Triple Higgs Boson Production at the LHC in Extended Scalar Sectors



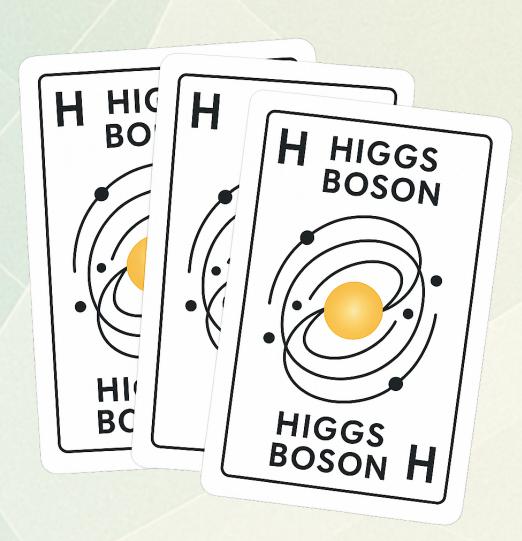


Andreas Papaefstathiou

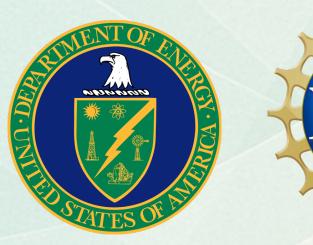
Kennesaw State University, GA, USA @ QCD@LHC 2025 [Sept. 8-12, 2025]



hhh at the LHC in Extended Scalar Sectors*



*Plus a digression into non-resonant hhh!





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Motivation: Measuring the Higgs Potential





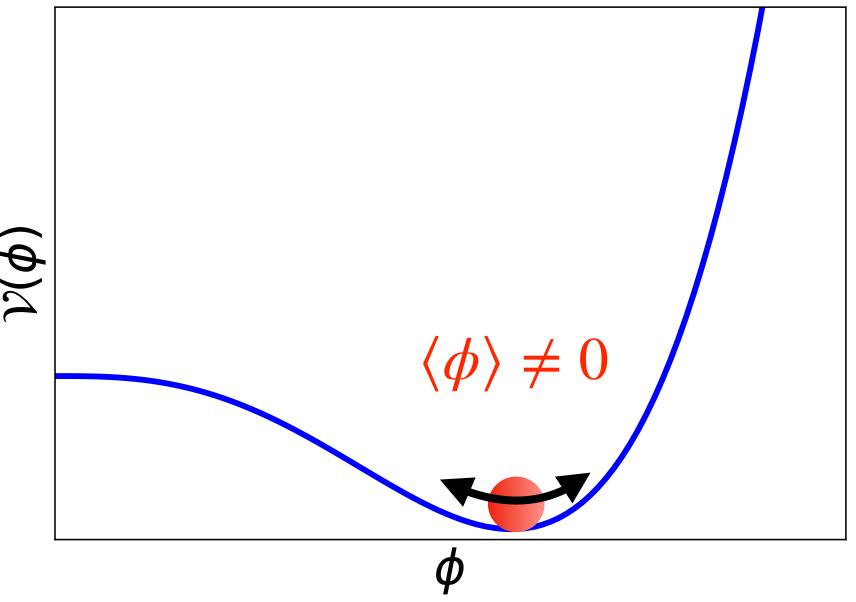
Motivation: Measuring the Higgs Potential

- In the SM, the Higgs field ϕ "sits" in a potential $\mathcal{V}(\phi) = \mathbf{o} |\phi|^2 + \mathbf{o} |\phi|^4$.
- Electroweak Symmetry Breaking:

$$\mathcal{V}(\langle \phi \rangle + h) = -h^2 + \blacktriangle h^3 + \blacksquare h^4$$

where *h* is the Higgs <u>boson</u>.



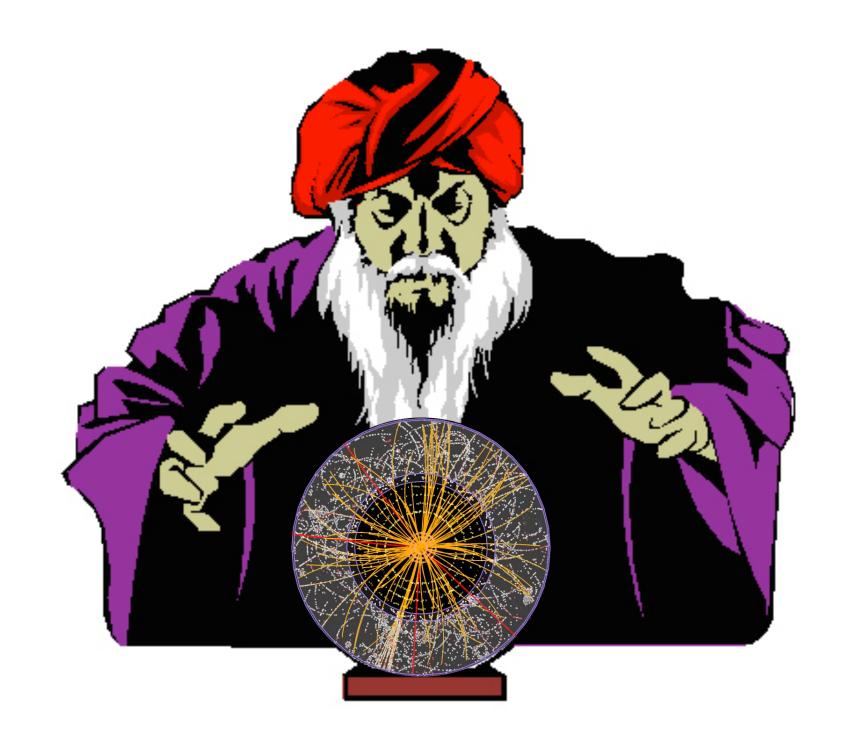


• Predicted in the SM via the Higgs boson's mass and the VEV:

$$\mathbf{S}_{\mathbf{M}} = \frac{1}{2} m_h^2 \; ;$$

$$= \frac{m_h^2}{8v^2}$$





Q: Can we probe hhh at the LHC?

And if so, what can we learn?

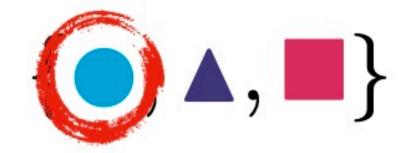


- Q: Can we verify that $\{\bullet, \blacktriangle, \blacksquare\} \approx \{\bullet, \blacktriangle, \blacksquare\}_{SM}$ at the LHC?
- We can certainly try! Most "direct" way is to produce on-shell Higgs bosons:

$$\{ullet, lacktriangle, lacktriangle\}$$



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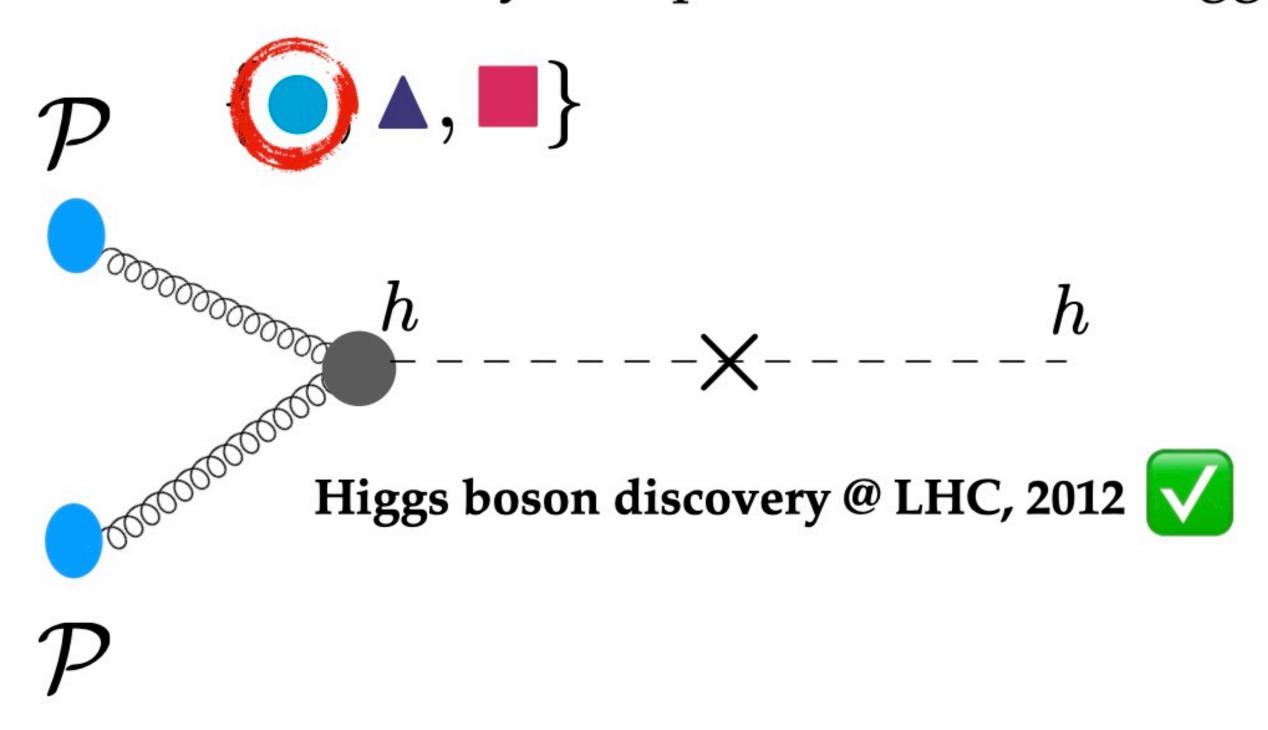


$$h$$
 $--- +$

Higgs boson discovery @ LHC, 2012

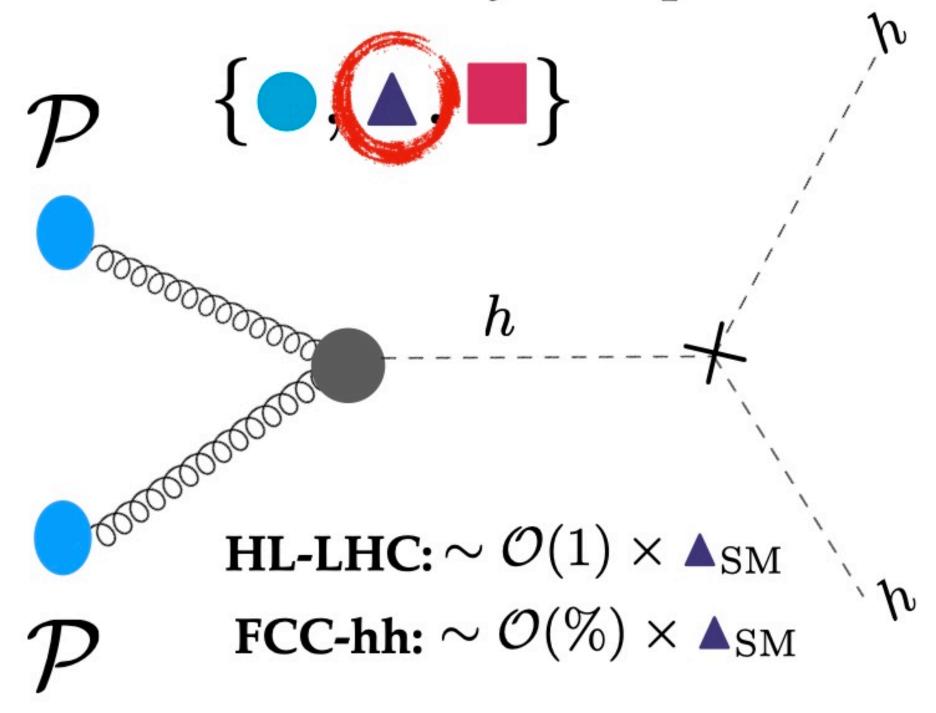


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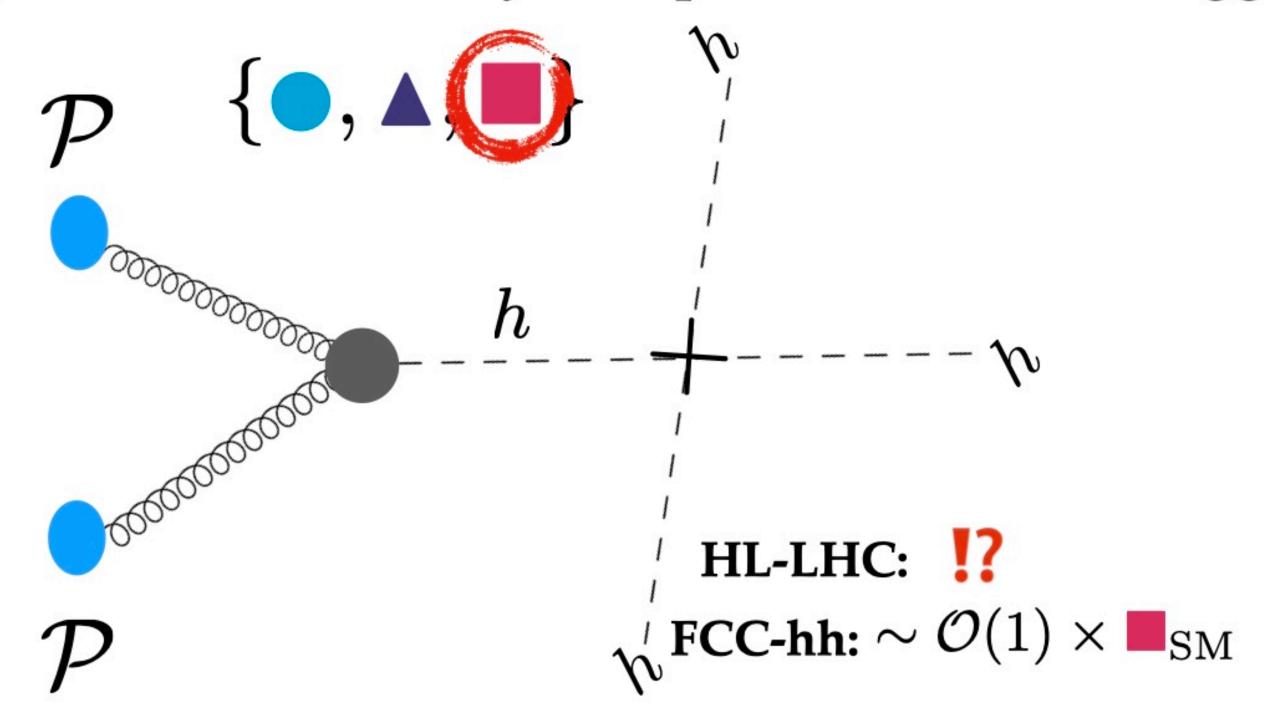


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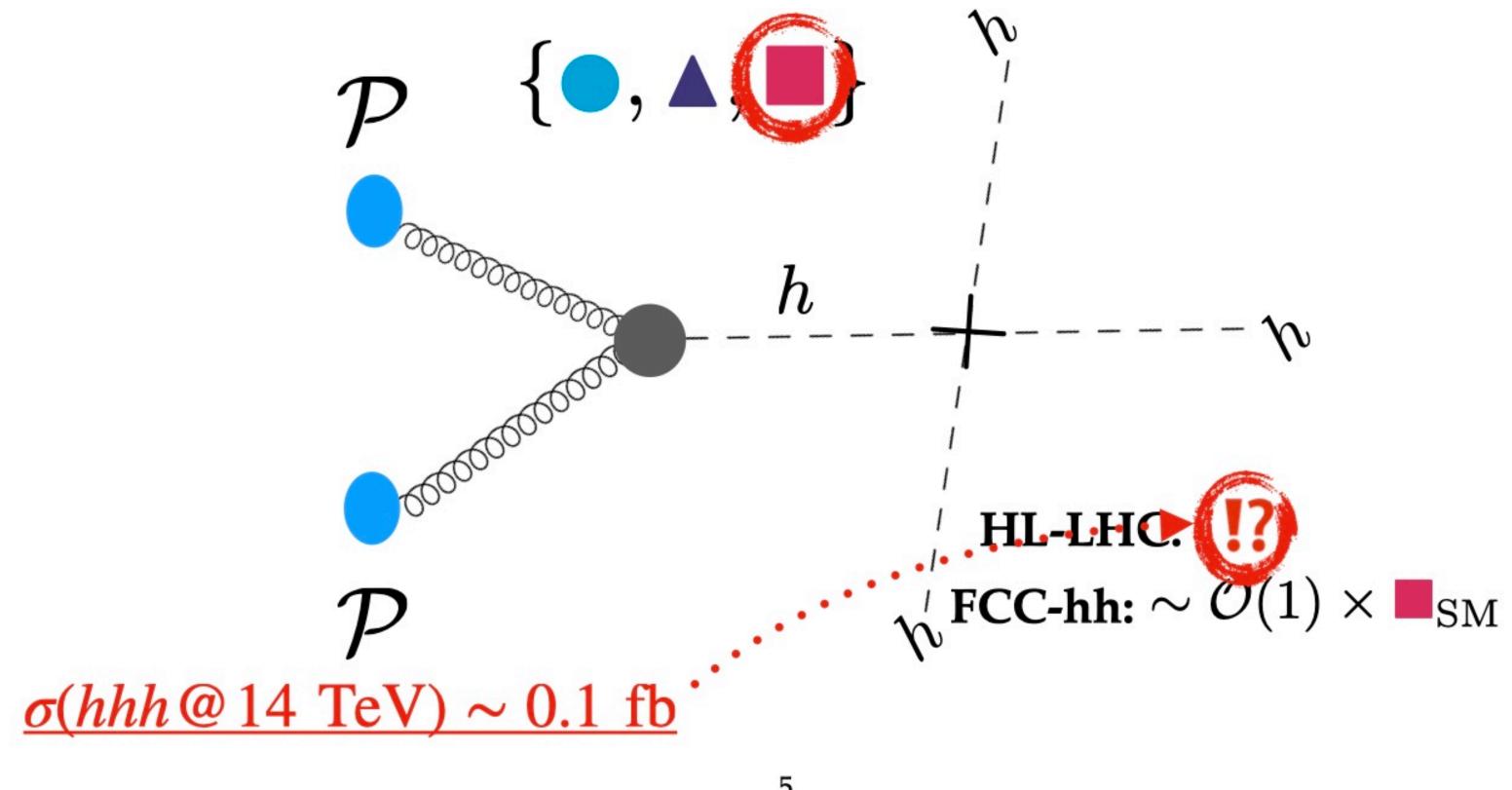


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• Anomalous couplings can enhance *hhh*! e.g. inspired by an Effective Field Theory.

[e.g. Stylianou, Weiglein, arXiv:2312.04646, ATLAS, arXiv:2411.02040, CMS-PAS-HIG-24-015]

ATLAS $\sqrt{s} = 13 \text{ TeV}, 126 \text{ fb}^{-1}$ $\sqrt{s} = 13 \text{ TeV}, 126 \text{ fb}^{-1}$



-200

-400

-600

-20

30

 κ_3

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[e.g. Stylianou, Weiglein, arXiv:2312.04646, ATLAS, arXiv:2411.02040, CMS-PAS-HIG-24-015] Exp. 68% CL ATLAS $\sqrt{s} = 13 \text{ TeV}, 126 \text{ fb}^{-1}$ Obs. 95% CL $^-$ HHH \rightarrow 6b Unitarity -200-600-20 30 κ_3

$$\mathcal{Z} \supset -\lambda_{\text{SM}} v \left(1 + c_3\right) h^3 - \frac{\lambda_{\text{SM}}}{4} \left(1 + d_4\right) h^4$$

$$\kappa_3$$

⇒ Cross section modifications (ratio to SM):

$$\sigma/\sigma_{\text{SM}}(13.6 \text{ TeV}) - 1 \approx 0.88c_3^2 - 0.82c_3 - 0.32c_3^3$$

$$-0.17c_3d_4 - 0.09d_4 + 0.05c_3^2d_4$$

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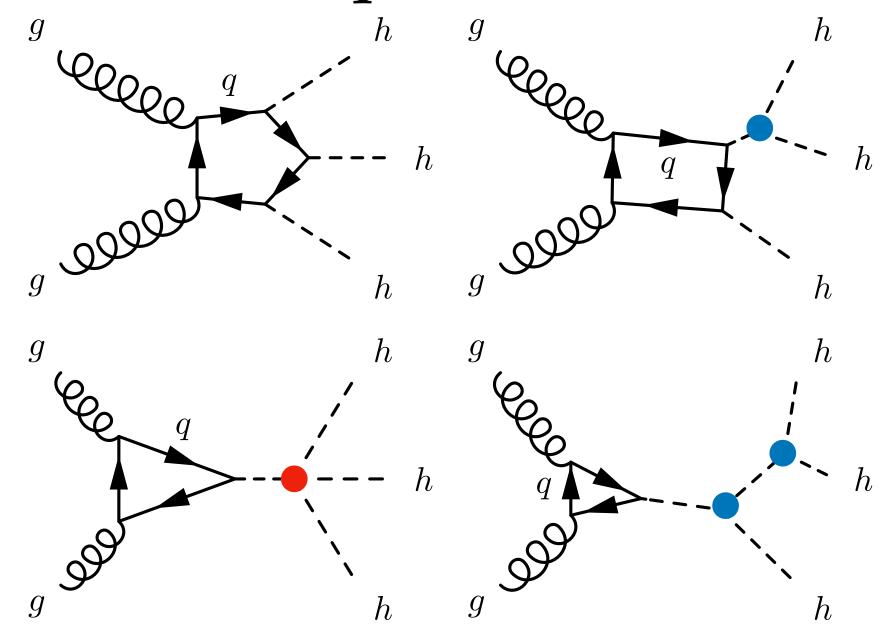
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Note: For **additional** anomalous couplings, e.g. $t\bar{t}h^3$, see: [AP, Tetlalmatzi-Xolocotzi, arXiv:2312.13562]

and: MadGraph5_aMC model at: https://gitlab.com/apapaefs/multihiggs_loop_sm

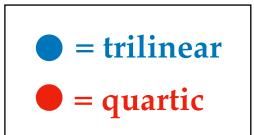


- Q: Which anomalous contributions to keep @ the matrix element-squared level?
- Effective Field Theory considerations: **not settled yet!** [→ see, e.g. discussions of arXiv:2201.04974, 2304.01968].
- Consider linear, quadratic or cubic truncations?

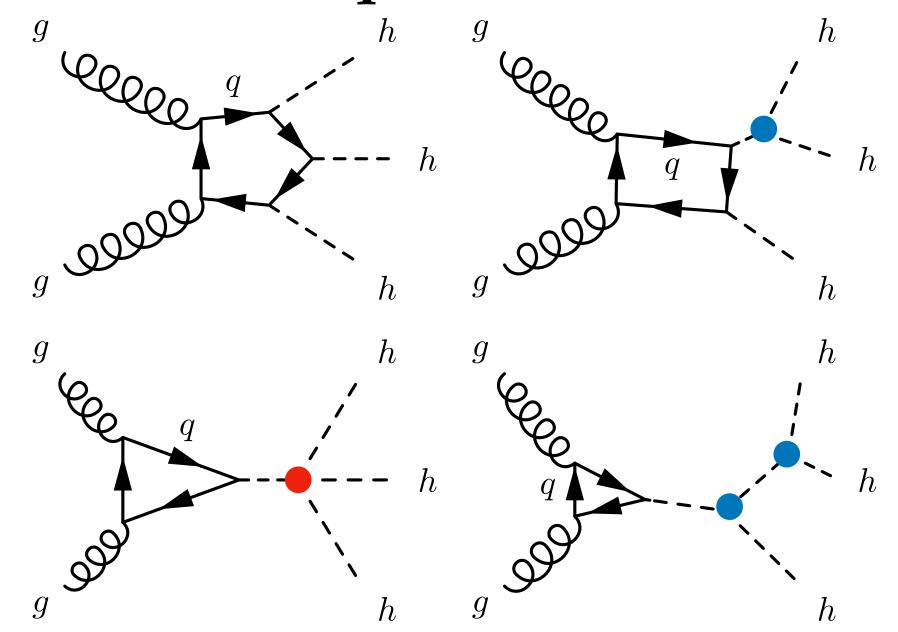


$$|\mathcal{M}|^2 \sim 1 + Ac_3 + Bd_4 \qquad \text{(linear)}$$



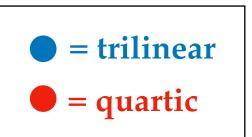


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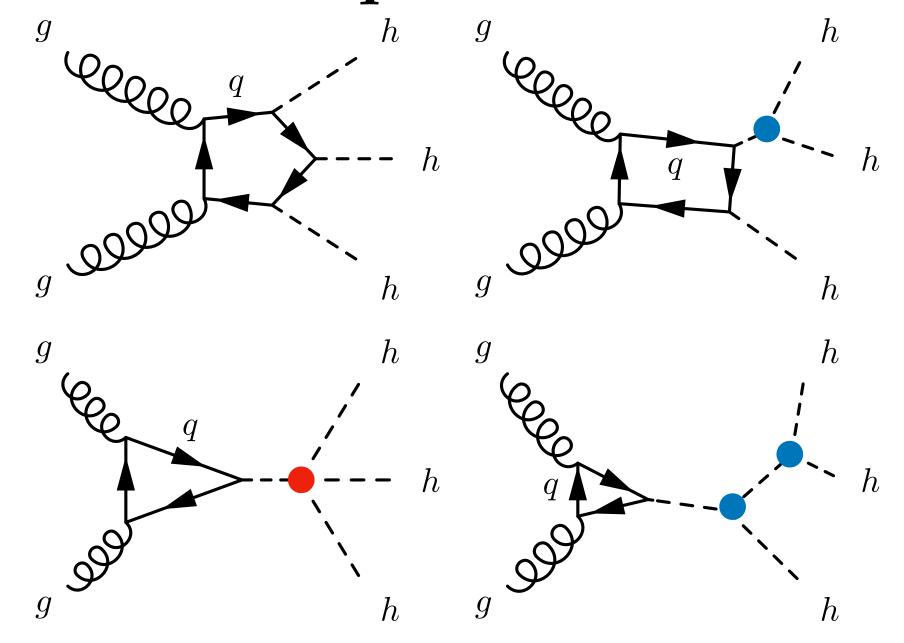


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 $+ Cd_4^2 + Dc_3d_4 + Ec_3^2$ (quadratic)



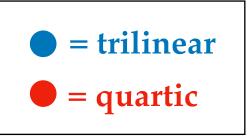


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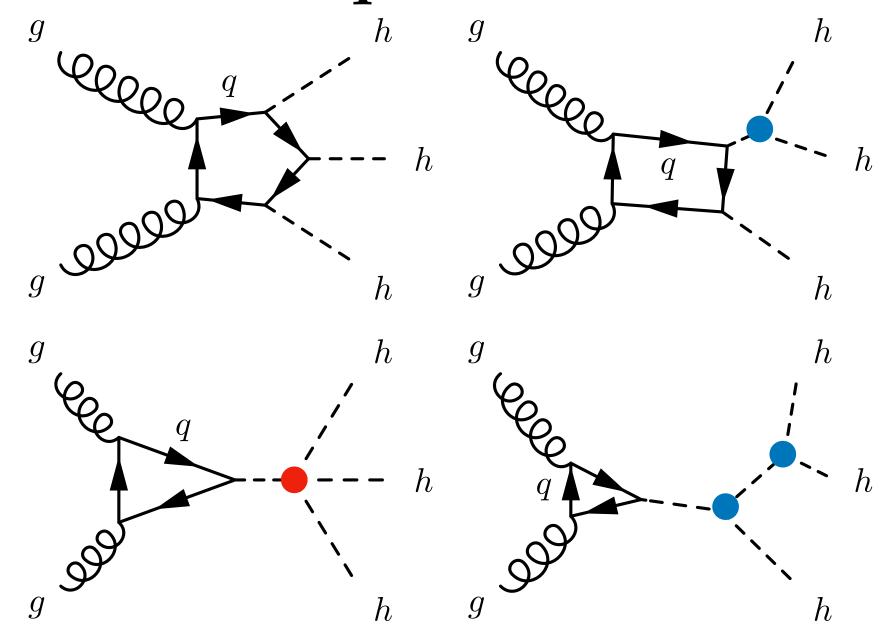


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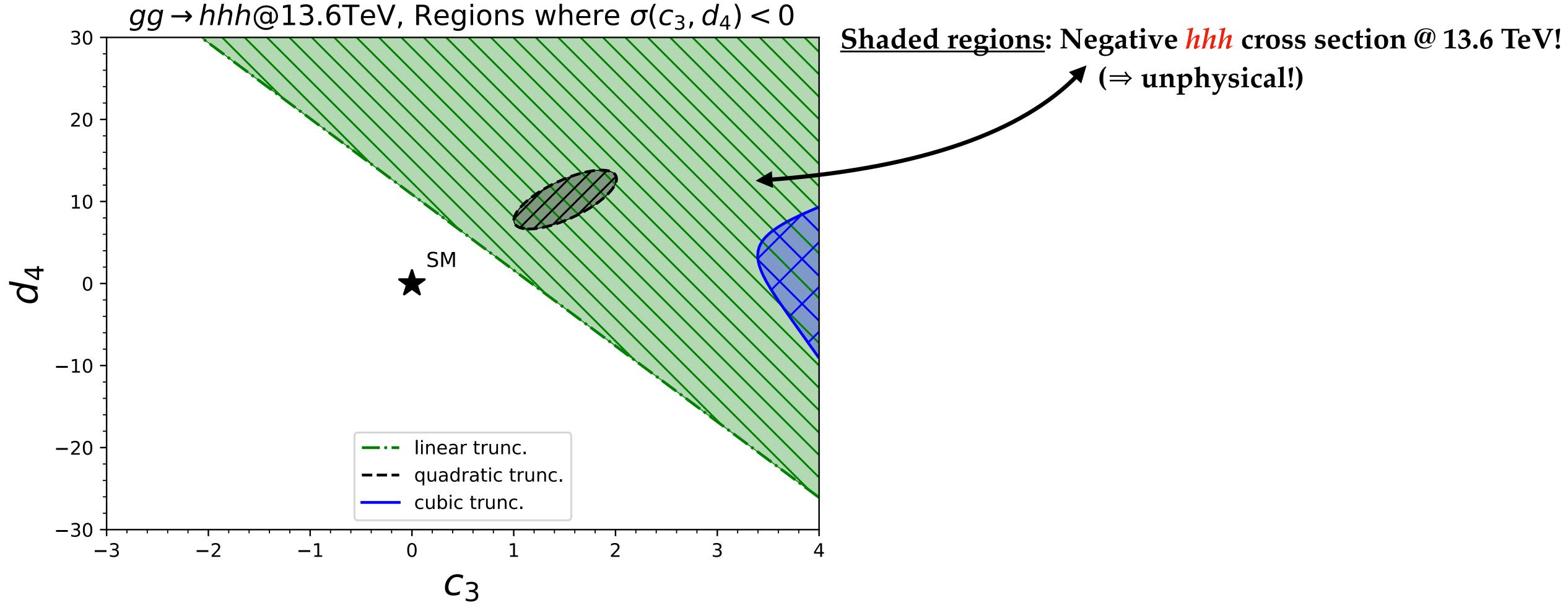
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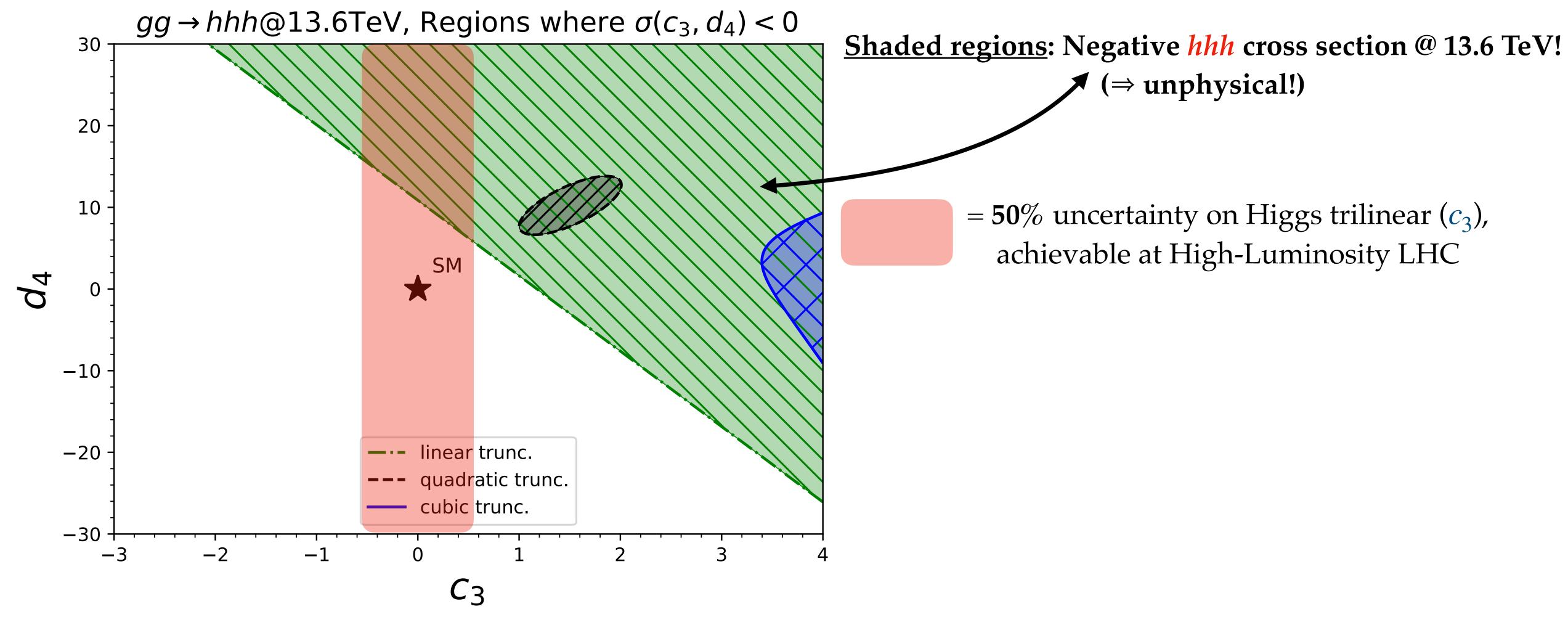


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 $+ c_3^4$ (no trunc.)

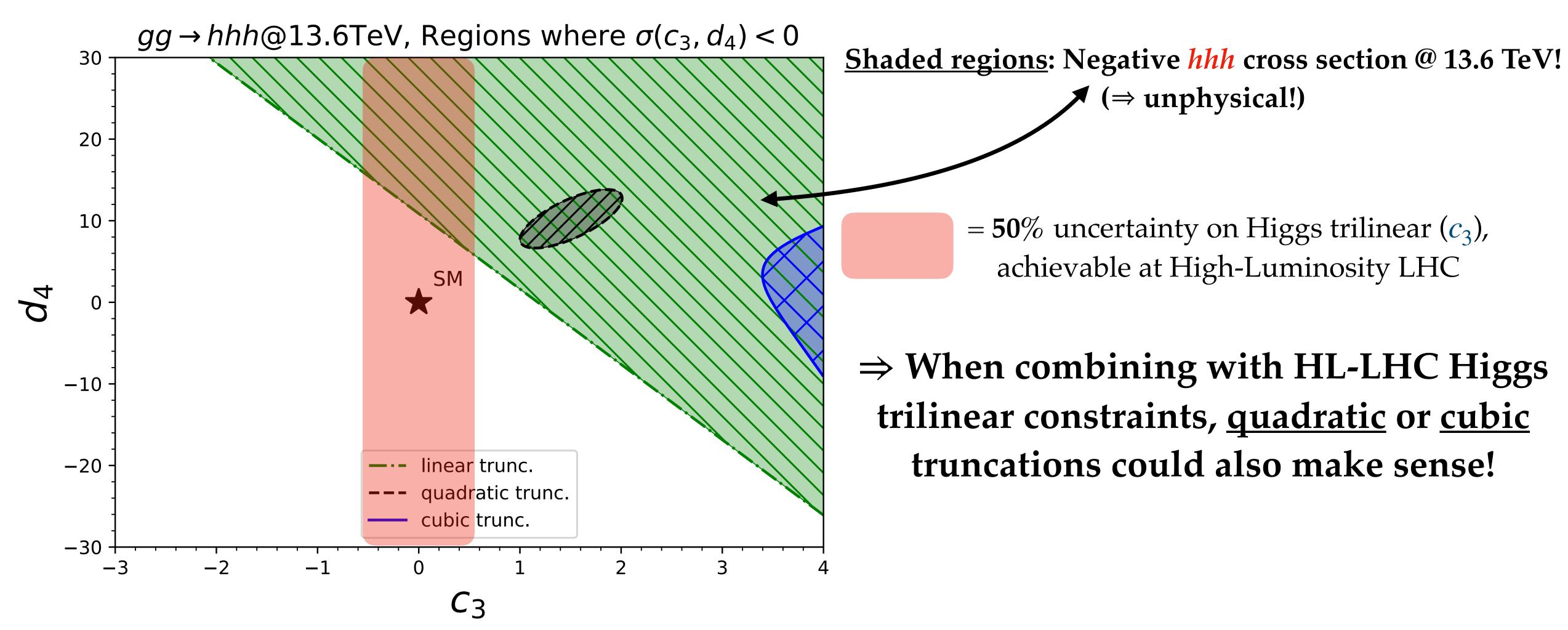




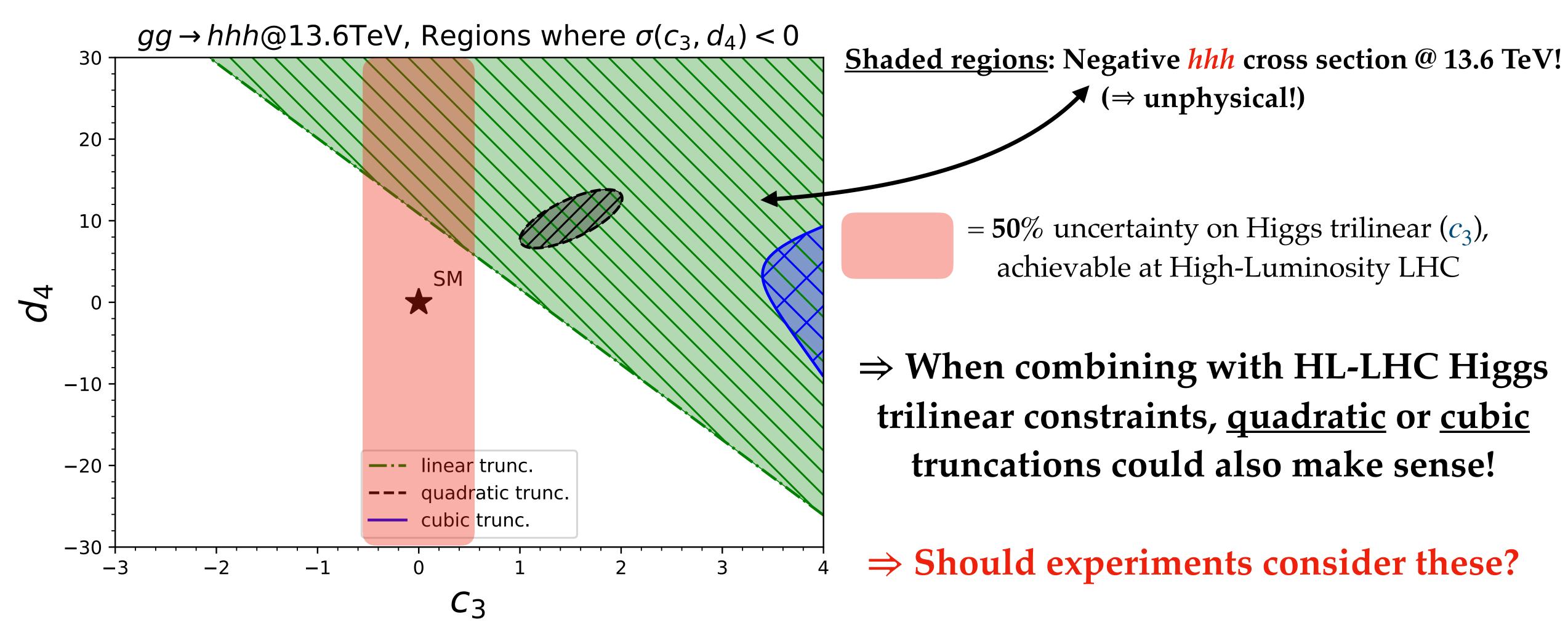














Extended Scalar Sectors and hhh





Extended Scalar Sectors: Why?

- Dark Matter (the scalar themselves, or portals to hidden sectors),
- First-order EW phase transitions (⇒ EW baryogenesis ⇒ matter-antimatter asymmetry), [e.g. AP, White, arXiv:2010.0059 & arXiv:2108.11394]
- & Model the scalar sector of more complicated models, e.g. SUSY.



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- & Model the scalar sector of more complicated models, e.g. SUSY.

$$\mathcal{V}(\phi,S) = \mathbf{O} \left|\phi\right|^2 + \mathbf{D} \left|\phi\right|^4$$
 Singlet \mathbf{NO} SM "charges"

Simplest extension to the SM: add ONE real singlet scalar field.

$$+ \bullet S^2 + \blacktriangle S^3 + \blacksquare S^4$$

$$+ \blacktriangle |\phi|^2 S + \blacksquare |\phi|^2 S^2 \leftarrow \text{``Portal'' interactions.}$$

$$+ S \times (\text{Hidden Sector}) + \dots \leftarrow \text{Dark Matter?}$$



SM+One Real Singlet [=xSM]

[e.g. AP, White, arXiv:2010.00597]

$$\mathcal{V}(\phi,S) = \bullet |\phi|^2 + \blacksquare |\phi|^4 + \bullet S^2 + \blacktriangle S^3 + \blacksquare S^4 + \blacktriangle |\phi|^2 S + \blacksquare |\phi|^2 S^2$$

Mass Eigenstates

$$\binom{h_1}{h_2} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \binom{h}{\chi}$$

 θ : mixing angle

$$h_1 \rightarrow$$
 "SM-like" Higgs boson.

 $h_2 \rightarrow$ new scalar resonance.

i.e. choose: $|\theta| \gtrsim 0$, and:

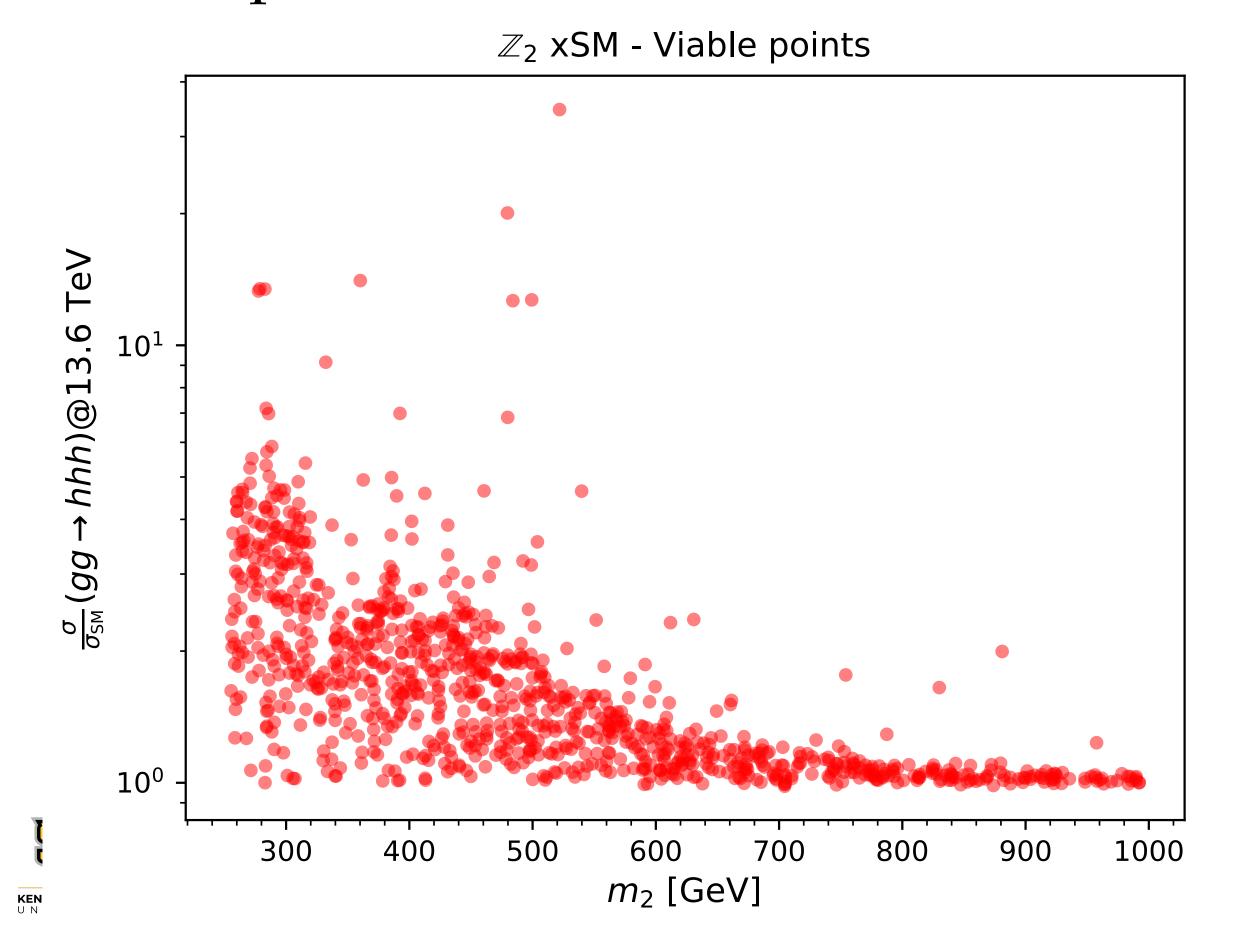
$$h_1 = h\cos\theta + \chi\sin\theta$$

$$h_2 = -h\sin\theta + \chi\cos\theta$$



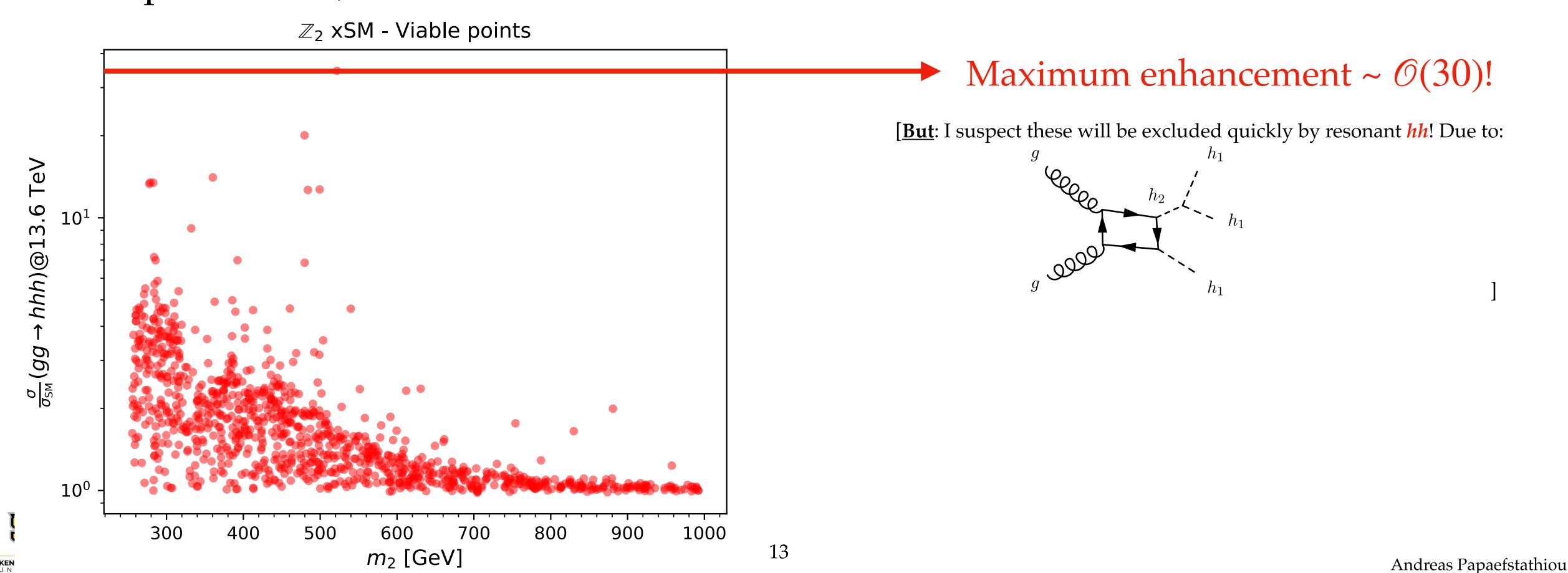
Resonant *hhh* in the SM+One Real Singlet [=xSM]

- What about *hhh* in the **xSM**? An example with \mathbb{Z}_2 symmetry: $\mathbb{Z}_2 : S \to -S$.
- Including: boundedness of potential + perturbativity + HiggsTools (i.e. experimental) constraints [Bahl, Biekötter, Heinemeyer, Li, Paasch, Weiglein, Wittbrodt, arXiv:2210.09332].



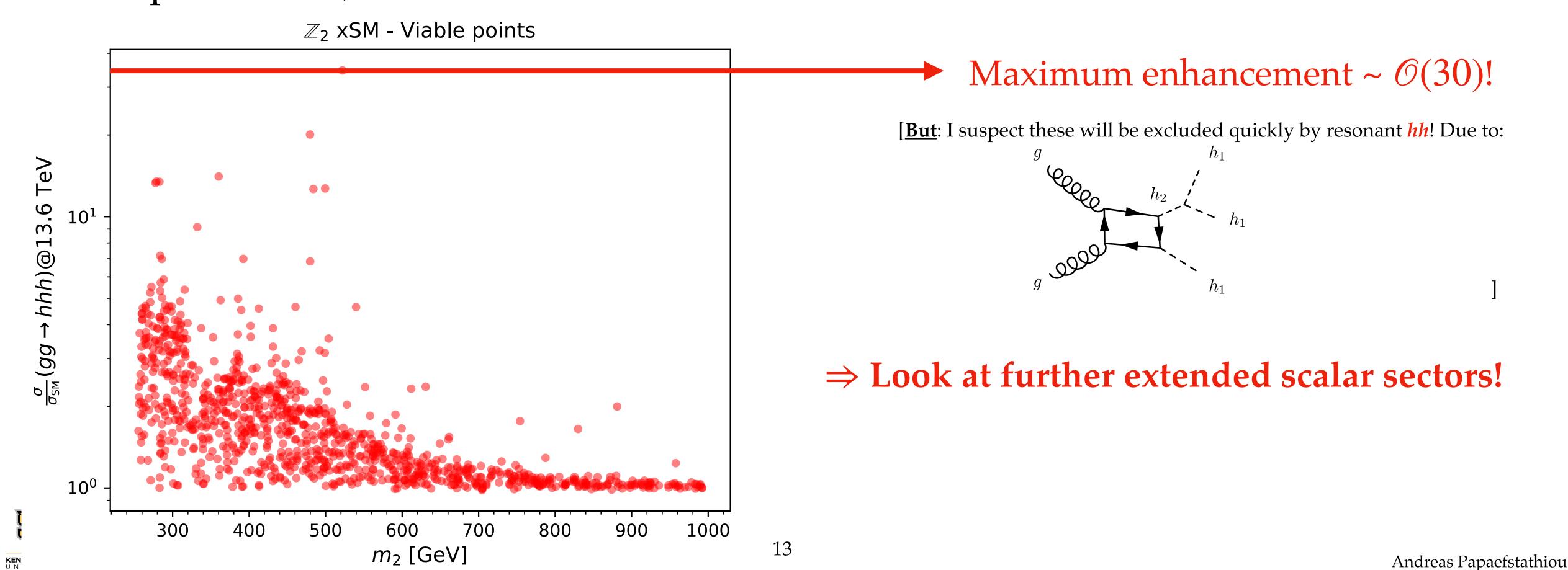
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SM + <u>Two Real Singlet Scalars [= TRSM]</u>

- Consider adding <u>two</u> real singlet scalar fields $S, X \rightarrow$ the TRSM.
- & impose discrete \mathbb{Z}_2 symmetries: $\mathbb{Z}_2^S: S \to -S, \ X \to X$

$$\mathbb{Z}_2^X:X\to -X,\ S\to S$$

⇒ TRSM Scalar Potential:

$$\mathcal{V}(\phi, S, X) = \bullet |\phi|^2 + \square |\phi|^4 + \bullet S^2 + \square S^4 + \bullet X^2 + \square X^4 + \square S^2 X^2 + \square |\phi|^2 S^2 + \square |\phi|^2 X^2$$



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SM + <u>Two Real Singlet Scalars [= TRSM]</u>

- Electroweak Symmetry Breaking in the TRSM:
 - \Rightarrow Three scalar bosons: $h_1, h_2, h_3 \rightarrow h_1 \approx$ SM-like "Higgs boson".
 - \Rightarrow *hhh* that may even be <u>detectable at the LHC!</u>

through:
$$pp \rightarrow h_3 \rightarrow h_2 h_1 \rightarrow h_1 h_1 h_1$$
 [Robens, Stefaniak, Wittbrodt, arXiv:1908.08554, AP, Robens, Tetlalmatzi-Xolocotzi, arXiv:2101.00037]



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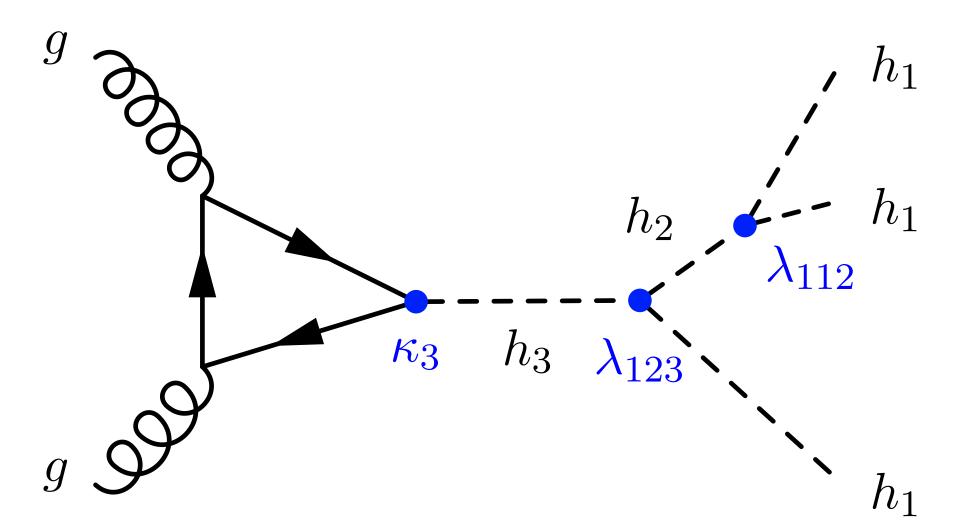
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→ Double-Resonant enhancement!





$$m_3 > m_2 + m_1, m_2 > 2m_1$$

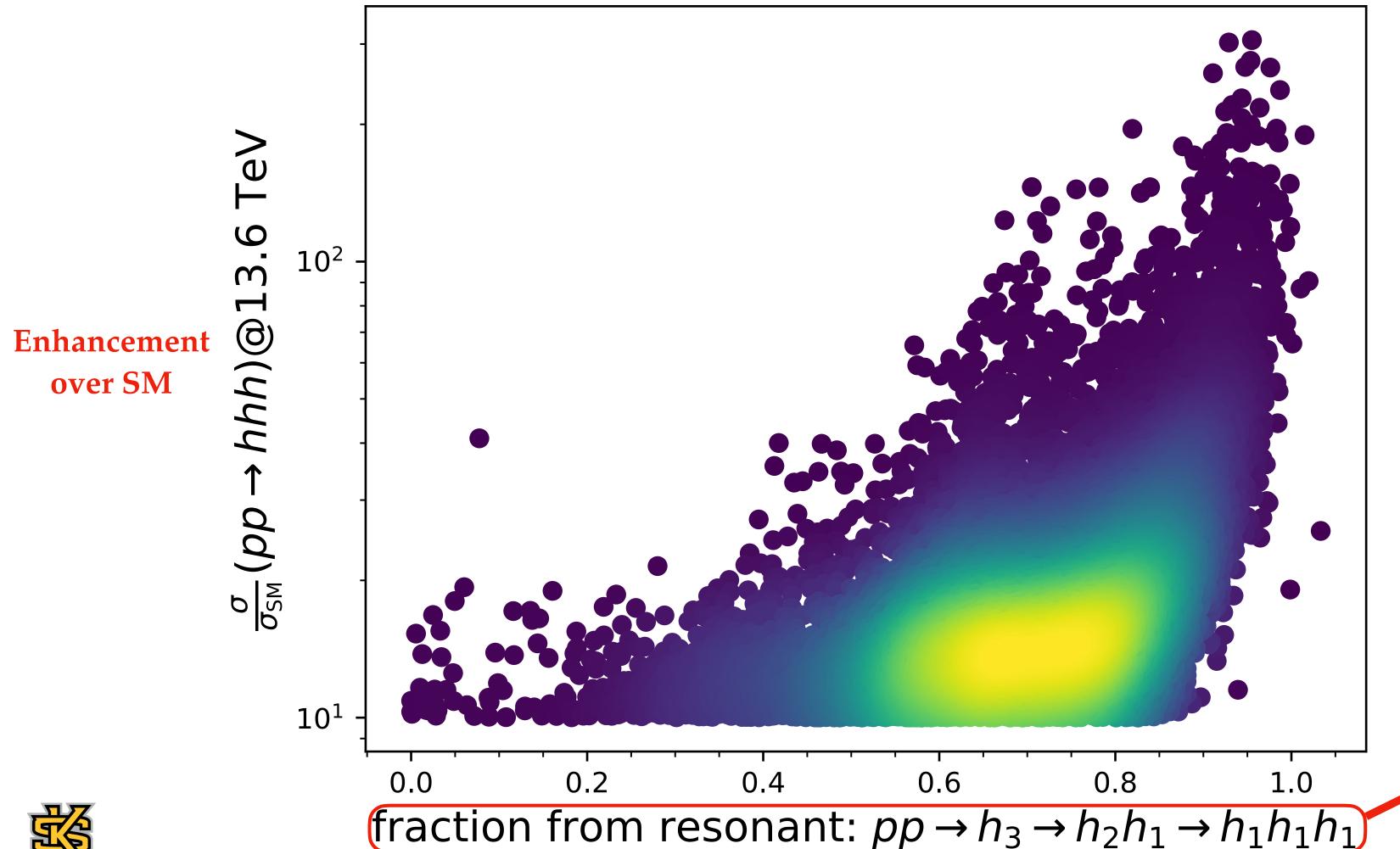


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Double-Resonant hhh in the TRSM

• Enhancement of $hhh \Leftrightarrow Large$ "fraction" of double-resonant process!

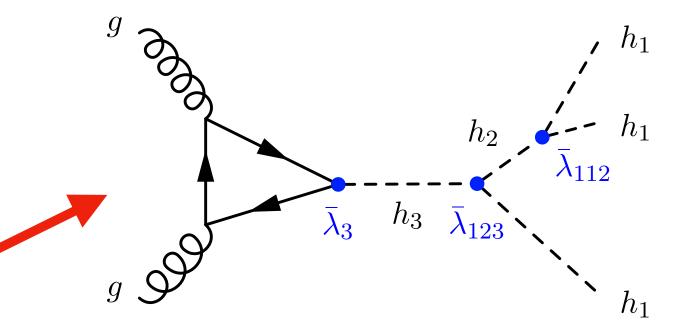
Viable points with $\sigma > 10 \times \sigma_{SM}(pp \rightarrow hhh) @13.6 \text{ TeV}$



[Karkout, AP, Postma, Tetlalmatzi-Xolocotzi, van de Vis, du Pree, arXiv:2404.12425]

[including boundedness of potential + perturbativity + HiggsTools constraints.]

Resonant Fraction (R.F.) = How much of the total cross section comes from...?

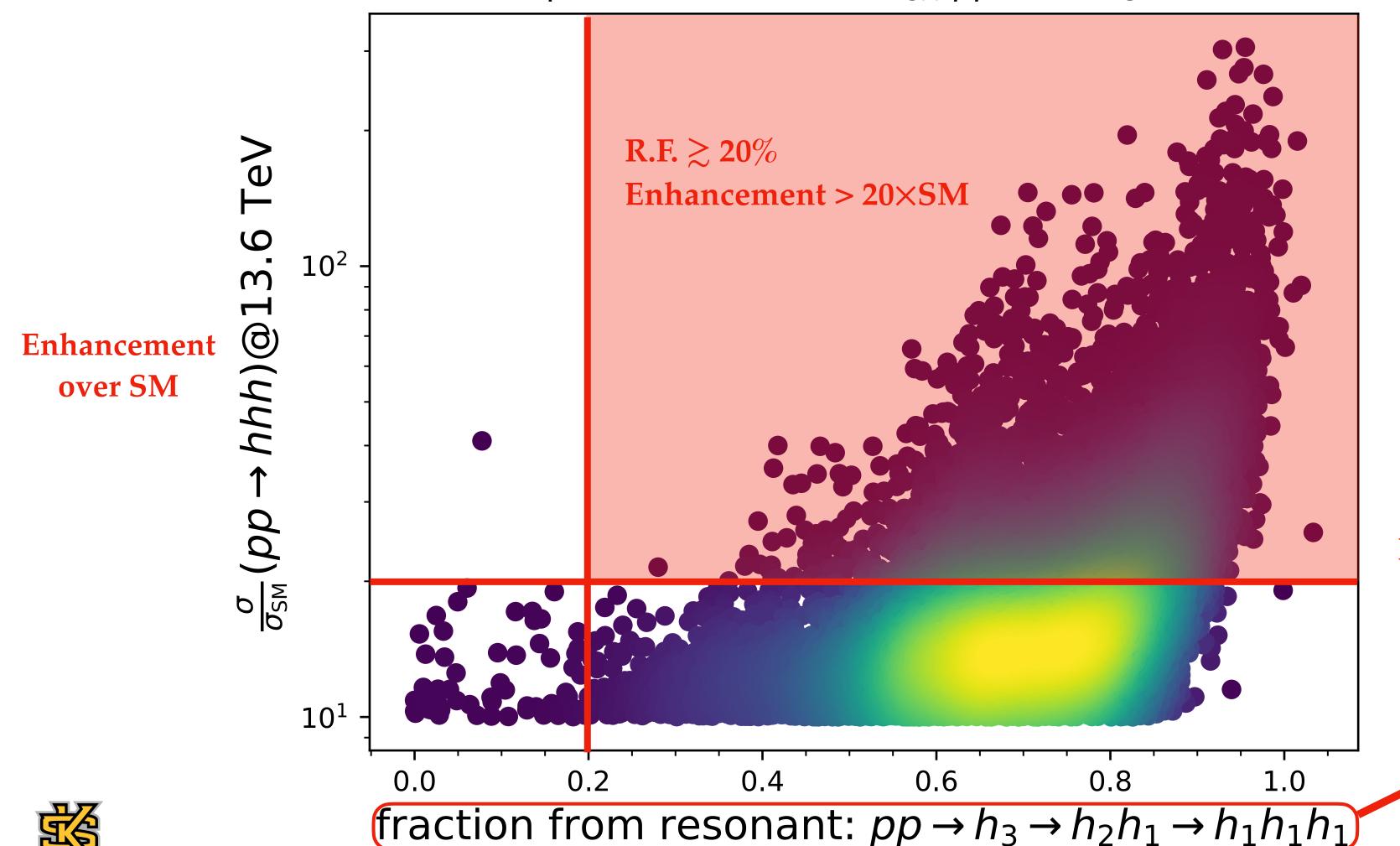




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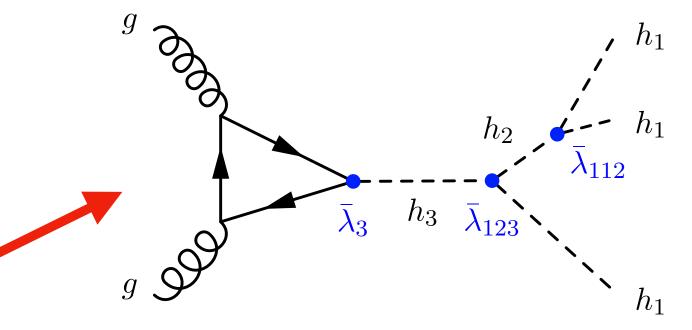
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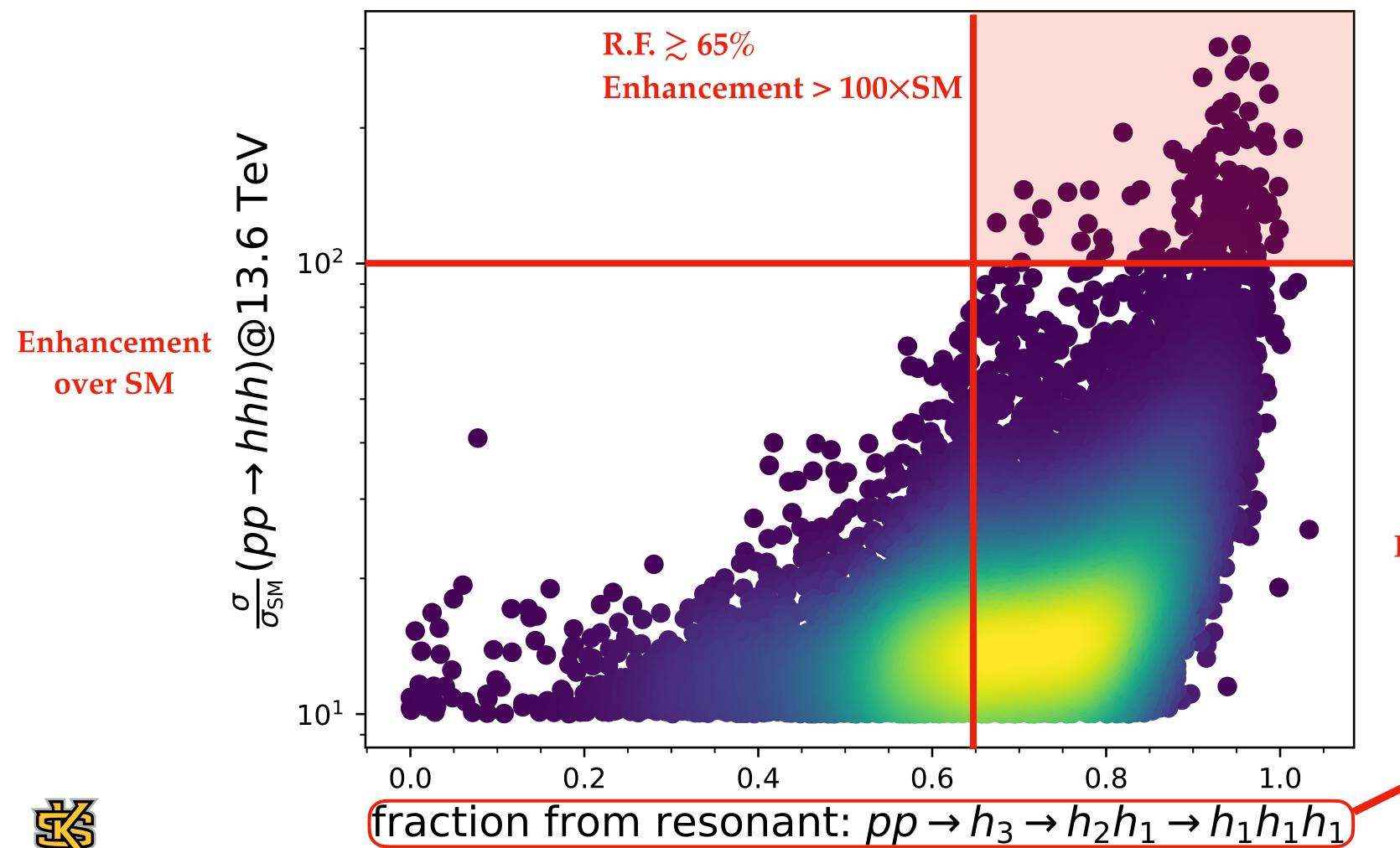
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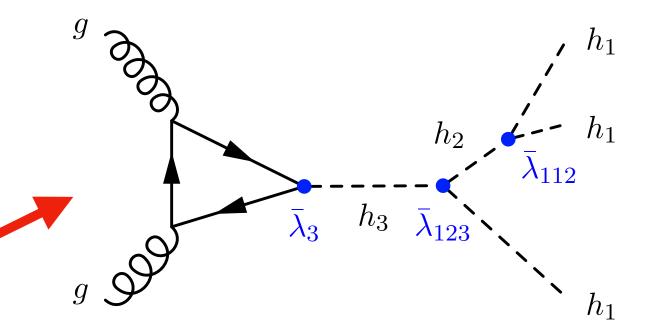
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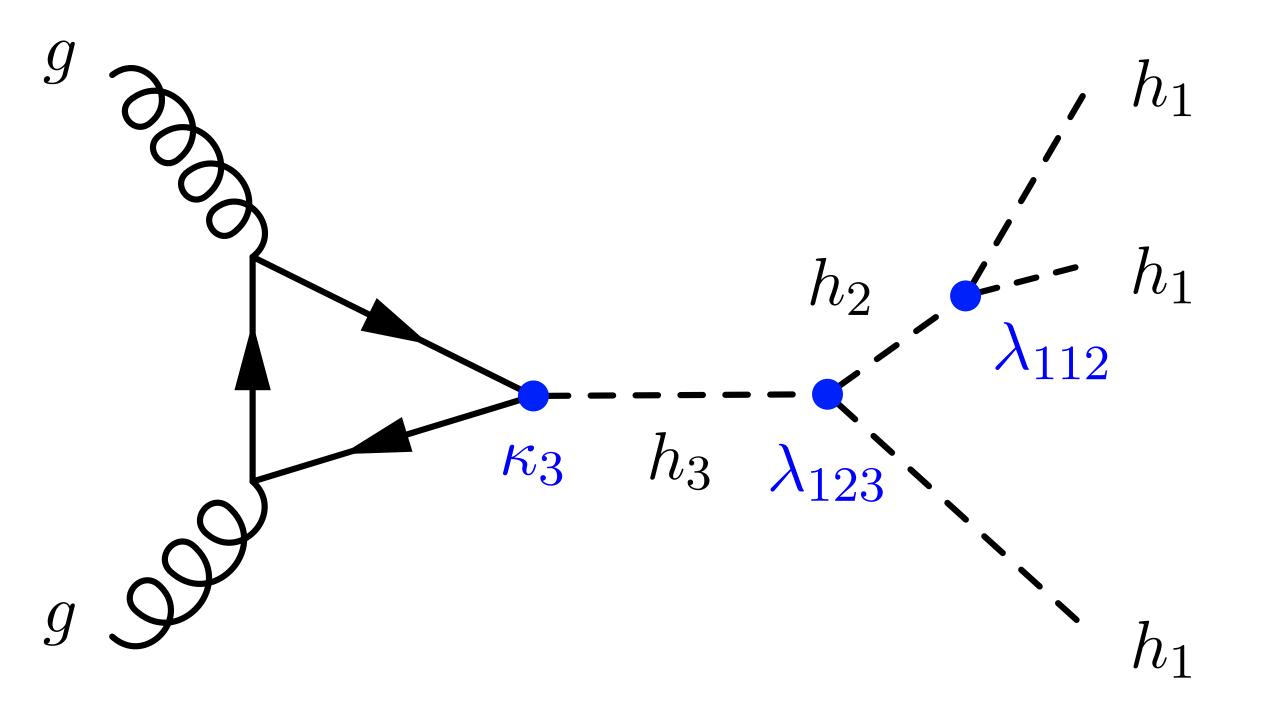


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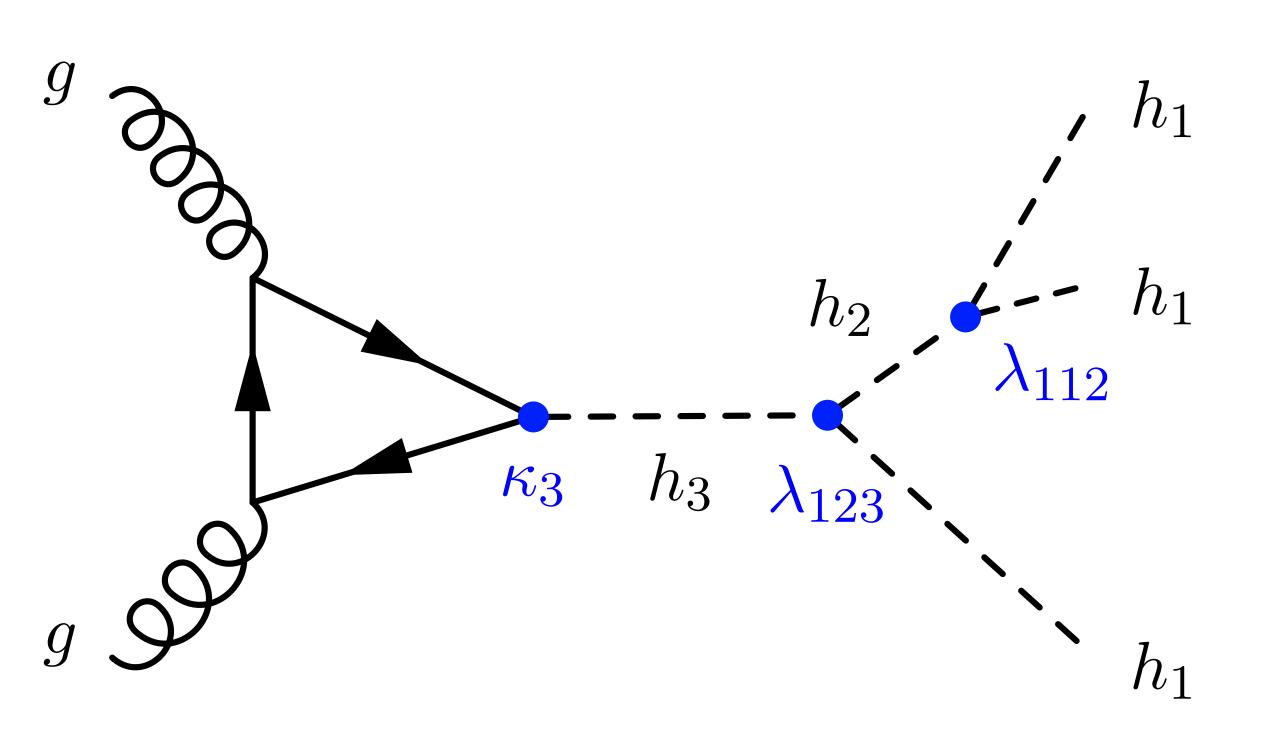
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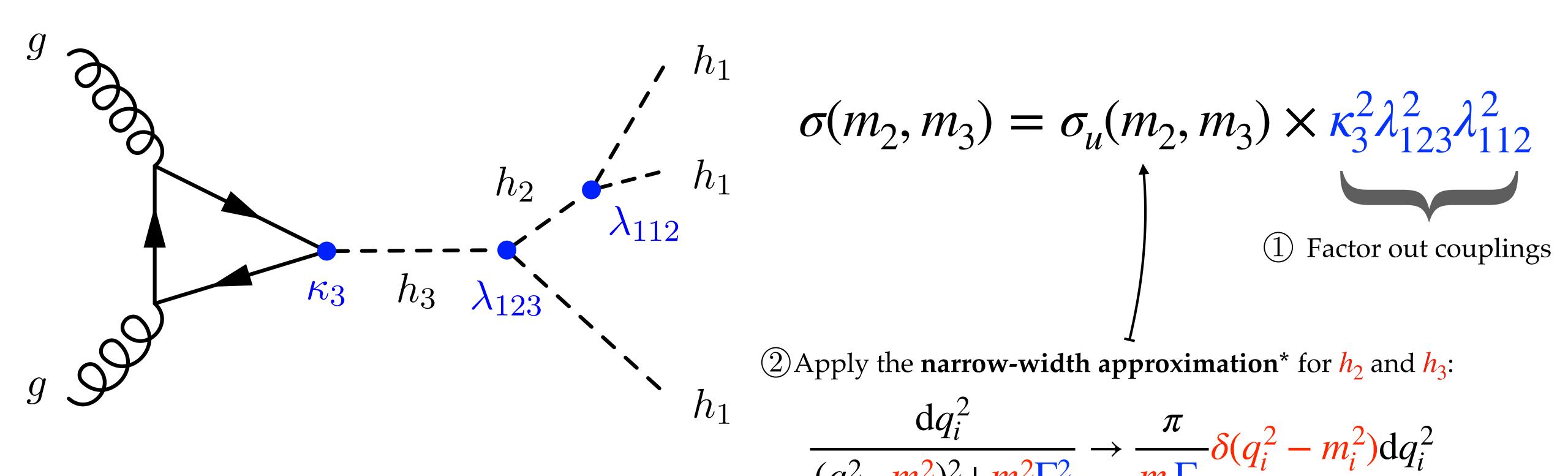




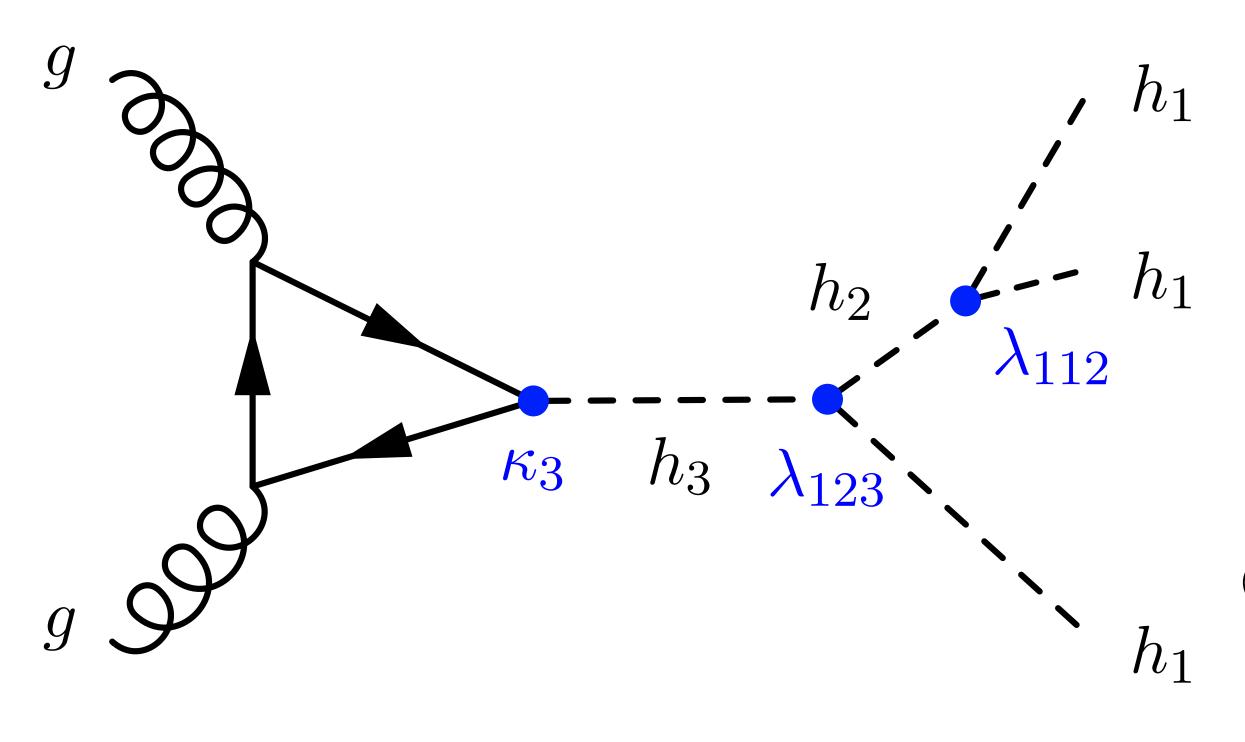


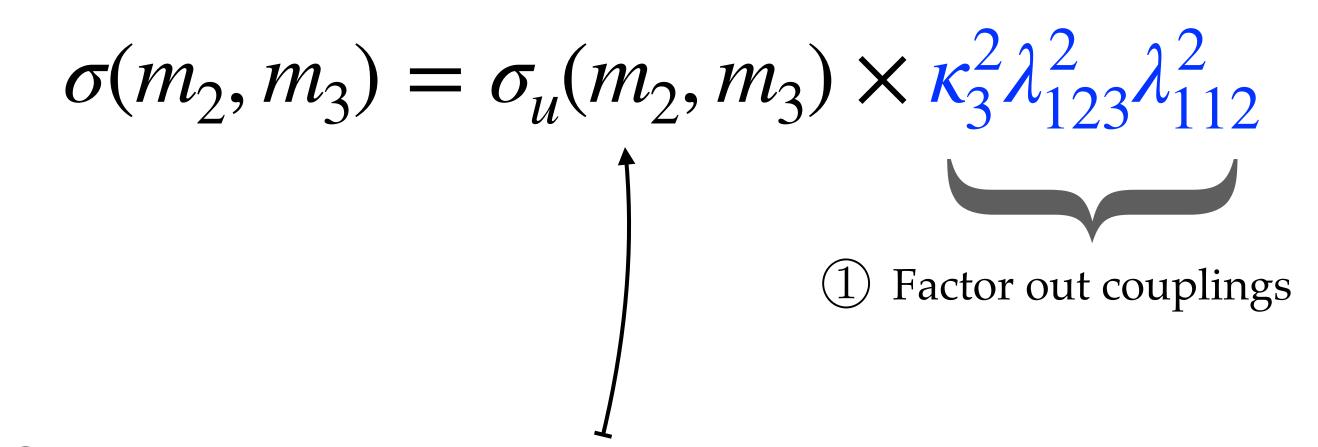
$$\sigma(m_2, m_3) = \sigma_u(m_2, m_3) \times \kappa_3^2 \lambda_{123}^2 \lambda_{112}^2$$
① Factor out couplings





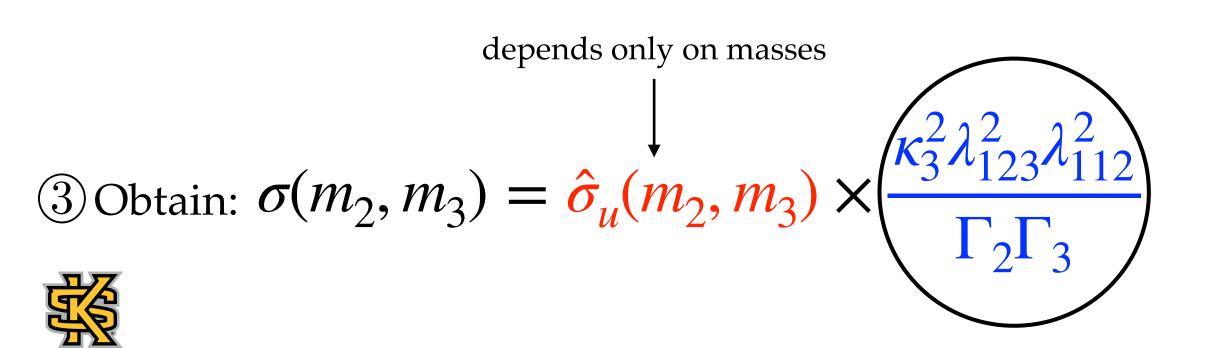


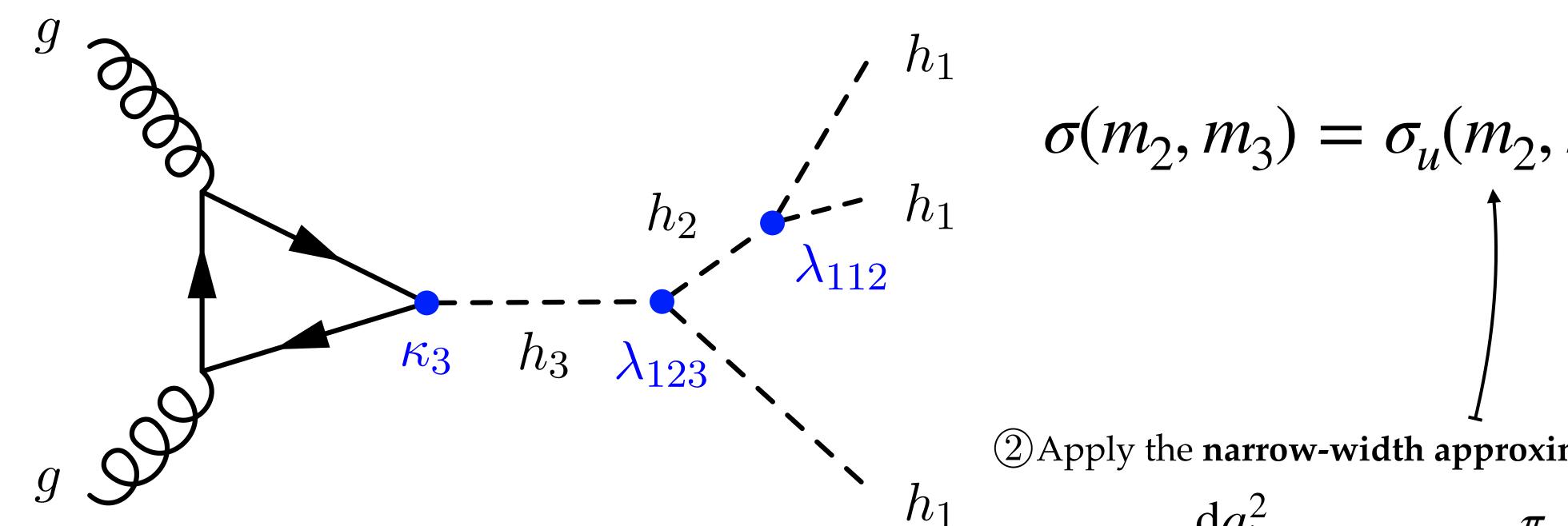


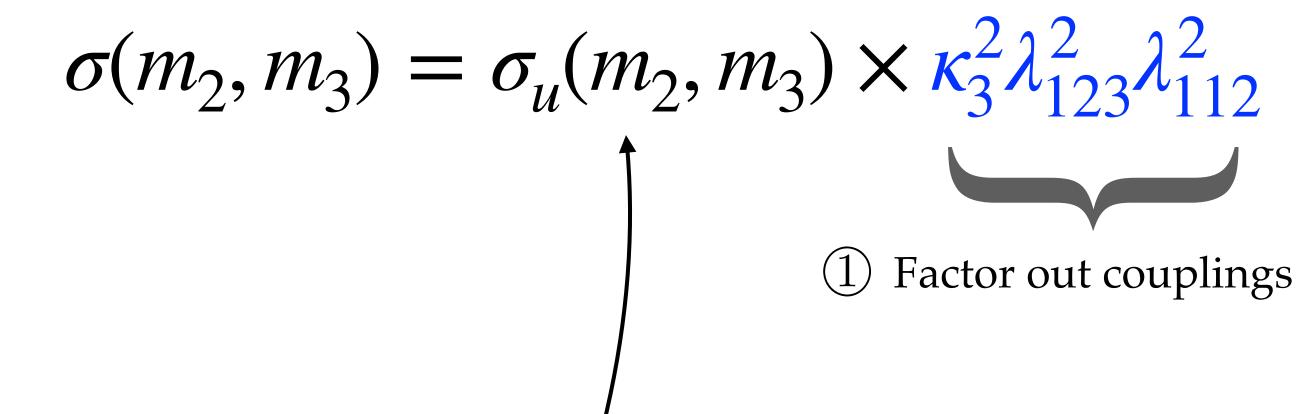


②Apply the narrow-width approximation* for h_2 and h_3 :

$$\frac{\mathrm{d}q_i^2}{(q_i^2 - m_i^2)^2 + m_i^2 \Gamma_i^2} \rightarrow \frac{\pi}{m_i \Gamma_i} \delta(q_i^2 - m_i^2) \mathrm{d}q_i^2$$







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4 Define: $\rho^2 \equiv \kappa_3^2 \lambda_{123}^2 \lambda_{112}^2 / (\Gamma_2 \Gamma_3)$

"rescaling factor"

What if...?

- Let's suppose two new scalars h_2 and h_3 are discovered (\checkmark):
 - $\Rightarrow m_2, m_3$ [and possibly] the widths Γ_2, Γ_3 would be known.



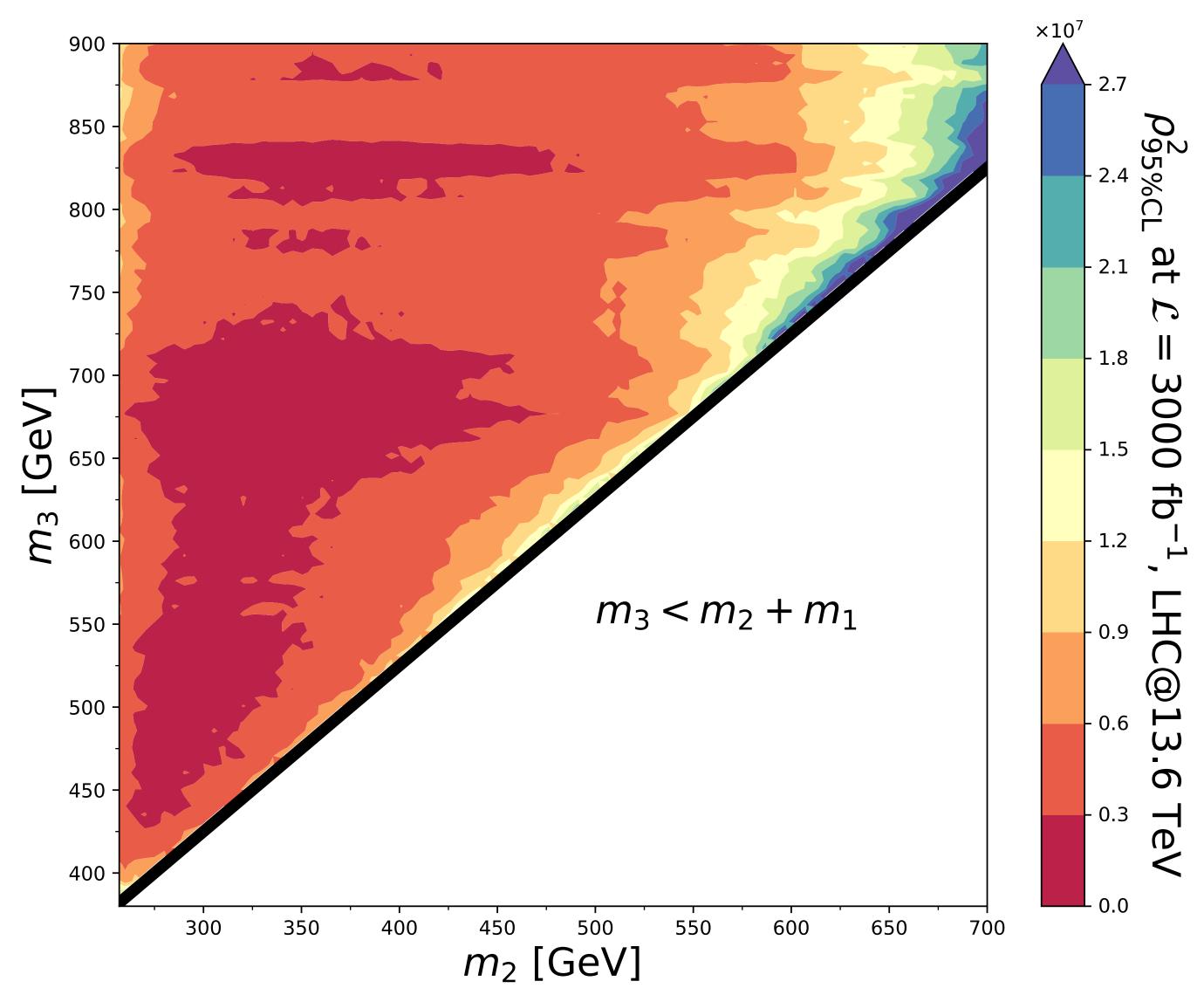
- *hhh* can provide relevant **information** on the theoretical parameter space.
 - → An important contribution to solving the <u>inverse problem!</u>
 - \Rightarrow through rescaling factor $\rho^2 = \kappa_3^2 \lambda_{123}^2 \lambda_{112}^2 / (\Gamma_2 \Gamma_3)$ (if narrow width!*)
- We derived constraints on ρ^2 via: $pp \to (b\bar{b})(b\bar{b})(b\bar{b}) \to 6$ b-jets.
 - → [~20% of the *hhh* final state.]

[AP, Tetlalmatzi-Xolocotzi, Zaro, arXiv:1909.09166,
AP, Robens, Tetlalmatzi-Xolocotzi, arXiv:2101.00037,
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*Narrow width: See appendix!



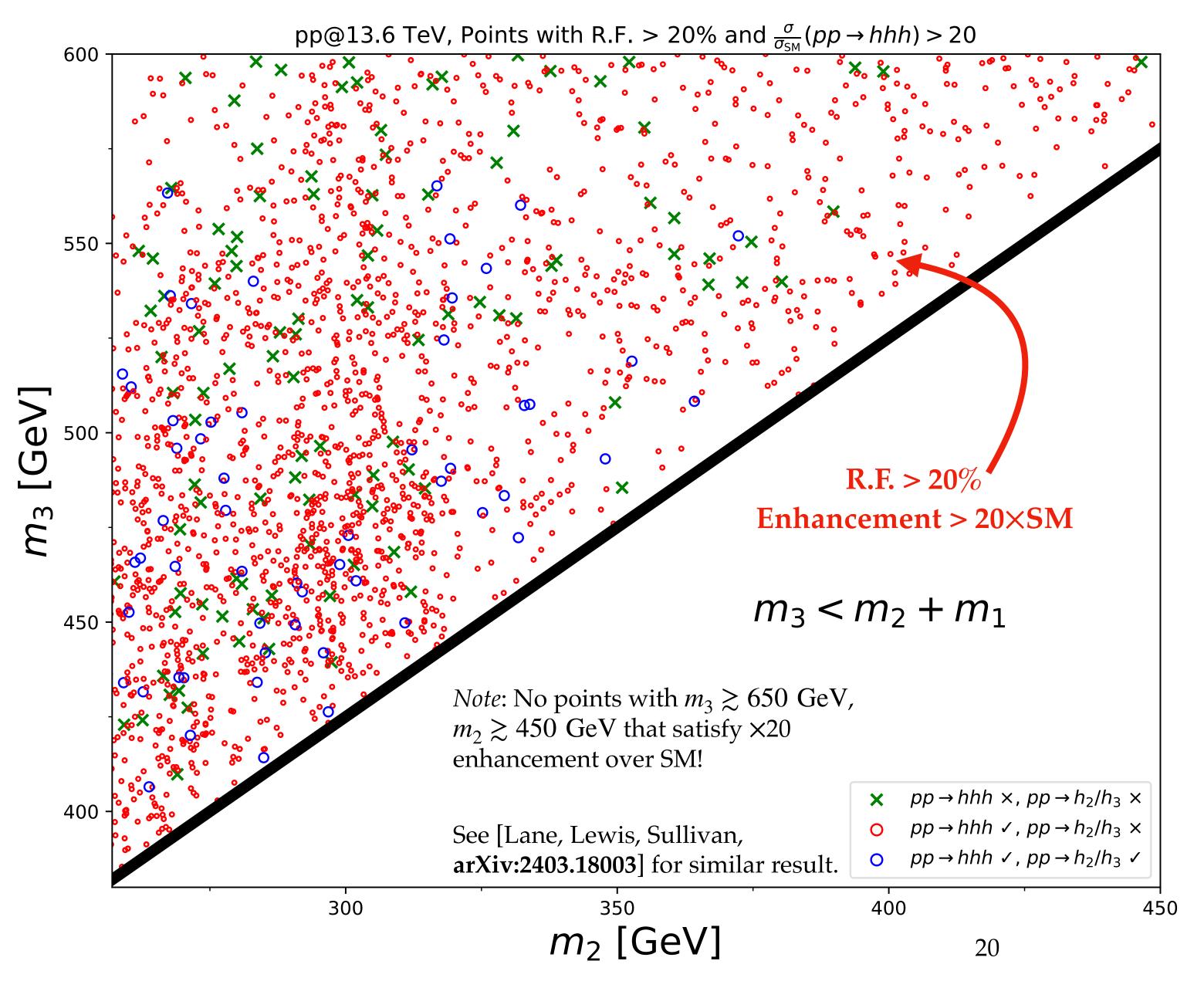
Constraints on $\rho^2 = \kappa_3^2 \lambda_{123}^2 \lambda_{112}^2 / (\Gamma_2 \Gamma_3)$ on the (m_2, m_3) -plane



95% C.L. Constraint on ρ^2 at HL-LHC.



Applied to TRSM Benchmark Points [R.F. > 20%, 20×SM enhancement]



HL-LHC results:

- X: Excluded **BOTH** by *hhh* & single h_2 and h_3 production,
- •: Excluded <u>ONLY</u> by single h_2 and h_3 production, <u>NOT</u> by *hhh*.
- •: NOT excluded @ HL-LHC
- (⇒ Future Colliders?)

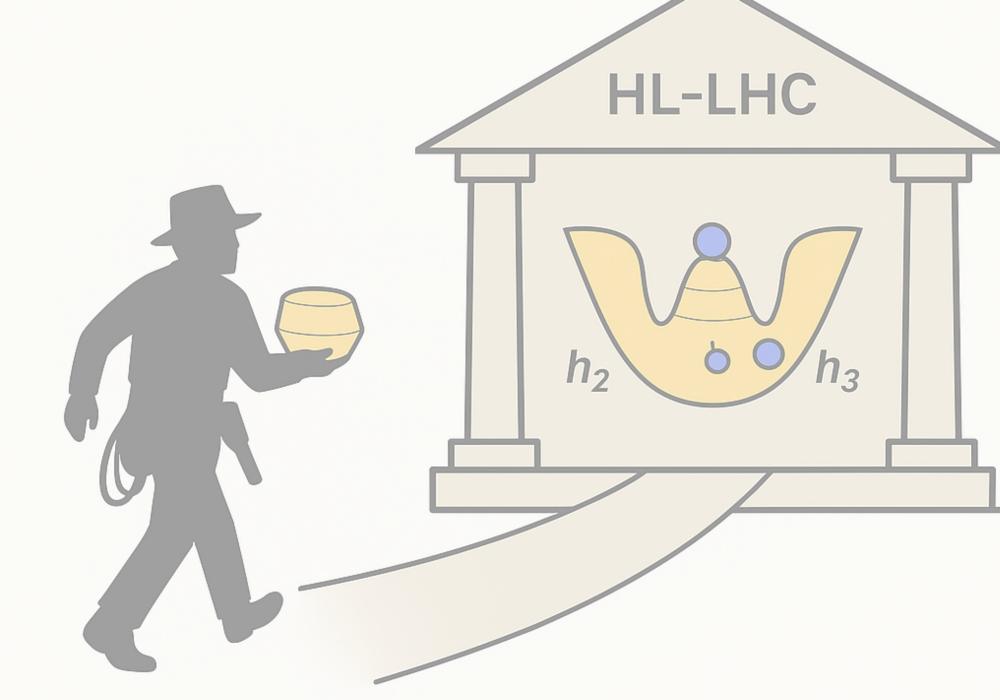
Notice: NO hhh exclusion without single h_2 and h_3 exclusion!

⇒ in TRSM, *hhh* is *unlikely* to be a "discovery" channel.

Conclusions & Outlook

- *hhh*: a direct probe of the Higgs quartic self-couping!
- But: @LHC: Non-resonant hhh: extremely challenging,
 - \rightarrow even with large anomalous couplings $(c_3, d_4)!$

& digging out the quartic self-coupling will be hard!



- → **Truncation** of the cross section to be considered within the context of EFTs (?)
- Extended scalar sectors can enhance *hhh* → observable at the LHC!

e.g.: <u>TRSM</u> \rightarrow Two <u>new</u> scalars \rightarrow double-resonant enhancement: $pp \rightarrow h_3 \rightarrow h_2 h_1 \rightarrow h_1 h_1 h_2$

- \rightarrow Information about the nature of extended scalar sectors (\sim Inverse problem).
- Future directions: non- \mathbb{Z}_2 symmetric TRSM, examine the Electroweak Phase Transition, Dark Matter, investigate further-extended scalar sectors [...].

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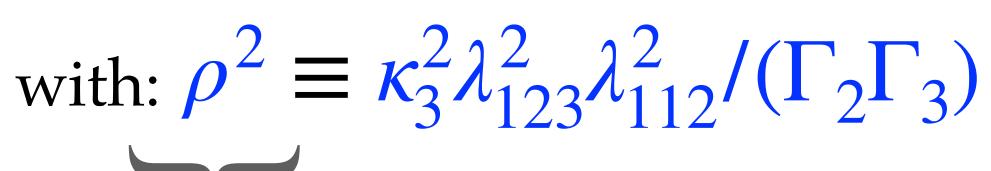
APPENDICES



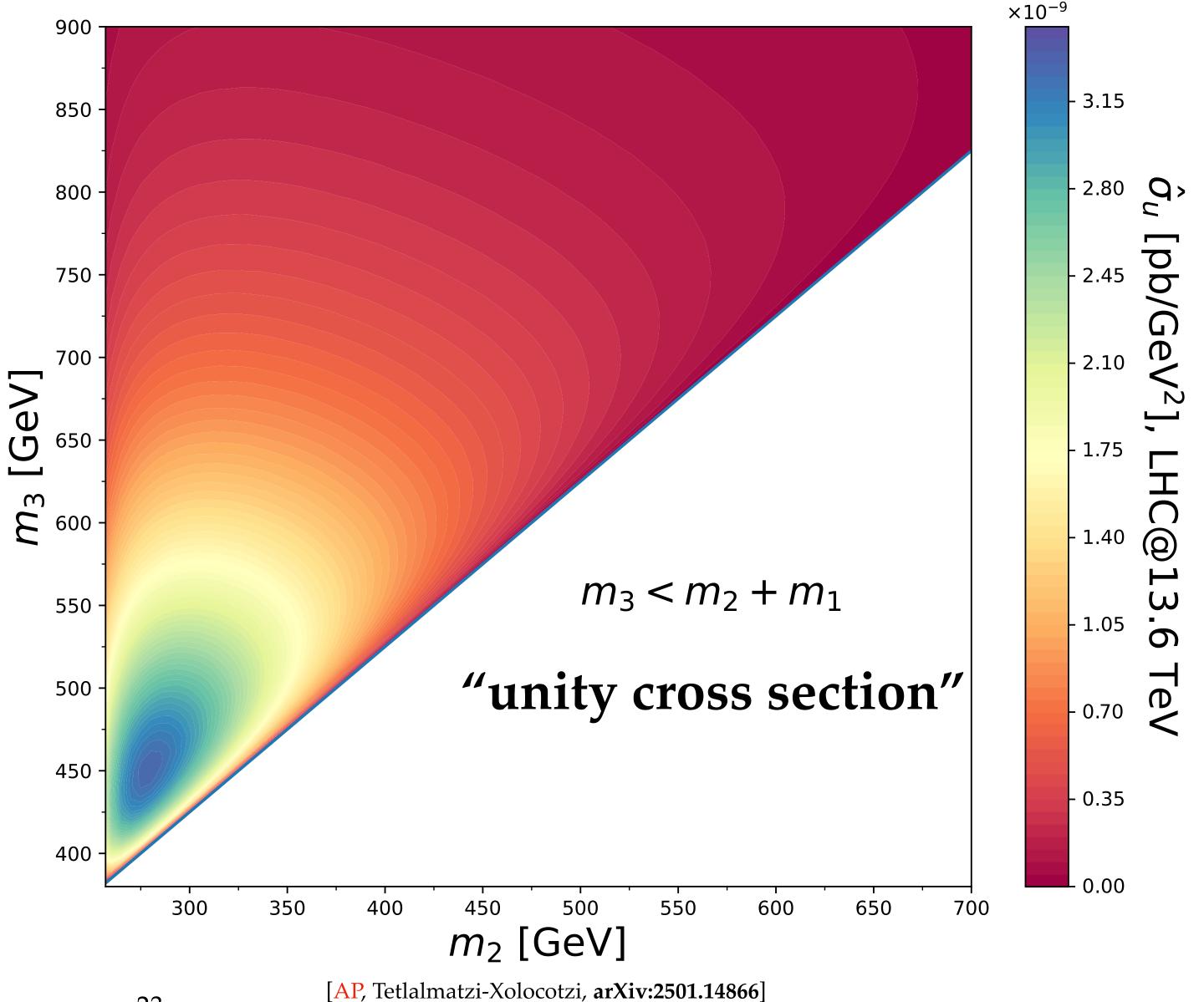
$$\sigma(m_2, m_3) = \hat{\sigma}_u(m_2, m_3) \times \rho^2$$

"unity cross section":

depends only on m_2 , m_3 . Derived once and for all! (at fixed collider energy)



"rescaling factor": couplings and widths





TRSM Benchmarks from:

[Karkout, AP, Postma, Tetlalmatzi-Xolocotzi, van de Vis, du Pree, arXiv:2404.12425]

Benchmark quantities relevant for double-resonant triple Higgs boson production

Name	m_2	m_3	Γ_2	Γ_3	κ_3	λ_{123}	λ_{112}	$ \begin{array}{c c} [\text{GeV}^2] \\ \rho^2 \\ [\times 10^6] \end{array} $	[pb/GeV 2] $\hat{\sigma}_u$ [$\times 10^{-9}$]
BM0	259.0	495.0	0.003514	3.927	0.1854	-191.8	8.167	6.11	2.018
BM1	270.6	444.7	0.5078	2.586	0.1571	-204.3	67.52	3.574	3.408
BM2	268.6	452.7	0.3805	3.142	0.1741	-203.6	57.78	3.509	3.165
BM3	272.6	480.7	0.2009	4.758	0.2024	-224.6	41.39	3.703	2.908
BM4	269.0	409.8	0.2836	1.995	0.1713	-180.3	48.89	4.031	2.663
BM5	269.1	486.9	0.0003346	2.017	0.1527	103.3	-2.477	2.264	2.805
BM6	259.2	577.0	0.0006274	5.79	0.1908	196.3	-3.701	5.289	1.108
BM7	283.7	575.0	0.001056	5.587	0.1884	193.5	-3.578	2.885	1.711
BM8	264.3	469.3	0.3916	2.941	0.1746	-144.3	55.88	1.721	2.789
BM9	266.5	461.9	0.3092	2.042	0.1635	142.8	39.98	1.381	3.29
BM10	259.2	399.7	0.2188	0.9312	0.1463	121.2	35.41	1.936	2.159

Including:

- EXP constraints through HiggsTools.
- TH constraints: perturbativity & boundedness from below.



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Including:

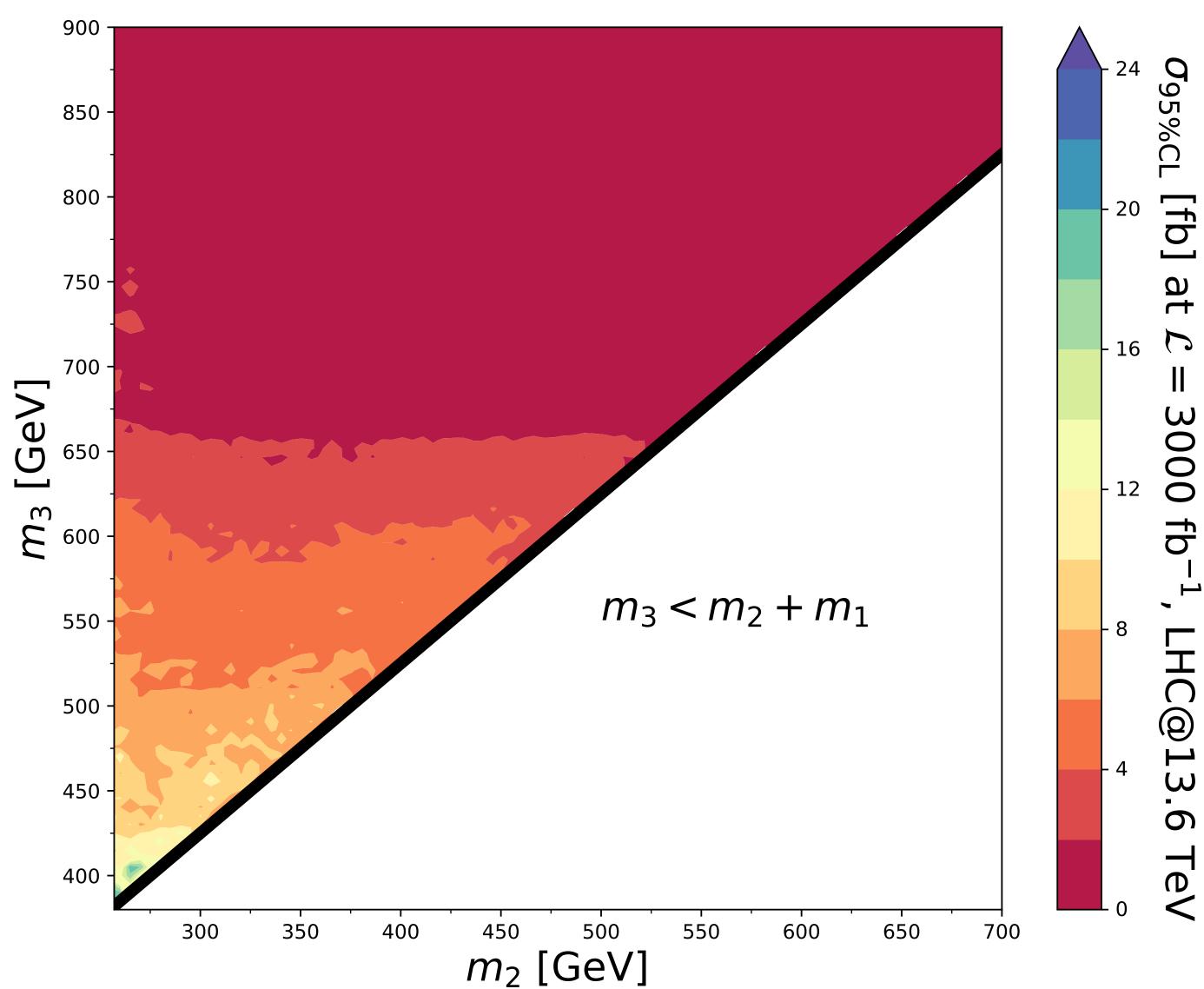
- EXP constraints through HiggsTools.
- TH constraints: perturbativity & boundedness from below.







Constraints on the Cross Section



95% C.L. Constraint on the cross section at HL-LHC.



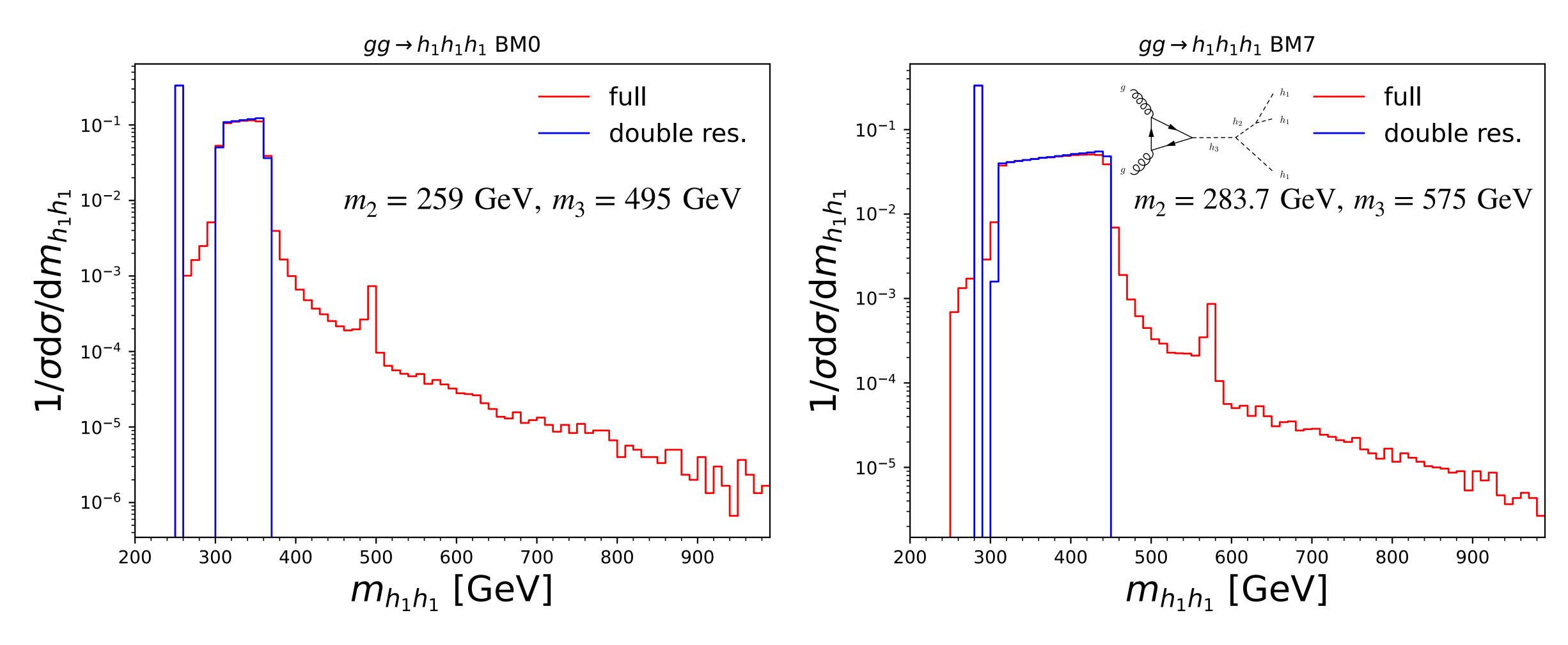
Comment: Validity of the Narrow-Width Approximation

• In the TRSM: h_2 and h_3 are constrained to possess small mixing angles.

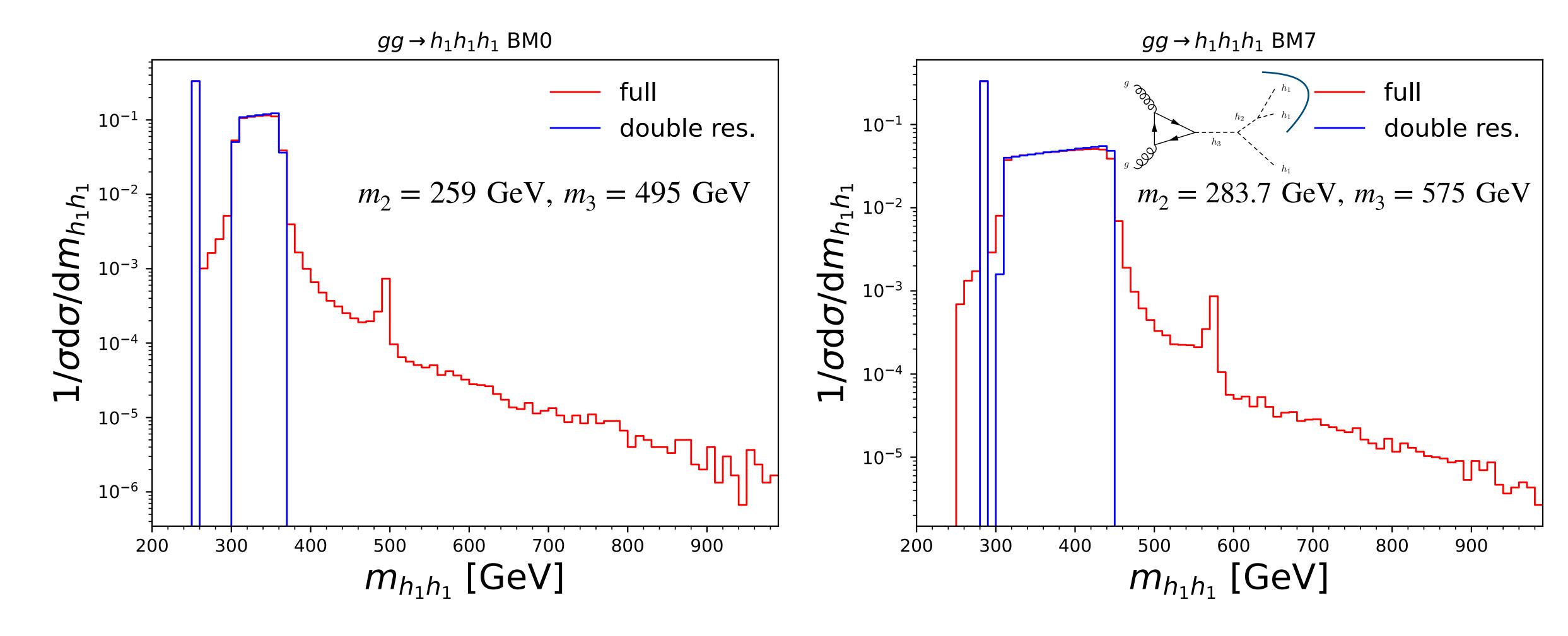
(e.g. we know this from h_1 signal strength)

- \Rightarrow Contributions to width from $h_{2,3} \rightarrow f\bar{f}$, VV are small!
- In general: Can increase width of h_2 and h_3 through scalar-to-scalar decays \Rightarrow no guarantee in generic models for the narrow width.
- If h_2 and h_3 are already discovered, Γ_2 , Γ_3 would be known (or limited) \Rightarrow the narrow-width approximation validity should be checked!
- TL;DR: The narrow width approximation is OK due to mixing constraints in the TRSM, but this statement is somewhat model dependent! [***see appendix for a case study!]

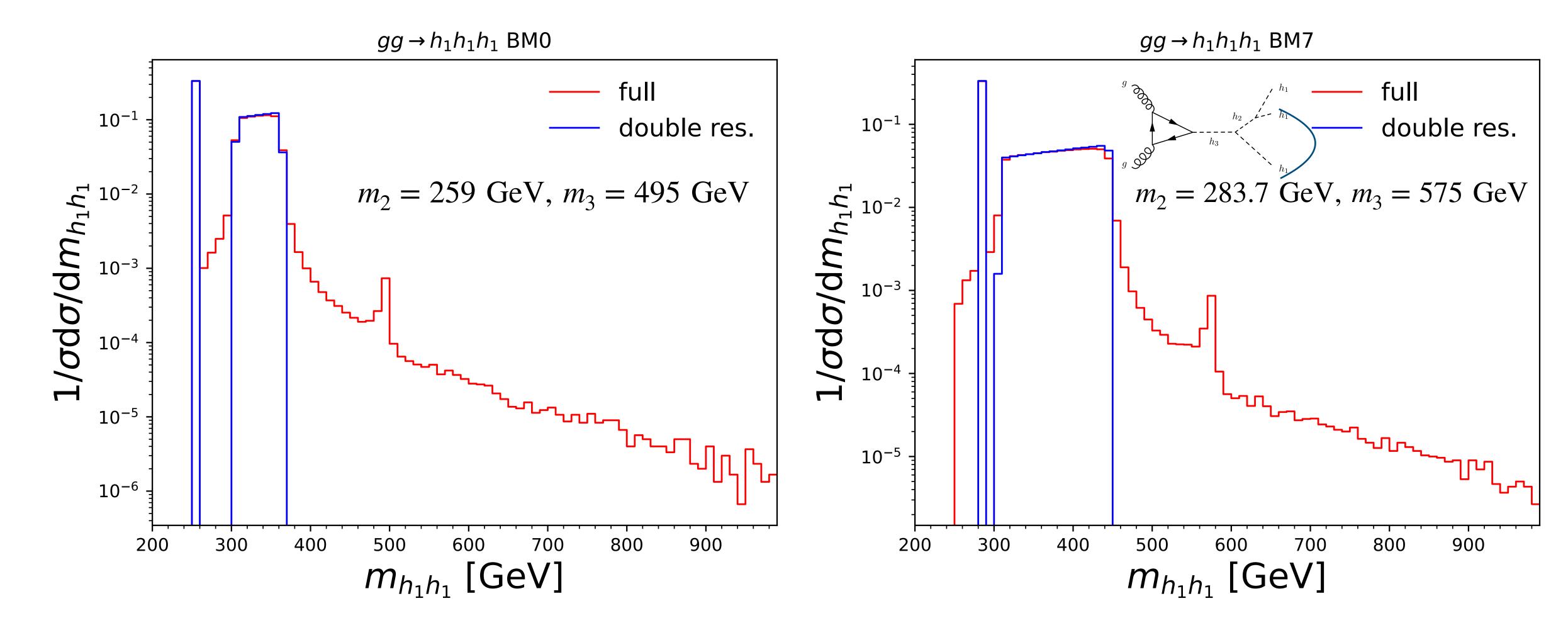




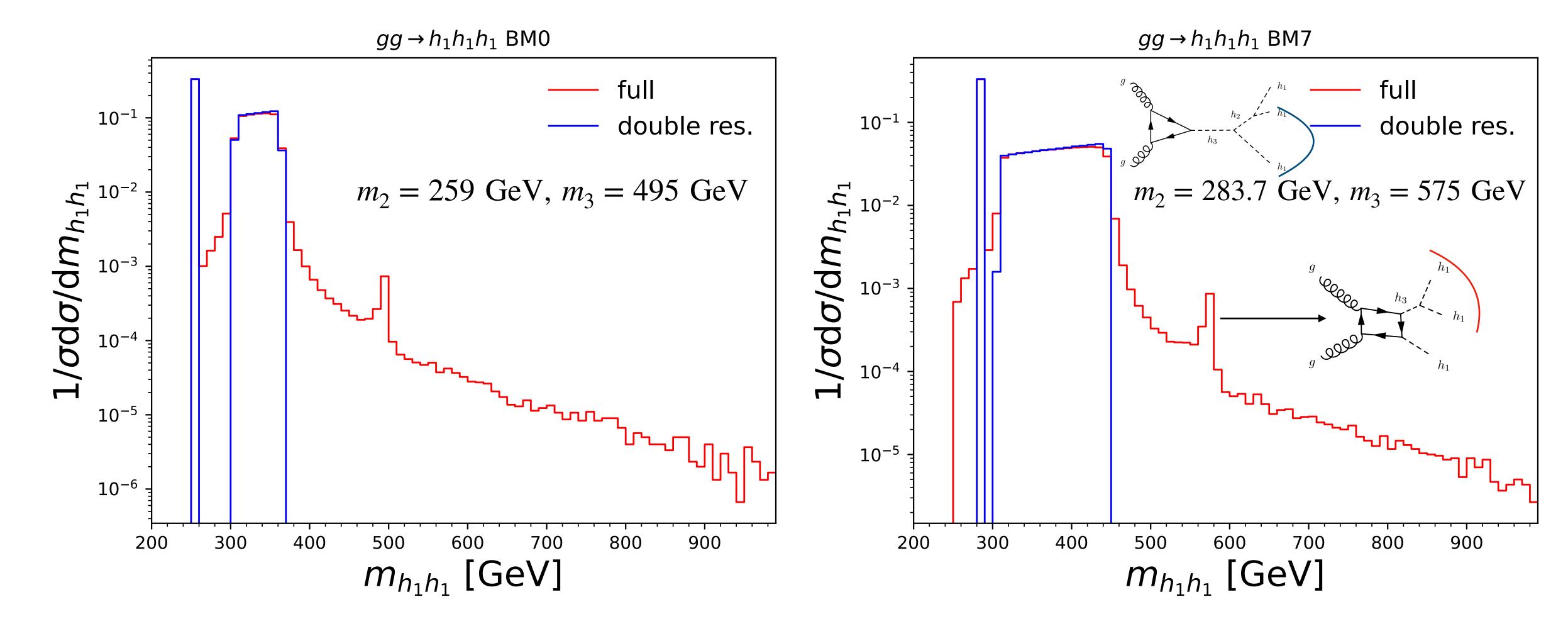






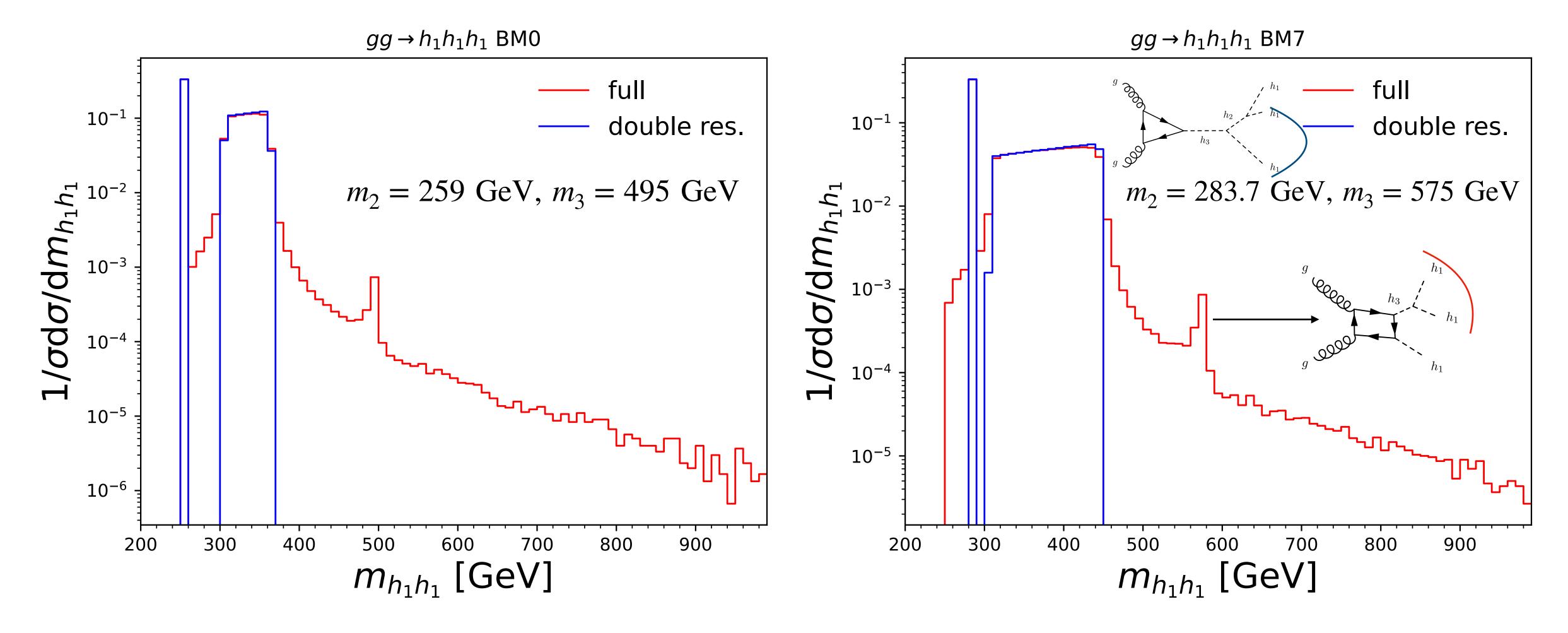








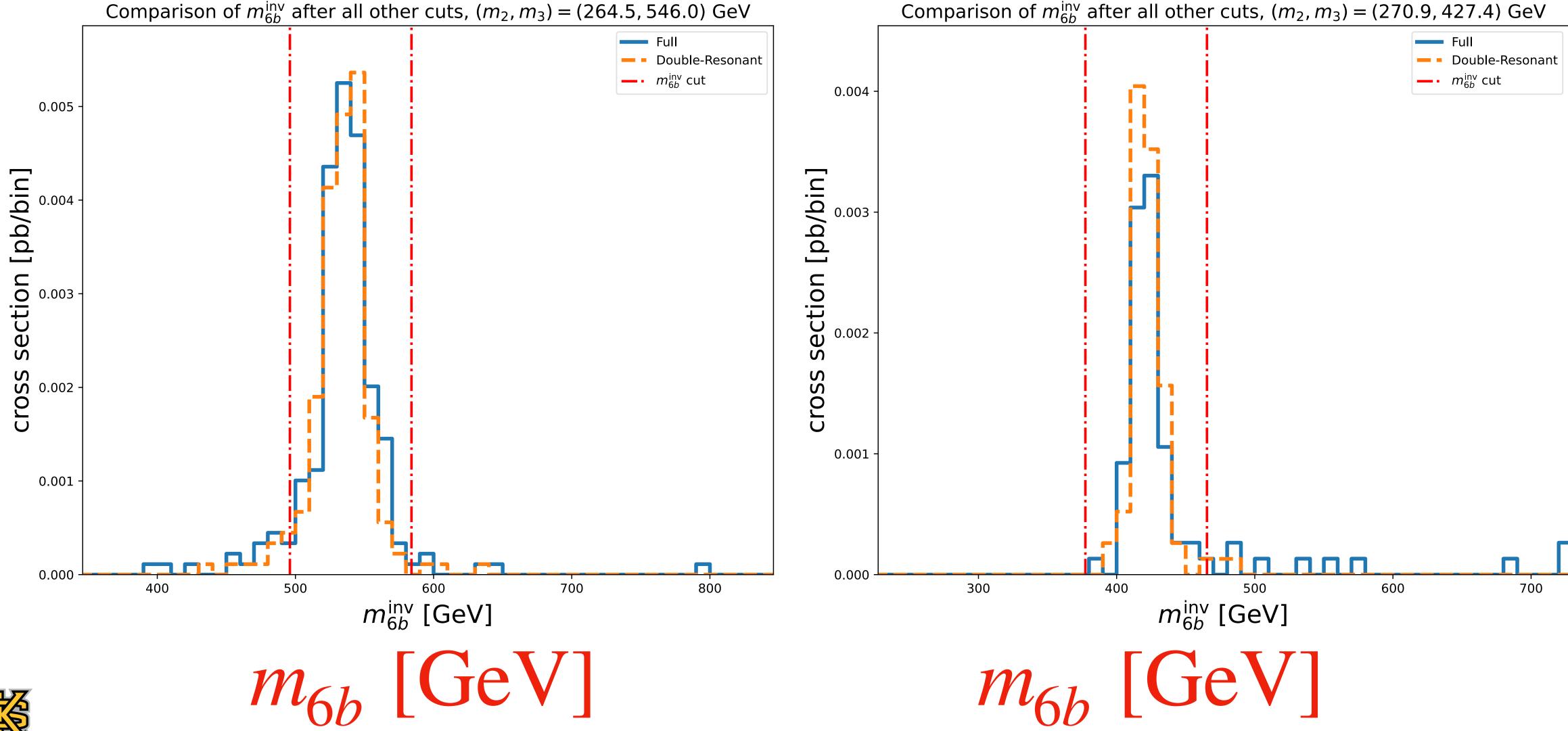
(any) two Higgs invariant mass distributions in *hhh* ("parton level", all combinations)





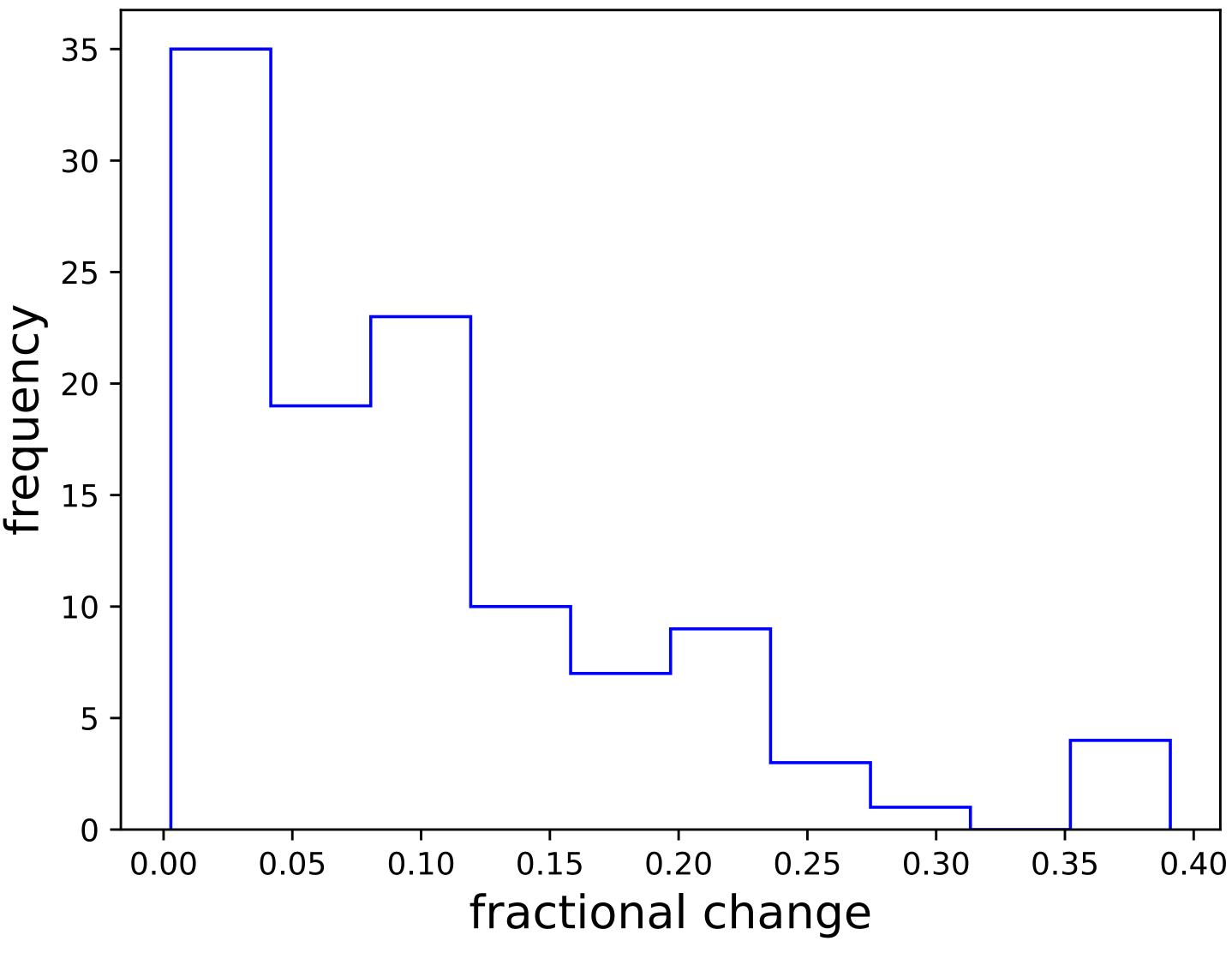
→ Bulk of cross section from double-resonant process.

Sample 6 b-jet invariant mass distributions in our analysis (all other cuts applied):



Fractional change in significance for excluded points

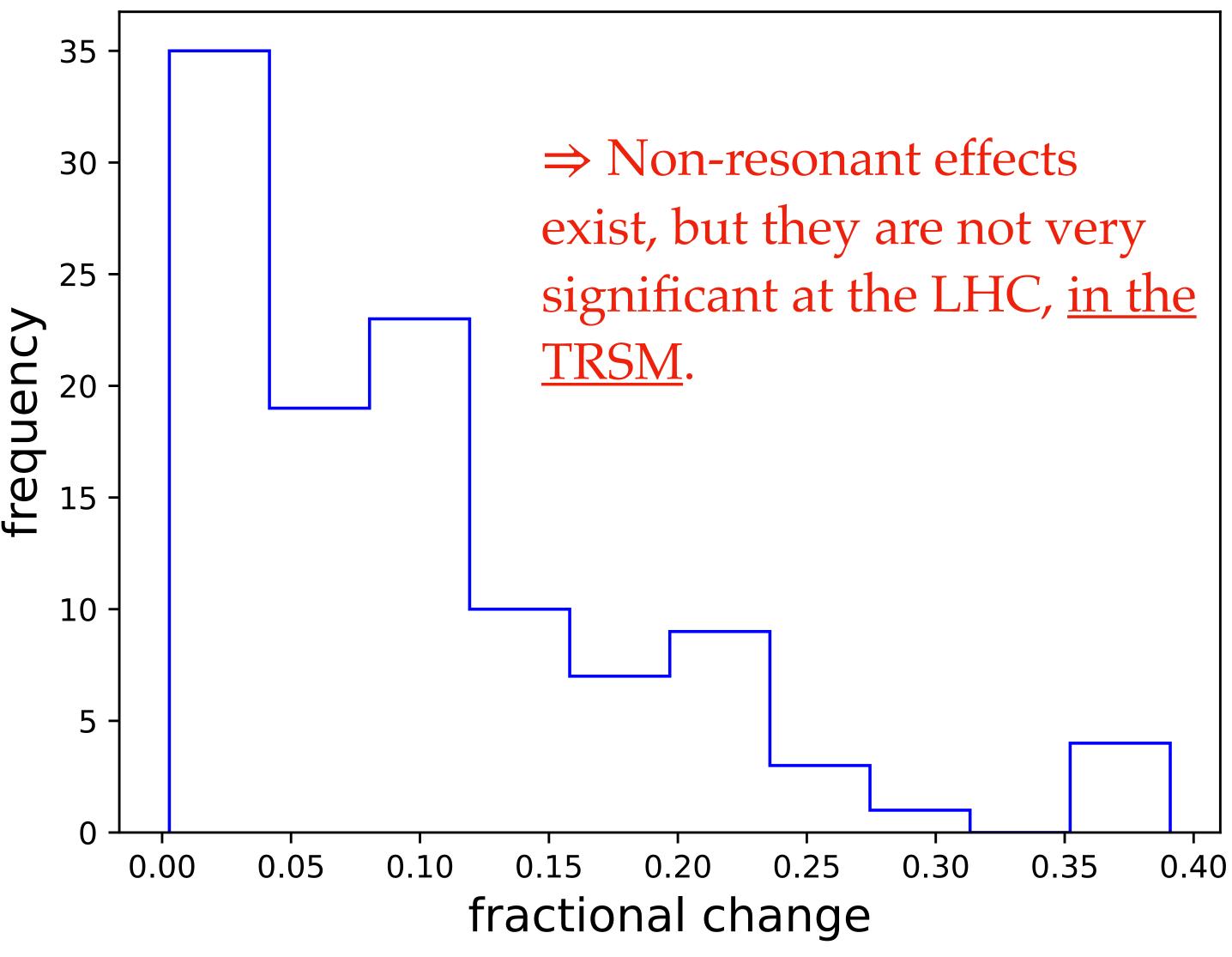
Number of Parameter
Space Points in our
Scan
that can be excluded at
95% C.L.





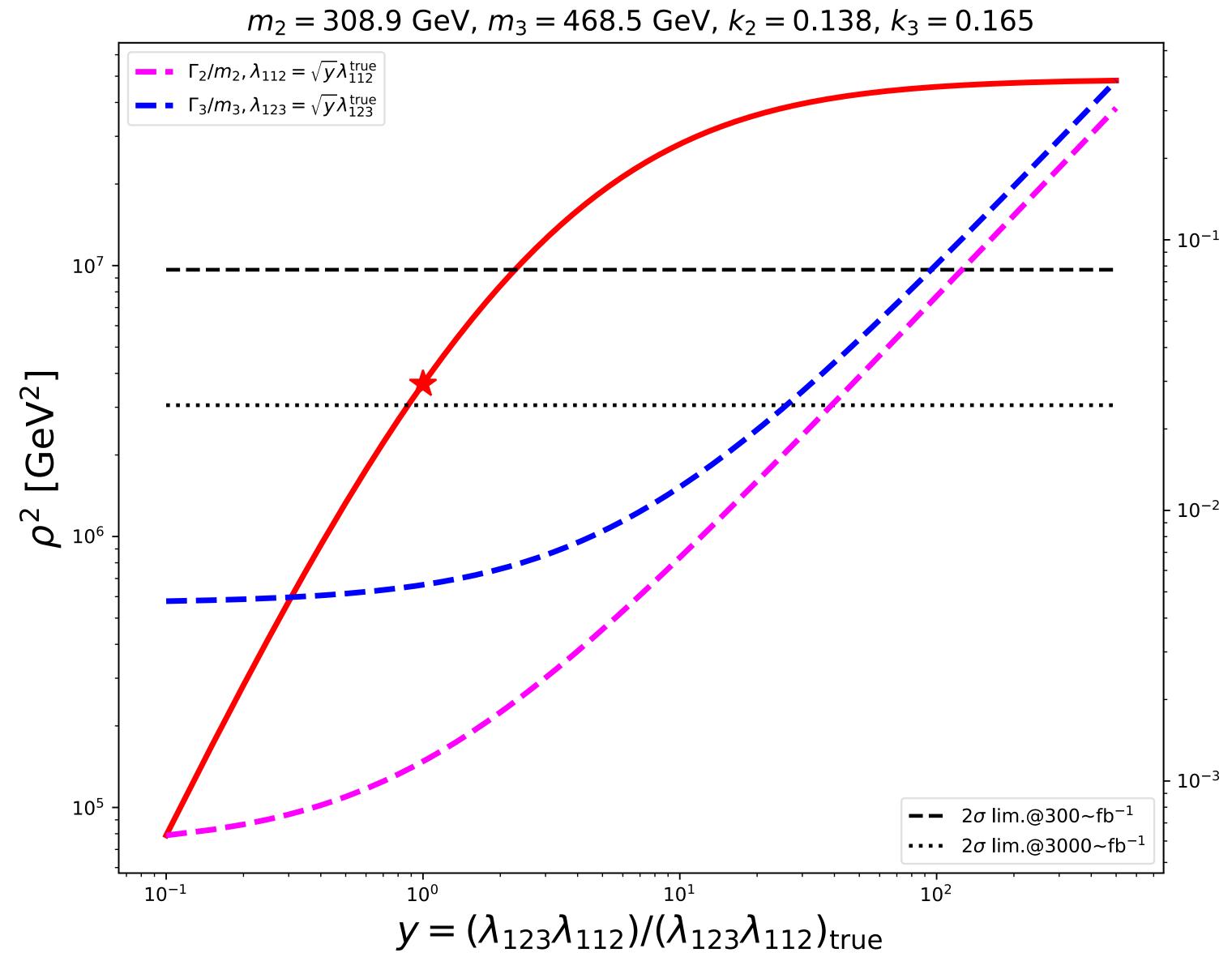
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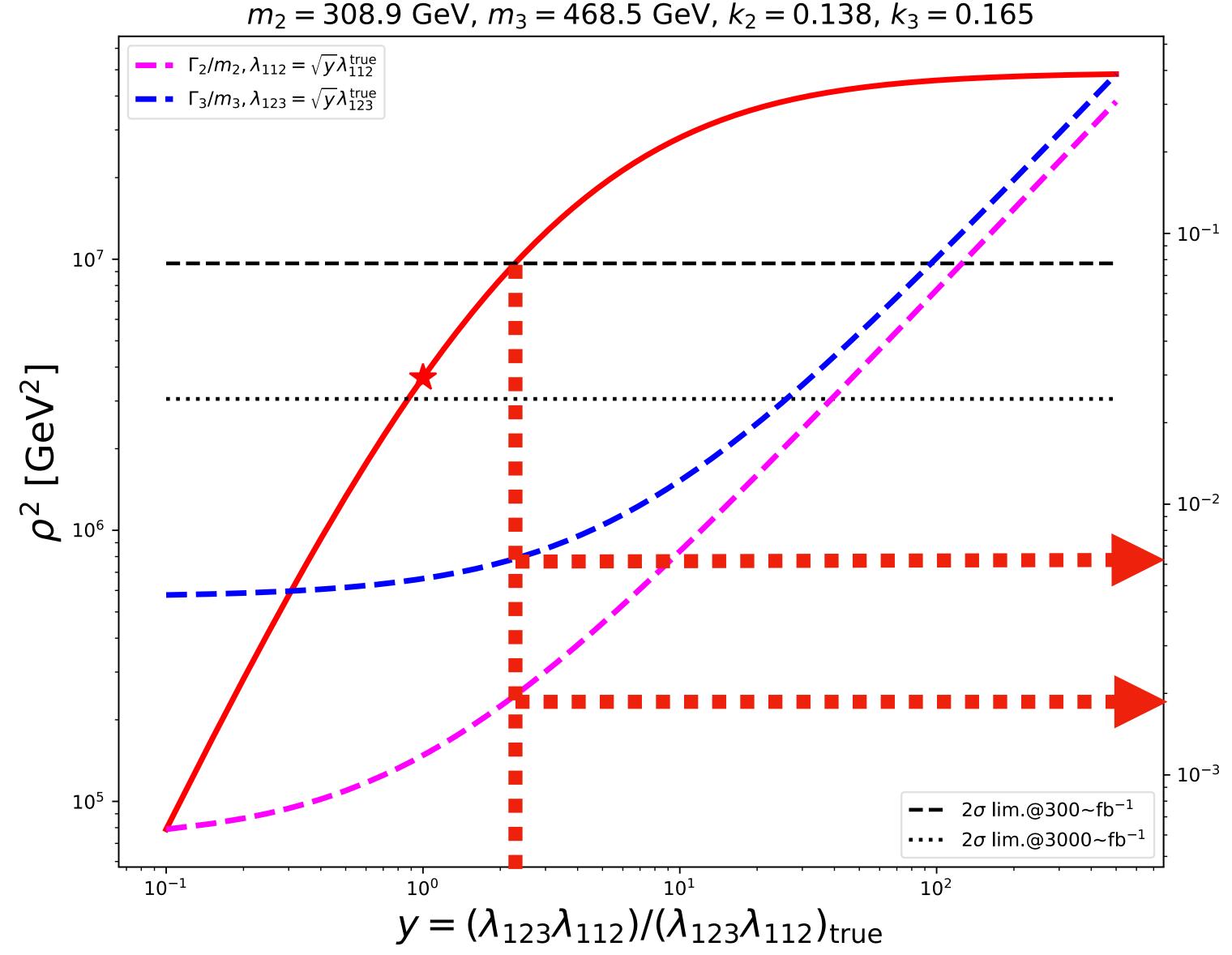
The Narrow Width Approximation: An Example



- Pick a parameter space point, fix: m_2 , m_3 and scalar coupling λ_{113} .
- Rescale: $\lambda_{112} = \sqrt{y} \lambda_{112}^{\text{true}}$ and $\lambda_{123} = \sqrt{y} \lambda_{123}^{\text{true}}$.
- Plot: ρ^2 , Γ_2/m_2 and Γ_3/m_3 versus y.
- Red star = true parameter point value.
- Plot pheno analysis limits (dashed: 300 fb⁻¹, dotted: 3000 fb⁻¹).
- For both limits, NWA approximation is valid!



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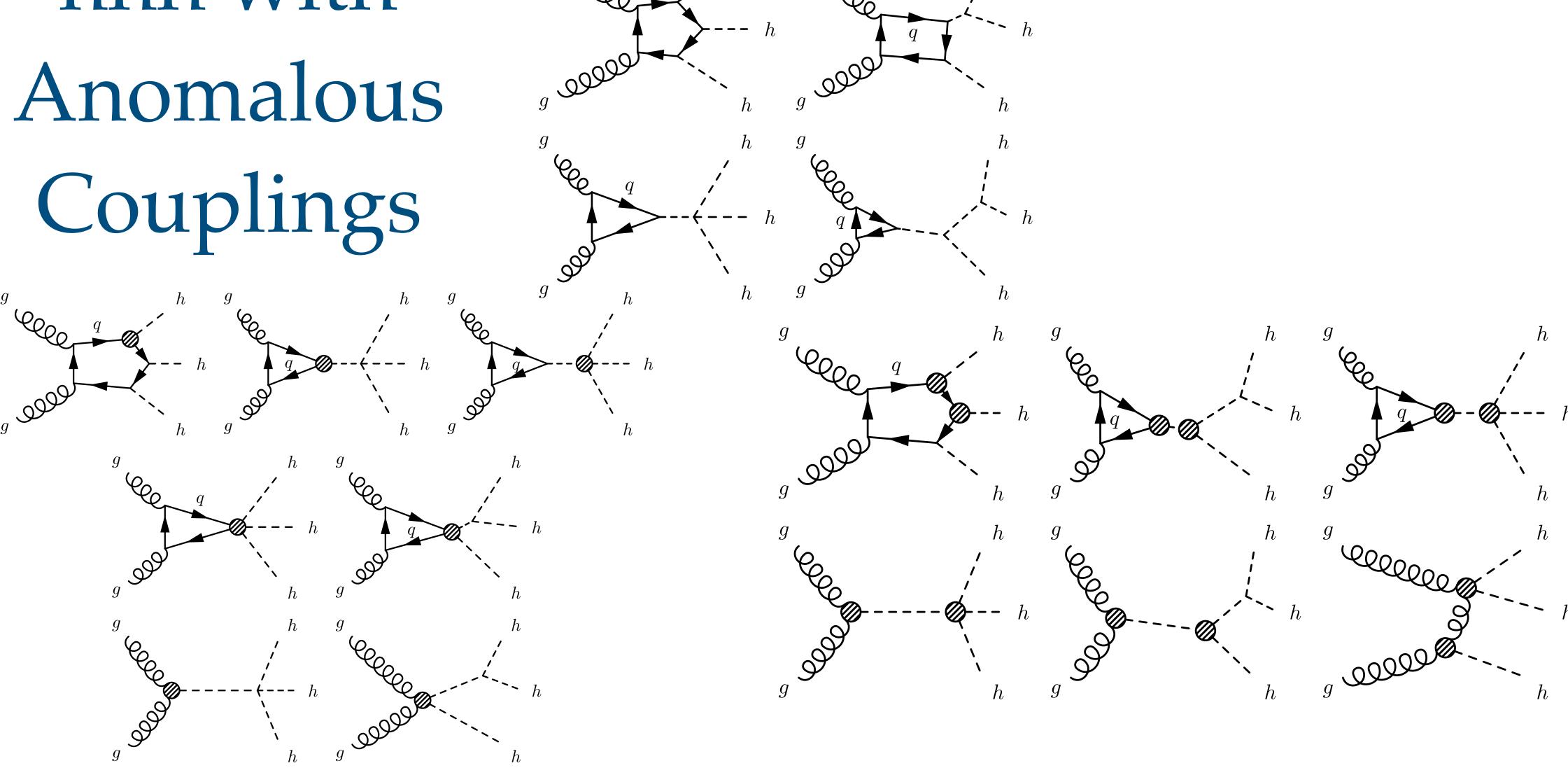


TRSM Monte Carlo Event Generation

- We have implemented a MadGraph5_aMC@NLO (MG5_aMC) "loop" model for the TRSM:
 - MG5_aMC input parameters: the three mixing angles, two masses/widths and all the scalar couplings (only 7 are independent in TRSM).
 - Comes with a **Python script** that:
 - allows conversion of M_2 , M_3 + three mixing angles + two VEVs to the MG5_aMC model input,
 - calculates several single-production cross sections, branching ratios, widths,
 - and writes associated MG5_aMC parameter card (param_card.dat) automatically.
 - Get it at: https://gitlab.com/apapaefs/twosinglet.
 - [AP, Tania Robens, Gilberto Tetlalmatzi-Xolocotzi, arXiv:2101.00037]



hhh with Couplings





hhh: Final states

Assume: K-factor = 2.

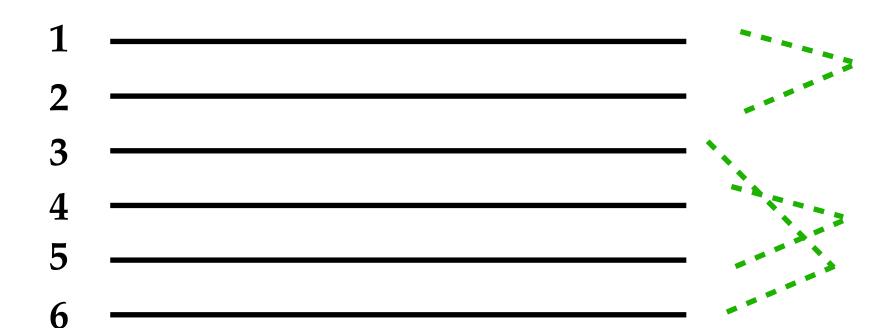
[Maltoni, Vryonidou, Zaro, 1408.6542]

$hhh \rightarrow \text{final state}$	$\mathrm{BR}~(\%)$	$N_{ m 20ab^{-1}}$	
$\overline{(bb)(bb)(bb)}$	19.21	22207	
$(b\overline{b})(b\overline{b})(WW_{1\ell})$	7.20	8328	
$(b \overline{b}) (b \overline{b}) (au ar{ au})$	6.31	• — • •	Fuks, Kim, Lee, 1510.07697,
$(b \overline{b}) (au ar{ au}) (WW_{1\ell})$	1.58	1824 Full	ks, Kim, Lee, 1704.04298.
$(b\overline{b})(b\overline{b})(WW_{2\ell})$	0.98	1128	
$(b\overline{b})(WW_{1\ell})(WW_{1\ell})$	0.90		Kilian, Sun, Yan, Zhao, Zhao,
$(b\overline{b})(\tau\overline{\tau})(\tau\overline{\tau})$	0.69	799	02.03554.
$(b\overline{b})(b\overline{b})(\gamma\gamma)$	0.23		<u>AP</u> , Sakurai, 1508.06524, Chen,
[AP, Sakurai,	1508.06524]	151	n, Zhao, Zhao, Zhong, 10.04013, Fuks, Kim, Lee, 10.07697. Andreas Papaefstathiou

- What can we learn about the anomalous couplings via **hhh** at 13.6 TeV?
- Begin by using the 6 b-jet final state!
- 1. Require 6 tagged b-jets.



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1	
2	
3	
4	
5	
6	



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3. For each pairing construct:

$$\chi^2 = \sum_{qr \in \text{pairings } I} (M_{qr} - m_h^2)^2$$

≡ sum of squared differences from Higgs mass (~125 GeV)



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 \Rightarrow 4. Pairing that gives minimum χ^2 determines "reconstructed Higgs boson".



The 6b final state, analysis

observable

cut

 h^i_r \rightarrow Higgs boson candidates

```
\begin{array}{lll} p_{T,b} & > 45 \ \mathrm{GeV} \\ |\eta_b| & < 3.2 \\ \Delta R_{b,b} & > 0.3 \\ p_T(h_r^i) & > [170,120,0] \ \mathrm{GeV}, \ i = 1,2,3 \\ \chi^2_{\min} & < 17 \ \mathrm{GeV} \\ \Delta m_{\min, \ \mathrm{mid, \ max}} & < 8,8,11 \ \mathrm{GeV} & \qquad \qquad \text{the three terms in } \chi^2_{\min}. \\ \Delta R(h_r^i,h_r^j) & < [3.5,3.5,3.5], \ (i,j) = [(1,2),(1,3),(2,3)] \\ \Delta R_{bb}(h_r^i) & < [3.5,3.5,3.5], \ i = 1,2,3 \end{array}
```



signal/backgrounds after analysis

Process	σ _{GEN} (pb)	$\sigma_{\rm NLO} \times {\rm BR} \ ({\rm pb})$	$oldsymbol{arepsilon}_{ ext{analysis}}$	$N_{20 \text{ ab}^{-1}}^{\mathrm{cuts}}$
hhh (SM)	2.88×10^{-3}	1.06×10^{-3}	0.0131	278
$\overline{ ext{QCD}\ (bar{b})(bar{b})(bar{b})}$	26.15	52.30	2.6×10^{-5}	27116
$qar{q} ightarrow hZZ ightarrow h(bar{b})(bar{b})$	8.77×10^{-4}	4.99×10^{-4}	1.8×10^{-4}	~ 2
qar q o ZZZ o (bar b)(bar b)	7.95×10^{-4}	7.95×10^{-4}	1.2×10^{-5}	< 1
ggF $hZZ ightarrow h(bar{b})(bar{b})$	1.08×10^{-4}	1.23×10^{-4}	$\mathcal{O}(10^{-3})$	~ 2
ggF $ZZZ ightarrow (bar{b})(bar{b})$	1.36×10^{-5}	2.73×10^{-5}	2×10^{-5}	≪ 1
$h(bar{b})(bar{b})$	1.46×10^{-2}	1.66×10^{-2}	5.4×10^{-4}	179
$hh(bar{ar{b}})$	1.40×10^{-4}	9.11×10^{-5}	2.8×10^{-4}	~ 1
$hhZ \stackrel{\cdot}{ o} hh(bar{b})$	4.99×10^{-3}	1.61×10^{-3}	7.2×10^{-4}	23
$hZ(bar{b}) ightarrow h(bar{b})(bar{b})$	9.08×10^{-3}	1.03×10^{-2}	1.4×10^{-4}	29
$ZZ(bar{b}) ightarrow (bar{b})(bar{b})(bar{b})$	2.87×10^{-2}	5.74×10^{-2}	1×10^{-5}	11
$Z(bar{b})(bar{b}) ightarrow (bar{b})(bar{b})(bar{b})$	0.93	1.87	3×10^{-5}	1121
Σ backgrounds				2.8×10^{4}



Reducible backgrounds

process	σ _{GEN} (pb)	$\sigma_{\text{GEN}} \times \mathscr{P}(6 \ b - \text{jets}) \text{ (pb)}$
$(bar{b})(bar{b})(car{c})$	76.8	0.768
$(bar{b})(car{c})(car{c})$	75.6	0.00756
$(c\bar{c})(c\bar{c})(c\bar{c})$	22.5	22.5×10^{-5}
$(bar{b})(bar{b})(jj)$	1.32×10^{4}	1.32
$(bar{b})(jj)(jj)$	9.79×19^{5}	0.00979
(jj)(jj)(jj)	1.37×10^6	1.37×10^{-6}

c.f. $\sigma_{GEN}(6b) = 26.15 \text{ pb}$



$$\mathcal{P}_{j \to b} = 0.1$$

$$\mathcal{P}_{j \to b} = 0.01$$

⇒ Assuming perfect b-tagging +
identical analysis efficiency to QCD 6b:

→~10% contribution from reducible backgrounds.

for P(b-tagging) = 0.8:

 \rightarrow ~30% contribution.



TRSM hhh — 6b analysis details

Introduce two observables:
$$\chi^{2,(4)} = \sum_{qr \in I} \left(M_{qr} - M_1 \right)^2$$

$$\chi^{2,(6)} = \sum_{qr \in J} \left(M_{qr} - M_1 \right)^2$$

 \rightarrow constructed from different pairings of 4 and 6 b-tagged jets, M_{qr} is the invariant mass of the pairing qr.



Monte Carlo Implementation of Anomalous Couplings

- Get the MG5_aMC model at: https://gitlab.com/apapaefs/multihiggs_loop_sm.
- [A <u>patch</u> to MG5_aMC to enable Loop × Tree is included].
- Can generate events either at:
 - SM^2 + interference of [SM × One-Insertion diagrams], i.e.: $|\mathcal{M}|^2 = |\mathcal{M}_{SM}|^2 + 2\text{Re}\{\mathcal{M}_{SM}^*\mathcal{M}_{1-\text{ins.}}\} \propto 1 + c_i$

$$|\mathcal{M}|^2 = |\mathcal{M}_{SM}|^2 + 2\text{Re}\{\mathcal{M}_{SM}^*\mathcal{M}_{1-\text{ins.}}\} \propto 1 + c_i$$

or

• SM² + interference of [SM \times One or Two insertion diagrams] + [One Insertion]^2, i.e.:

$$|\mathcal{M}|^2 = |\mathcal{M}_{SM}|^2 + 2\text{Re}\{\mathcal{M}_{SM}^* \mathcal{M}_{1-\text{ins.}}\} + 2\text{Re}\{\mathcal{M}_{SM}^* \mathcal{M}_{2-\text{ins.}}\} + |\mathcal{M}_{1-\text{ins.}}|^2$$

$$\propto 1 + c_i + c_j c_k + c_\ell^2$$

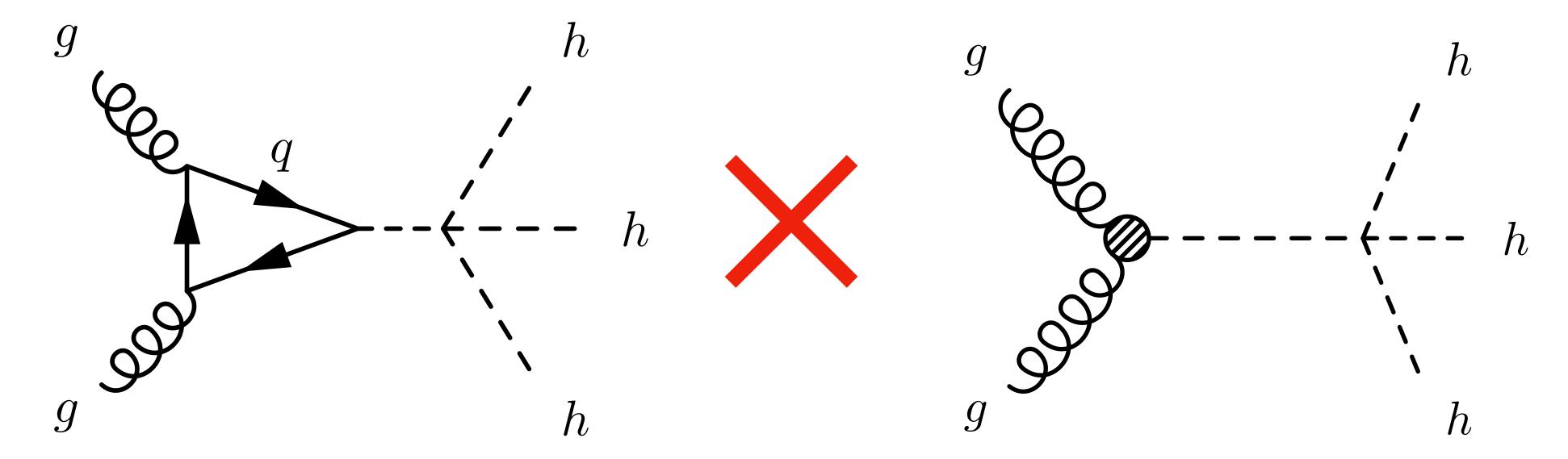


Monte Carlo Implementation of Anomalous Couplings

- We have implemented a MadGraph5_aMC@NLO "loop" model for $\mathscr{L}_{PhenoExp}$.
- Includes Loop × Tree level interference between the various diagrams.

[see: Hirschi, https://cp3.irmp.ucl.ac.be/projects/madgraph/wiki/LoopInducedTimesTree].

• e.g.:

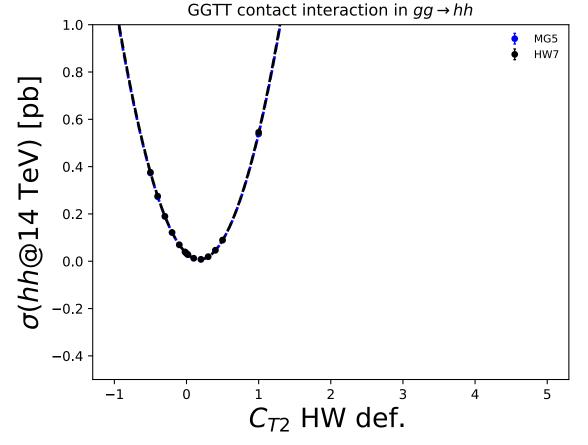




Model Validation

[AP, Tetlalmatzi-Xolocotzi, arXiv:2312.13562]

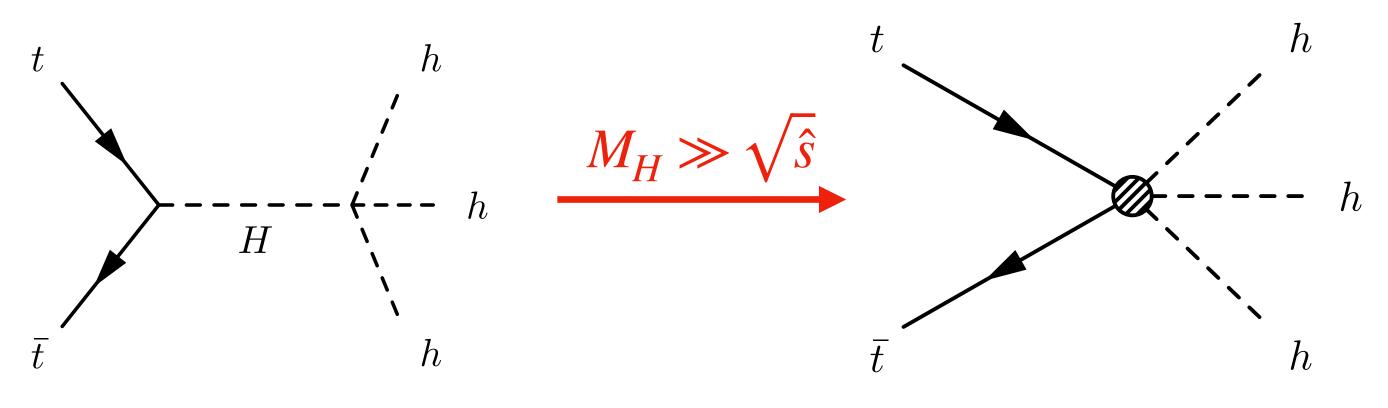
• Most couplings validated vs. a **Herwig** $7 pp \rightarrow hh$ implementation, e.g.:

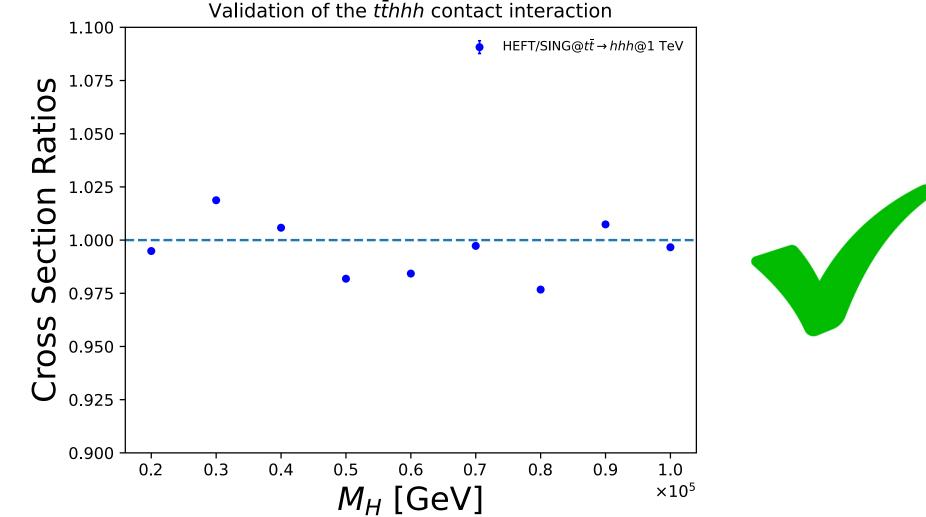




• The one "new" non-trivial coupling that appears, $\propto c_{t3} t\bar{t}h^3$ has been validated

via an "EFT" limit, in the $t\bar{t} \rightarrow hhh$ process:



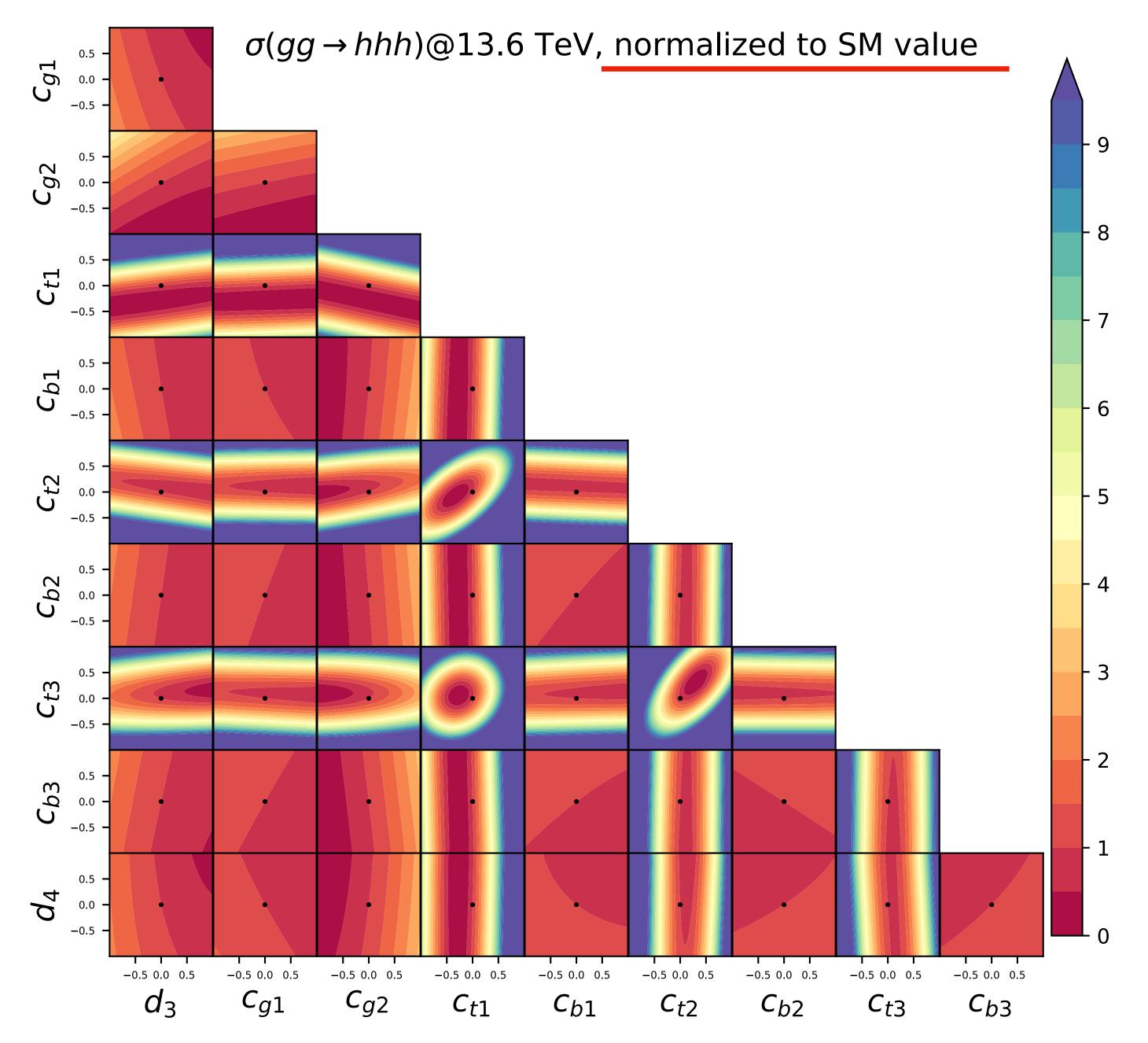




hhh Cross Sections @ 13.6 TeV

[AP, Tetlalmatzi-Xolocotzi, arXiv:2312.13562]

- Cross section as a multiple of the SM
- $(\sigma_{SM} \sim 0.04 \text{ fb at LO@13.6 TeV}).$
- In each 2D panel shown: all other coefficients set to zero!





Anomalous Couplings Constraints

- Other processes constrain (at LO) all coefficients except $\{c_{t3}, d_4\}$ (only in hhh).
- Projected constraints:

Percentage uncertainties						
	HL-LHC	FCC-hh	Ref.			
$\delta(d_3)$	50	5	[145] (table 12)			
$\delta(c_{g1})$	2.3	0.49	[145] (table 3)			
$\delta(c_{g2})$	5	1	[140] (Figure 12, right)			
$\delta(c_{t1})$	3.3	1.0	[145] (table 3)			
$\delta(c_{t2})$	30	10	[140] (Figure 12, right)			
$\delta(c_{b1})$	3.6	0.43	[145] (table 3)			
$\delta(c_{b2})$	30	10	assumed same as c_{t2}			

[See AP, Tetlalmatzi-Xolocotzi, arXiv:2312.13562 for the references]



Digression: Non-Resonant hhh @ LHC

• Anomalous couplings can enhance *hhh*! e.g. $\kappa_3 \neq 1, \kappa_4 \neq 1$, but also others!

$$\begin{split} \mathcal{L}_{\text{PhenoExp}} \supset -\lambda_{\text{SM}} v \left(1 + c_{3}\right) h^{3} - \frac{\lambda_{\text{SM}}}{4} \left(1 + d_{4}\right) h^{4} \\ + \frac{\alpha_{s}}{12\pi} \left(c_{g1} \frac{h}{v} - c_{g2} \frac{h^{2}}{2v^{2}}\right) G_{\mu\nu}^{a} G_{a}^{\mu\nu} \\ - \left[\frac{m_{t}}{v} \left(1 + c_{t1}\right) \bar{t}_{L} t_{R} h + \frac{m_{b}}{v} \left(1 + c_{b1}\right) \bar{b}_{L} b_{R} h + \text{h.c.}\right] \\ - \left[\frac{m_{t}}{v^{2}} c_{t2} \bar{t}_{L} t_{R} h^{2} + \frac{m_{b}}{v^{2}} c_{b2} \bar{b}_{L} b_{R} h^{2} + \text{h.c.}\right] \\ - \left[\frac{m_{t}}{v^{3}} \left(\frac{c_{t3}}{2}\right) \bar{t}_{L} t_{R} h^{3} + \frac{m_{b}}{v^{3}} \left(\frac{c_{b3}}{2}\right) \bar{b}_{L} b_{R} h^{3} + \text{h.c.}\right] \end{split}$$

[AP, Tetlalmatzi-Xolocotzi, arXiv:2312.13562]

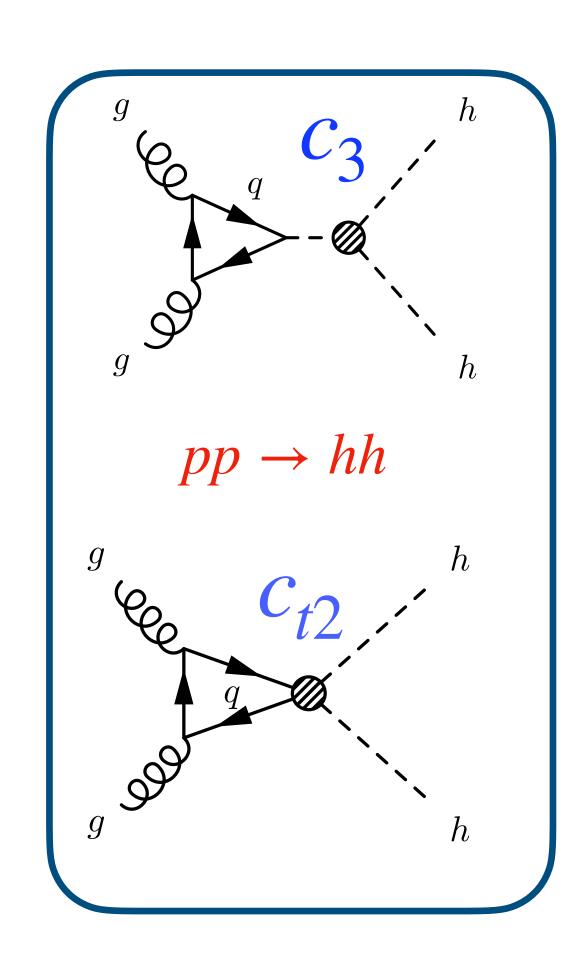
MadGraph5_aMC model at: https://gitlab.com/apapaefs/multihiggs_loop_sm



→ Here: Examine models with new scalar resonances!

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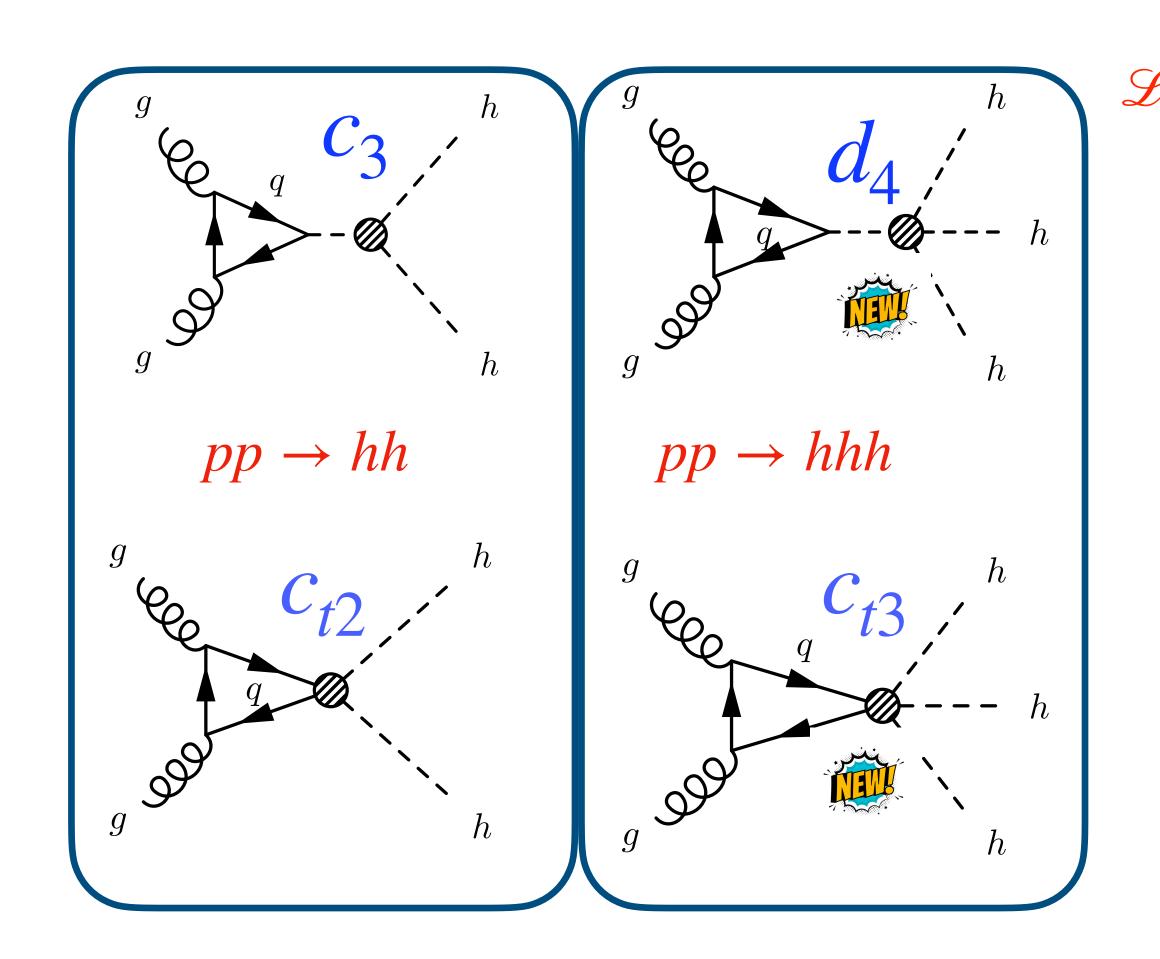
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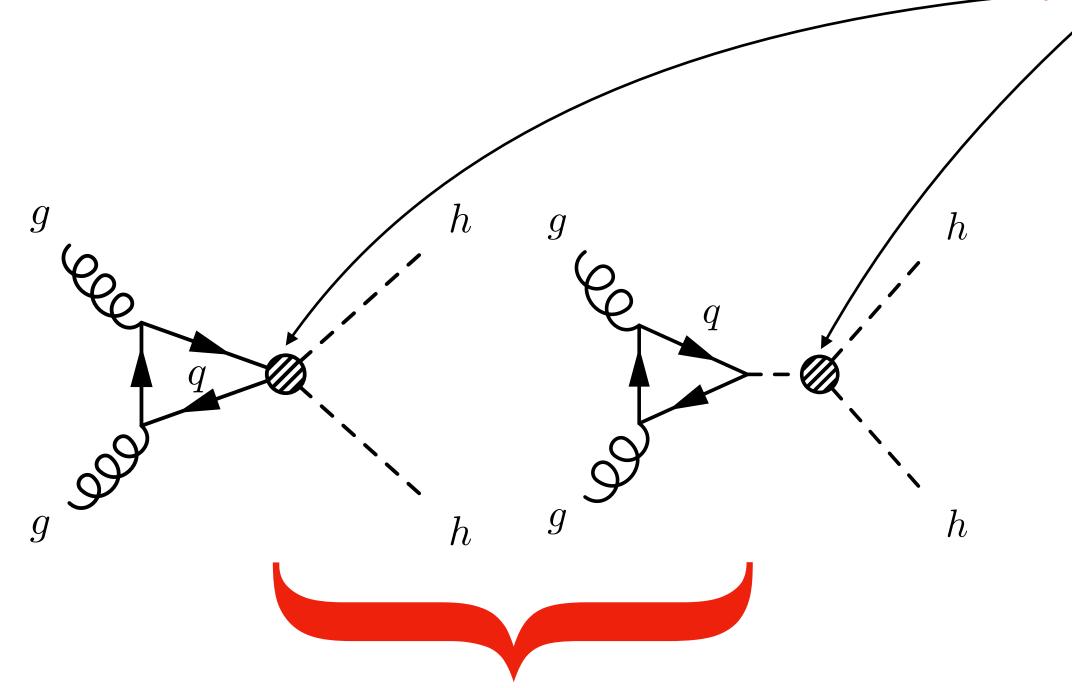
→ Here: Examine models with new scalar resonances!

• Focusing on a model with non-zero $\{c_{t2}, d_3, c_{t3}, d_4\}$:



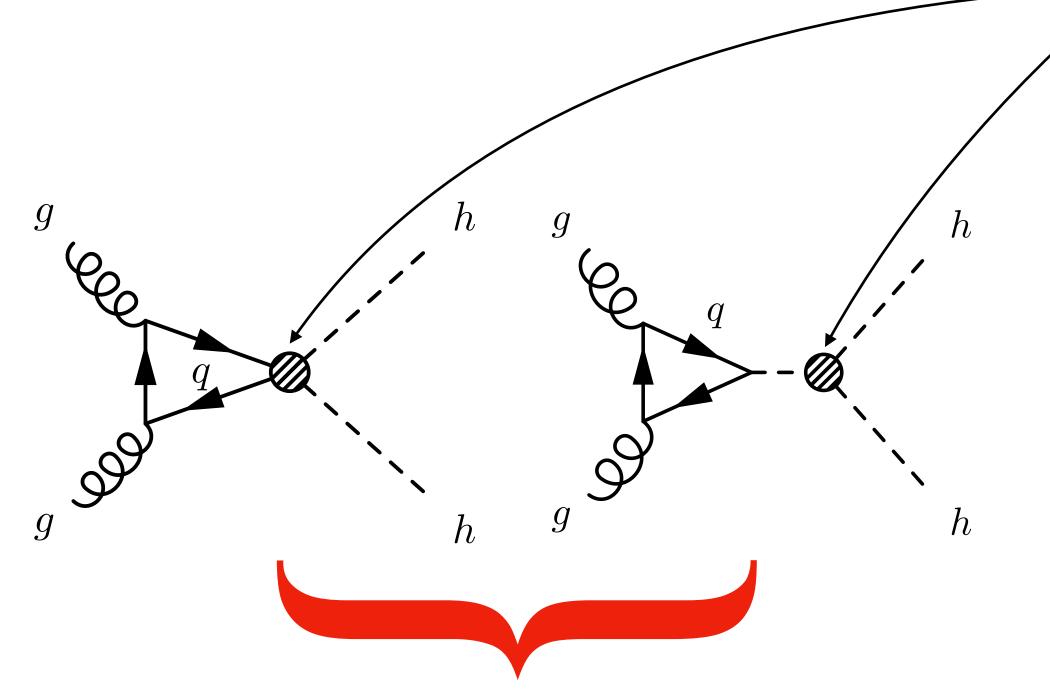
Anomalous Couplings Constraints

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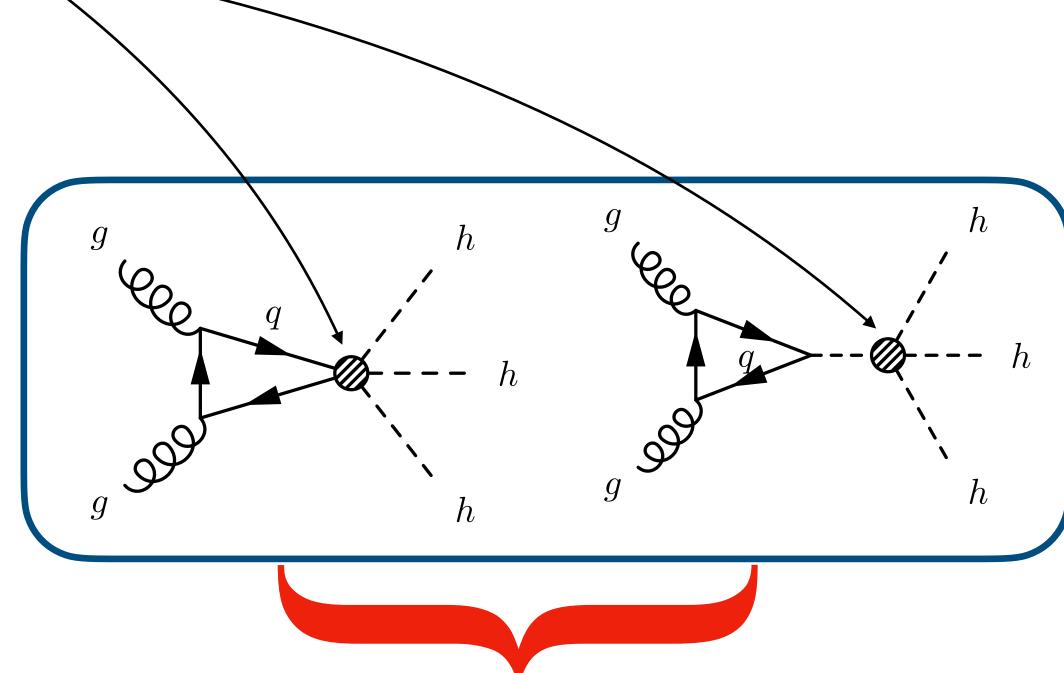


constrained by $pp \rightarrow hh$

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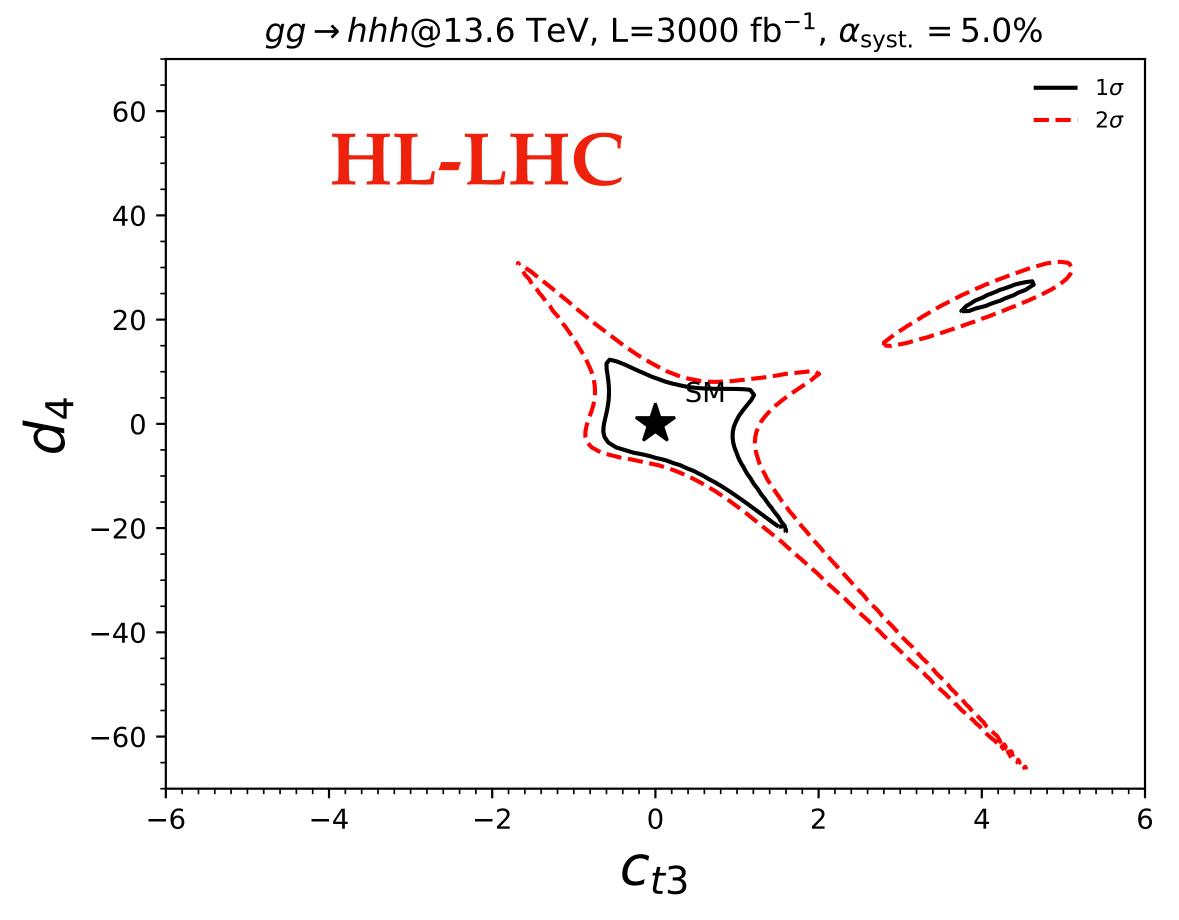


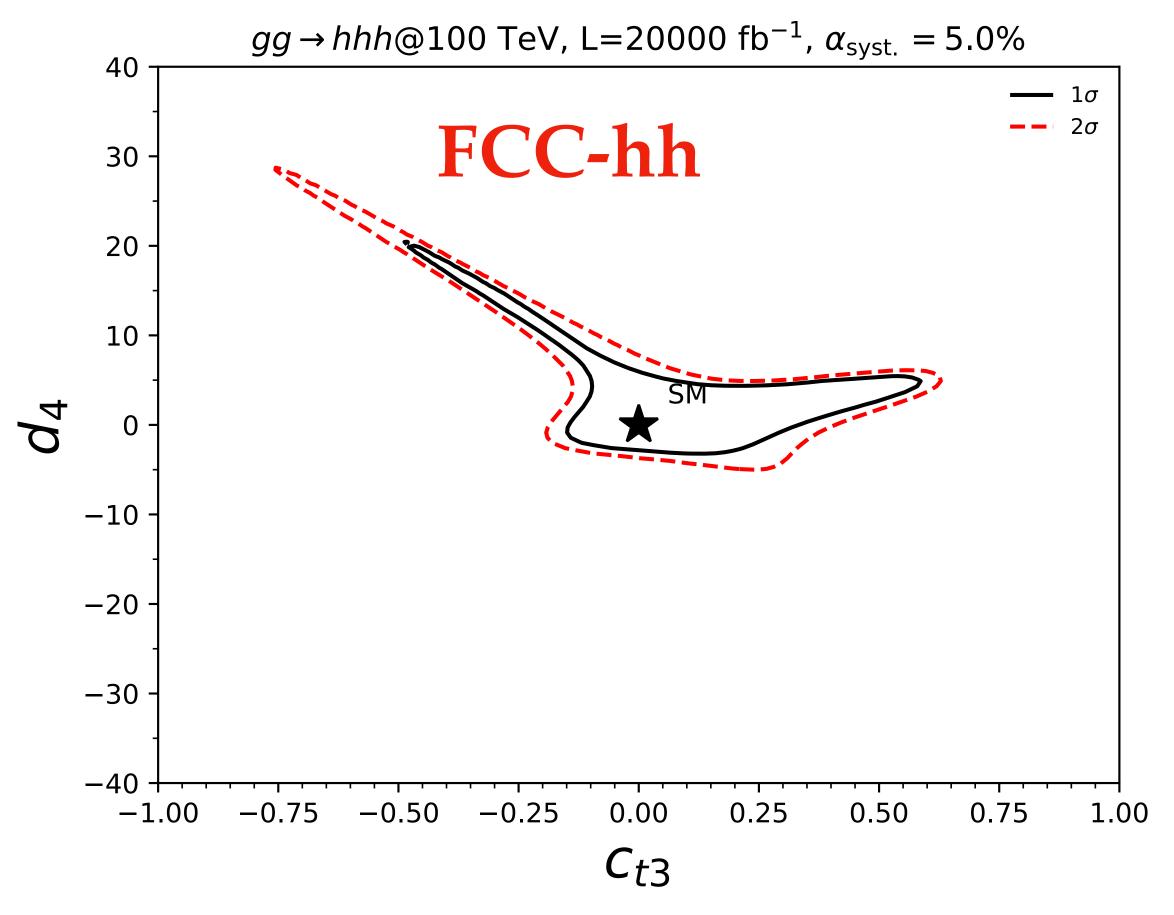


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Anomalous Couplings Constraints

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	HL-LHC 68%	HL-LHC 95%	FCC-hh 68%	FCC-hh 95%
d_4	[-6.6, 12.4]	[-10.0, 21.3]	[-3.9, 10.5]	[-10.6, 18.8]
c_{t3}	[-0.6, 1.1]	[-0.9, 3.6]	[-0.1, 0.3]	[-0.4, 0.6]

