($\Rightarrow$ EW baryogenesis $\Rightarrow$ **matter-antimatter asymmetry**),
[e.g. **AP**, White, 2010.0059 & 2108.11394]
- 🎩 & Model the scalar sector of more complicated models, e.g. **SUSY**.

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Simplest extension to the SM: add ONE real singlet scalar field.

$$\begin{aligned}\mathcal{V}(\phi, S) = & \text{●} |\phi|^2 + \text{■} |\phi|^4 \\ & + \text{●} S^2 + \text{▲} S^3 + \text{■} S^4 \\ & + \text{▲} |\phi|^2 S + \text{■} |\phi|^2 S^2 \leftarrow \text{“Portal” interactions.} \\ & \quad [|\phi|^2 \text{ is also an SM singlet!}] \\ & + S \times (\text{Hidden Sector}) + \dots \leftarrow \text{Dark Matter?}\end{aligned}$$

Singlet $\equiv$ NO SM “charges”

SM+One Real Singlet [=xSM]

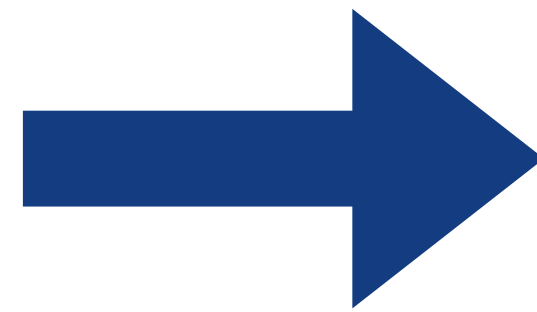
[e.g. AP, White, 2010.00597]

$$\mathcal{V}(\phi, S) = \text{●} |\phi|^2 + \text{■} |\phi|^4 + \text{●} S^2 + \text{▲} S^3 + \text{■} S^4 + \text{▲} |\phi|^2 S + \text{■} |\phi|^2 S^2$$

Mass Eigenstates

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} h \\ \chi \end{pmatrix}$$

θ : mixing angle



$h_1 \rightarrow$ “SM-like” Higgs boson.

$h_2 \rightarrow$ new scalar resonance.

i.e. choose: $|\theta| \gtrsim 0$, and:

$$h_1 = h \cos \theta + \chi \sin \theta$$

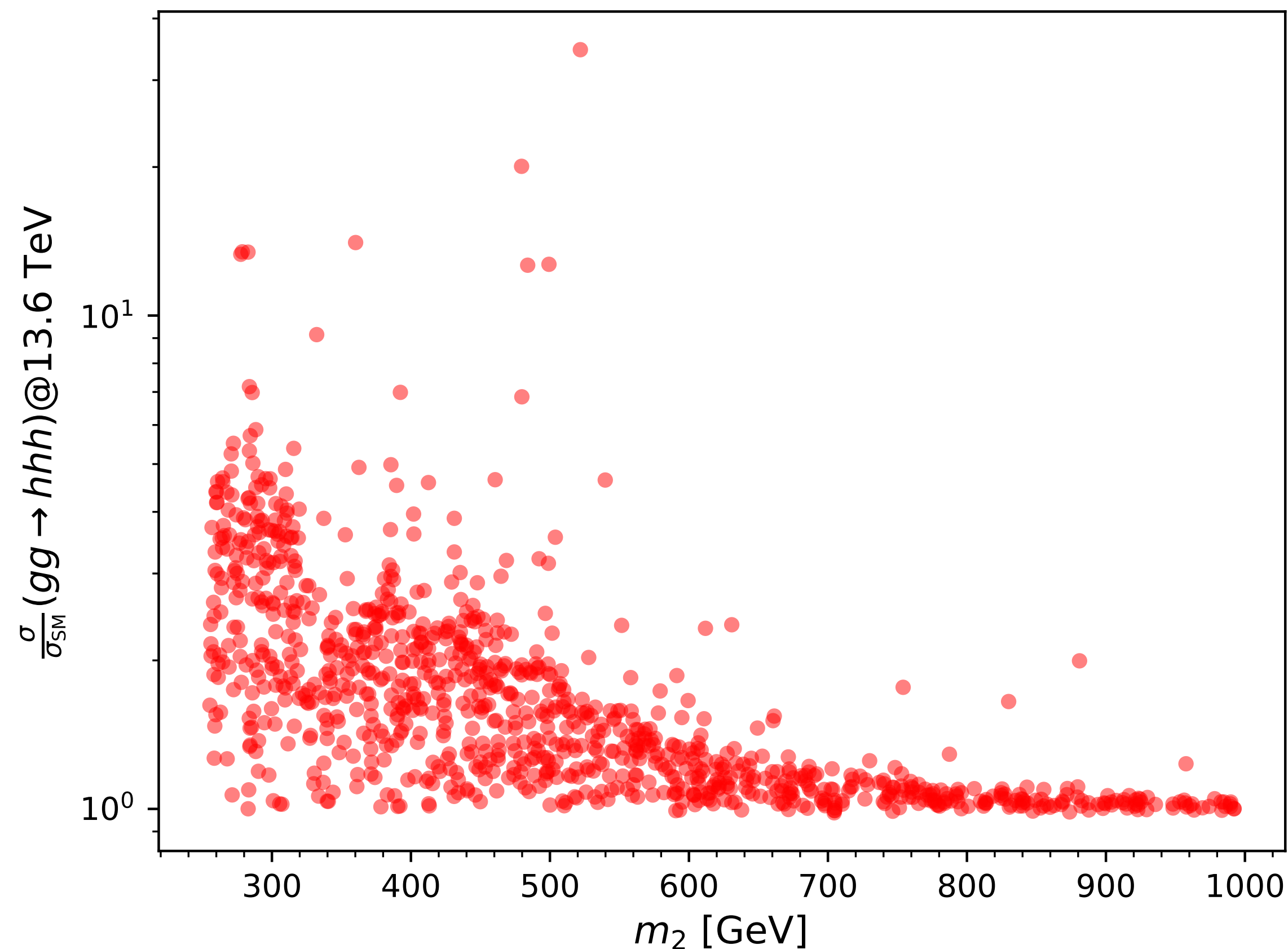
$$h_2 = -h \sin \theta + \chi \cos \theta$$

SM+One Real Singlet [=xSM]

👒 What about hhh in the xSM? An example with $\mathbb{Z}_2$ symmetry: $\mathbb{Z}_2 : S \rightarrow -S$.

👒 Including: boundedness of potential + perturbativity + HiggsTools (i.e. experimental) constraints [Bahl, Biekötter, Heinemeyer, Li, Paasch, Weiglein, Wittbrodt, 2210.09332].

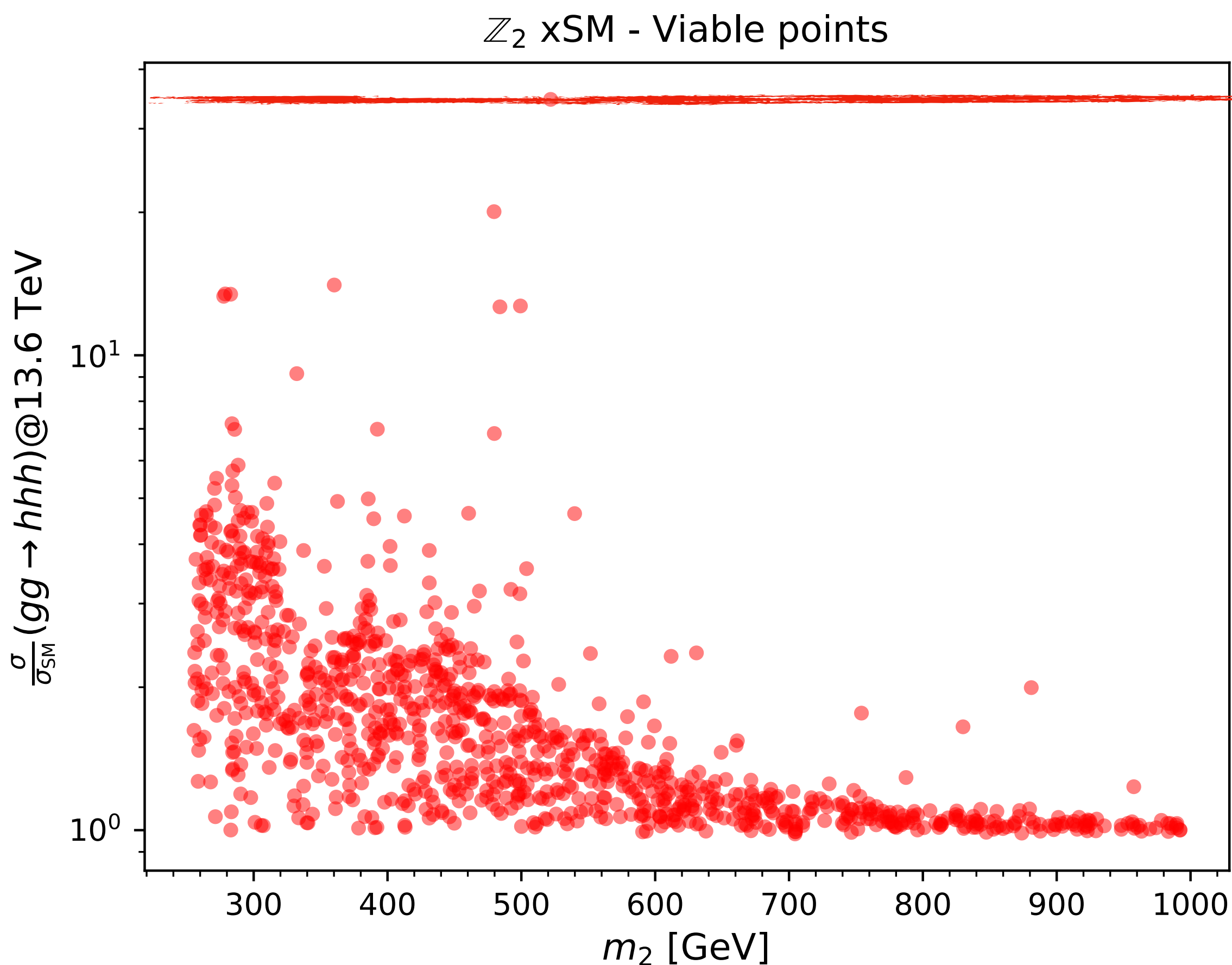
$\mathbb{Z}_2$ xSM - Viable points



SM+One Real Singlet [=xSM]

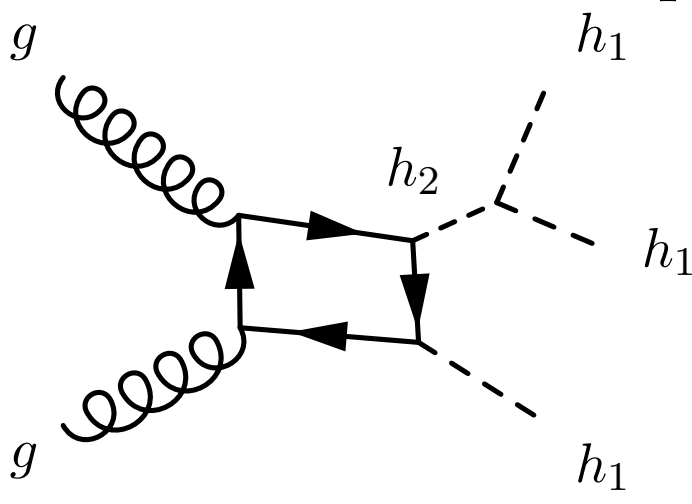
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Maximum enhancement $\sim \mathcal{O}(30)$!

[**But:** I suspect these will be excluded quickly by resonant *hh*! Due to:

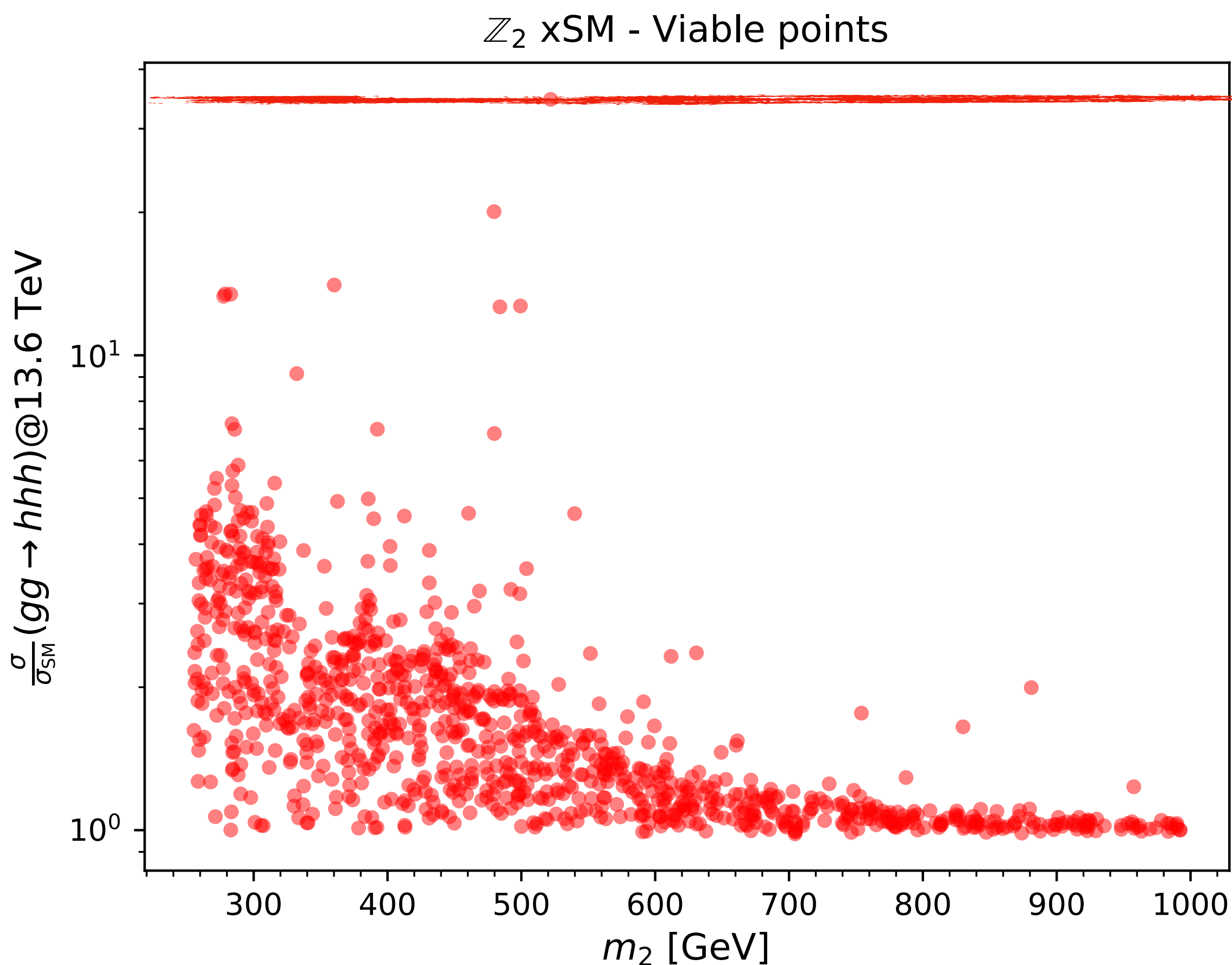


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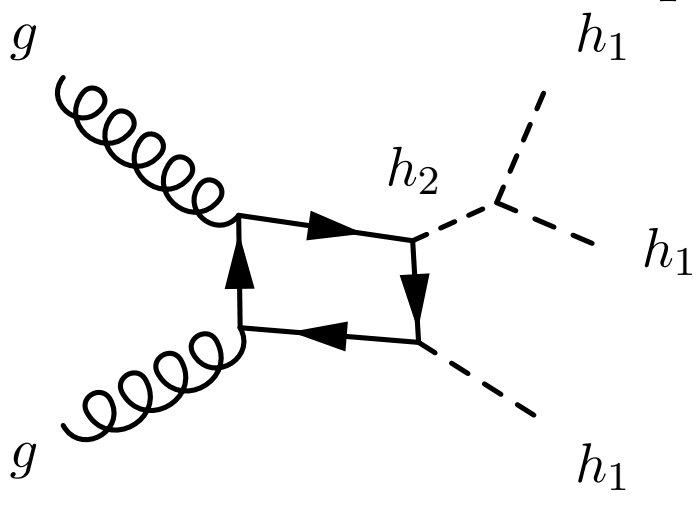
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Maximum enhancement $\sim \mathcal{O}(30)$!

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⇒ Look at further extended scalar sectors!

SM + Two Real Singlet Scalars [= TRSM]

🎩 Consider adding two real singlet scalar fields $S, X \rightarrow$ the **TRSM**.

🎩 & impose discrete $\mathbb{Z}_2$ symmetries:

$$\mathbb{Z}_2^S : S \rightarrow -S, \quad X \rightarrow X$$
$$\mathbb{Z}_2^X : X \rightarrow -X, \quad S \rightarrow S$$

$\Rightarrow$ **TRSM** Scalar Potential:

$$\begin{aligned} \mathcal{V}(\phi, S, X) = & \bullet |\phi|^2 + \blacksquare |\phi|^4 + \bullet S^2 + \blacksquare S^4 + \bullet X^2 + \blacksquare X^4 \\ & + \blacksquare S^2 X^2 \\ & + \blacksquare |\phi|^2 S^2 + \blacksquare |\phi|^2 X^2 \end{aligned}$$

SM + Two Real Singlet Scalars [= TRSM]

👒 Electroweak Symmetry Breaking in the TRSM:

⇒ Three scalar bosons: $h_1, h_2, h_3 \rightarrow h_1 \approx \text{SM-like "Higgs boson"}$.

⇒ hhh that may even be detectable at the LHC!

through: $pp \rightarrow h_3 \rightarrow h_2 h_1 \rightarrow h_1 h_1 h_1$

[Robens, Stefaniak, Wittbrodt, 1908.08554,
AP, Robens, Tetlalmatzi-Xolocotzi, 2101.00037]

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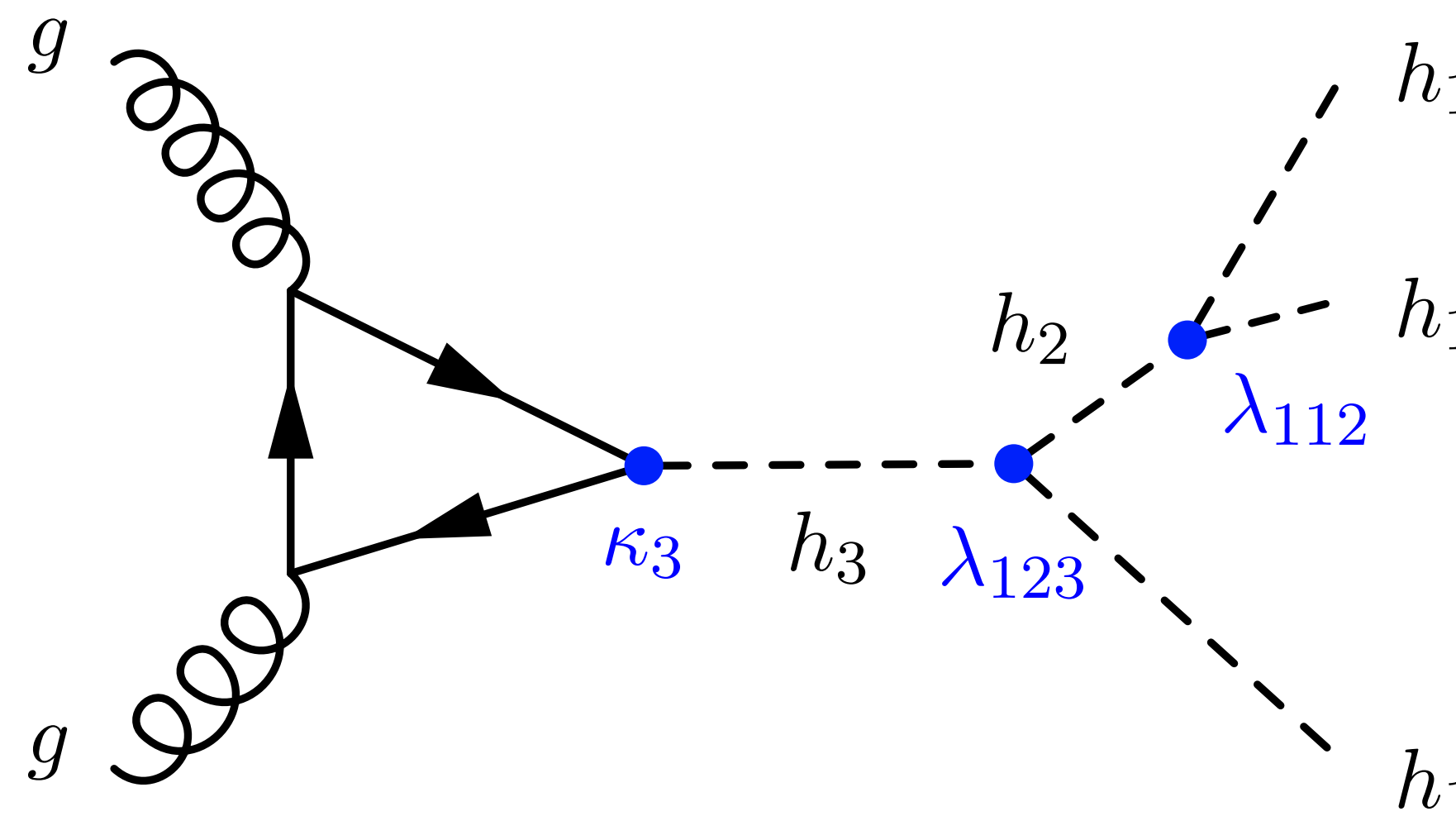
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→ Double-Resonant enhancement!



Requires:
 $m_3 > m_2 + m_1, m_2 > 2m_1$

Double-Resonant Enhancement in the TRSM

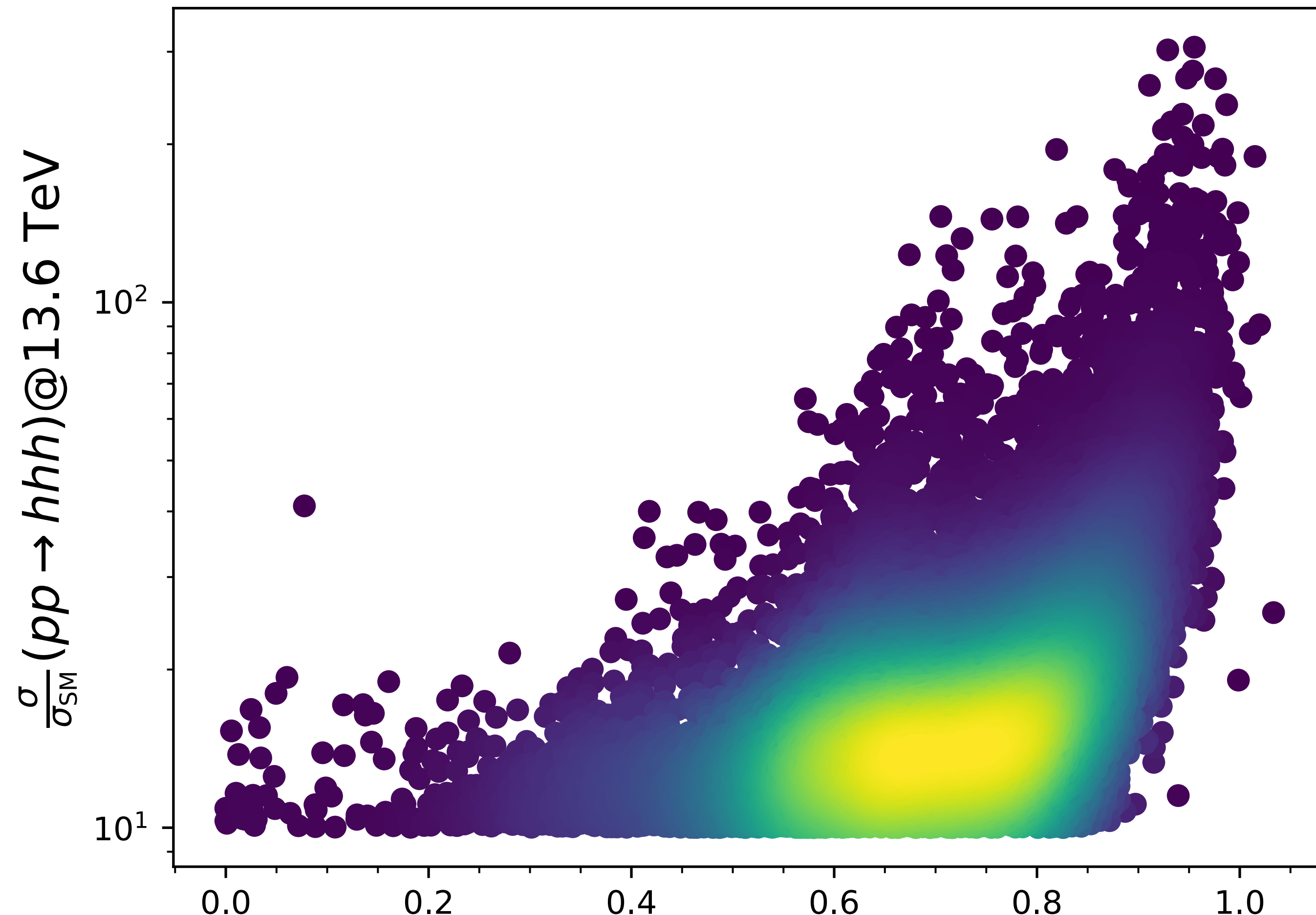
🎩 Enhancement of hhh $\Leftrightarrow$ Large “fraction” of double-resonant process!

Viable points with $\sigma > 10 \times \sigma_{\text{SM}}(pp \rightarrow hhh)@13.6 \text{ TeV}$

[Karkout, AP, Postma, Tetlalmatzi-Xolocotzi,
van de Vis, du Pree, 2404.12425]

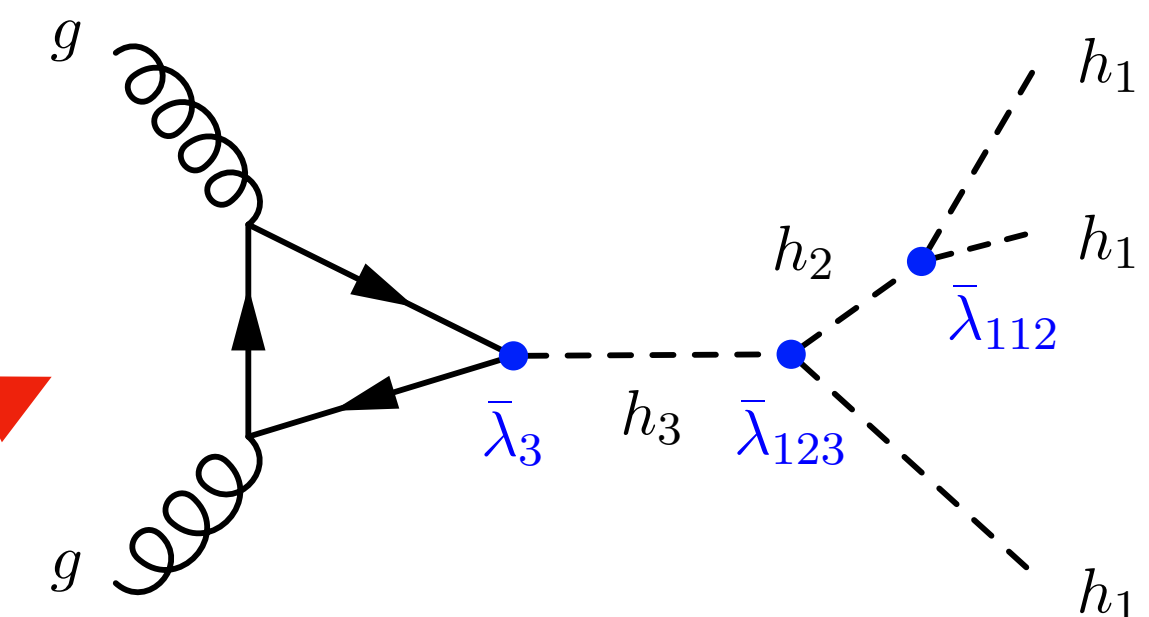
[including boundedness of potential +
perturbativity + **HiggsTools** constraints.]

Enhancement
over SM



fraction from resonant: $pp \rightarrow h_3 \rightarrow h_2 h_1 \rightarrow h_1 h_1 h_1$

Resonant Fraction (R.F.) =
How much of the total cross section comes from... ?

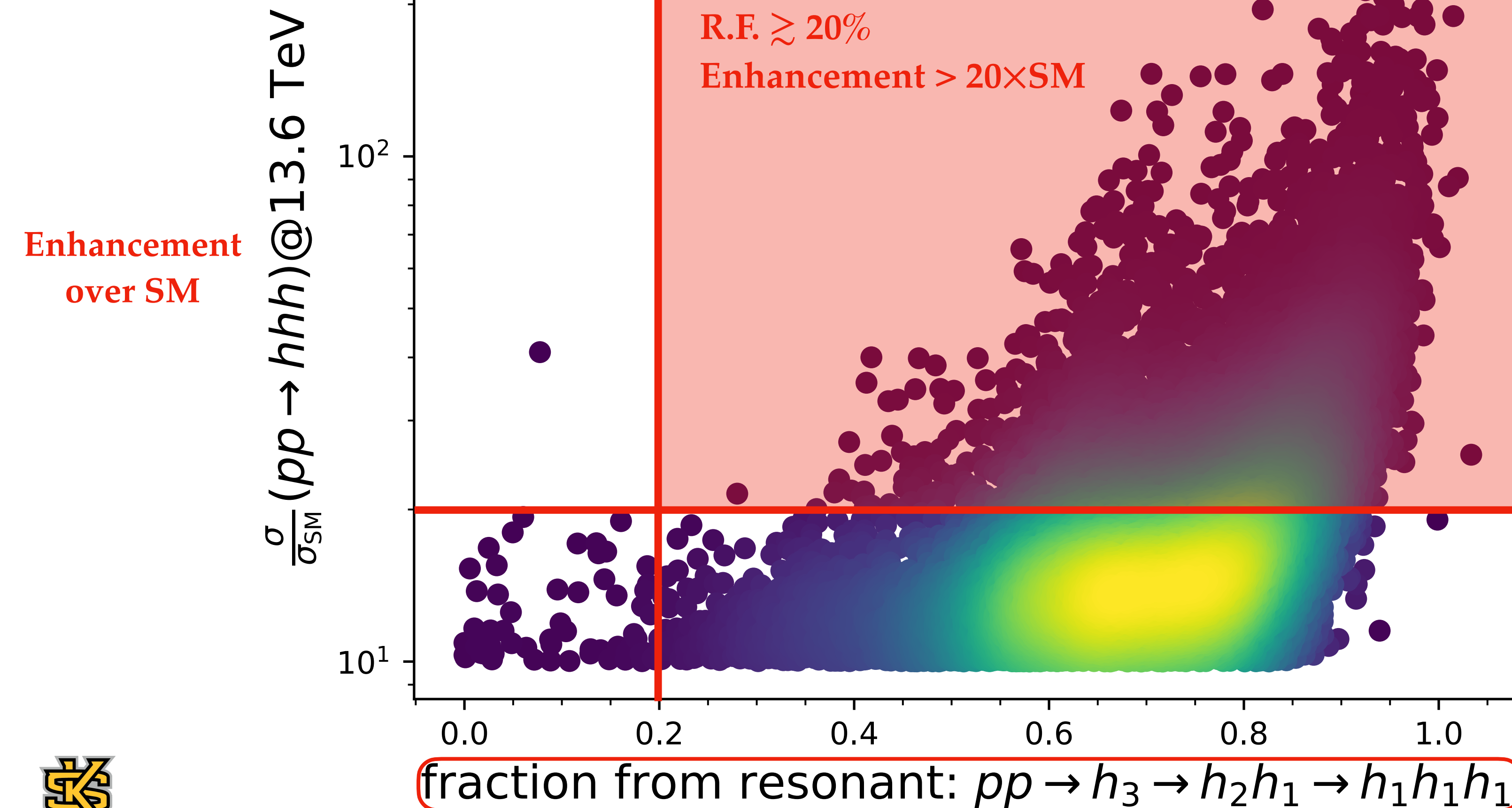


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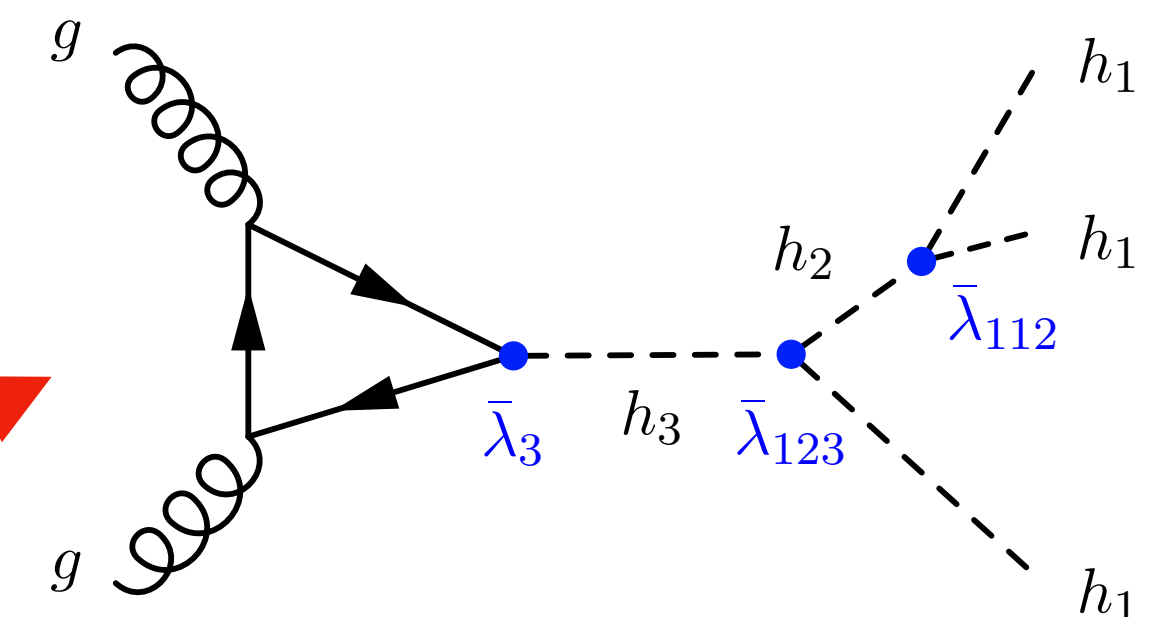
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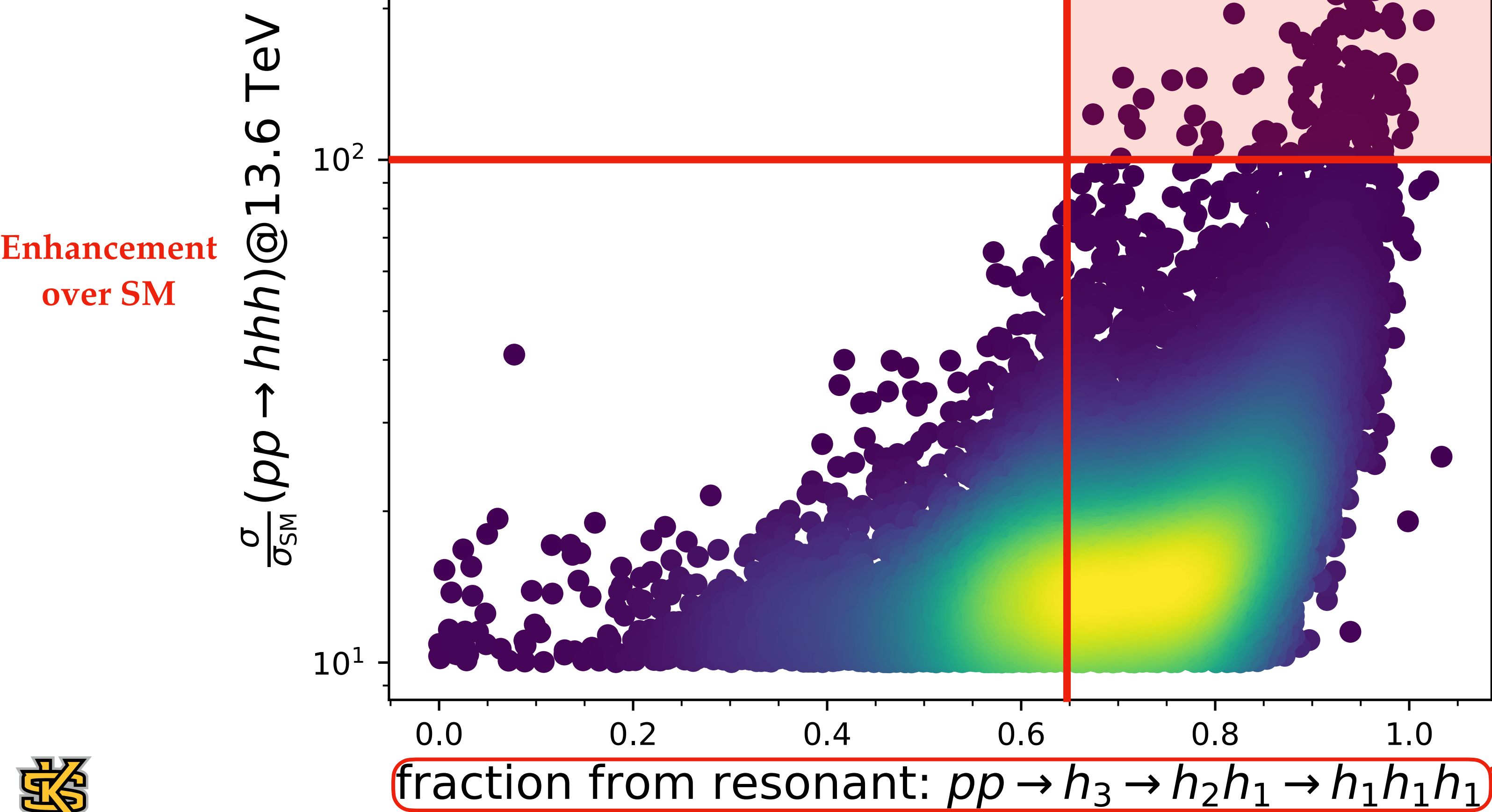
Resonant Fraction (R.F.) =
How much of the total cross section comes from... ?



Double-Resonant Enhancement in the TRSM

🎩 Enhancement of hhh $\Leftrightarrow$ Large “fraction” of double-resonant process!

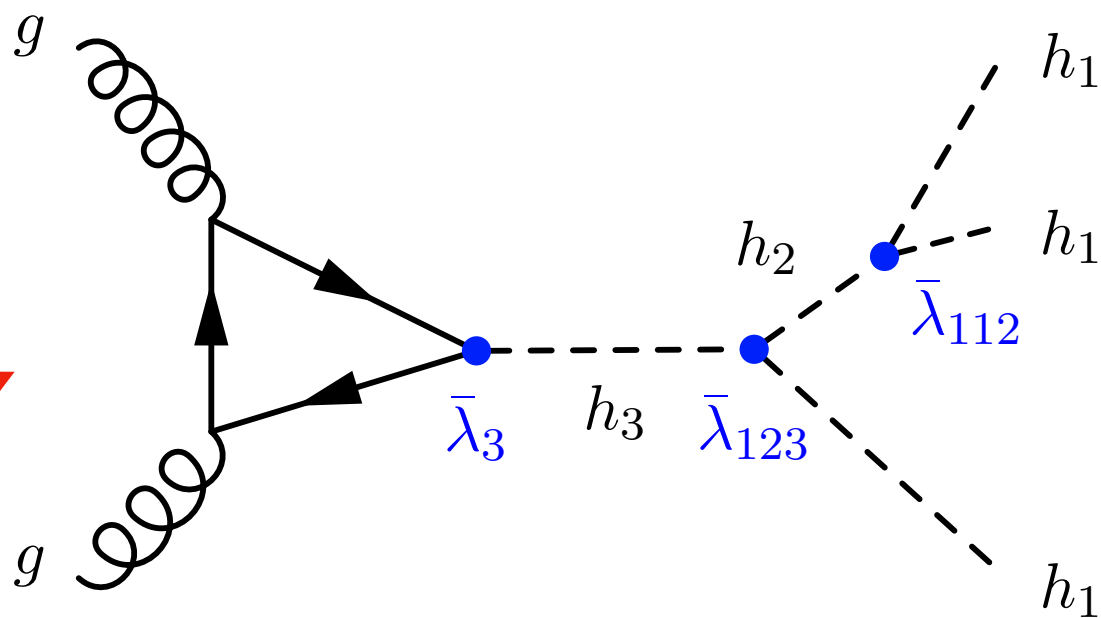
Viable points with $\sigma > 10 \times \sigma_{\text{SM}}(pp \rightarrow hhh)@13.6 \text{ TeV}$



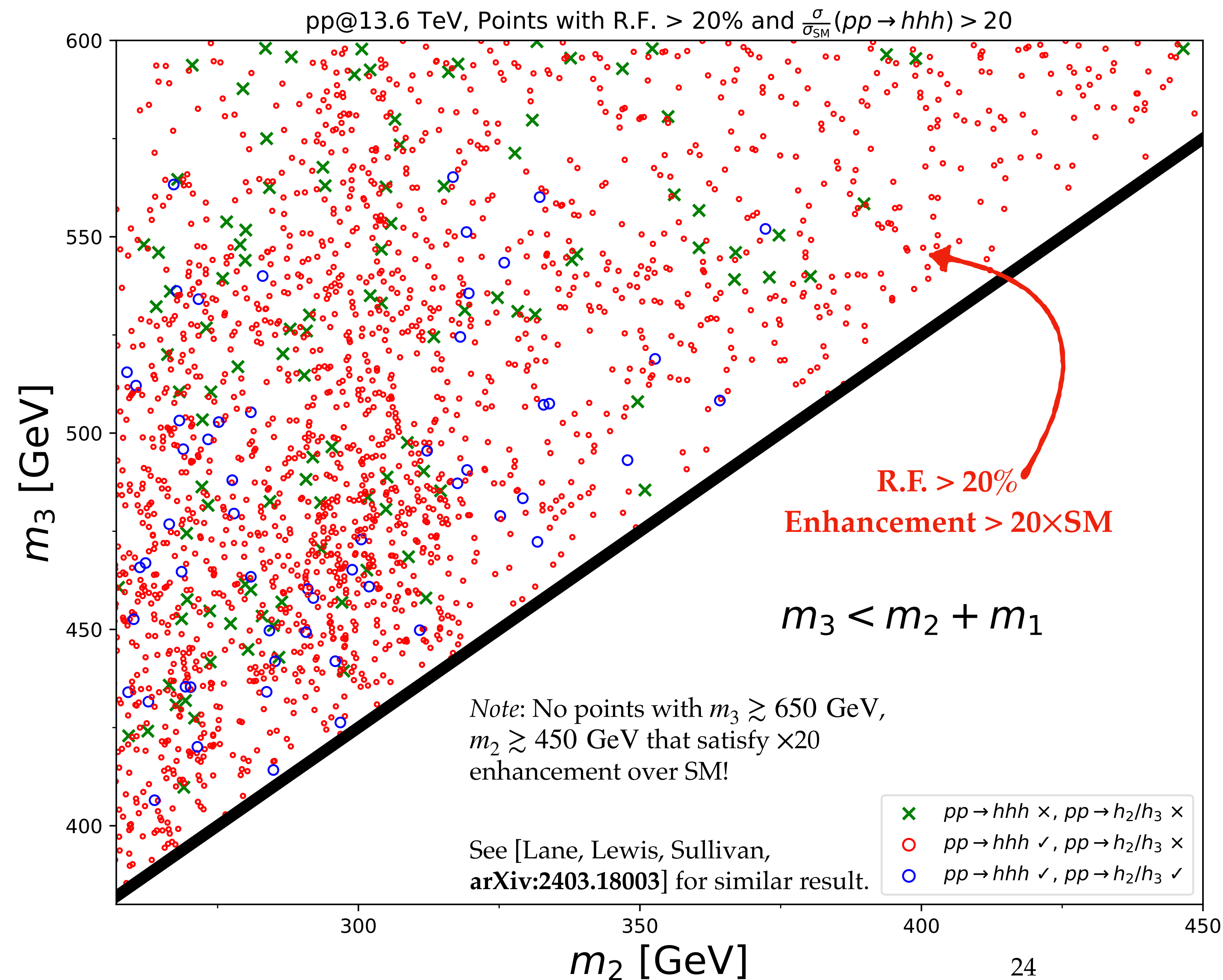
[Karkout, AP, Postma, Tetlalmatzi-Xolocotzi, van de Vis, du Pree, 2404.12425]

[including boundedness of potential + perturbativity + **HiggsTools** constraints.]

Resonant Fraction (R.F.) =
How much of the total cross section comes from... ?



Pheno of TRSM Benchmark Points @ HL-LHC



HL-LHC results:

$\circ$: Excluded ONLY by single h_2 and h_3 production, NOT by hhh ,

$\times$: Excluded ALSO by hhh ,

$\circ$: NOT excluded @ HL-LHC

($\Rightarrow$ Future Colliders?)

Notice: NO hhh exclusion without single h_2 and h_3 exclusion!

$\Rightarrow$ in TRSM, hhh is *unlikely* to be a “discovery” channel [$\sim$ *but: inverse problem?*]

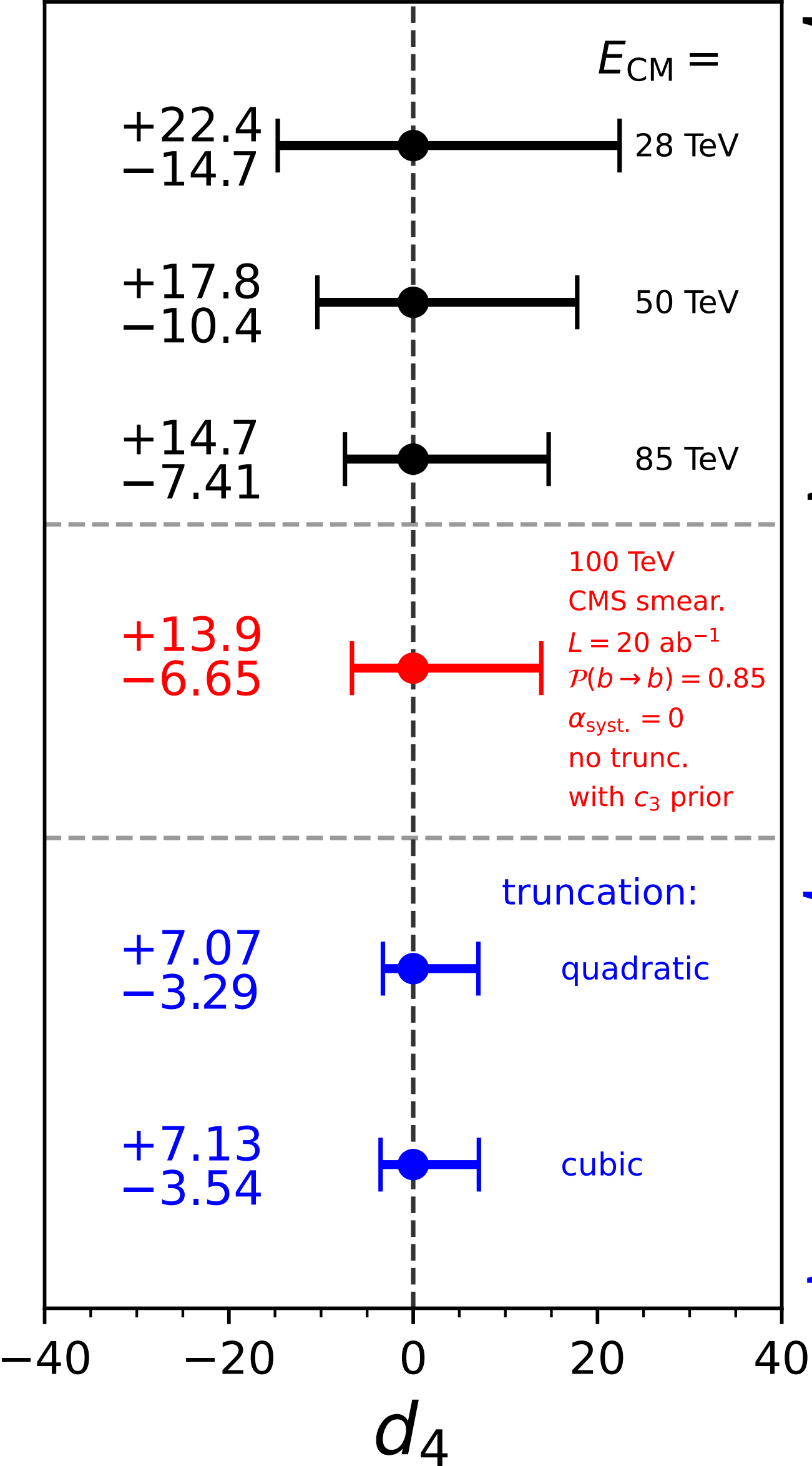
PART IV: FUTURE PROSPECTS



Future Hadron Colliders

👒 Proton colliders:
(six b-jet final state)

Results Summary : CM Energy & Truncation



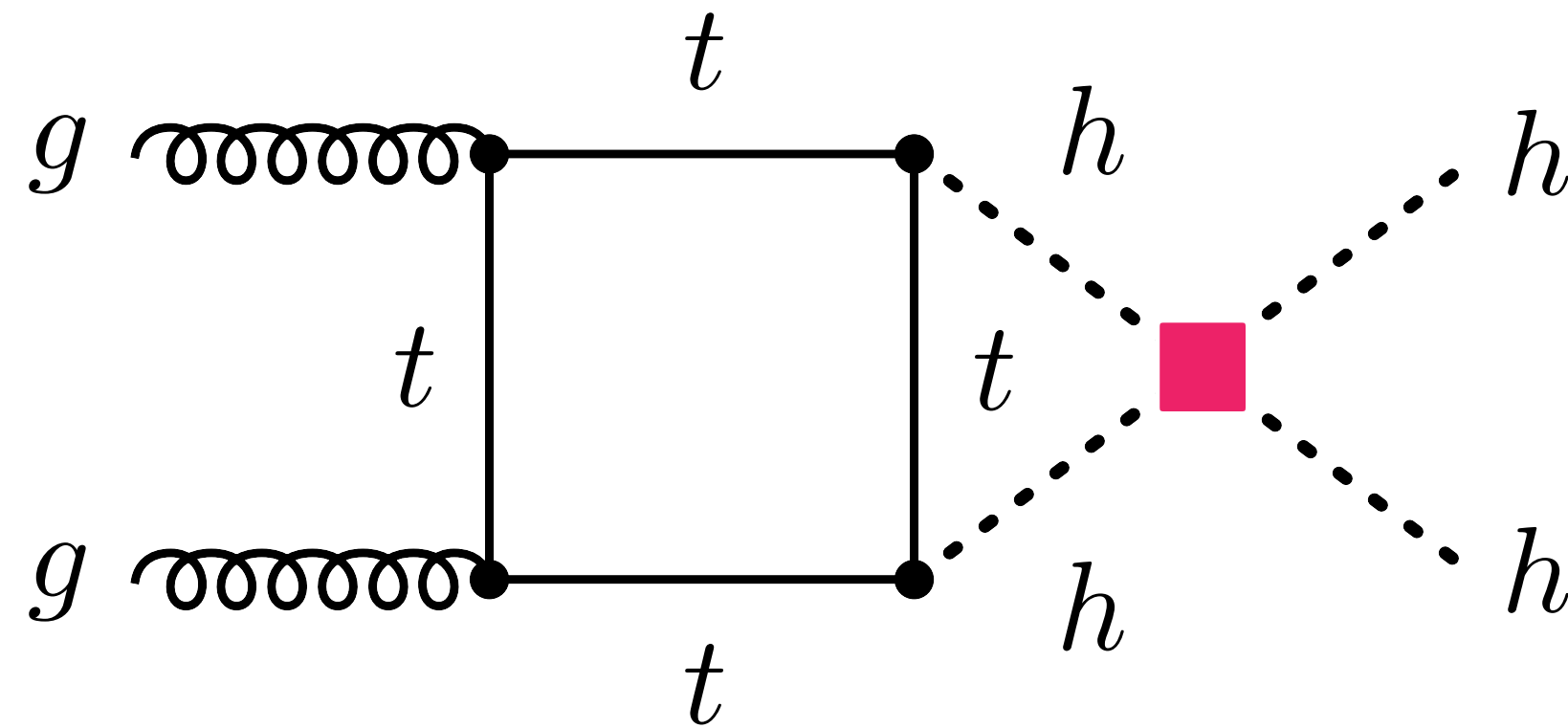
pp collider energy variation
(85 $\rightarrow$ 50 $\rightarrow$ 28 TeV)

hhh 100 TeV / 20 ab^{-1} (no truncation) +
combination with trilinear constraint
($\delta c_3 = 5\%$)

cubic/quadratic truncation of ME^2

Indirect Constraints on the Quartic

🎩 Obtain indirect constraints on Higgs boson's quartic (λ_4) via hh production, e.g. through electroweak corrections:



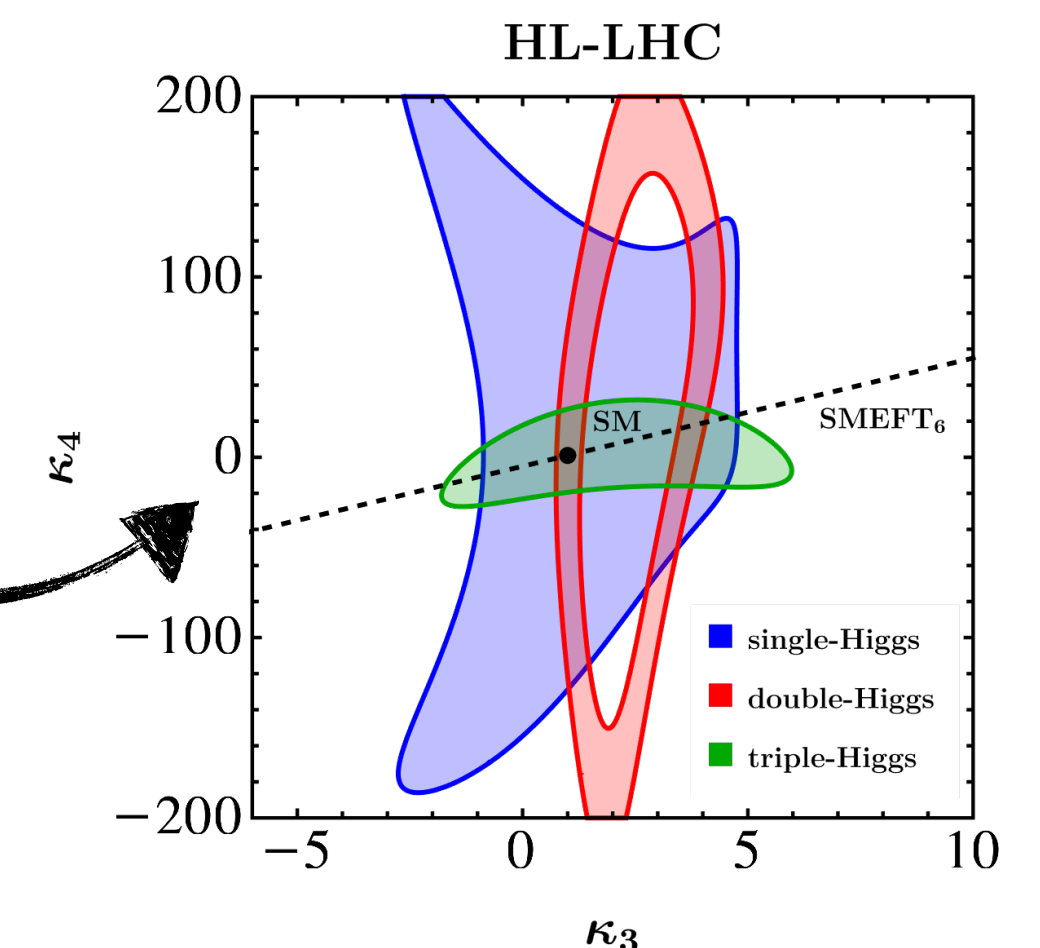
[Bizoń, Haisch, Rottoli, 1810.04665, Li Si, Wang, Zhang, Zhao, 2407.14716
Heinrich, Jones, Kerner, Stone, Vesten, 2407.04653]

$$\lambda_4/\lambda_4^{\text{SM}} \sim [-5, 14]$$

[For $\lambda_3 = \lambda_3^{\text{SM}}$]

[But note: Full EW corrections are only available for the SM!]

🎩 **See also:** [Haisch, Sankar, Zanderighi, 2505.20463] for single Higgs constraints.



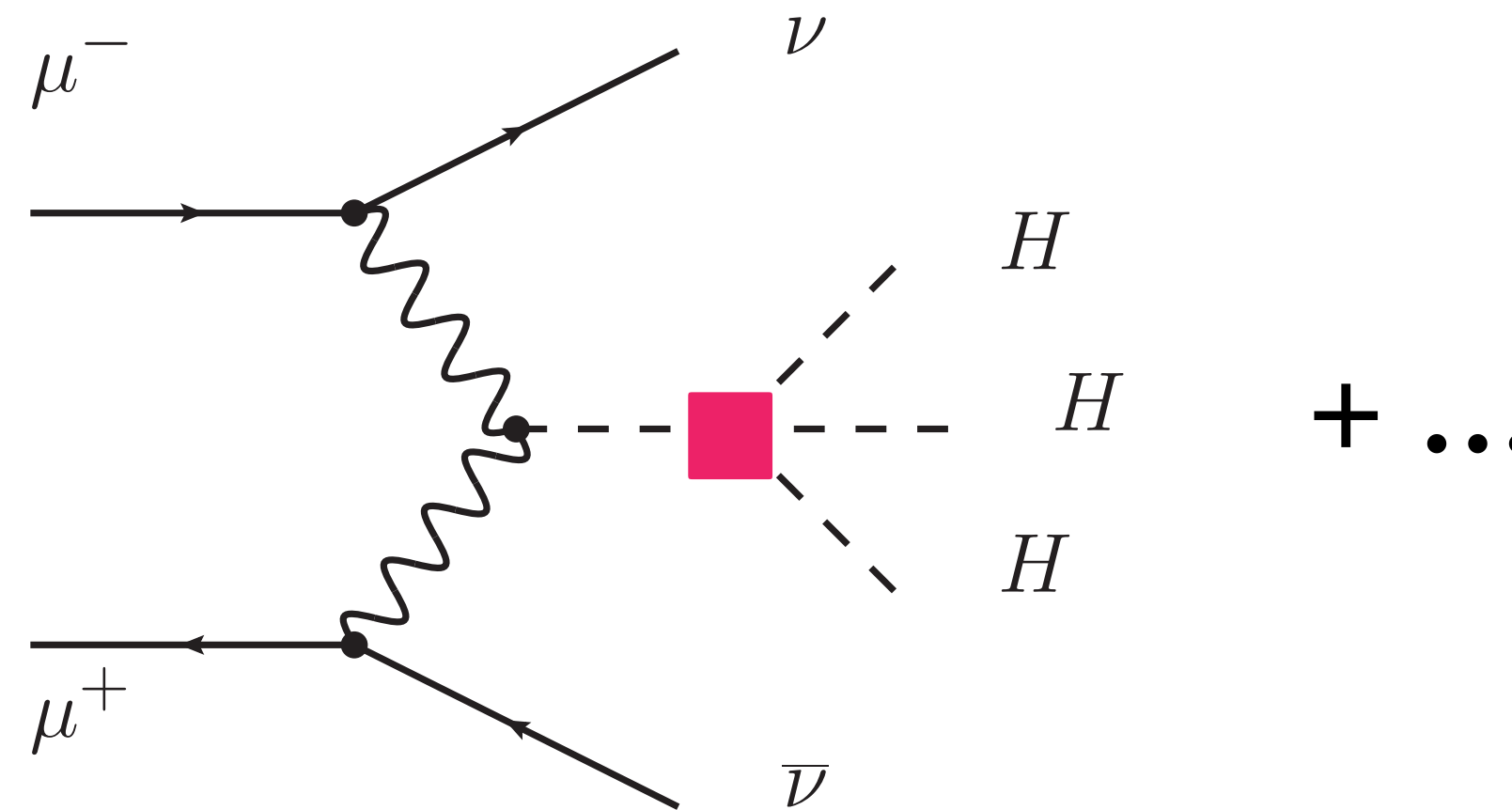
Lepton Colliders

🎩 FCC-ee: below threshold for Zhh production (that would allow indirect constraints).

🎩 ILC-500/4 ab^{-1} : (associated) multi-Higgs production $\rightarrow \lambda_4/\lambda_4^{\text{SM}} \sim [-5, 14]$. $[\lambda_3 = \lambda_3^{\text{SM}}]$

[Maltoni, Pagano, Zhao, 1802.07616,
Liu, Lyu, Ren, Zhu, 1803.04359]

🎩 Muon Collider, through VBF:



e.g. for $\sqrt{s} = 14 \text{ TeV}$, 33 ab^{-1} : $\lambda_4/\lambda_4^{\text{SM}} \sim [0.65, 1.55]$ $[\lambda_3 = \lambda_3^{\text{SM}}]$

↖ [Chiesa, Maltoni, Mantani, Mele, Piccinini, Zhao, 2003.13628]

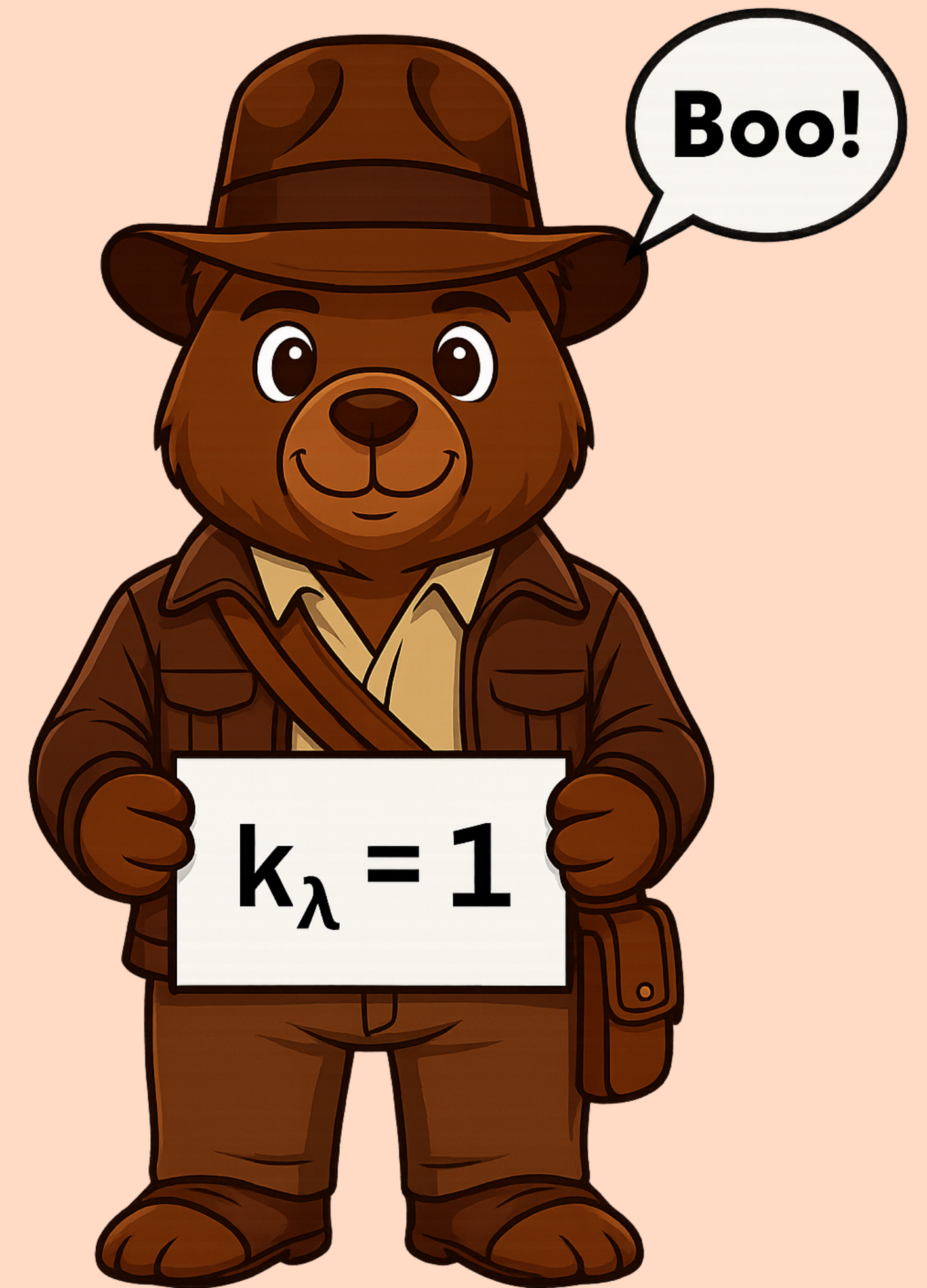
CONCLUSIONS

- 🎩 hhh : a direct probe of the Higgs quartic self-coupling!
- 🎩 **But:** @LHC: **Non-resonant** hhh : extremely challenging,
- 🎩 $\rightarrow$ *even* with large anomalous couplings (c_3 , d_4)!
- 🎩 & digging out the quartic self-coupling will be *hard*!
- 🎩 **Recommendation:** Experiments should consider:
 - $\rightarrow$ **Truncation** of the cross section within the context of EFTs (?).
 - $\rightarrow$ **More anomalous couplings.**
- 🎩 **Extended scalar sectors can enhance $hhh \leadsto$ observable at the LHC!**
- 🎩 e.g.: TRSM $\rightarrow$ Two new scalars $\rightarrow$ **double-resonant enhancement:**
 - $\rightarrow$ hhh (probably) **not a discovery channel!**
 - $\rightarrow$ **Information on nature of extended scalar sectors ($\leadsto$ Inverse problem).**



CONCLUSIONS

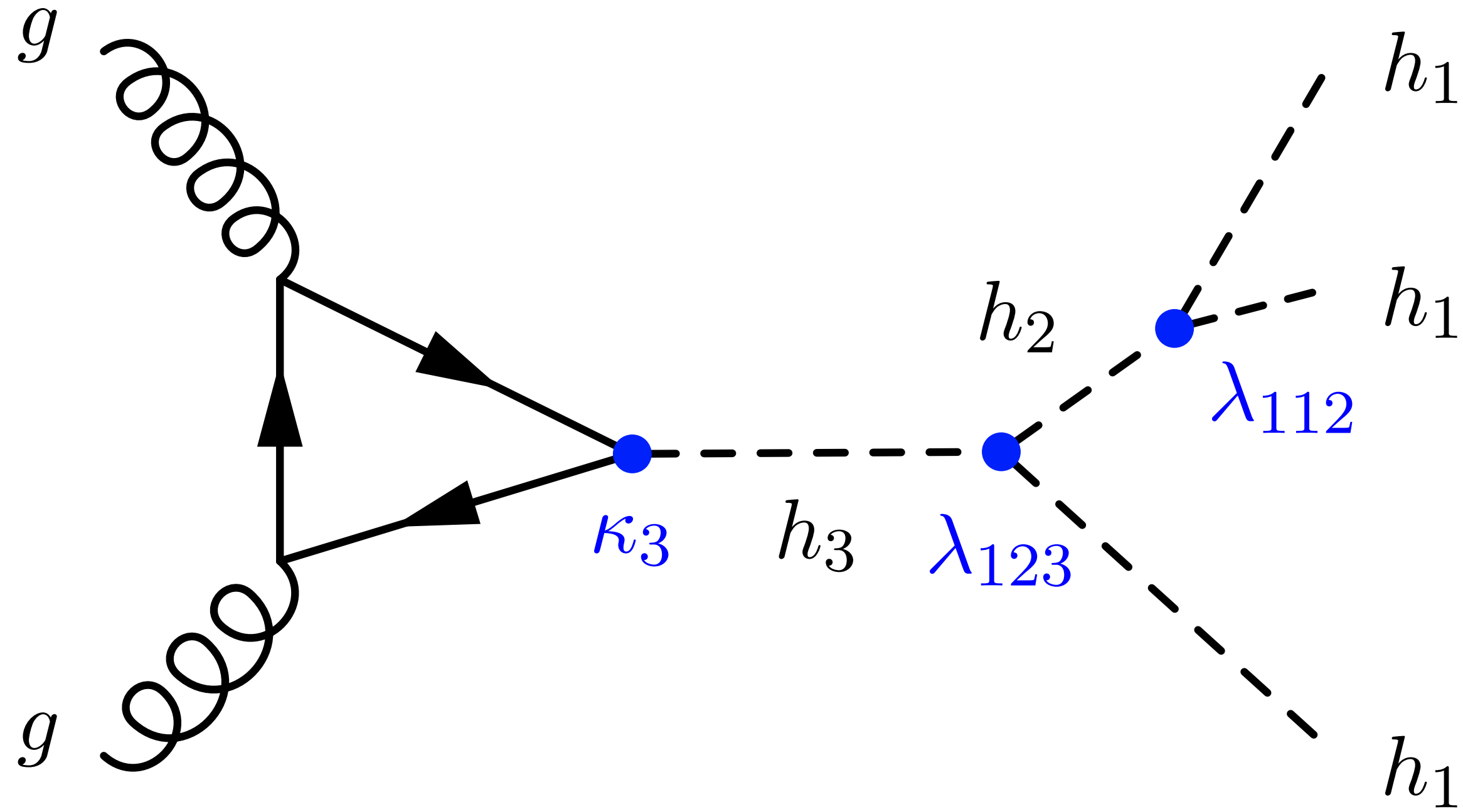
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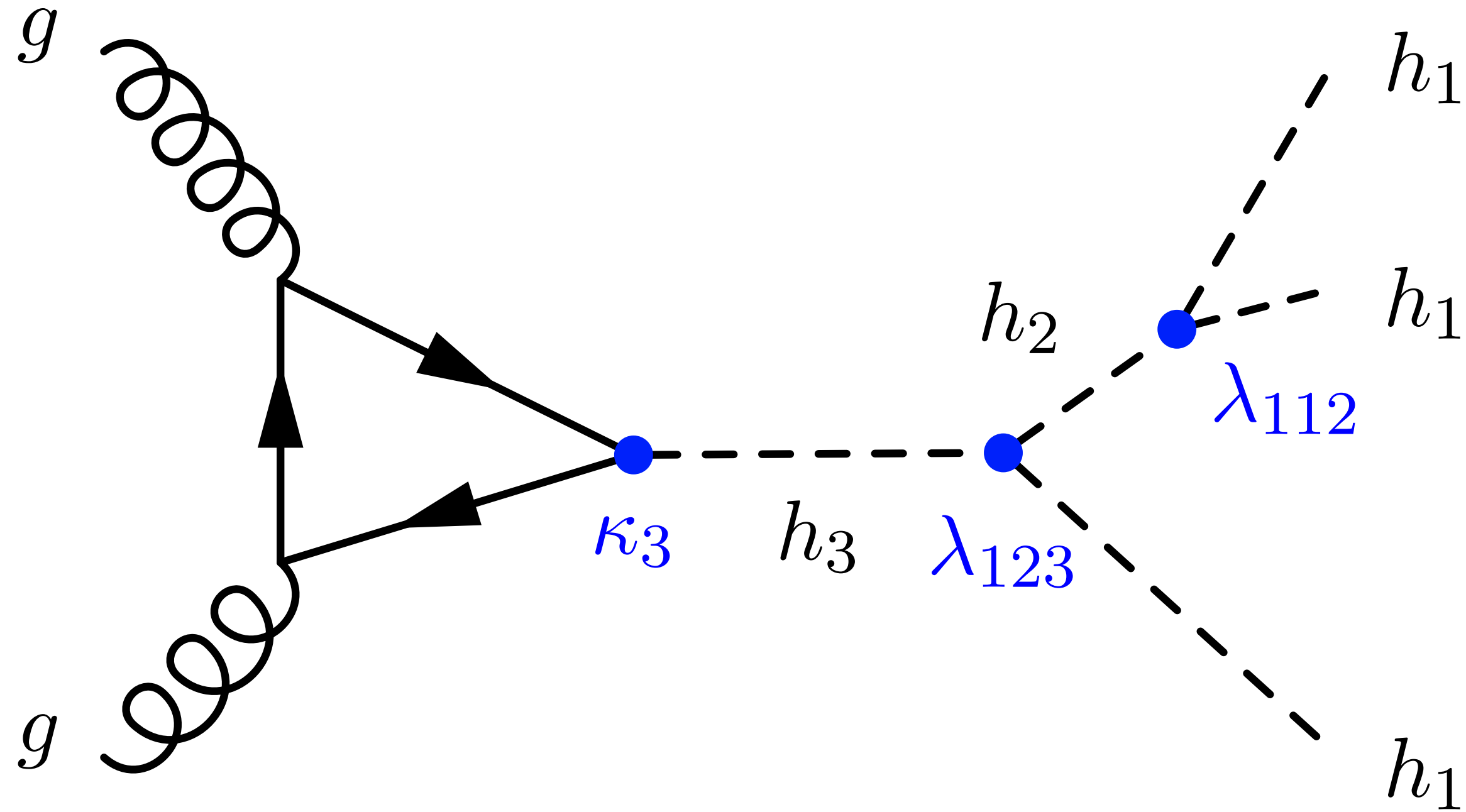
THANKS! QUESTIONS?

APPENDICES

Anatomy of Double-Resonant *hhh*



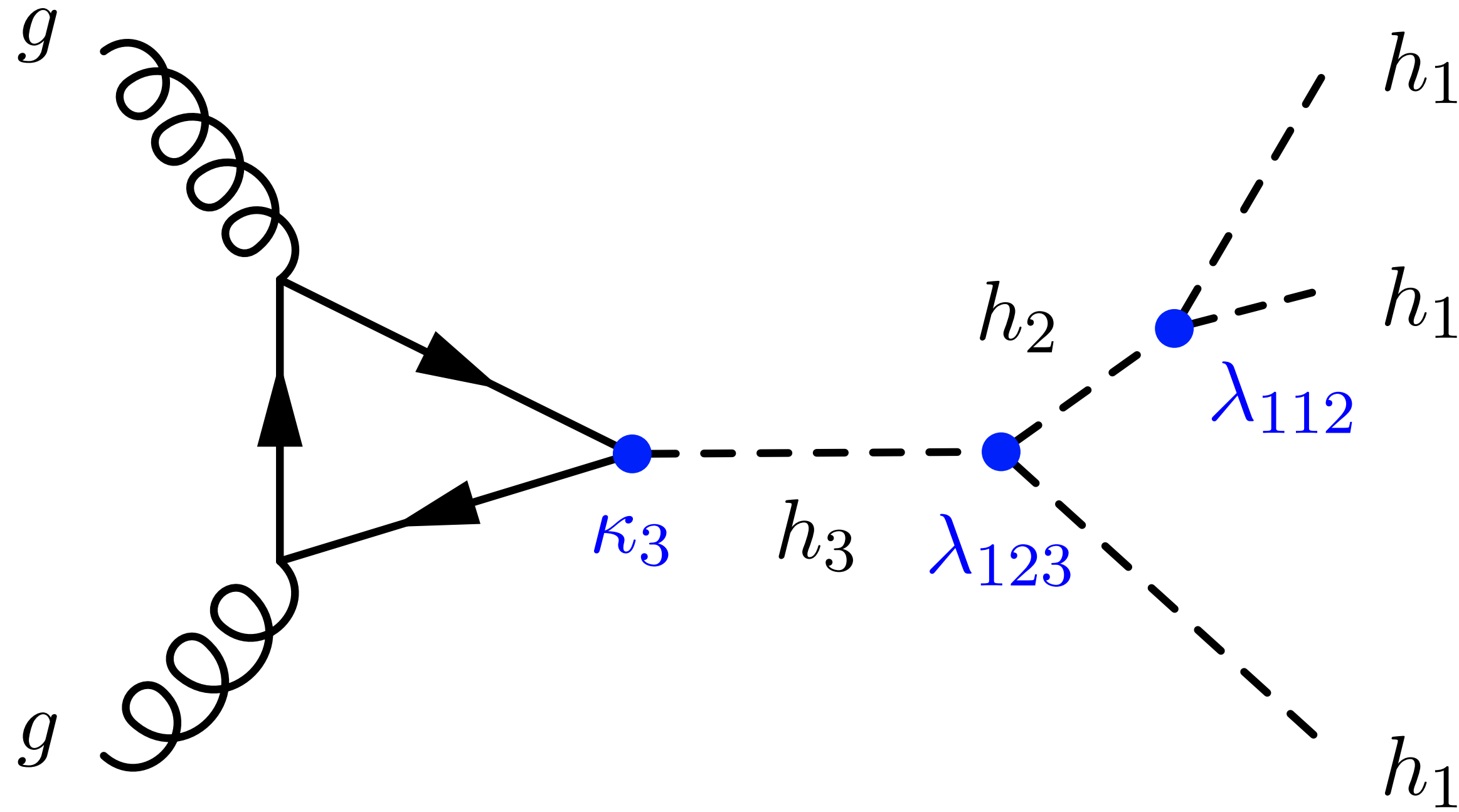
Anatomy of Double-Resonant *hhh*



$$\sigma(m_2, m_3) = \sigma_u(m_2, m_3) \times \underbrace{\kappa_3^2 \lambda_{123}^2 \lambda_{112}^2}_{\text{Factor out couplings}}$$

① Factor out couplings

Anatomy of Double-Resonant *hhh*



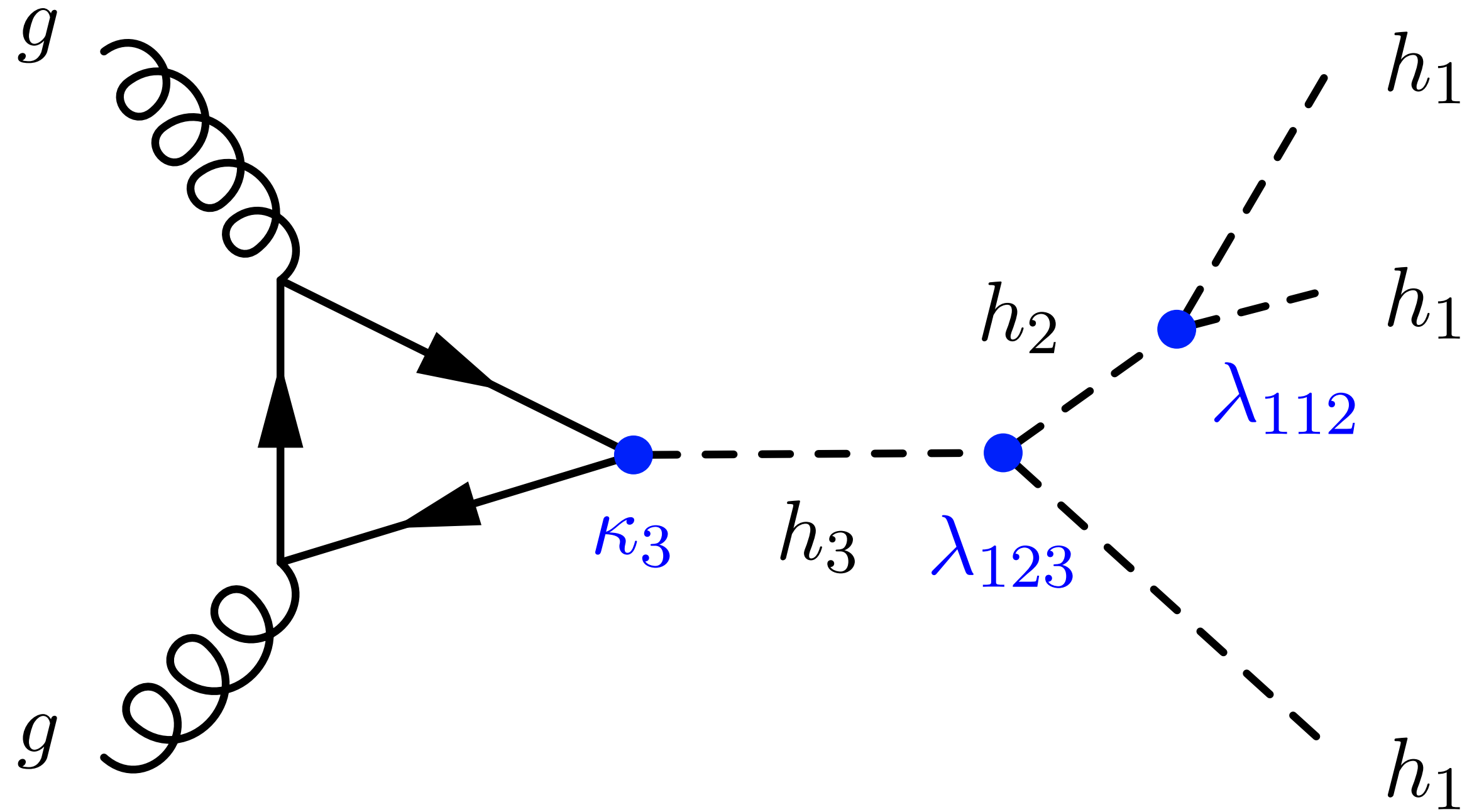
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① Factor out couplings

② Apply the narrow-width approximation* for h_2 and h_3 :

$$\frac{dq_i^2}{(q_i^2 - m_i^2)^2 + m_i^2 \Gamma_i^2} \rightarrow \frac{\pi}{m_i \Gamma_i} \delta(q_i^2 - m_i^2) dq_i^2$$

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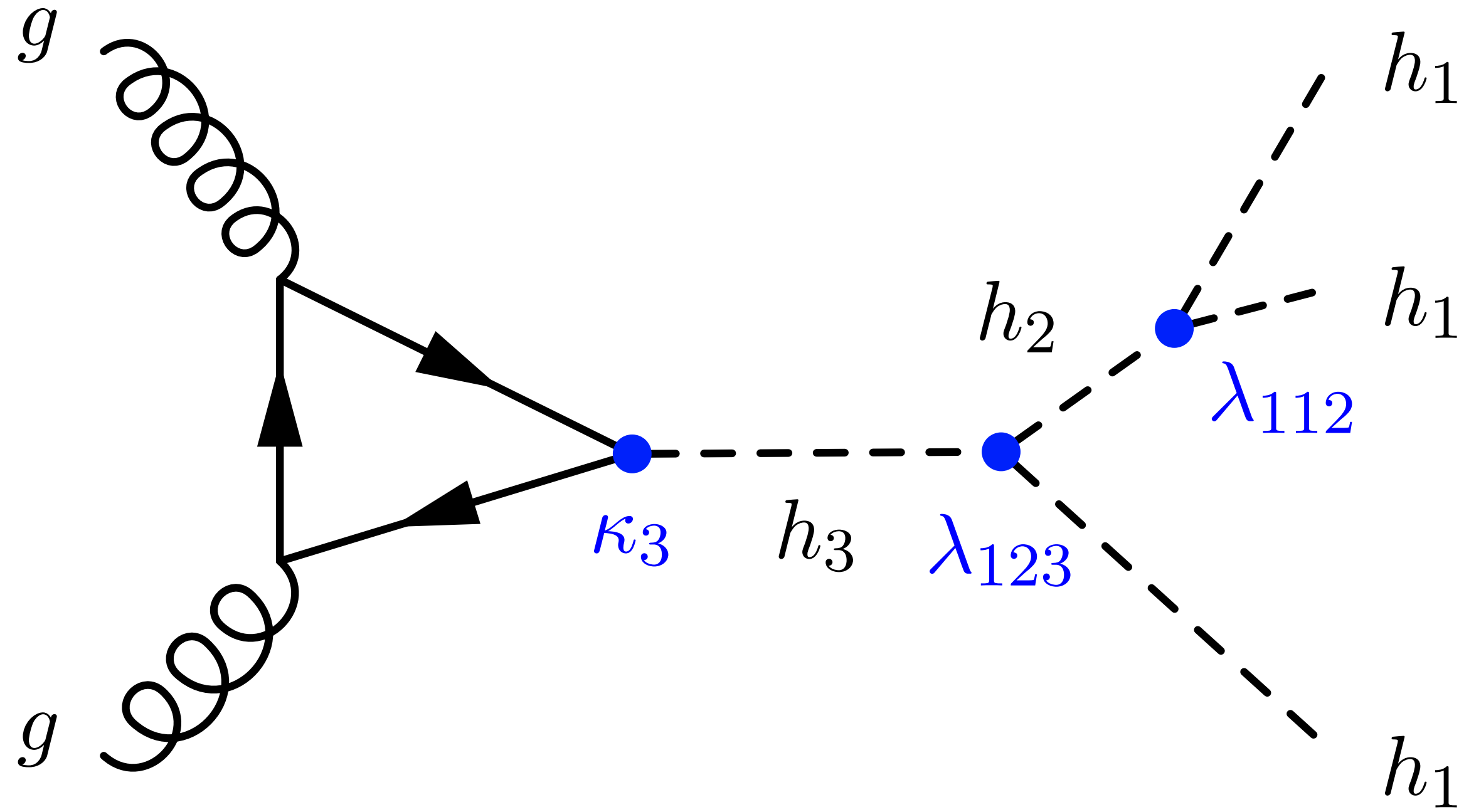
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depends only on masses

③ Obtain: $\sigma(m_2, m_3) = \hat{\sigma}_u(m_2, m_3) \times \left(\frac{\kappa_3^2 \lambda_{123}^2 \lambda_{112}^2}{\Gamma_2 \Gamma_3} \right)$

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④ Define: $\rho^2 \equiv \kappa_3^2 \lambda_{123}^2 \lambda_{112}^2 / (\Gamma_2 \Gamma_3)$

“rescaling factor”

*Narrow width: See appendix!

Andreas Papaefstathiou

What if...?

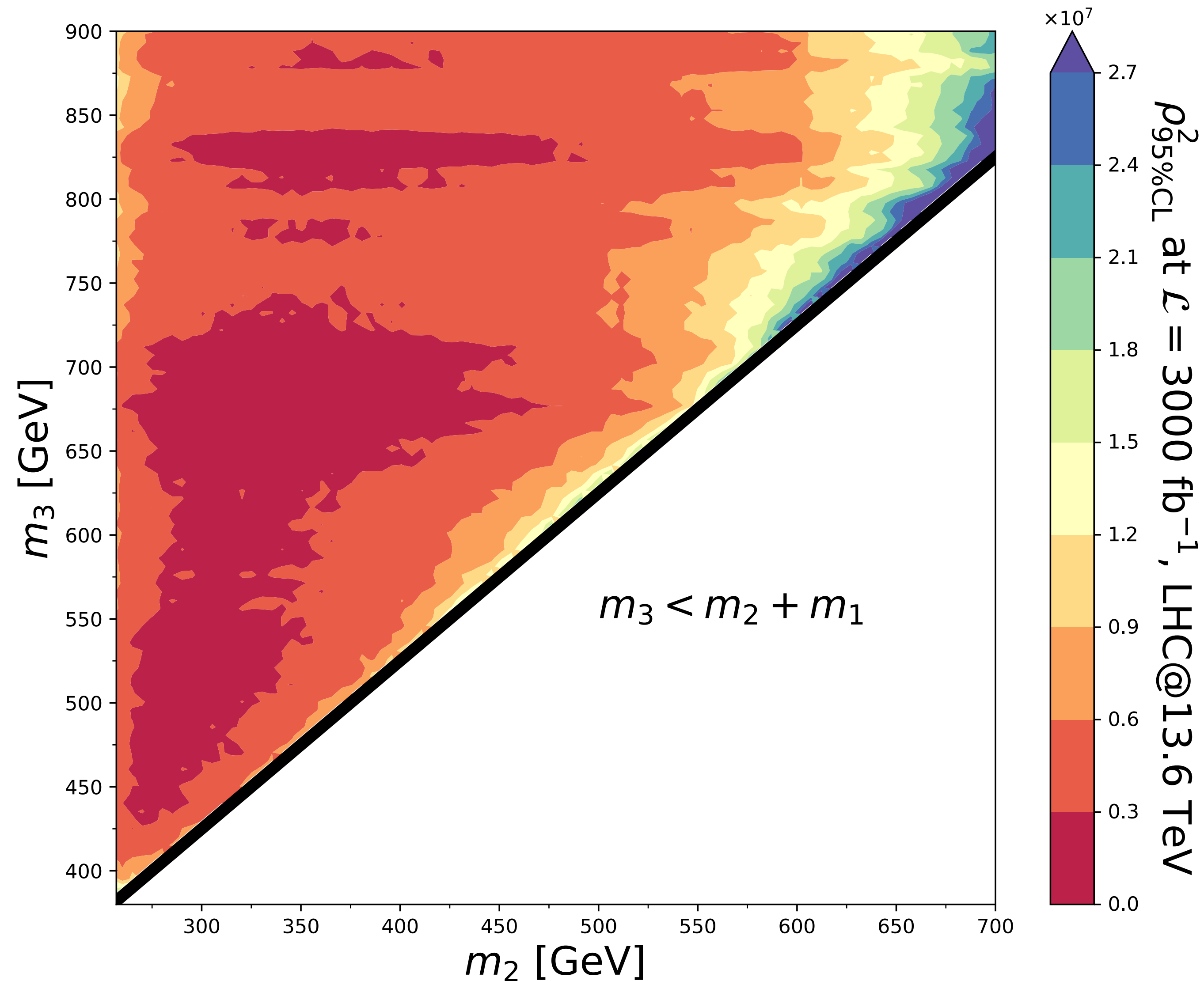
- Let's suppose two new scalars h_2 and h_3 are discovered (🍾🍾):
 - ➡ m_2, m_3 [and possibly] the widths Γ_2, Γ_3 would be known.
- hhh can provide relevant **information** on the theoretical parameter space.
 - ➡ An important contribution to solving the inverse problem!
 - ➡ through rescaling factor $\rho^2 = \kappa_3^2 \lambda_{123}^2 \lambda_{112}^2 / (\Gamma_2 \Gamma_3)$ (if narrow width!*)
- We derived **constraints** on ρ^2 via: $pp \rightarrow (b\bar{b})(b\bar{b})(b\bar{b}) \rightarrow 6 \text{ b-jets}$.
 - ➡ [$\sim 20\%$ of the hhh final state.]



[AP, Tetlalmatzi-Xolocotzi, Zaro,
arXiv:1909.09166,
AP, Robens, Tetlalmatzi-Xolocotzi,
arXiv:2101.00037,
AP, Tetlalmatzi-Xolocotzi,
arXiv:2501.14866]

*Narrow width: See appendix!

Constraints on $\rho^2 = \kappa_3^2 \lambda_{123}^2 \lambda_{112}^2 / (\Gamma_2 \Gamma_3)$ on the (m_2, m_3) -plane



95% C.L. Constraint on ρ^2
at HL-LHC.

[AP, Tetlalmatzi-Xolocotzi, arXiv:2501.14866]

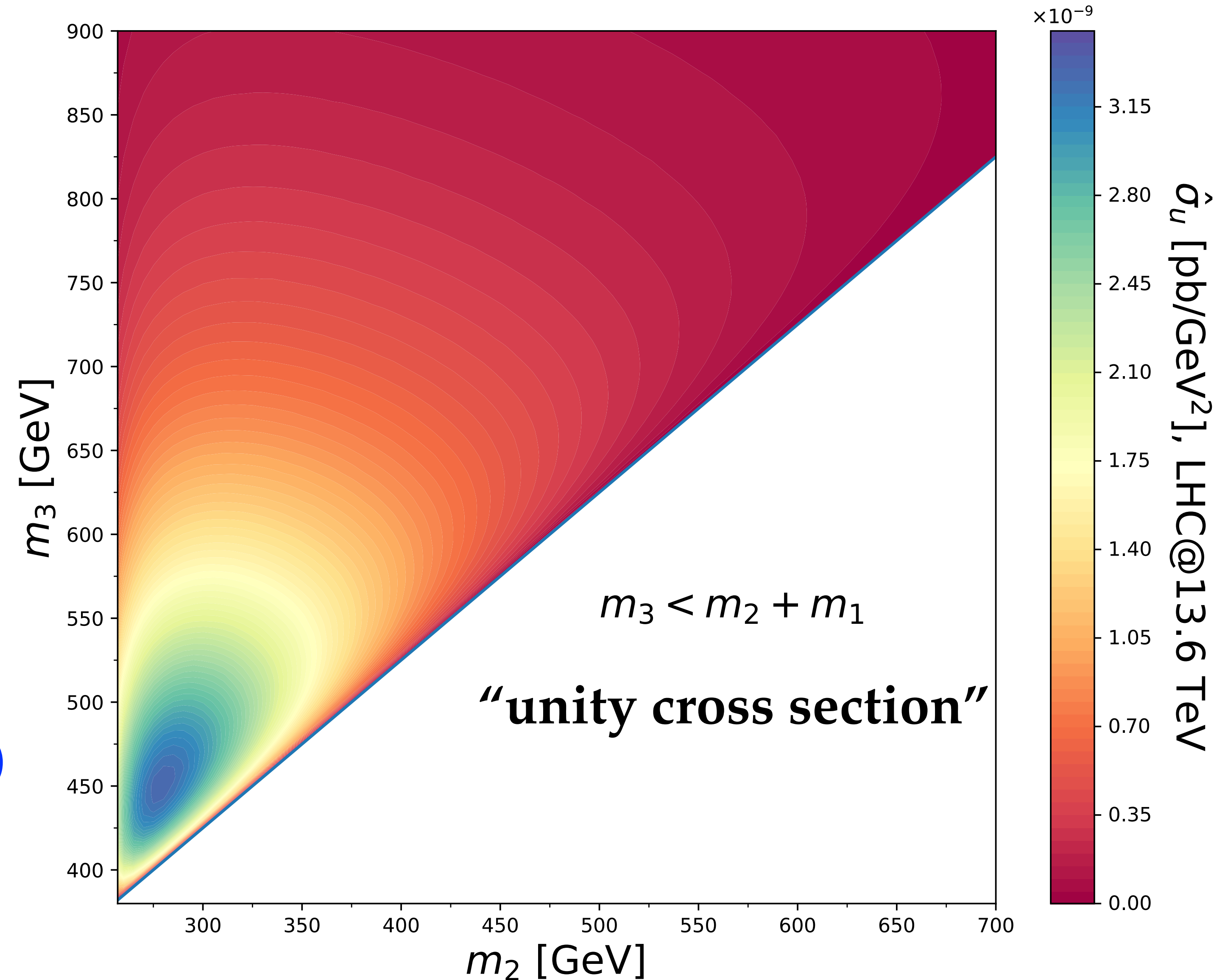
Anatomy of Double-Resonant *hhh*

$$\sigma(m_2, m_3) = \underbrace{\hat{\sigma}_u(m_2, m_3)}_{\text{"unity cross section"}} \times \rho^2$$

“unity cross section”:
depends only on *m₂*, *m₃*.
Derived once and for all!
(at fixed collider energy)

with: $\rho^2 \equiv \kappa_3^2 \lambda_{123}^2 \lambda_{112}^2 / (\Gamma_2 \Gamma_3)$

“rescaling factor”:
couplings and widths



[AP, Tetlalmatzi-Xolocotzi, arXiv:2501.14866]

TRSM Benchmarks from: [Karkout, AP, Postma, Tetlalmatzi-Xolocotzi, van de Vis, du Pree, **arXiv:2404.12425**]

Benchmark quantities relevant for double-resonant triple Higgs boson production									
Name	m_2	m_3	Γ_2	Γ_3	κ_3	λ_{123}	λ_{112}	$\begin{matrix} [\text{GeV}^2] \\ \rho^2 \\ [\times 10^6] \end{matrix}$	$\begin{matrix} [\text{pb/GeV}^2] \\ \hat{\sigma}_u \\ [\times 10^{-9}] \end{matrix}$
BM0	259.0	495.0	0.003514	3.927	0.1854	-191.8	8.167	6.11	2.018
BM1	270.6	444.7	0.5078	2.586	0.1571	-204.3	67.52	3.574	3.408
BM2	268.6	452.7	0.3805	3.142	0.1741	-203.6	57.78	3.509	3.165
BM3	272.6	480.7	0.2009	4.758	0.2024	-224.6	41.39	3.703	2.908
BM4	269.0	409.8	0.2836	1.995	0.1713	-180.3	48.89	4.031	2.663
BM5	269.1	486.9	0.0003346	2.017	0.1527	103.3	-2.477	2.264	2.805
BM6	259.2	577.0	0.0006274	5.79	0.1908	196.3	-3.701	5.289	1.108
BM7	283.7	575.0	0.001056	5.587	0.1884	193.5	-3.578	2.885	1.711
BM8	264.3	469.3	0.3916	2.941	0.1746	-144.3	55.88	1.721	2.789
BM9	266.5	461.9	0.3092	2.042	0.1635	142.8	39.98	1.381	3.29
BM10	259.2	399.7	0.2188	0.9312	0.1463	121.2	35.41	1.936	2.159

- Including:
- **EXP constraints** through HiggsTools.
 - **TH constraints:** perturbativity & boundedness from below.

[AP, Tetlalmatzi-Xolocotzi, **arXiv:2501.14866**]

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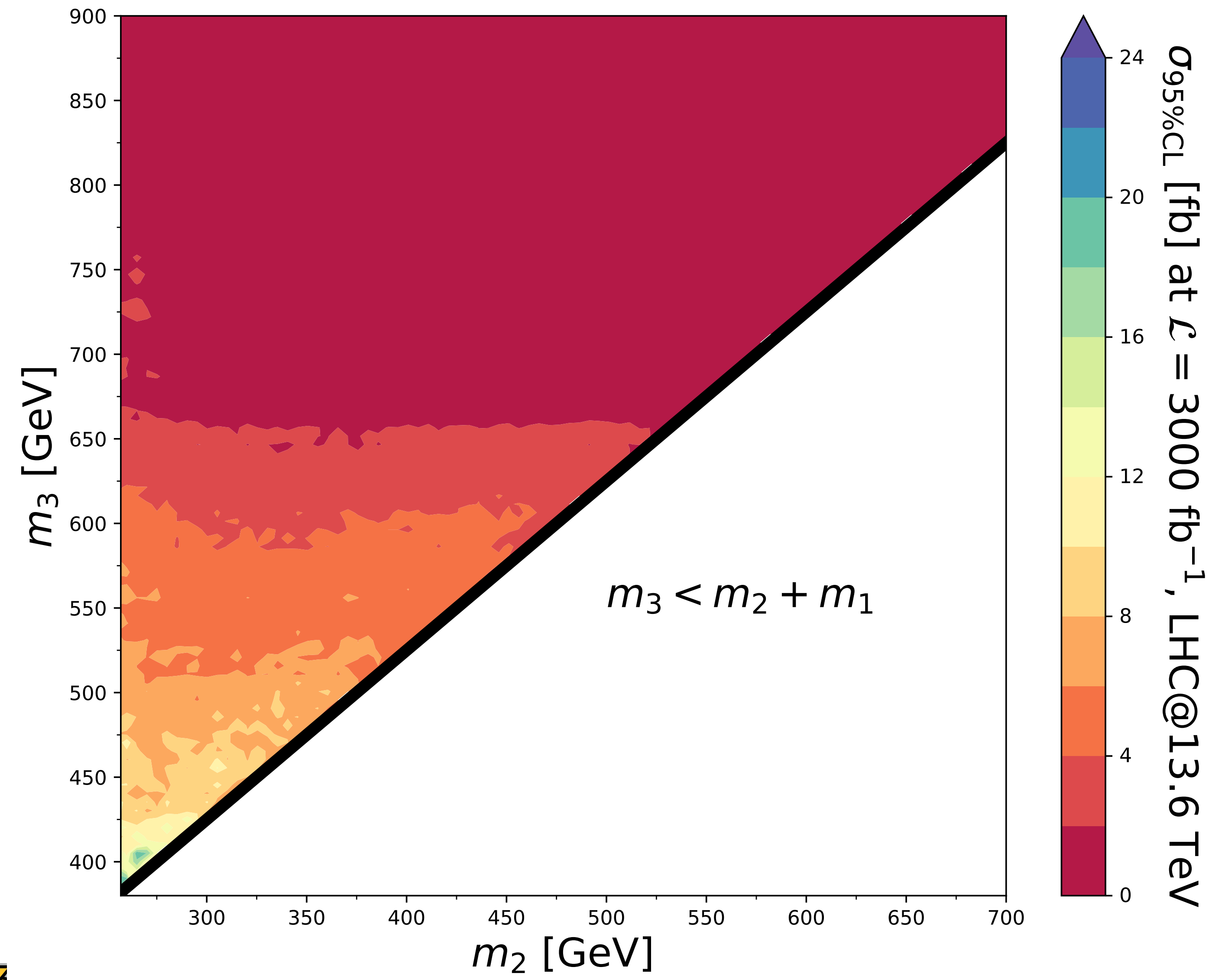
Including:

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➡ $\sigma(13.6 \text{ TeV}) \gtrsim \mathcal{O}(10) \text{ fb}$
[AP, Tetlalmatzi-Xolocotzi, arXiv:2501.14866]

➡ **Narrow width OK!**

Constraints on the Cross Section



95% C.L. Constraint on
the cross section
at HL-LHC.

[AP, Tetlalmatzi-Xolocotzi, arXiv:2501.14866]

Comment: Validity of the Narrow-Width Approximation

- In the TRSM: h_2 and h_3 are **constrained** to possess small mixing angles.

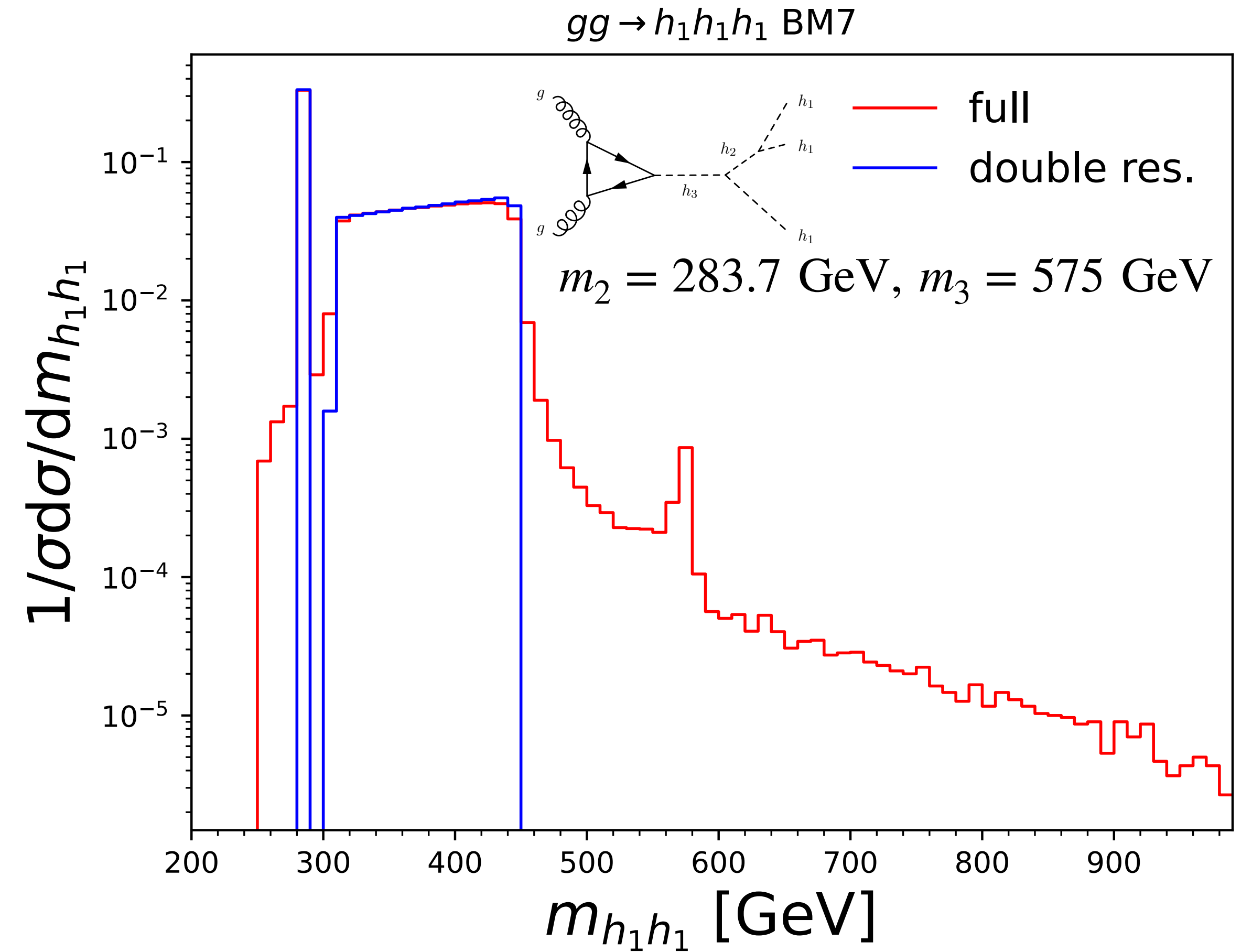
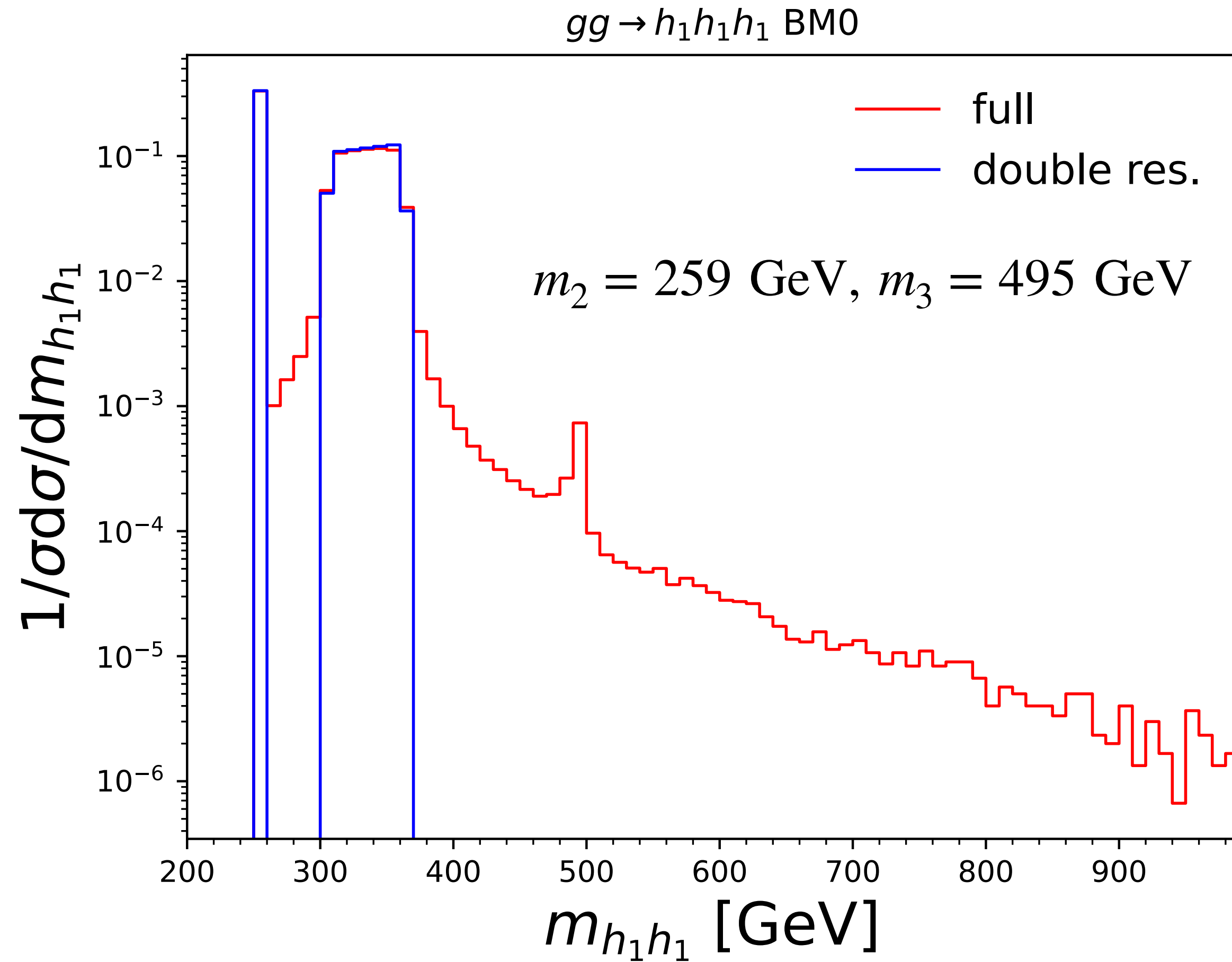
(e.g. we know this from h_1 signal strength)

⇒ Contributions to width from $h_{2,3} \rightarrow f\bar{f}, VV$ are small!

- In general: Can increase width of h_2 and h_3 through scalar-to-scalar decays ⇒ no guarantee in generic models for the narrow width.
- If h_2 and h_3 are already discovered, Γ_2, Γ_3 would be known (or limited) ⇒ the narrow-width approximation validity should be checked!
- **TL;DR**: The narrow width approximation is OK due to mixing constraints in the TRSM, but this statement is somewhat model dependent! [***see appendix for a case study!]

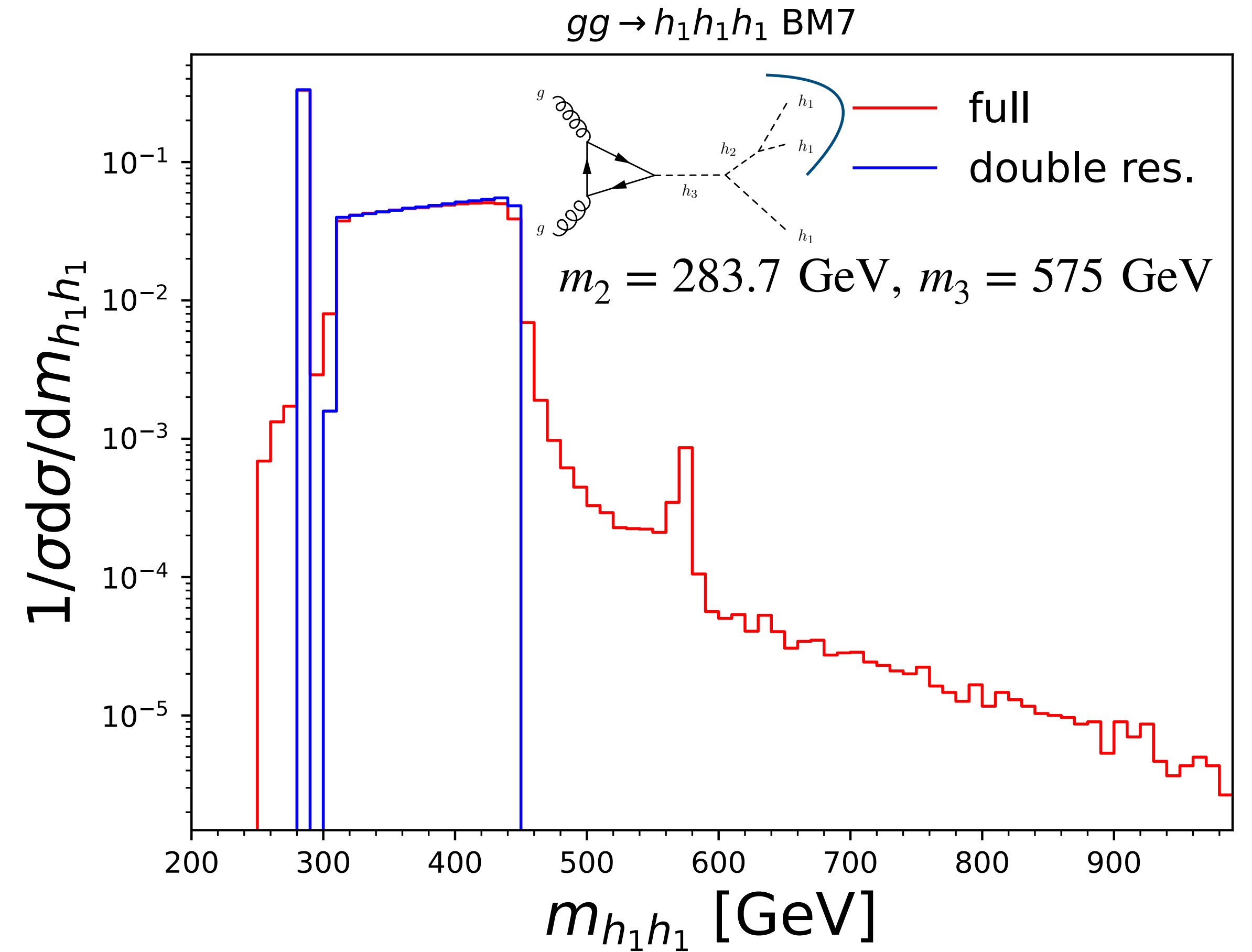
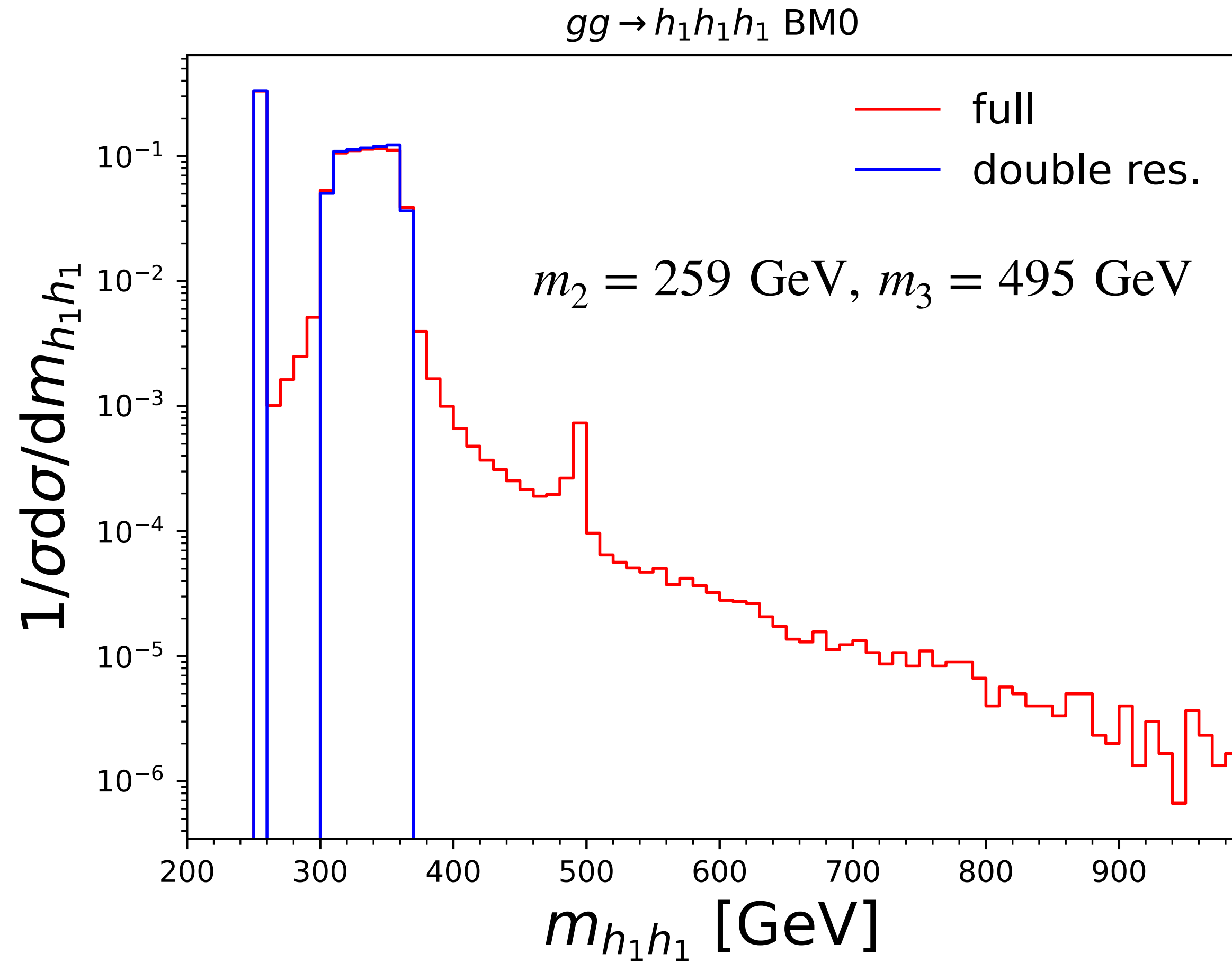
Non-Resonant Effects in hhh in the TRSM

(any) two Higgs invariant mass distributions in hhh (“parton level”, all combinations)



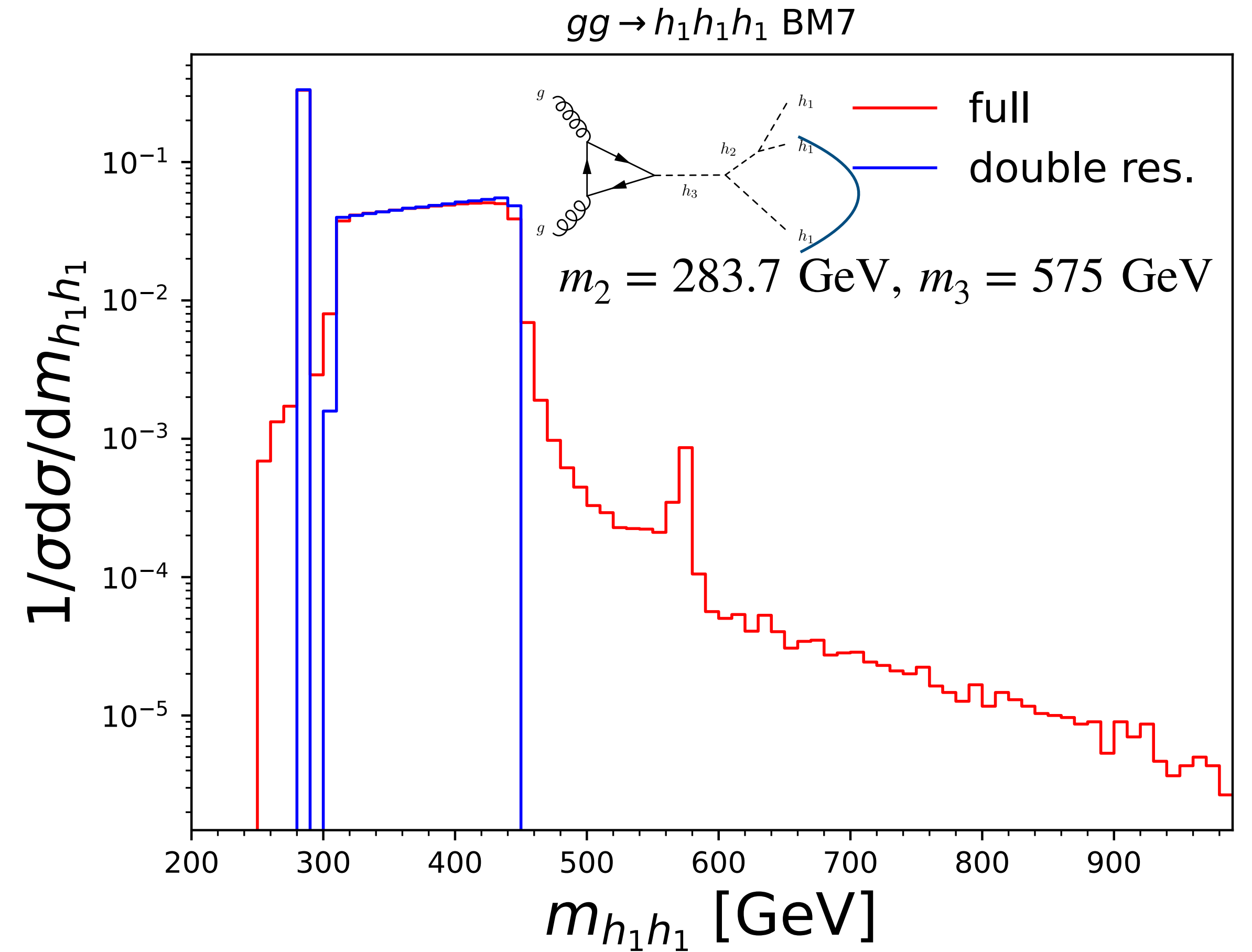
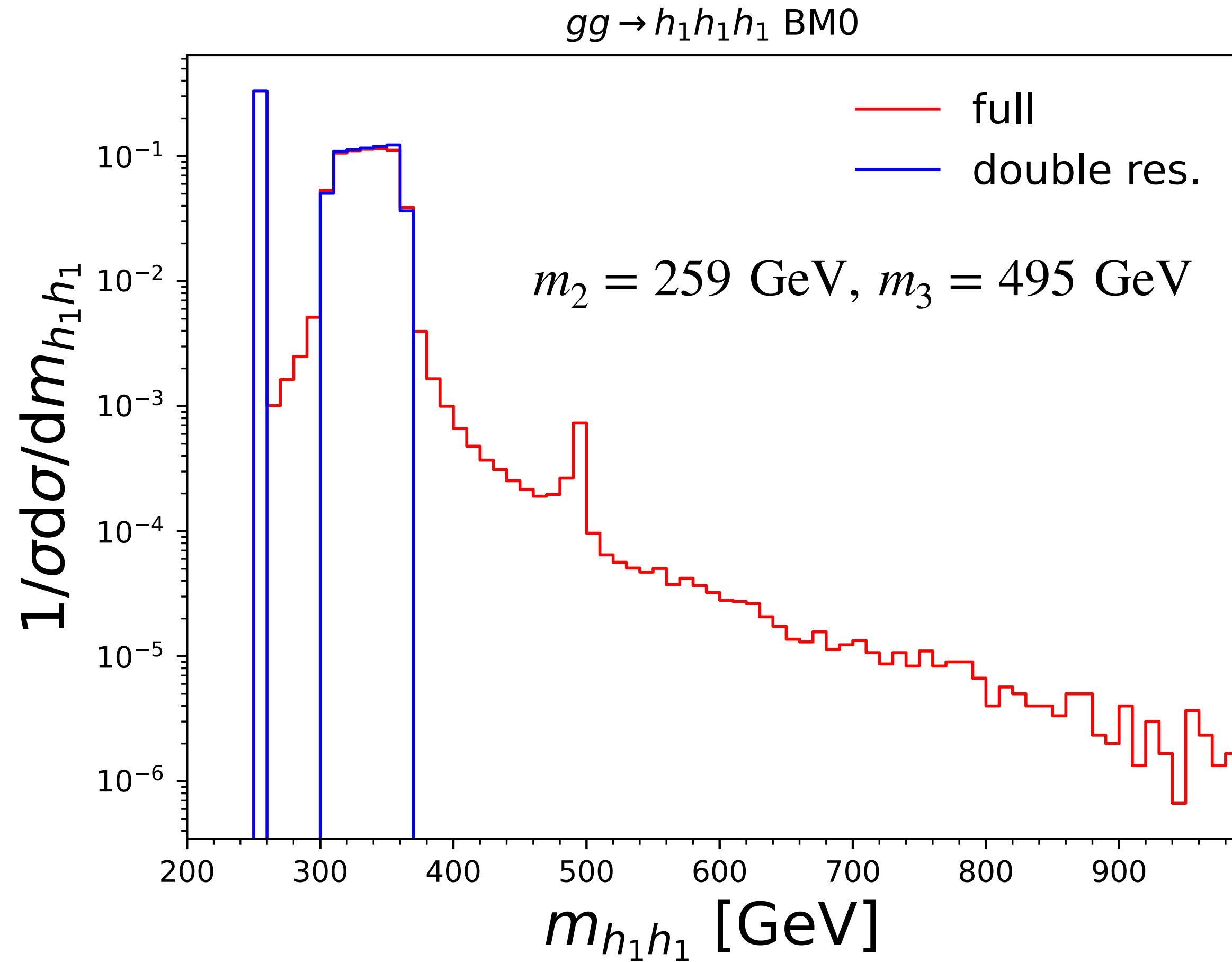
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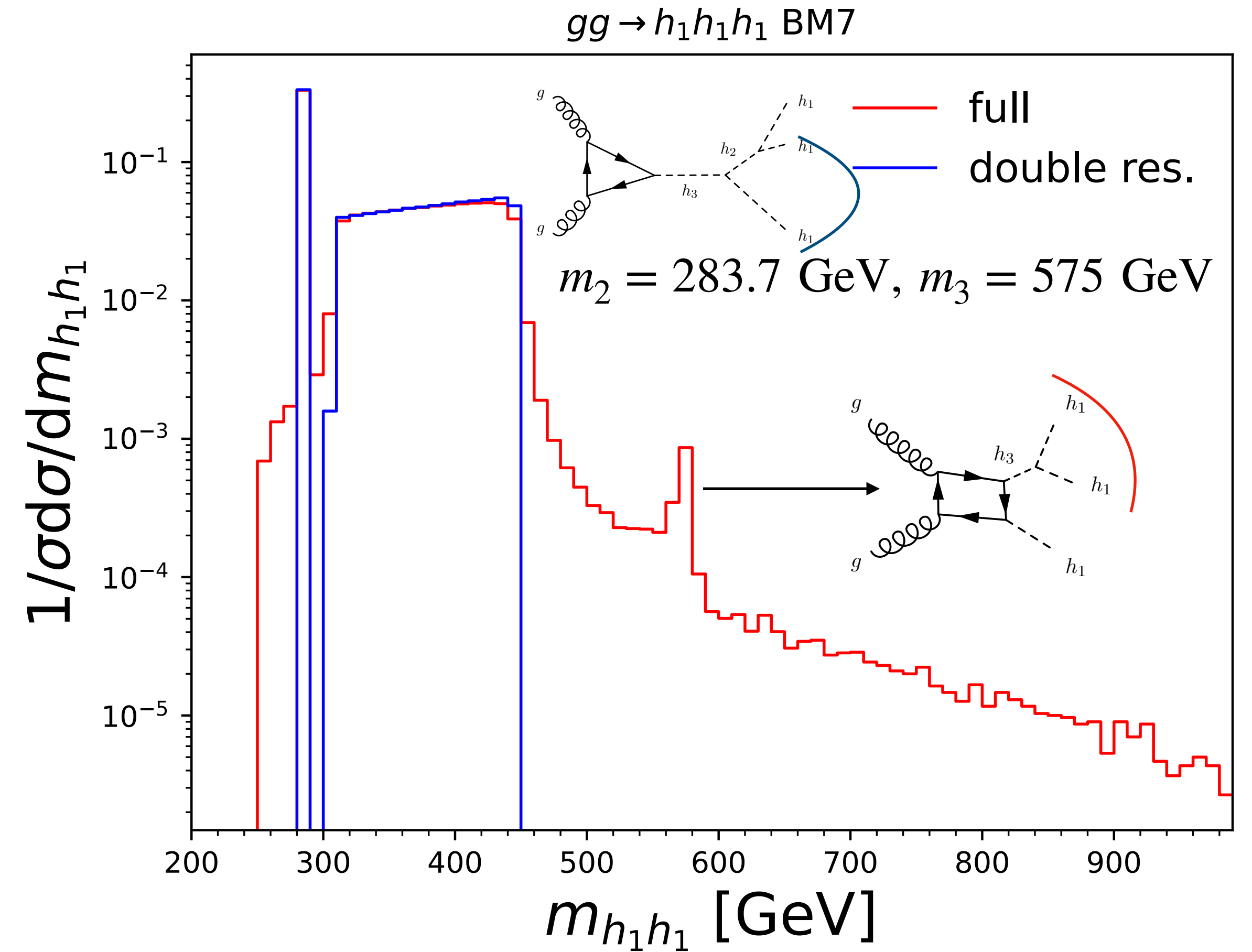
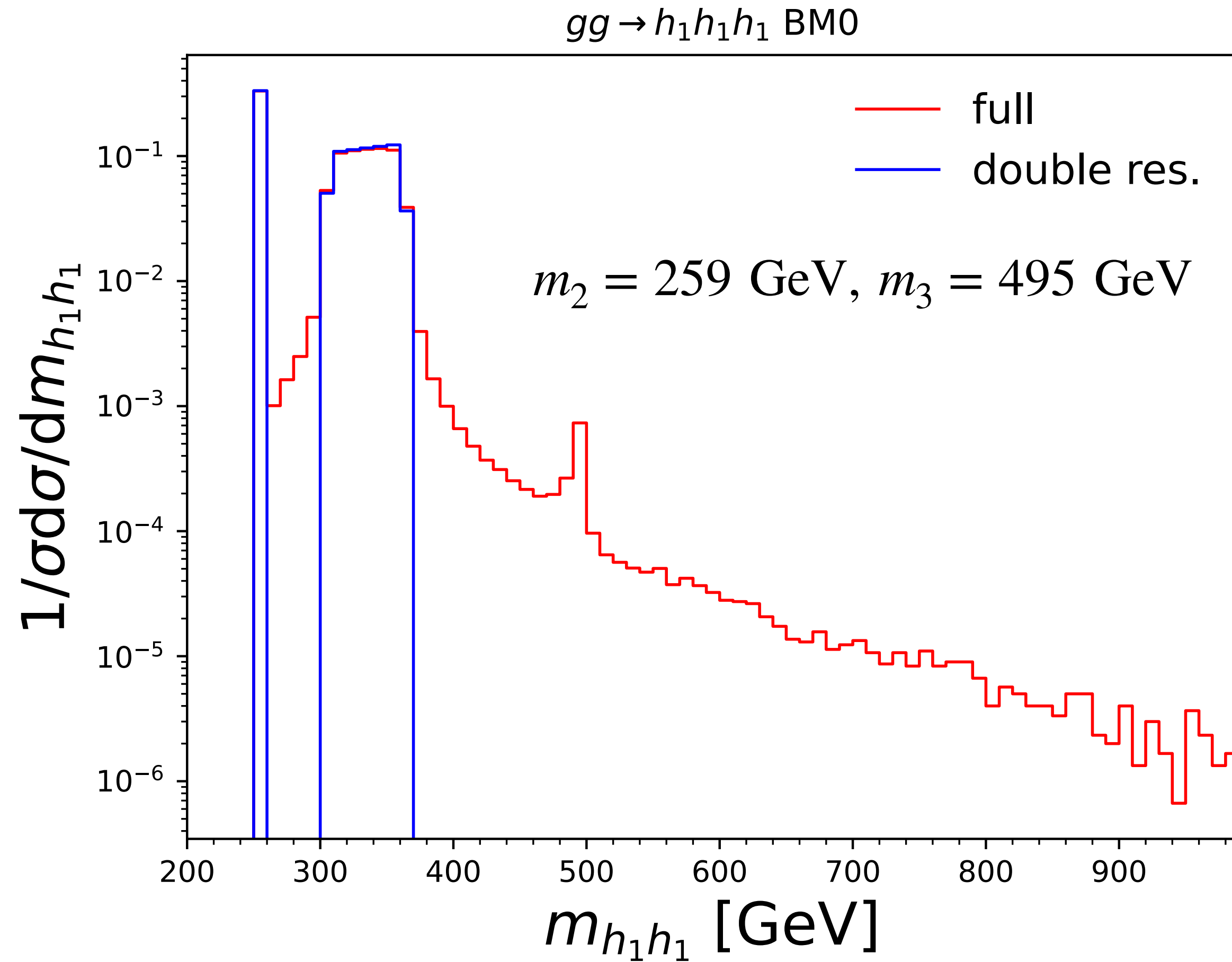
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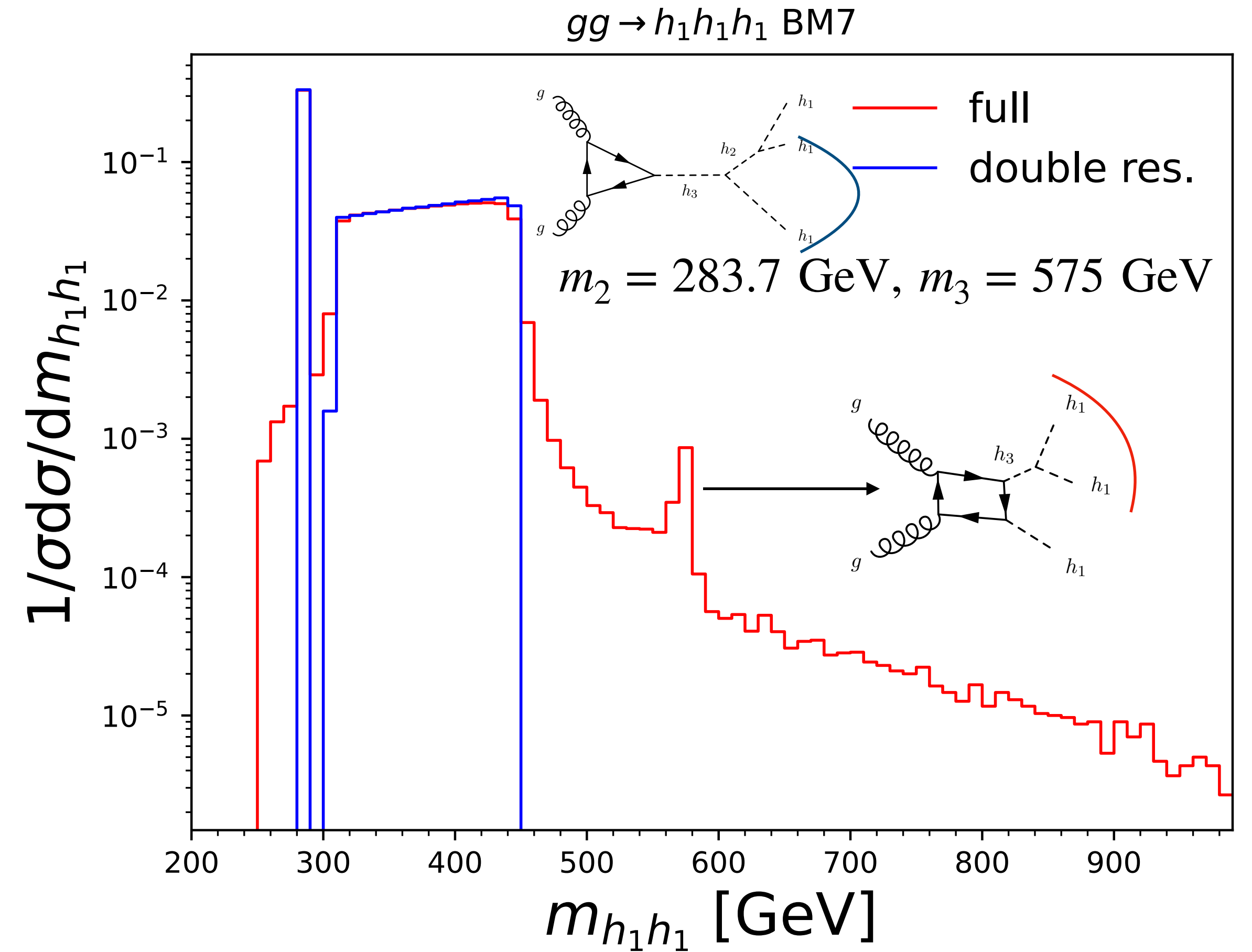
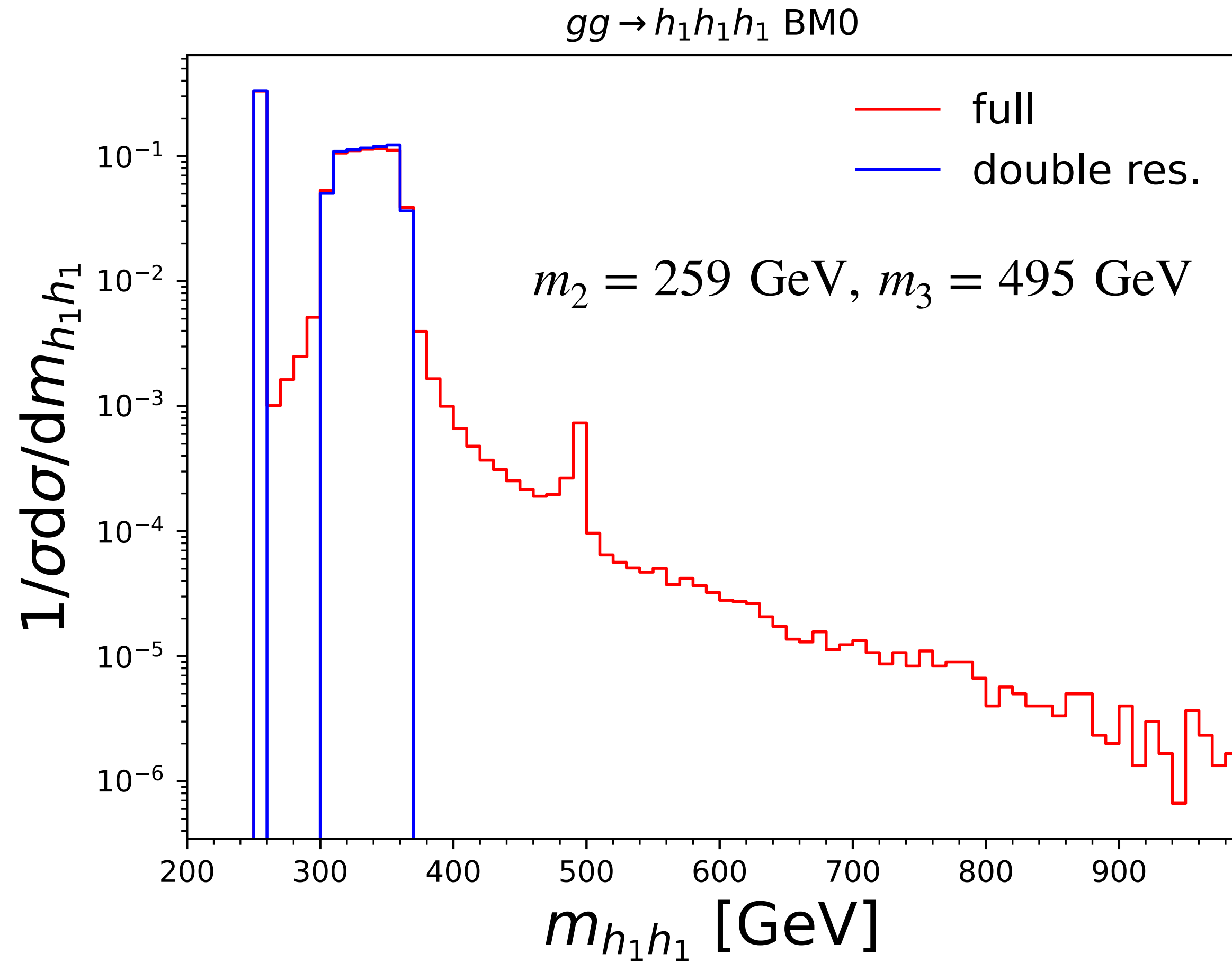
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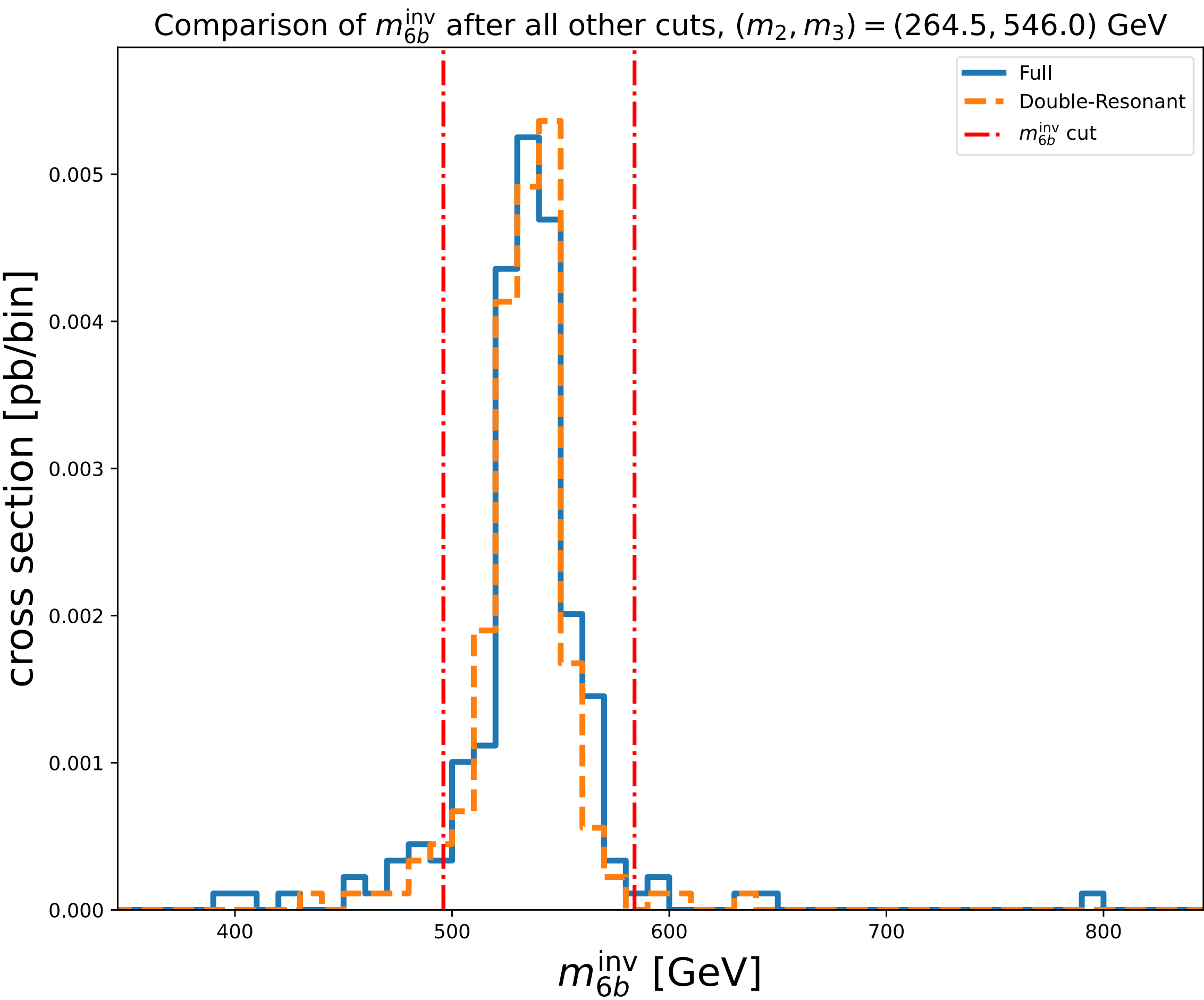
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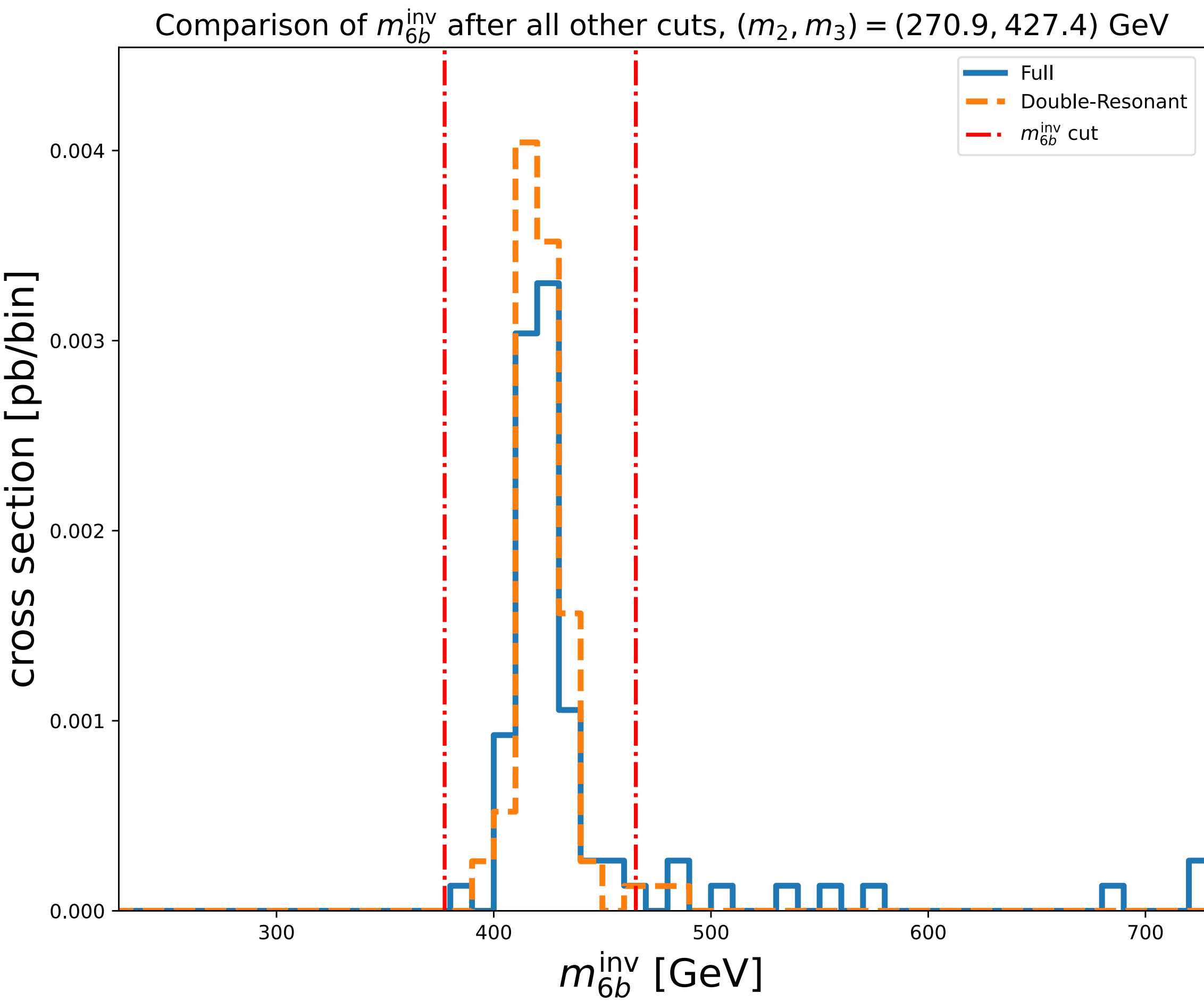
➡ Bulk of cross section from double-resonant process.

Non-Resonant Effects in *hhh* in the TRSM

Sample 6 *b*-jet invariant mass distributions in our analysis (all other cuts applied):



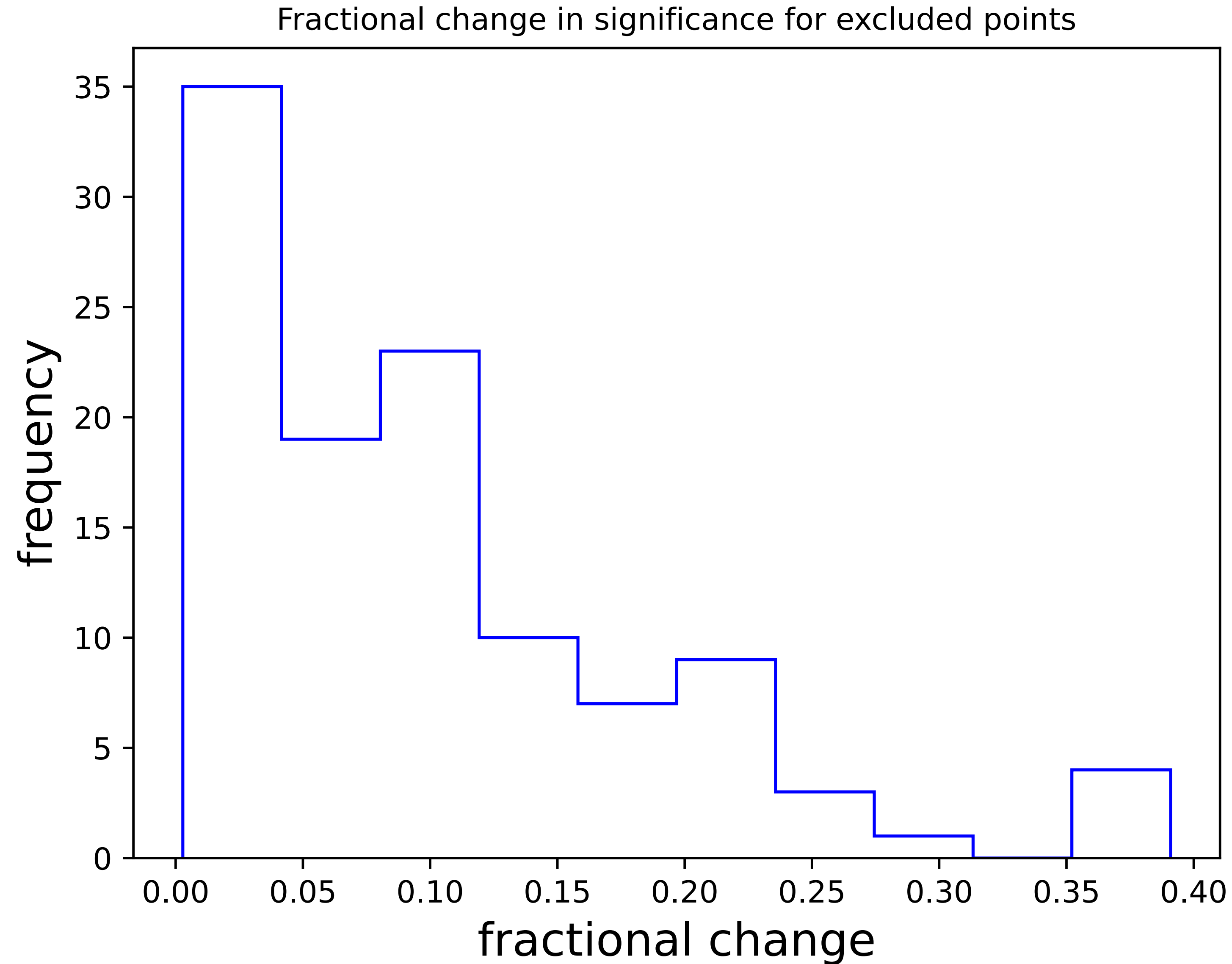
m_{6b} [GeV]



m_{6b} [GeV]

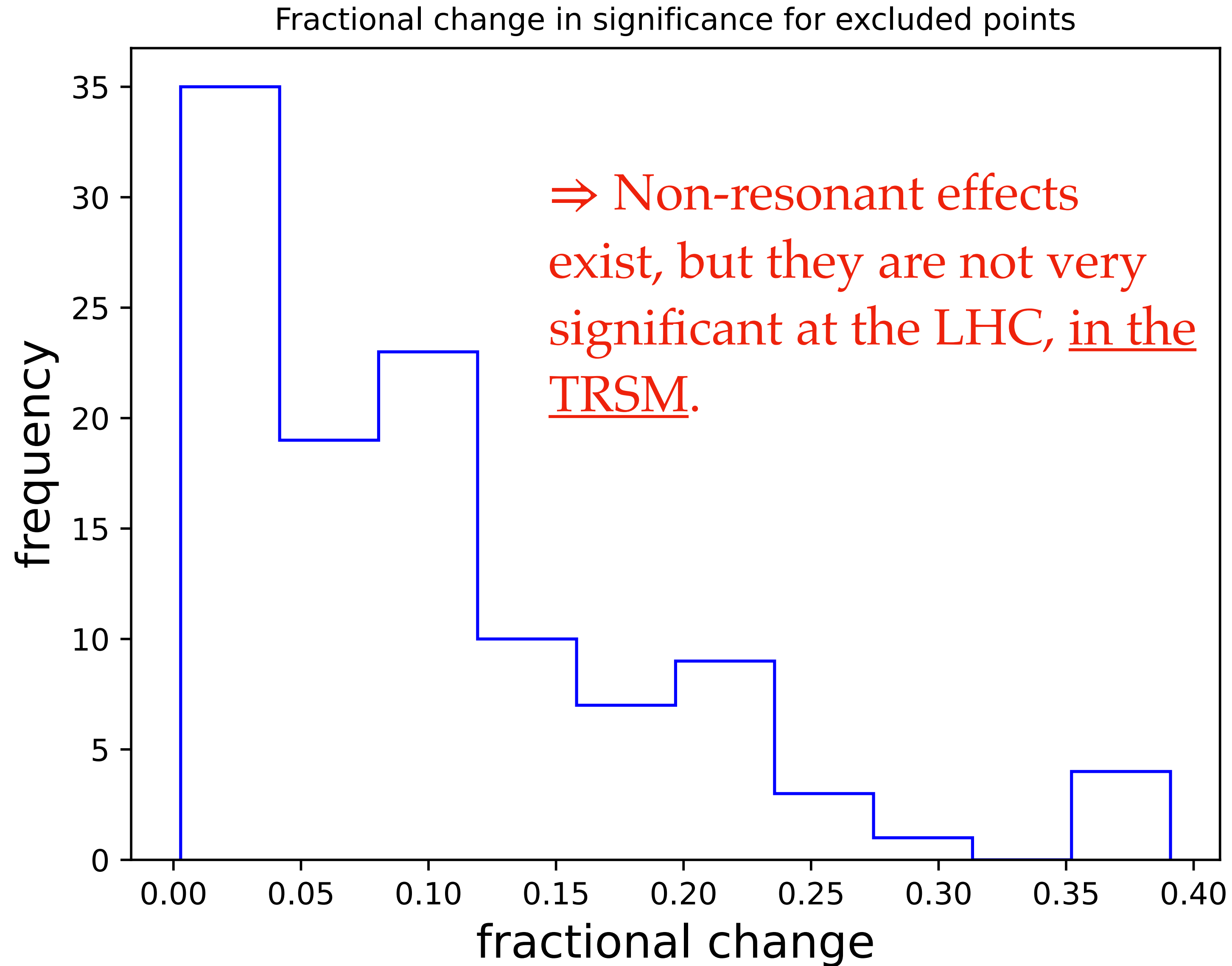
Non-Resonant Effects in *hhh* in the TRSM

= Number of Parameter
Space Points in our
Scan
that can be excluded at
95% C.L.



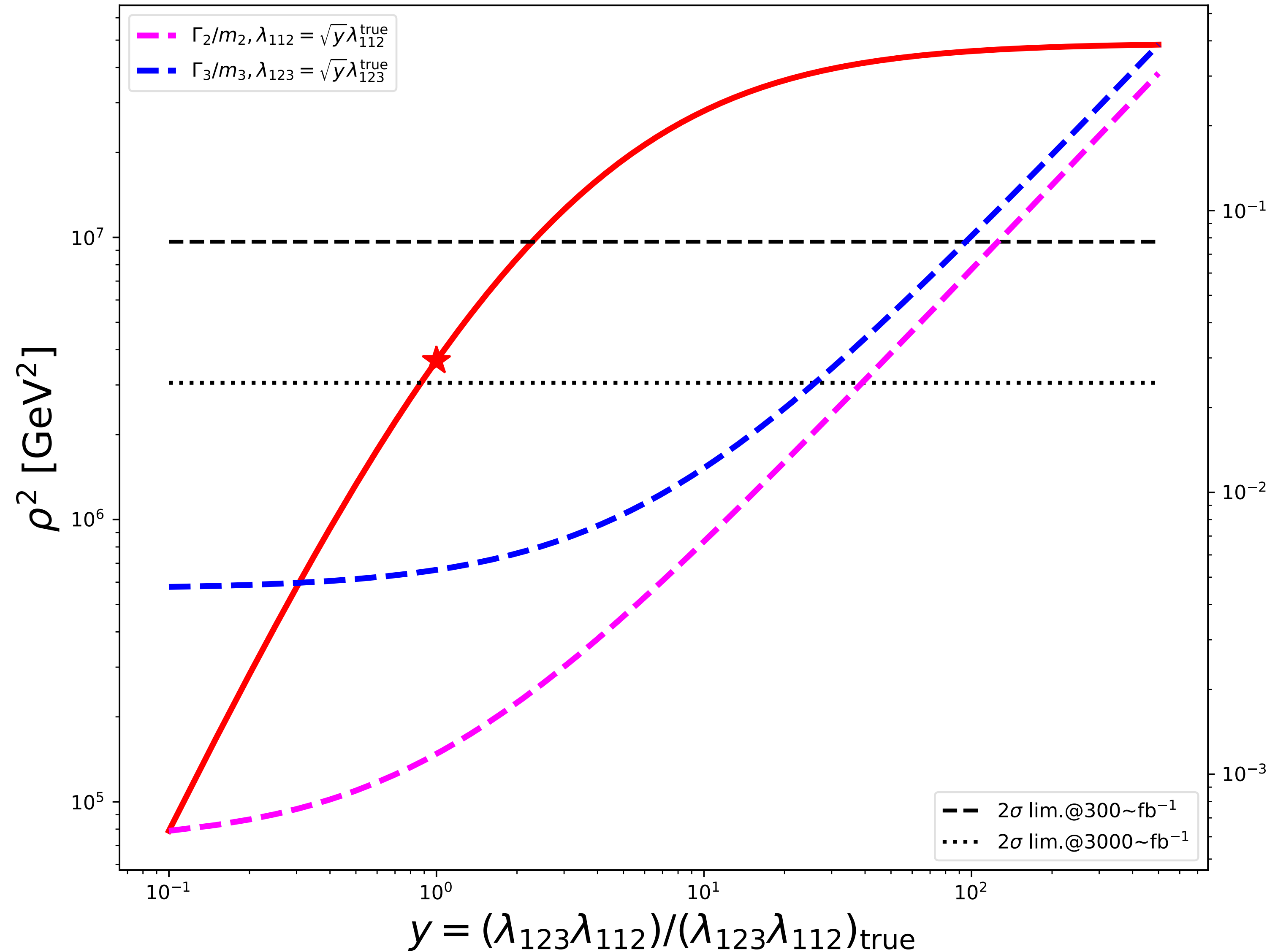
Non-Resonant Effects in *hhh* in the TRSM

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The Narrow Width Approximation: An Example

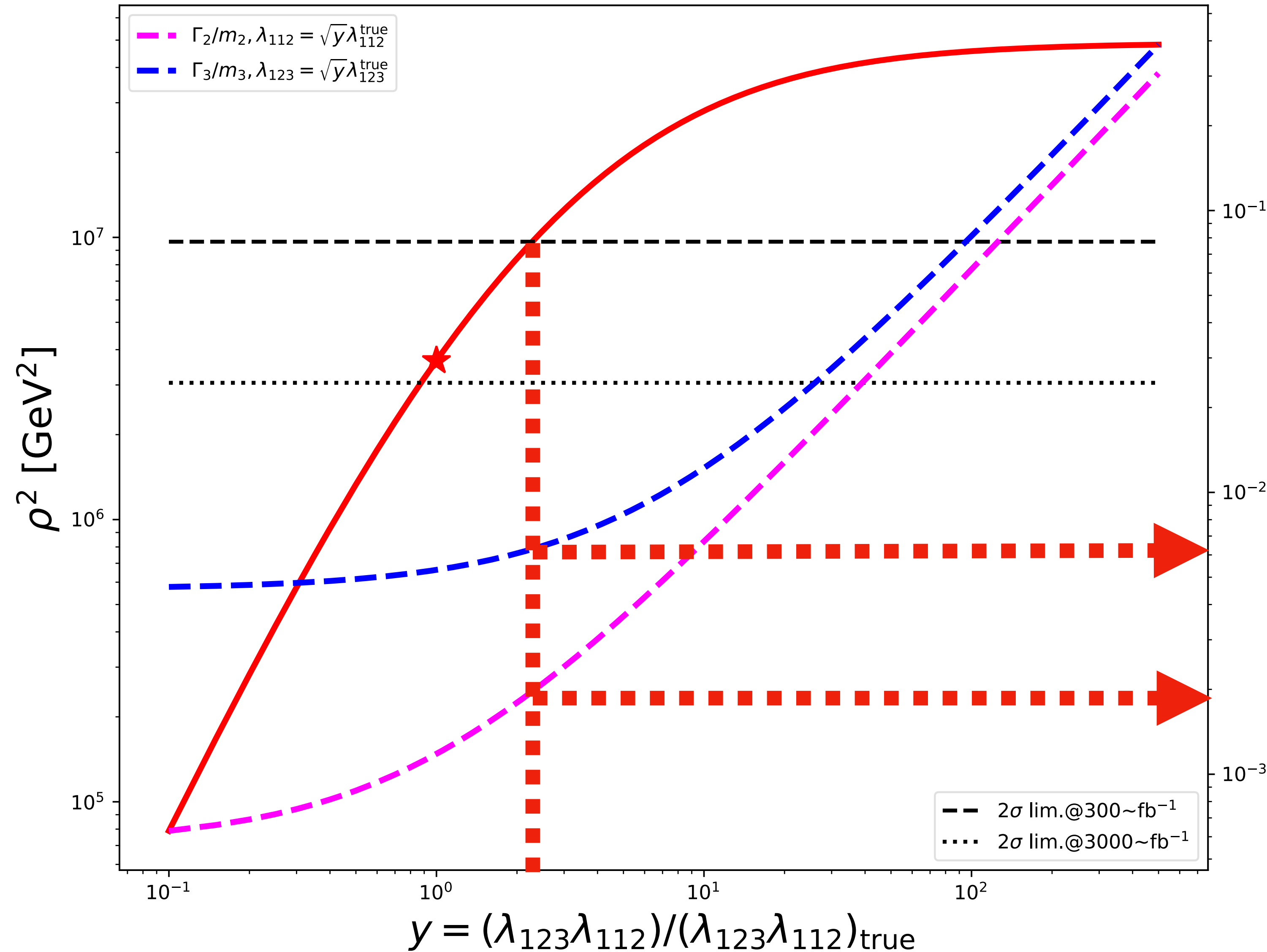
$$m_2 = 308.9 \text{ GeV}, m_3 = 468.5 \text{ GeV}, k_2 = 0.138, k_3 = 0.165$$



- Pick a parameter space point, **fix:** m_2, m_3 and scalar coupling λ_{113} .
- **Rescale:** $\lambda_{112} = \sqrt{y}\lambda_{112}^{\text{true}}$ and $\lambda_{123} = \sqrt{y}\lambda_{123}^{\text{true}}$.
- **Plot:** ρ^2 , Γ_2/m_2 and Γ_3/m_3 versus y .
- **Red star** = **true** parameter point value.
- Plot pheno analysis limits (dashed: 300 fb⁻¹, dotted: 3000 fb⁻¹).
- **For both limits, NWA approximation is valid!**

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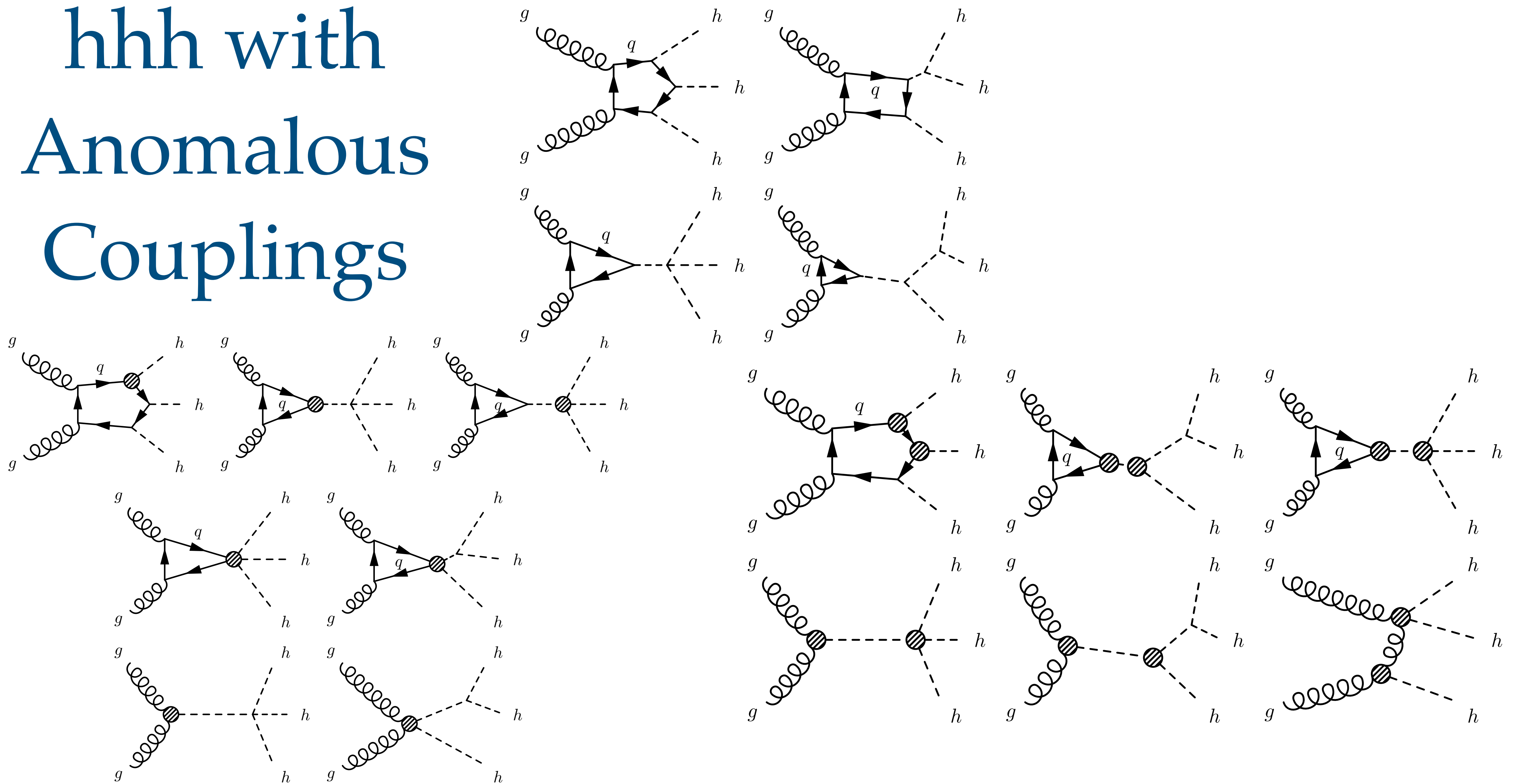
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TRSM Monte Carlo Event Generation

- We have implemented a MadGraph5_aMC@NLO (MG5_aMC) “loop” model for the TRSM:
 - **MG5_aMC input parameters:** the three mixing angles, two masses / widths and **all** the scalar couplings (only 7 are independent in TRSM).
 - Comes with a **Python script** that:
 - allows conversion of M_2, M_3 + three mixing angles + two VEVs to the MG5_aMC model input,
 - calculates several single-production cross sections, branching ratios, widths,
 - and writes associated MG5_aMC parameter card (**param_card.dat**) automatically.
- **Get it at:** <https://gitlab.com/apapaefs/twosinglet>.

[[AP](#), Tania Robens, Gilberto Tetlalmatzi-Xolocotzi, arXiv:2101.00037]

hhh with Anomalous Couplings



hhh: Final states

Assume: K-factor = 2.

[Maltoni, Vryonidou, Zaro, 1408.6542]

$hhh \rightarrow$ final state	BR (%)	$N_{20\text{ab}}^{-1}$	
$(b\bar{b})(b\bar{b})(b\bar{b})$	19.21	22207	
$(b\bar{b})(b\bar{b})(WW_{1\ell})$	7.20	8328	
$(b\bar{b})(b\bar{b})(\tau\bar{\tau})$	6.31	7297	→ Fuks, Kim, Lee, 1510.07697, Fuks, Kim, Lee, 1704.04298.
$(b\bar{b})(\tau\bar{\tau})(WW_{1\ell})$	1.58	1824	
$(b\bar{b})(b\bar{b})(WW_{2\ell})$	0.98	1128	
$(b\bar{b})(WW_{1\ell})(WW_{1\ell})$	0.90	1041	→ Kilian, Sun, Yan, Zhao, Zhao, 1702.03554.
$(b\bar{b})(\tau\bar{\tau})(\tau\bar{\tau})$	0.69	799	
$(b\bar{b})(b\bar{b})(\gamma\gamma)$	0.23	263	→ <u>AP</u> , Sakurai, 1508.06524, Chen, Yan, Zhao, Zhao, Zhong, 1510.04013, Fuks, Kim, Lee, 1510.07697.

[AP, Sakurai, 1508.06524]

The 6b final state, analysis [[AP](#), Gilberto Tetlalmatzi-Xolocotzi, Marco Zaro, arXiv:1909.09166]

- What can we learn about the anomalous couplings via **hhh** at 13.6 TeV?
- Begin by using the 6 **b-jet final state!**

1. Require 6 tagged b-jets.

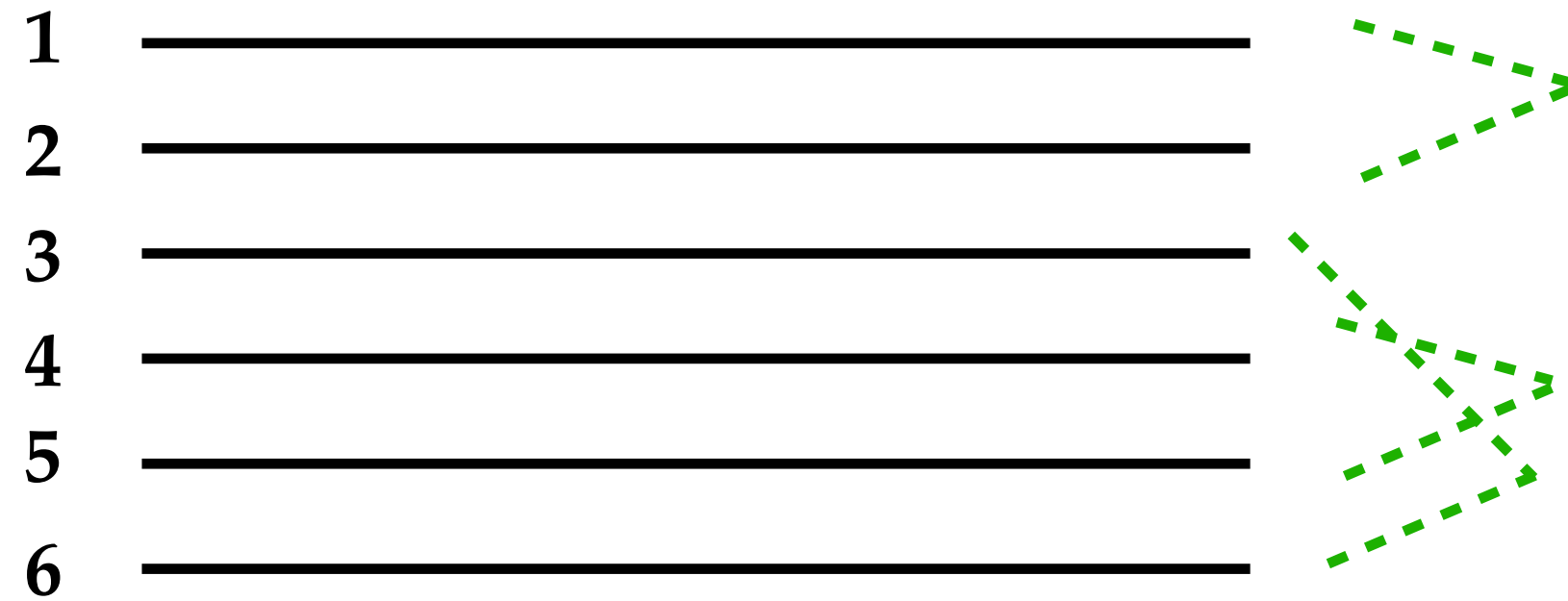
1	_____
2	_____
3	_____
4	_____
5	_____
6	_____

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- 3. For each pairing construct:

$$\chi^2 = \sum_{qr \in \text{pairings}} (M_{qr} - m_h^2)^2$$

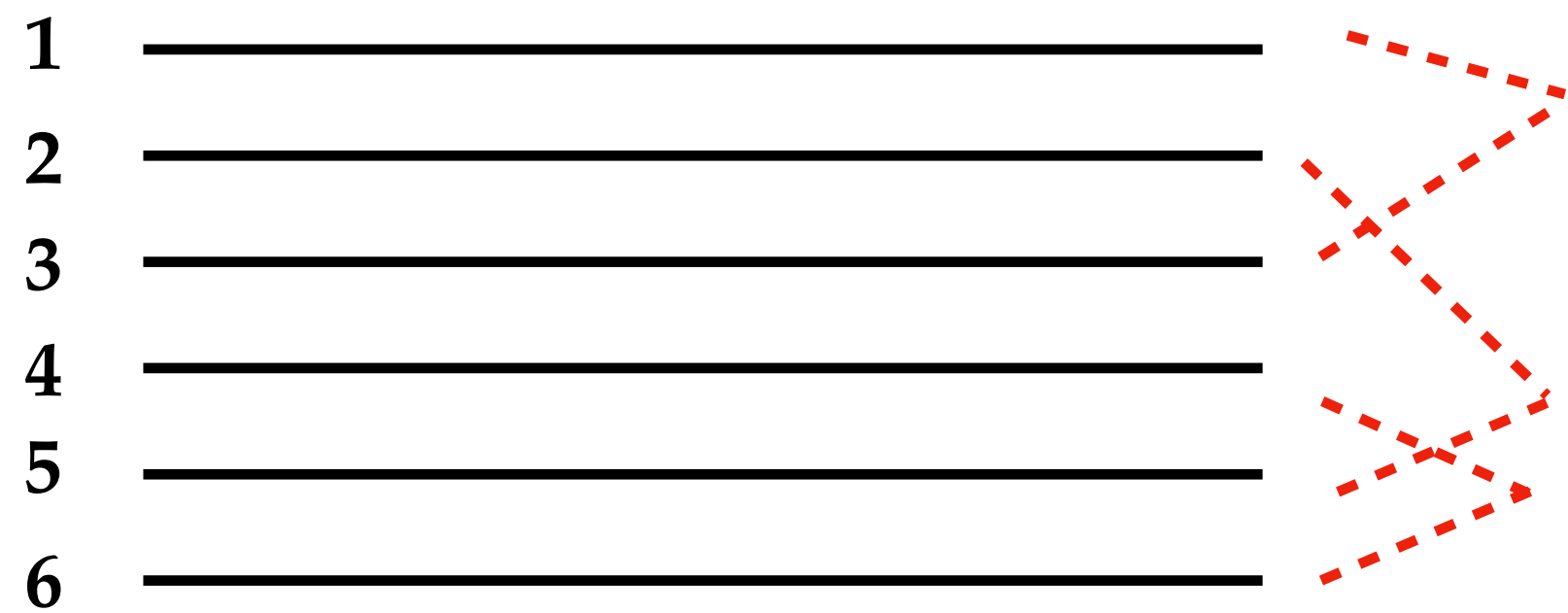
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⇒ 4. Pairing that gives minimum χ^2 determines “reconstructed Higgs boson”.

$$\chi_{\min}^2$$

The 6b final state, analysis

$$h_r^i \rightarrow \text{Higgs boson candidates}$$

observable

cut

$$p_{T,b}$$

$$> 45 \text{ GeV}$$

$$|\eta_b|$$

$$< 3.2$$

$$\Delta R_{b,b}$$

$$> 0.3$$

$$p_T(h_r^i)$$

$$> [170, 120, 0] \text{ GeV}, i = 1, 2, 3$$

$$\chi_{\min}^2$$

$$< 17 \text{ GeV}$$

$$\Delta m_{\min, \text{mid}, \text{max}}$$

$$< 8, 8, 11 \text{ GeV} \quad \leftarrow \text{the three terms in } \chi_{\min}^2.$$

$$\Delta R(h_r^i, h_r^j)$$

$$< [3.5, 3.5, 3.5], (i, j) = [(1, 2), (1, 3), (2, 3)]$$

$$\Delta R_{bb}(h_r^i)$$

$$< [3.5, 3.5, 3.5], i = 1, 2, 3$$

signal/backgrounds after analysis

Process	σ_{GEN} (pb)	$\sigma_{\text{NLO}} \times \text{BR}$ (pb)	$\epsilon_{\text{analysis}}$	$N_{20 \text{ ab}^{-1}}^{\text{cuts}}$
hhh (SM)	2.88×10^{-3}	1.06×10^{-3}	0.0131	278
QCD $(b\bar{b})(b\bar{b})(b\bar{b})$	26.15	52.30	2.6×10^{-5}	27116
$q\bar{q} \rightarrow hZZ \rightarrow h(b\bar{b})(b\bar{b})$	8.77×10^{-4}	4.99×10^{-4}	1.8×10^{-4}	~ 2
$q\bar{q} \rightarrow ZZZ \rightarrow (b\bar{b})(b\bar{b})$	7.95×10^{-4}	7.95×10^{-4}	1.2×10^{-5}	< 1
ggF $hZZ \rightarrow h(b\bar{b})(b\bar{b})$	1.08×10^{-4}	1.23×10^{-4}	$\mathcal{O}(10^{-3})$	~ 2
ggF $ZZZ \rightarrow (b\bar{b})(b\bar{b})$	1.36×10^{-5}	2.73×10^{-5}	2×10^{-5}	$\ll 1$
$h(b\bar{b})(b\bar{b})$	1.46×10^{-2}	1.66×10^{-2}	5.4×10^{-4}	179
$hh(b\bar{b})$	1.40×10^{-4}	9.11×10^{-5}	2.8×10^{-4}	~ 1
$hhZ \rightarrow hh(b\bar{b})$	4.99×10^{-3}	1.61×10^{-3}	7.2×10^{-4}	23
$hZ(b\bar{b}) \rightarrow h(b\bar{b})(b\bar{b})$	9.08×10^{-3}	1.03×10^{-2}	1.4×10^{-4}	29
$ZZ(b\bar{b}) \rightarrow (b\bar{b})(b\bar{b})(b\bar{b})$	2.87×10^{-2}	5.74×10^{-2}	1×10^{-5}	11
$Z(b\bar{b})(b\bar{b}) \rightarrow (b\bar{b})(b\bar{b})(b\bar{b})$	0.93	1.87	3×10^{-5}	1121
Σ backgrounds				2.8×10^4

Reducible backgrounds

process	σ_{GEN} (pb)	$\sigma_{\text{GEN}} \times \mathcal{P}(6\ b\text{-jets})$ (pb)
$(b\bar{b})(b\bar{b})(c\bar{c})$	76.8	0.768
$(b\bar{b})(c\bar{c})(c\bar{c})$	75.6	0.00756
$(c\bar{c})(c\bar{c})(c\bar{c})$	22.5	22.5×10^{-5}
$(b\bar{b})(b\bar{b})(jj)$	1.32×10^4	1.32
$(b\bar{b})(jj)(jj)$	9.79×10^5	0.00979
$(jj)(jj)(jj)$	1.37×10^6	1.37×10^{-6}

c.f. $\sigma_{\text{GEN}}(6b) = 26.15\text{ pb}$

↑
applied:

$$\mathcal{P}_{c \rightarrow b} = 0.1$$

$$\mathcal{P}_{j \rightarrow b} = 0.01$$

⇒ Assuming perfect b-tagging + identical analysis efficiency to QCD 6b:

→ **~10% contribution from reducible backgrounds.**

for $P(\text{b-tagging}) = 0.8$:

→ **~30% contribution.**

TRSM hhh $\rightarrow$ 6b analysis details

Introduce two observables: $\chi^{2,(4)} = \sum_{qr \in I} \left(M_{qr} - M_1 \right)^2$

$$\chi^{2,(6)} = \sum_{qr \in J} \left(M_{qr} - M_1 \right)^2$$

$\rightarrow$ constructed from different pairings of 4 and 6 b-tagged jets, M_{qr} is the invariant mass of the pairing qr .

Monte Carlo Implementation of Anomalous Couplings

- Get the **MG5_aMC model** at: https://gitlab.com/apapaefs/multihiggs_loop_sm.
- [A patch to MG5_aMC to enable **Loop × Tree** is included].
- Can generate events either at:

- **SM²** + interference of [**SM × One-Insertion** diagrams], i.e.:

$$|\mathcal{M}|^2 = |\mathcal{M}_{\text{SM}}|^2 + 2\text{Re}\{\mathcal{M}_{\text{SM}}^* \mathcal{M}_{1\text{-ins.}}\} \propto 1 + c_i$$

or

- **SM²** + interference of [**SM × One or Two** insertion diagrams] + [**One Insertion**]², i.e.:

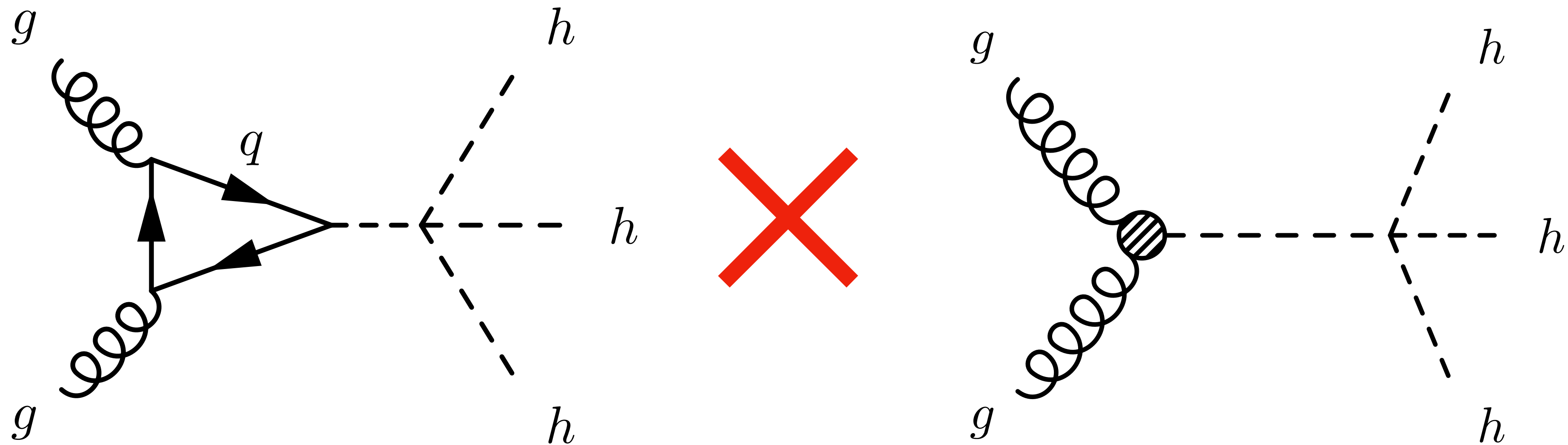
$$|\mathcal{M}|^2 = |\mathcal{M}_{\text{SM}}|^2 + 2\text{Re}\{\mathcal{M}_{\text{SM}}^* \mathcal{M}_{1\text{-ins.}}\} + 2\text{Re}\{\mathcal{M}_{\text{SM}}^* \mathcal{M}_{2\text{-ins.}}\} + |\mathcal{M}_{1\text{-ins.}}|^2 \\ \propto 1 + c_i + c_j c_k + c_\ell^2$$

Monte Carlo Implementation of Anomalous Couplings

- We have implemented a MadGraph5_aMC@NLO “loop” model for $\mathcal{L}_{\text{PhenoExp}}$.
- Includes Loop $\times$ Tree level interference between the various diagrams.

[see: Hirschi, <https://cp3.irmp.ucl.ac.be/projects/madgraph/wiki/LoopInducedTimesTree>].

- e.g.:



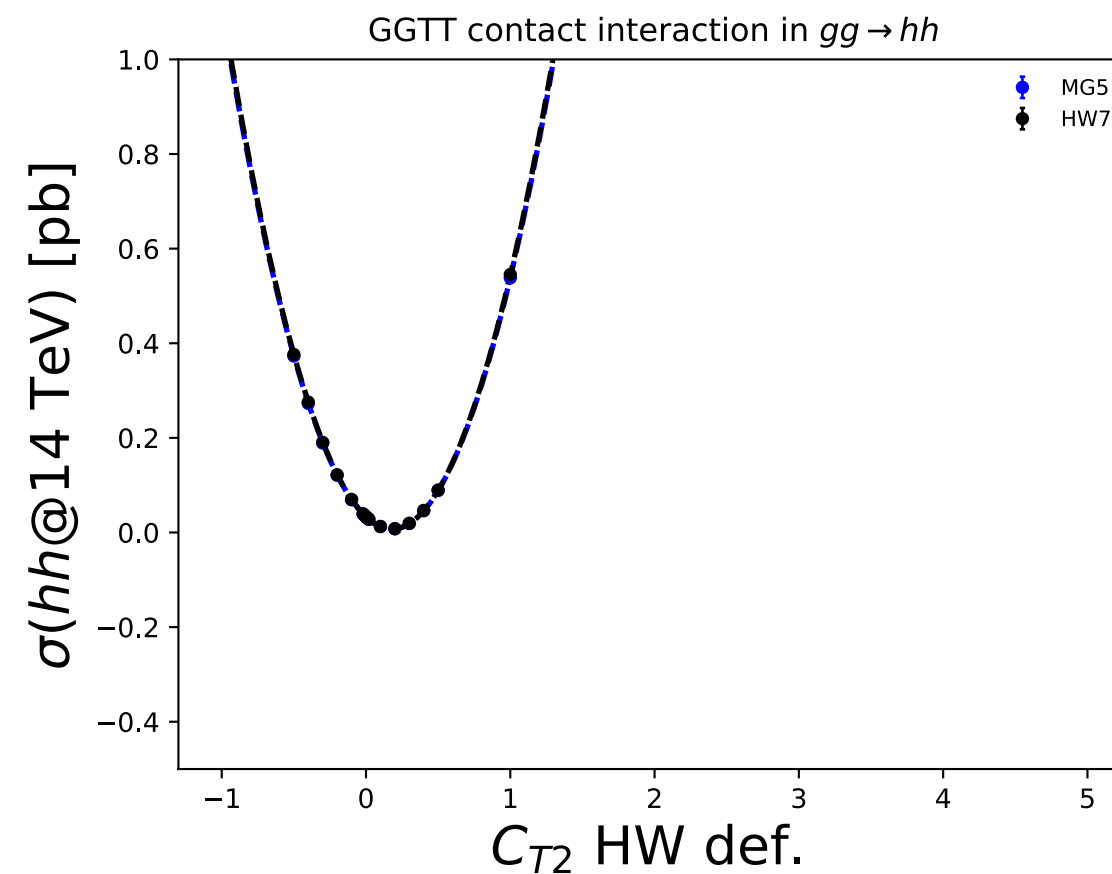
[AP, Tetlalmatzi-Xolocotzi, arXiv:2312.13562]

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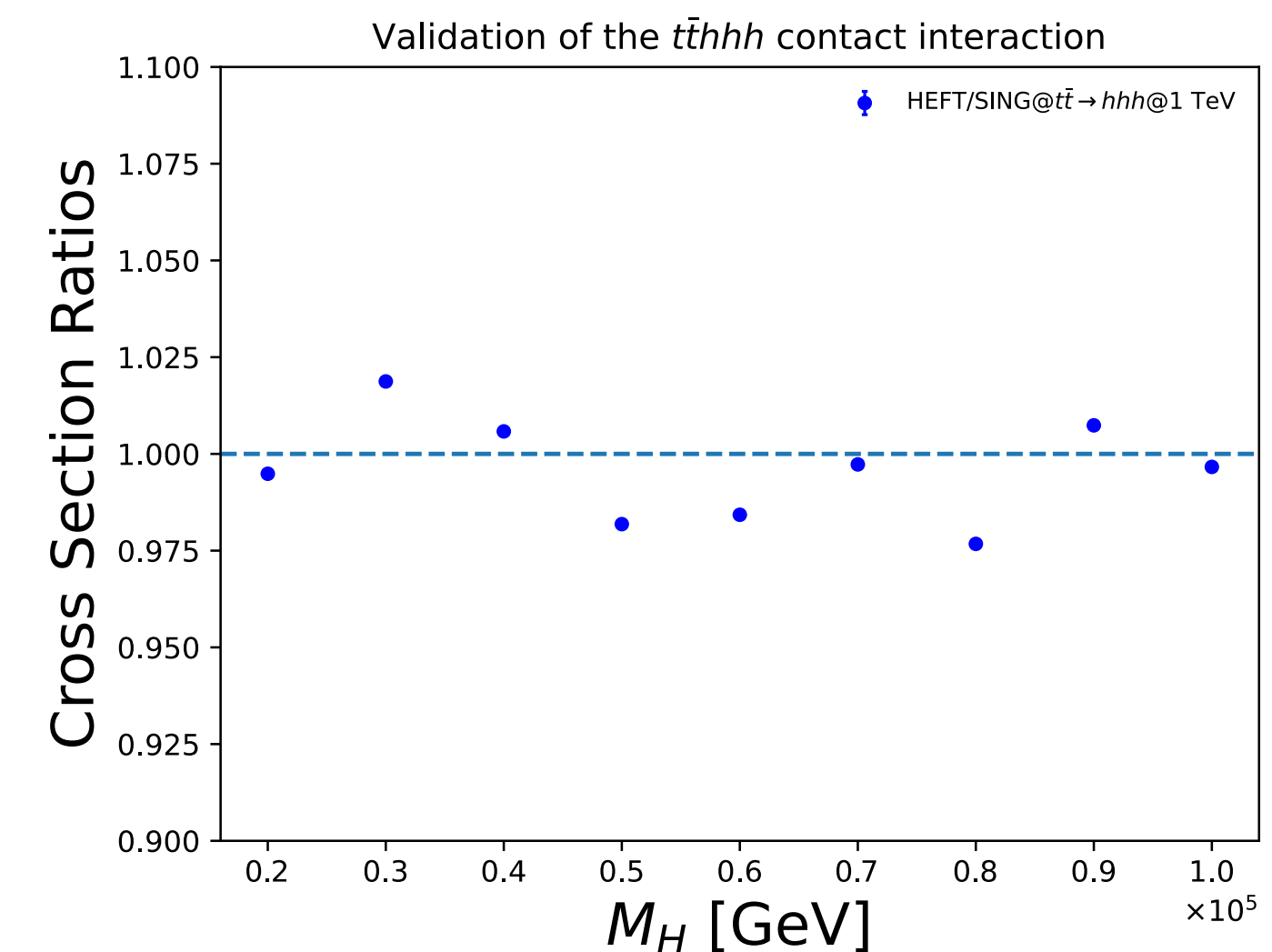
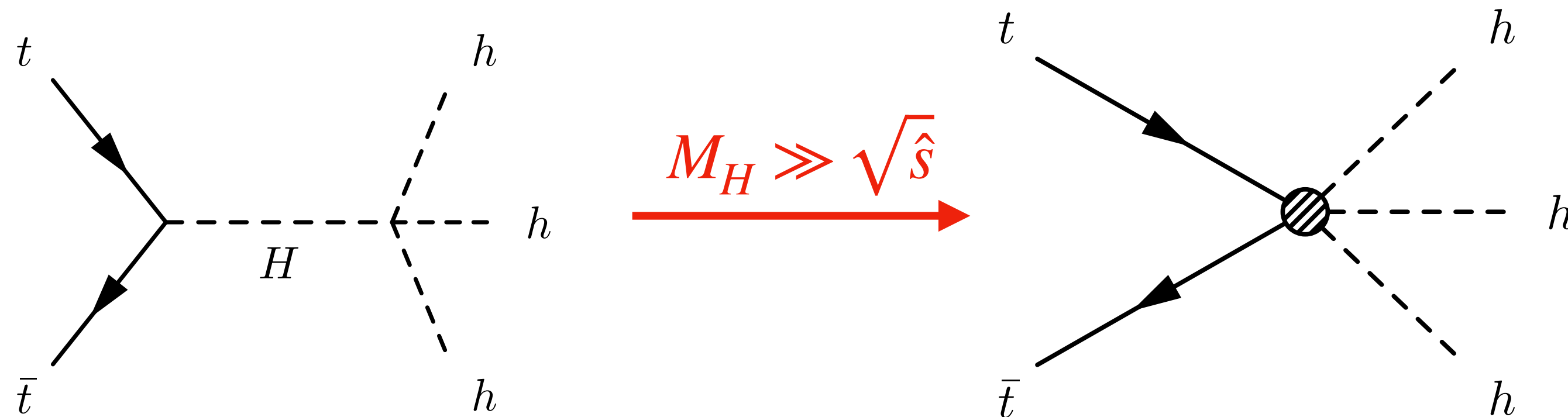
Model Validation

[AP, Tetlalmatzi-Xolocotzi, arXiv:2312.13562]

- Most couplings validated vs. a Herwig 7 $pp \rightarrow hh$ implementation, e.g.:



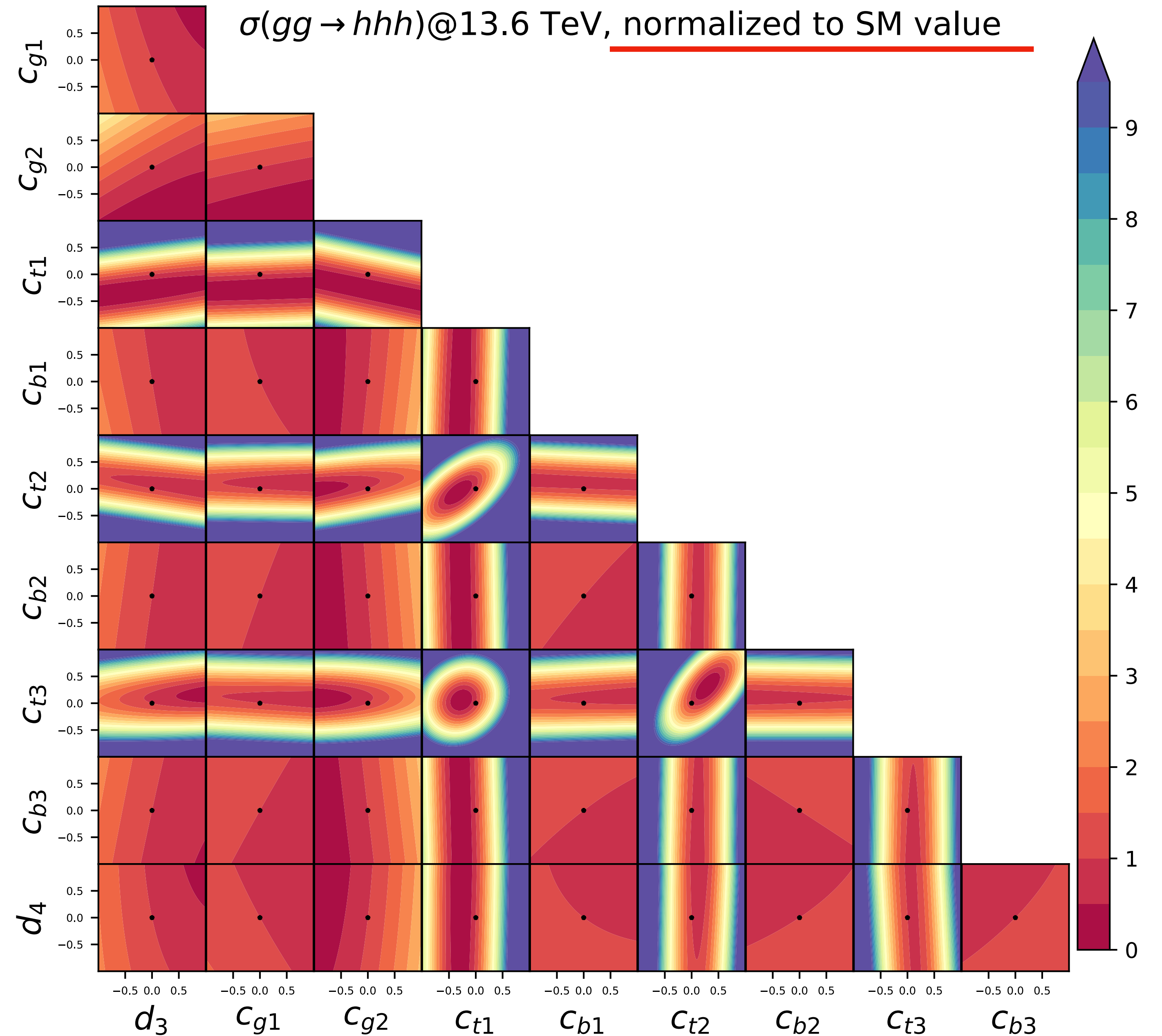
- The one “new” non-trivial coupling that appears, $\propto c_{t3} t\bar{t}h^3$ has been validated via an “EFT” limit, in the $t\bar{t} \rightarrow hhh$ process:



hhh Cross Sections @ 13.6 TeV

[AP, Tetlalmatzi-Xolocotzi, arXiv:2312.13562]

- Cross section as a multiple of the SM
- ($\sigma_{\text{SM}} \sim 0.04 \text{ fb}$ at LO@13.6 TeV).
- In each 2D panel shown: **all other coefficients set to zero!**



Anomalous Couplings Constraints

- Other processes constrain (at LO) all coefficients except $\{c_{t3}, d_4\}$ (**only in hhh**).
- **Projected constraints:**

Percentage uncertainties			
	HL-LHC	FCC-hh	Ref.
$\delta(d_3)$	50	5	[145] (table 12)
$\delta(c_{g1})$	2.3	0.49	[145] (table 3)
$\delta(c_{g2})$	5	1	[140] (Figure 12, right)
$\delta(c_{t1})$	3.3	1.0	[145] (table 3)
$\delta(c_{t2})$	30	10	[140] (Figure 12, right)
$\delta(c_{b1})$	3.6	0.43	[145] (table 3)
$\delta(c_{b2})$	30	10	assumed same as c_{t2}

[See **AP**, Tetlalmatzi-Xolocotzi, arXiv:2312.13562 for the references]

Digression: Non-Resonant hhh @ LHC

- Anomalous couplings can enhance hhh ! e.g. $\kappa_3 \neq 1, \kappa_4 \neq 1$, but also others!

$$\begin{aligned} \mathcal{L}_{\text{PhenoExp}} \supset & -\lambda_{\text{SM}} v (1+c_3) h^3 - \frac{\lambda_{\text{SM}}}{4} (1+d_4) h^4 \\ & + \frac{\alpha_s}{12\pi} \left(c_{g1} \frac{h}{v} - c_{g2} \frac{h^2}{2v^2} \right) G_{\mu\nu}^a G_a^{\mu\nu} \\ & - \left[\frac{m_t}{v} (1+c_{t1}) \bar{t}_L t_R h + \frac{m_b}{v} (1+c_{b1}) \bar{b}_L b_R h + \text{h.c.} \right] \\ & - \left[\frac{m_t}{v^2} c_{t2} \bar{t}_L t_R h^2 + \frac{m_b}{v^2} c_{b2} \bar{b}_L b_R h^2 + \text{h.c.} \right] \\ & - \left[\frac{m_t}{v^3} \left(\frac{c_{t3}}{2} \right) \bar{t}_L t_R h^3 + \frac{m_b}{v^3} \left(\frac{c_{b3}}{2} \right) \bar{b}_L b_R h^3 + \text{h.c.} \right] \end{aligned}$$

[AP, Tatlalmatzi-Xolocotzi, arXiv:2312.13562]

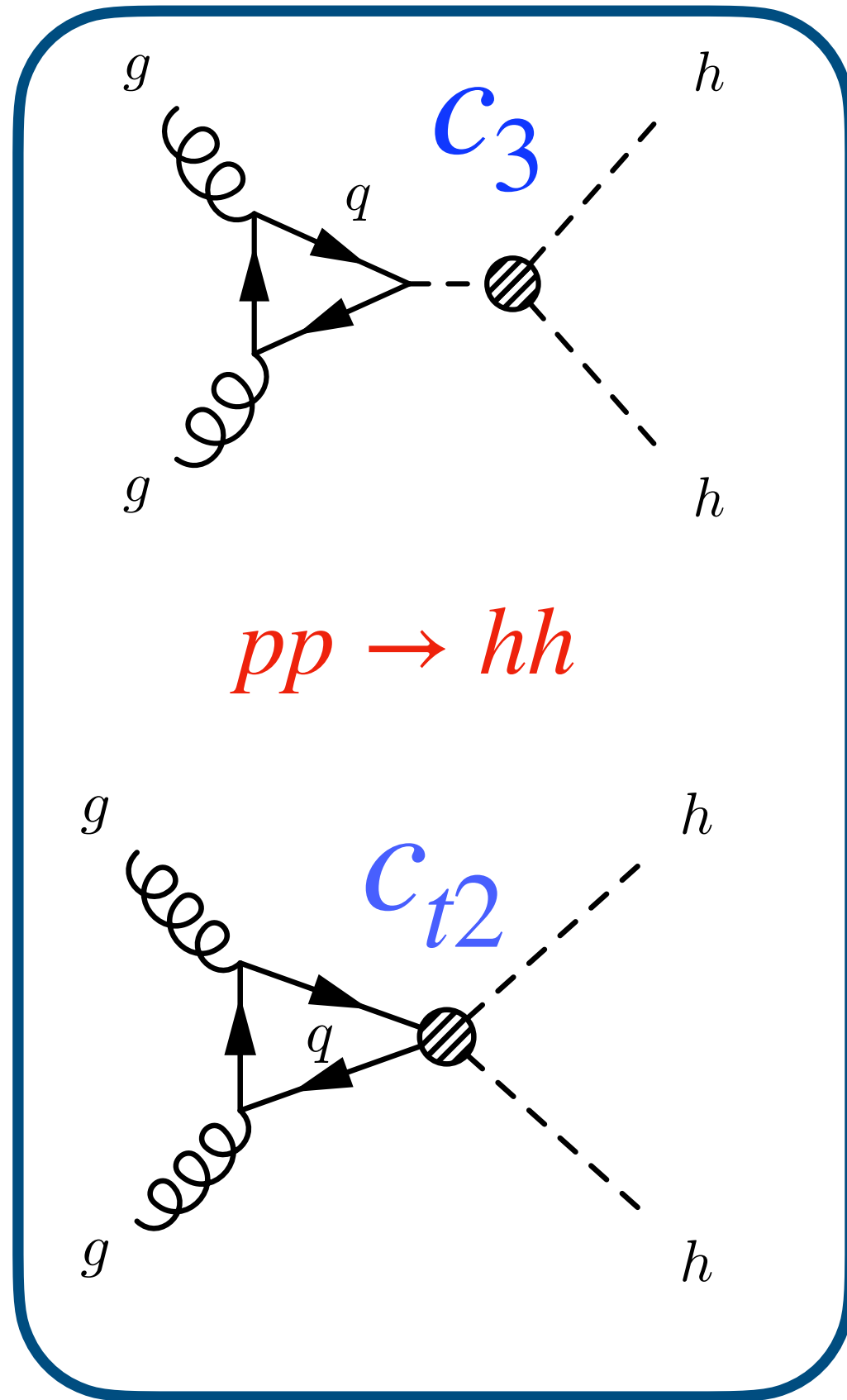
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➡ Here: Examine models with new scalar resonances!

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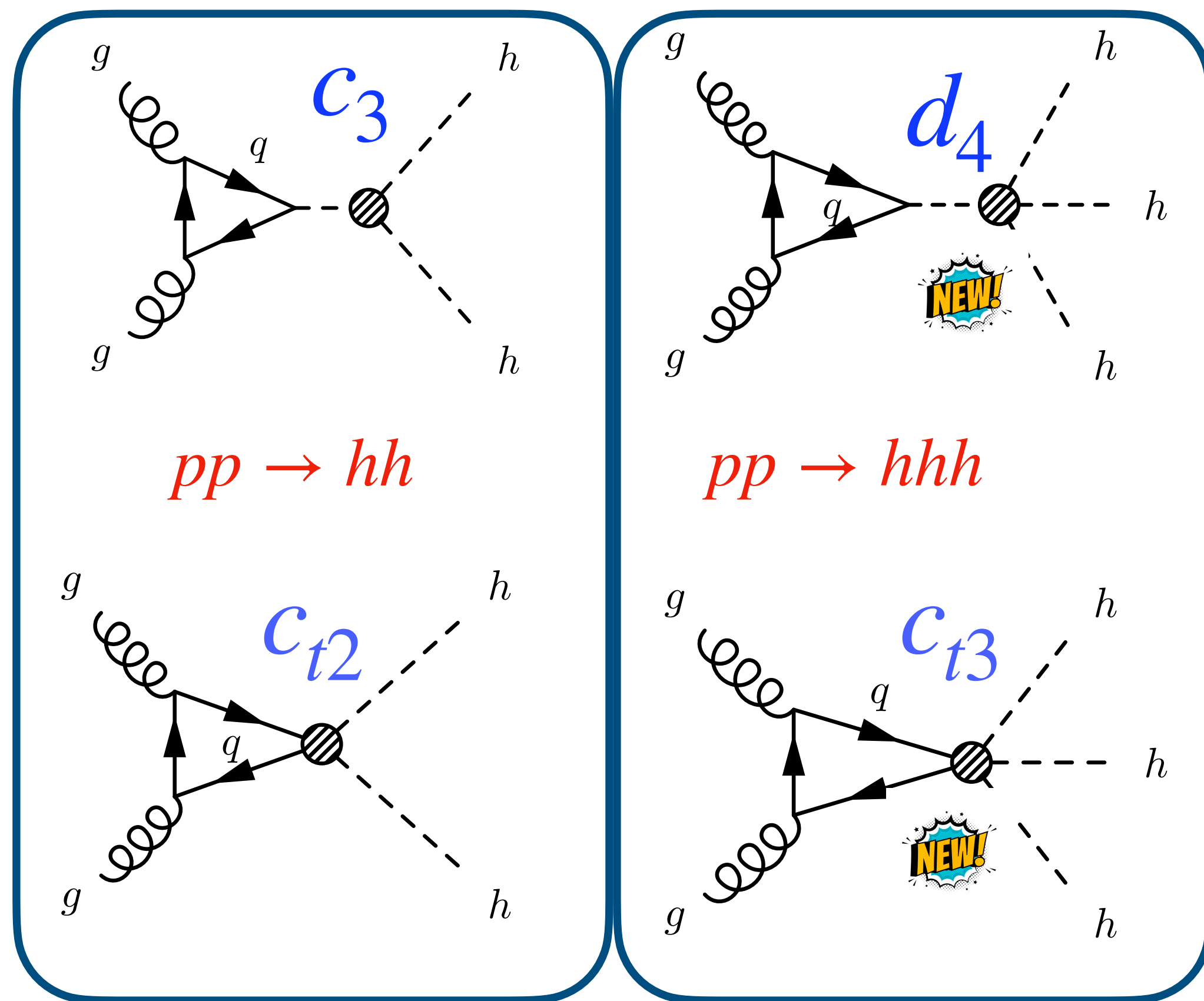
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Anomalous Couplings Constraints

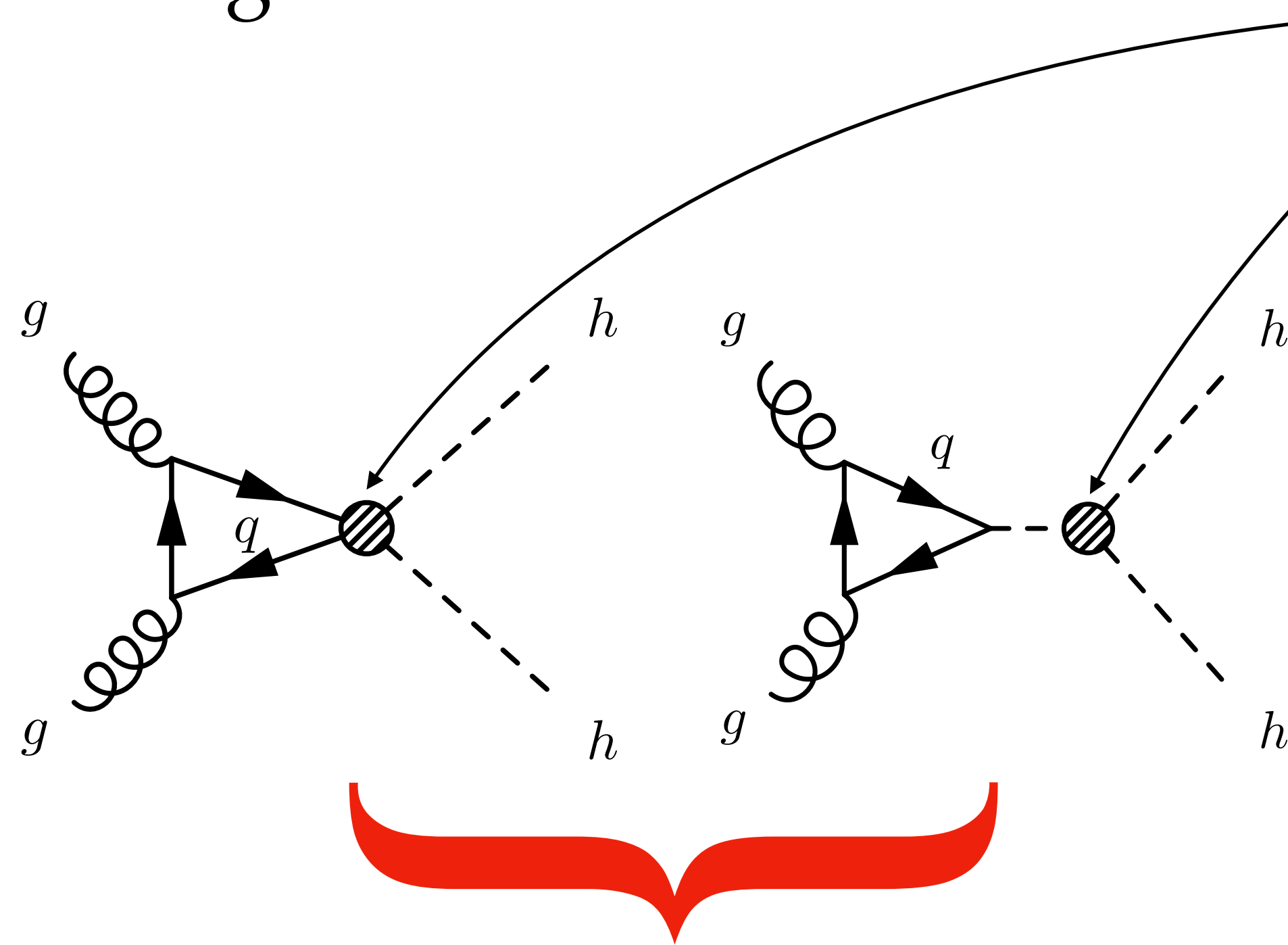
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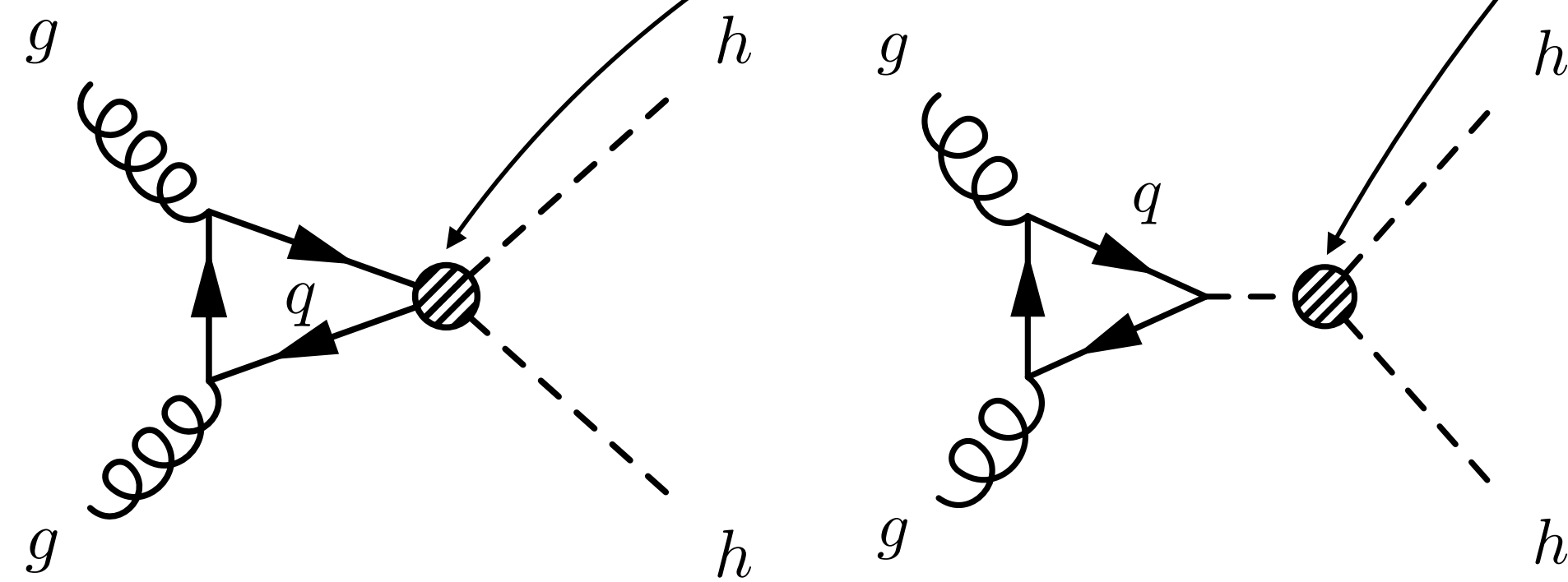


constrained by $pp \rightarrow hh$

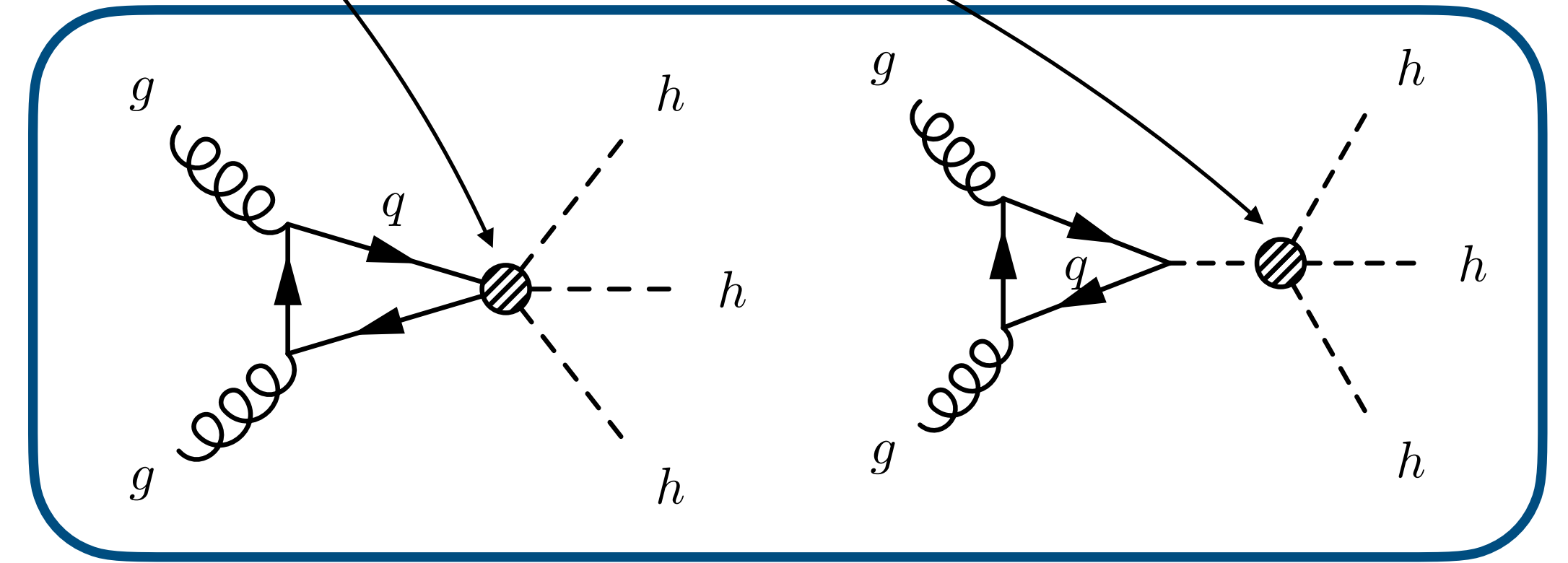
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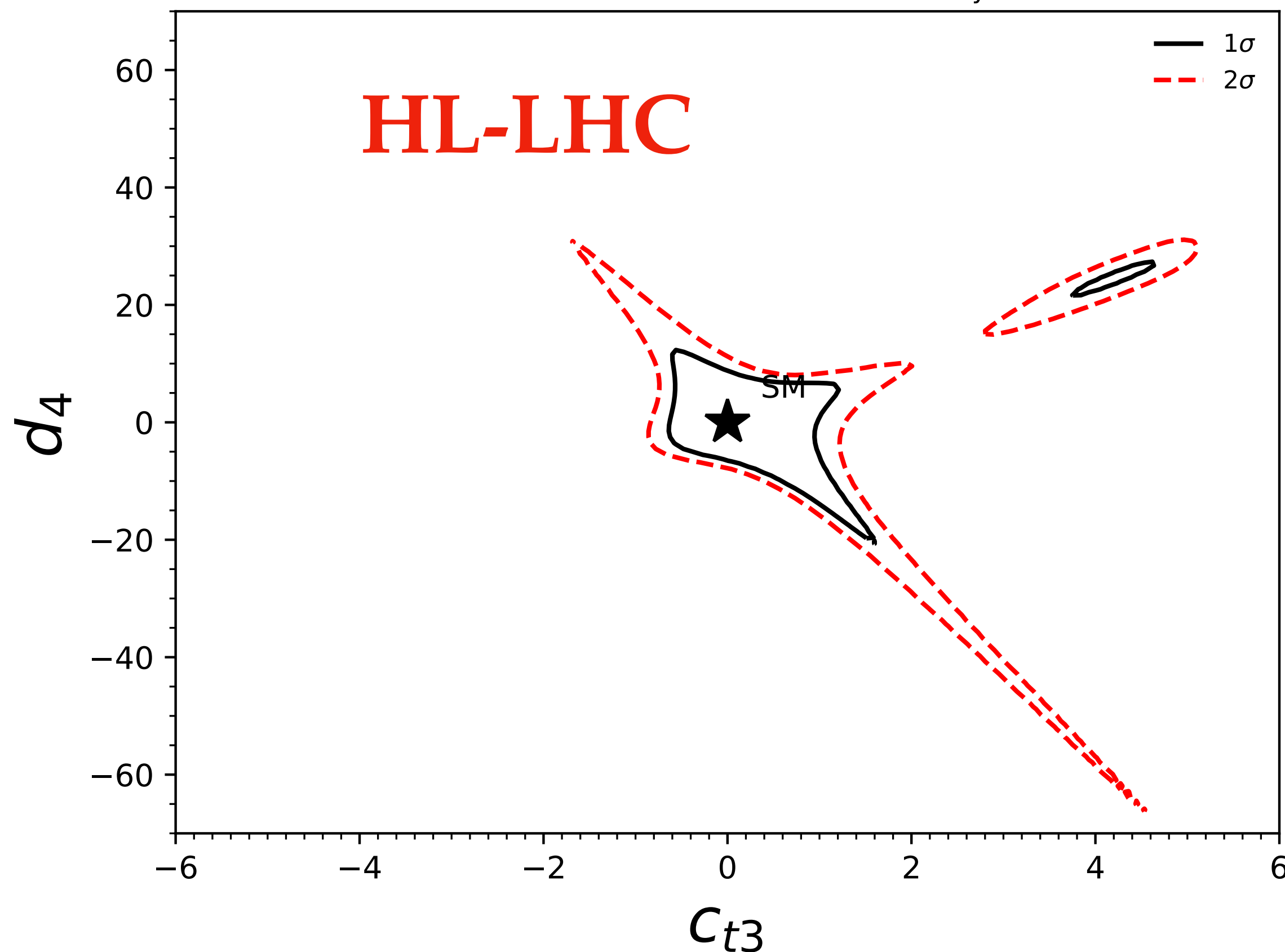
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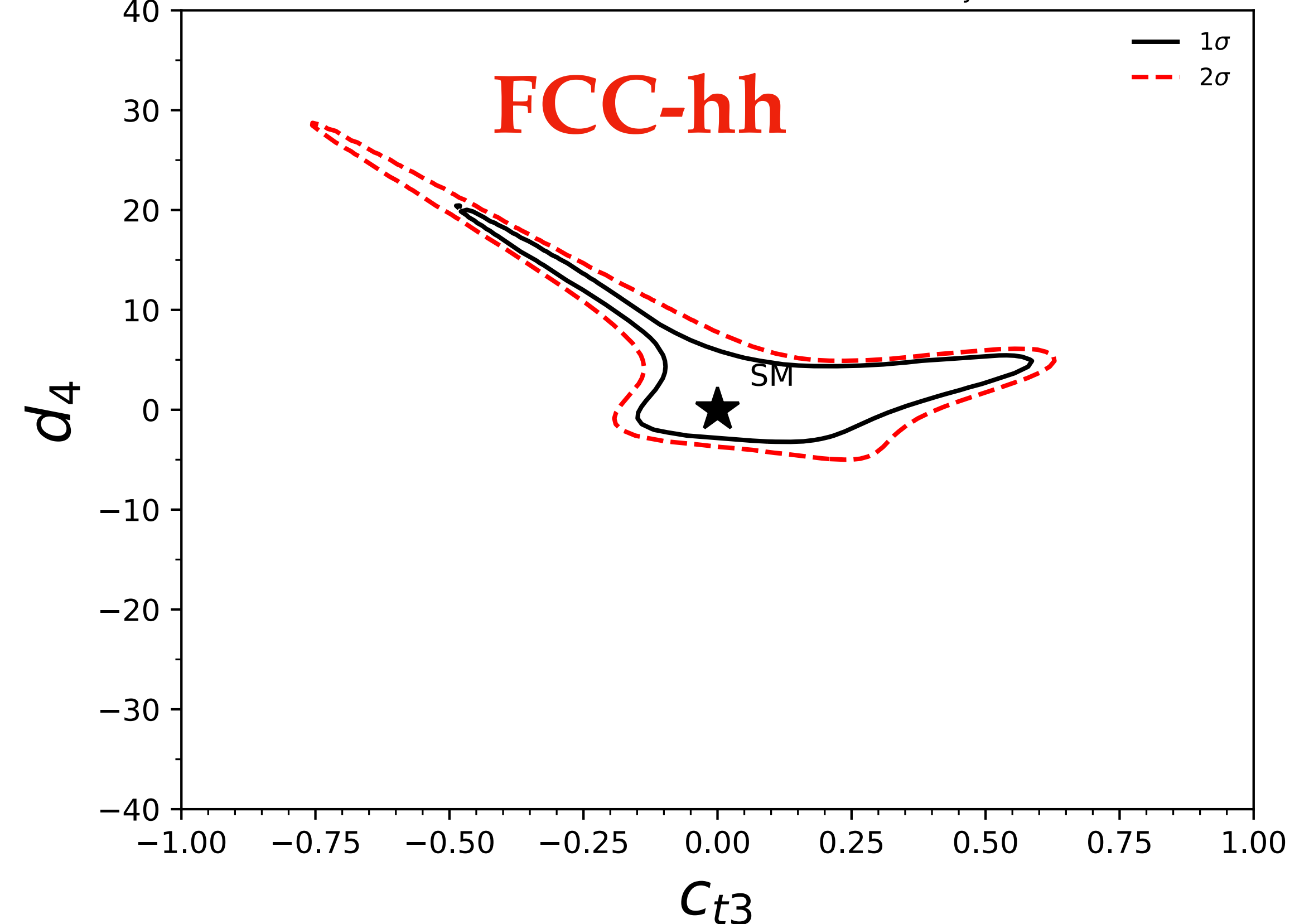
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- Focusing on a model with only $\{c_{t2}, d_3, c_{t3}, d_4\}$,
- Using the **6 b-jet final state**, and marginalizing over $\{c_{t2}, d_3\}$ within projected constraints:

$gg \rightarrow hhh@13.6 \text{ TeV}, L=3000 \text{ fb}^{-1}, \alpha_{\text{syst.}} = 5.0\%$



$gg \rightarrow hhh@100 \text{ TeV}, L=20000 \text{ fb}^{-1}, \alpha_{\text{syst.}} = 5.0\%$

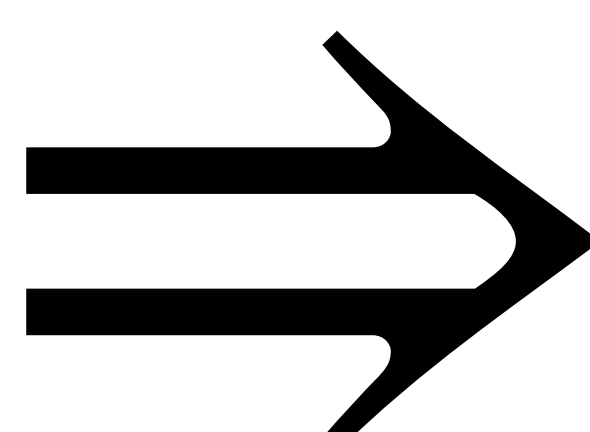


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	HL-LHC 68%	HL-LHC 95%	FCC-hh 68%	FCC-hh 95%
d_4	$[-6.6, 12.4]$	$[-10.0, 21.3]$	$[-3.9, 10.5]$	$[-10.6, 18.8]$
c_{t3}	$[-0.6, 1.1]$	$[-0.9, 3.6]$	$[-0.1, 0.3]$	$[-0.4, 0.6]$


$$c_{t3} \sim \mathcal{O}(0.1 - 1)$$
$$d_4 \sim \mathcal{O}(10)$$