Multi-Higgs Boson Production at Current and Future Colliders

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Kennesaw State University



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The Plan

01 The Higgs Potential: A Brief Introduction

02 The Higgs Potential: Electroweak Baryogenesis

03 Probing the Scalar Sector: Multi-Higgs Boson Production

04 Conclusions

SECTION 01

The Higgs Potential: A Brief Introduction

The Higgs Field's Potential

SM: $\phi \rightarrow$ complex doublet scalar field $\longrightarrow \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$

Potential: $\mathcal{V}(\phi) = \frac{1}{2}\mu^2 |\phi|^2 + \frac{1}{4}\lambda |\phi|^4$

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For $\lambda > 0$ and $\mu^2 > 0 \Rightarrow$ Minimum **at** origin,

or: $\mu^2 < 0 \Rightarrow$ Minima ***away*** from origin

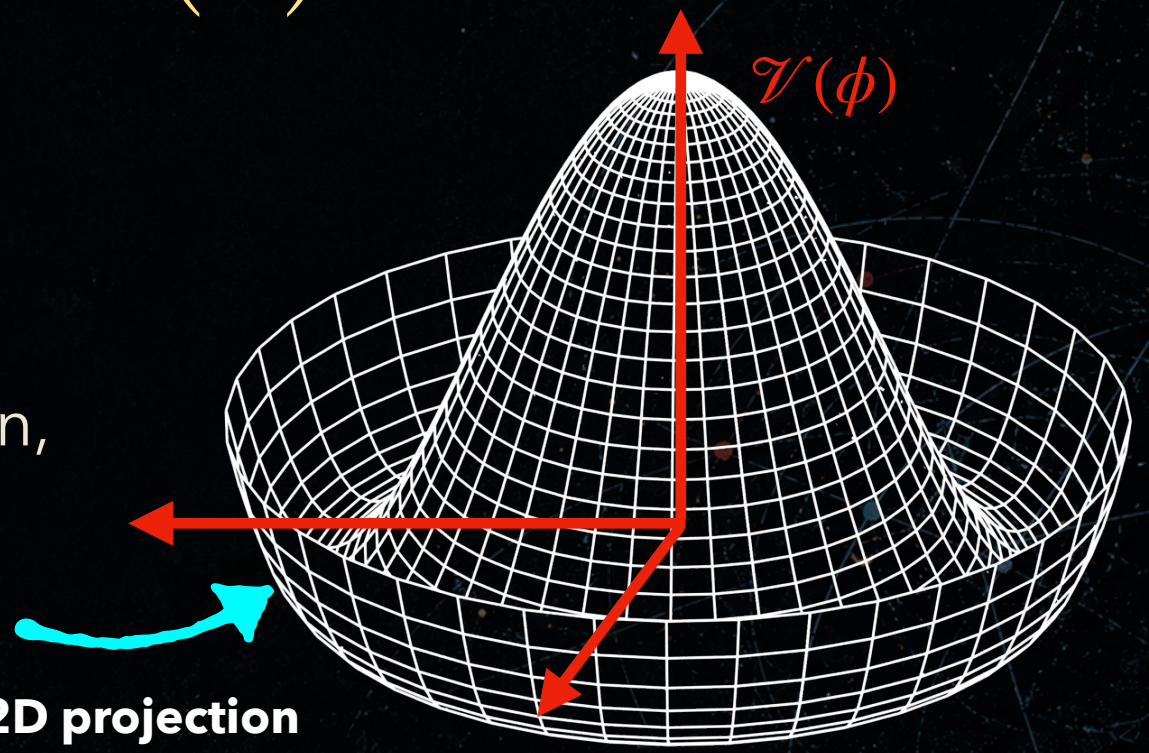
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The Higgs Boson's Potential

Higgs doublet ϕ acquires **Vacuum Expectation Value (VEV)**: $\phi \sim \langle\phi\rangle + h \longrightarrow h$ is the **Higgs boson**!

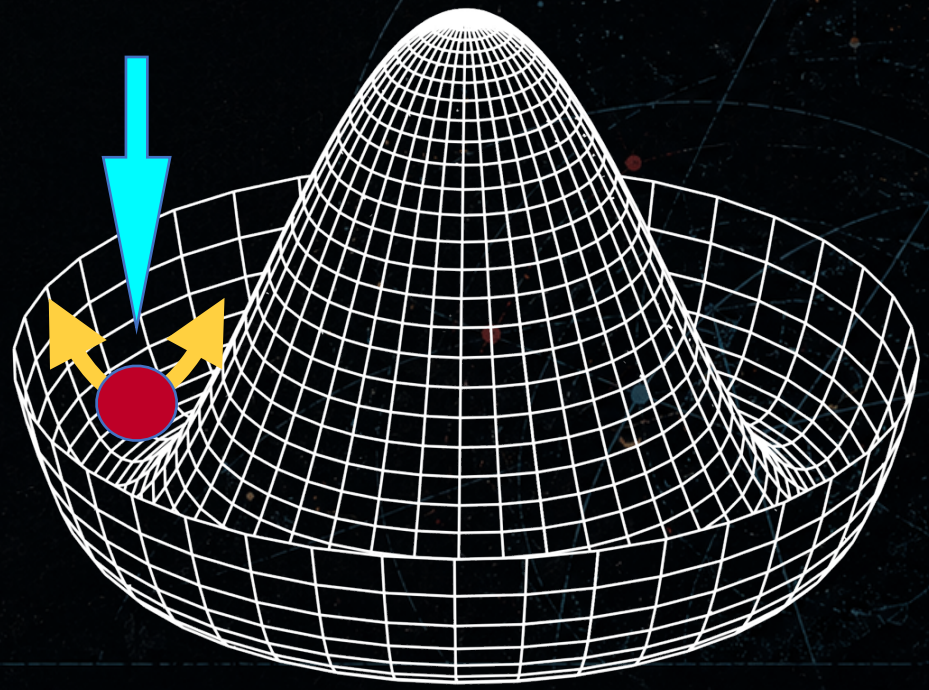
$$\Rightarrow \mathscr{V}\left(\langle\phi\rangle + h\right) = \frac{1}{2}m_h^2 h^2 + \frac{m_h^2}{2v}h^3 + \frac{m_h^2}{8v^2}h^4$$

where: $\langle\phi\rangle = v/\sqrt{2}$ and $v^2 = -\mu^2/\lambda \rightarrow v$ is **known** from low-energy measurements!

[& Gauge boson & Fermion masses!]

ElectroWeak Symmetry Breaking

Life takes place about one of the degenerate minima:



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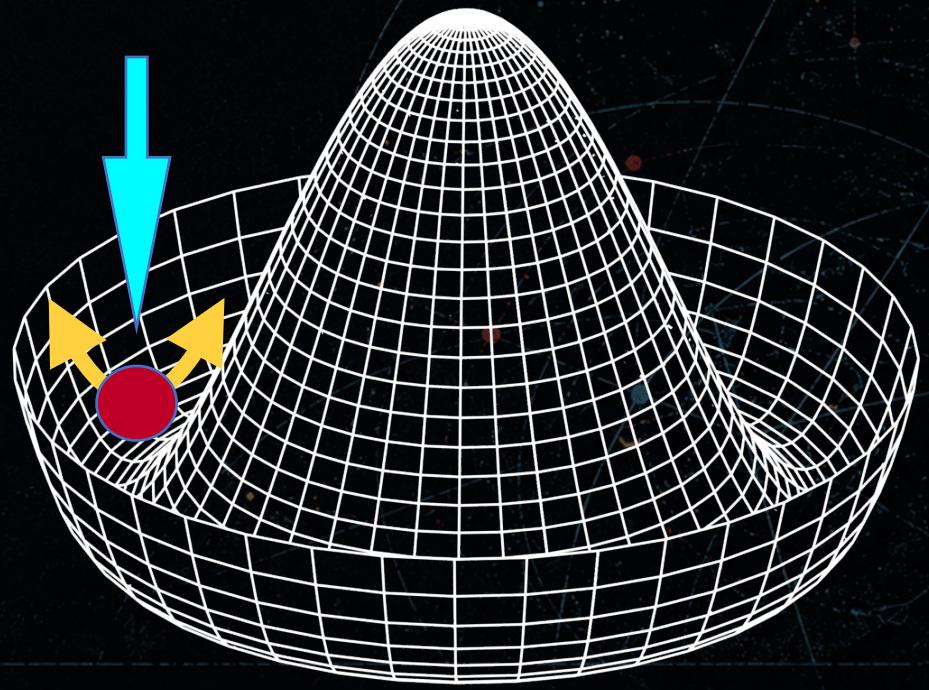
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SM **predictions**!

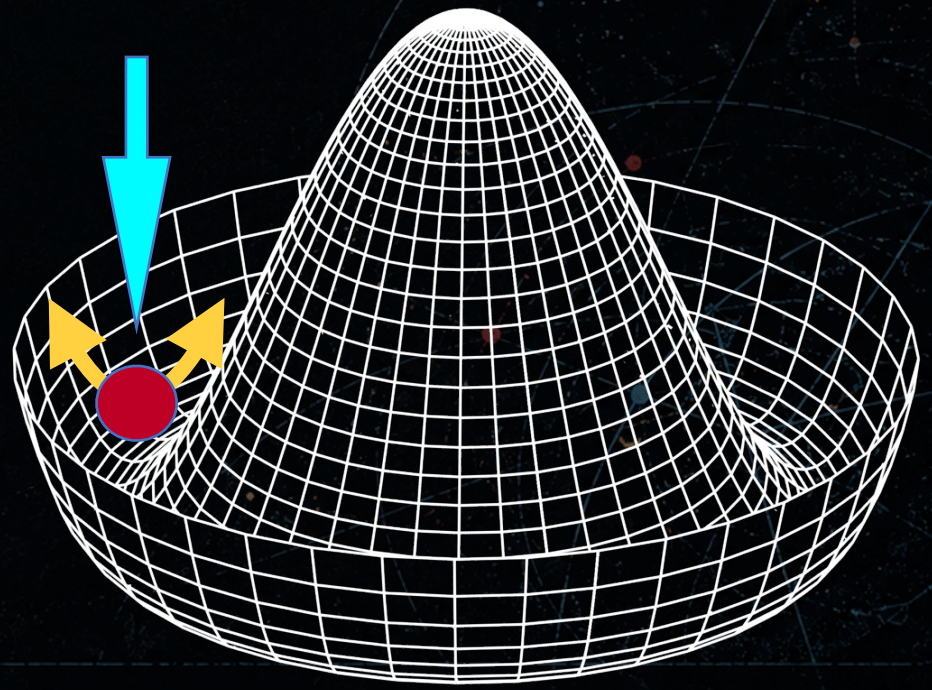
VERIFY!

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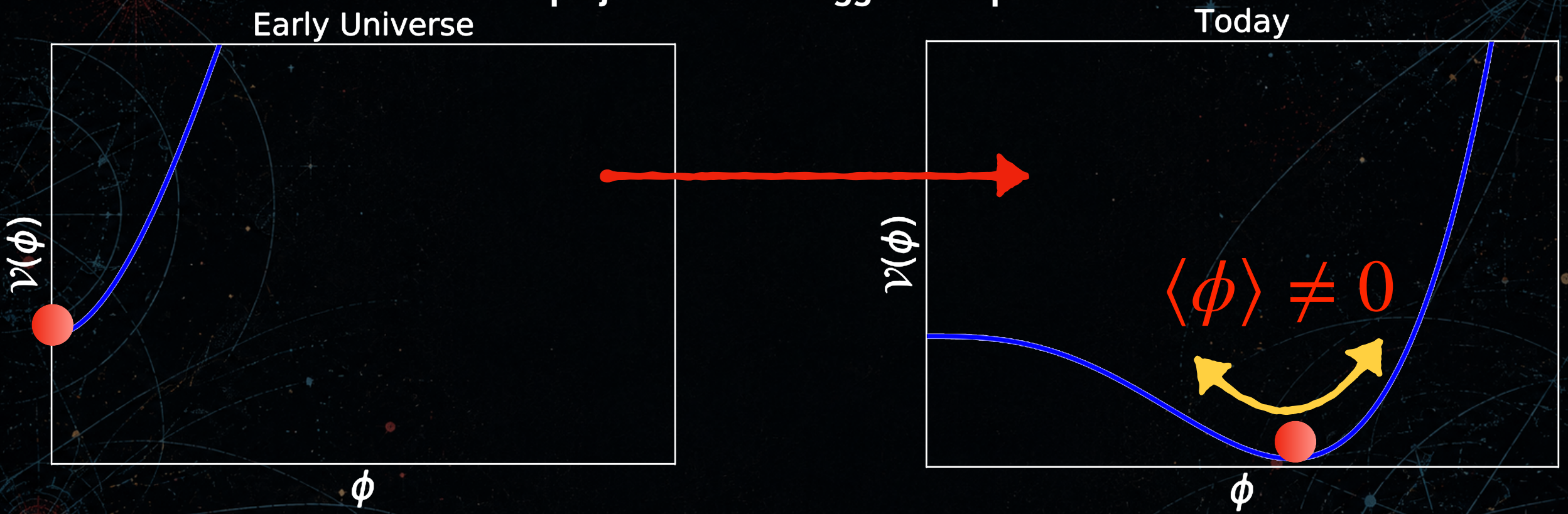


SECTION 02

The Higgs Potential: Electroweak Baryogenesis

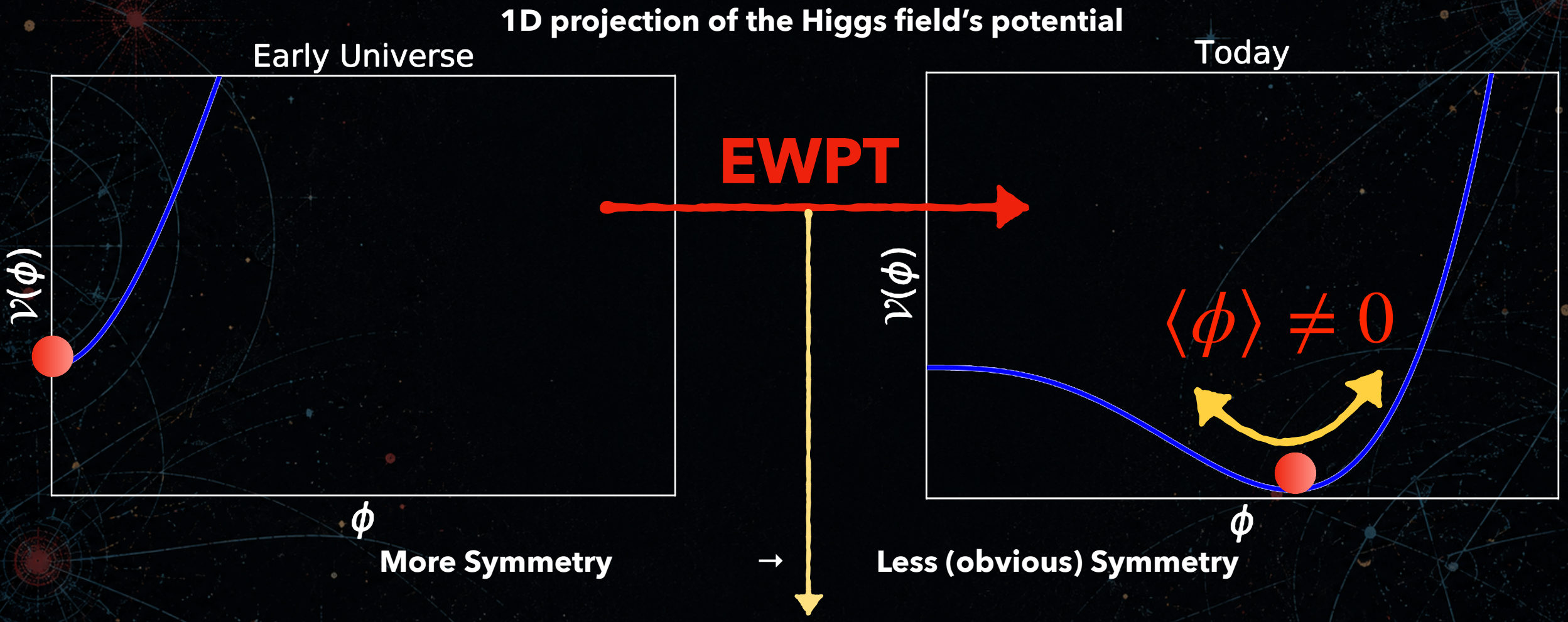
The ElectroWeak Phase Transition

1D projection of the Higgs field's potential

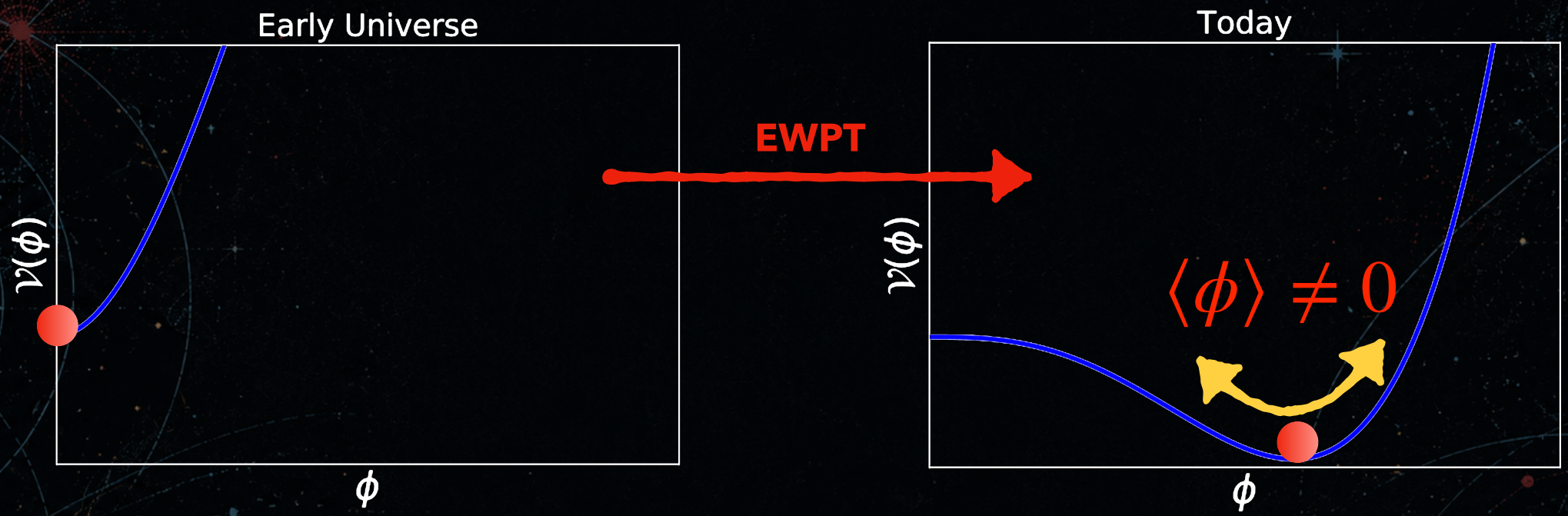


More Symmetry → **Less (obvious) Symmetry**

The ElectroWeak Phase Transition



The Nature of the ElectroWeak Phase Transition



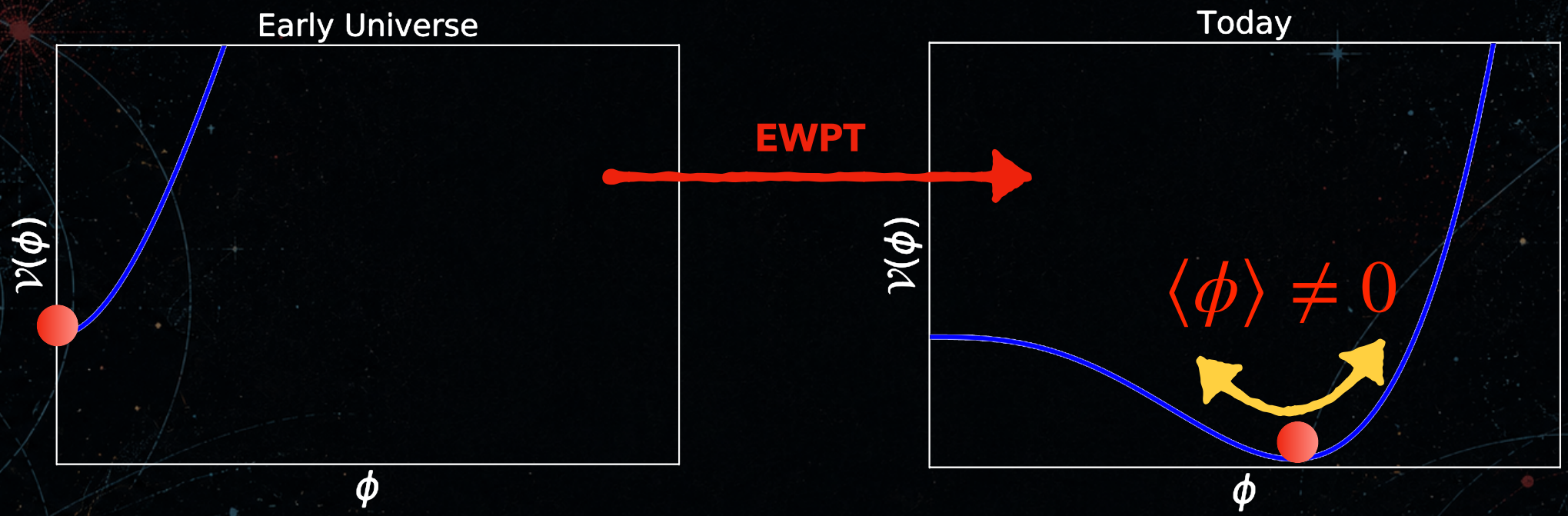
The **nature** of the EWPT is an important open question: i.e. its **order**:

Is it a **first-order transition**? (e.g. the boiling of water)



or a **second-order transition**? (e.g. the superfluid transition) or **cross-over**?

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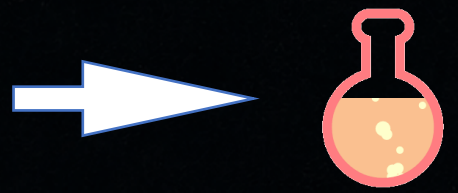
ElectroWeak Baryogenesis



May provide clues to the origin of the **matter-antimatter asymmetry**:

Was the asymmetry created ***during*** the EWPT? → **Electroweak Baryogenesis.**

A **pre-requisite**: **A first-order EWPT**!



But: this ***does not*** occur in the SM [Kajantie, Laine, Rummukainen, Shaposhnikov hep-ph/9605288].

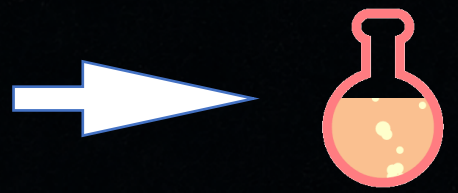
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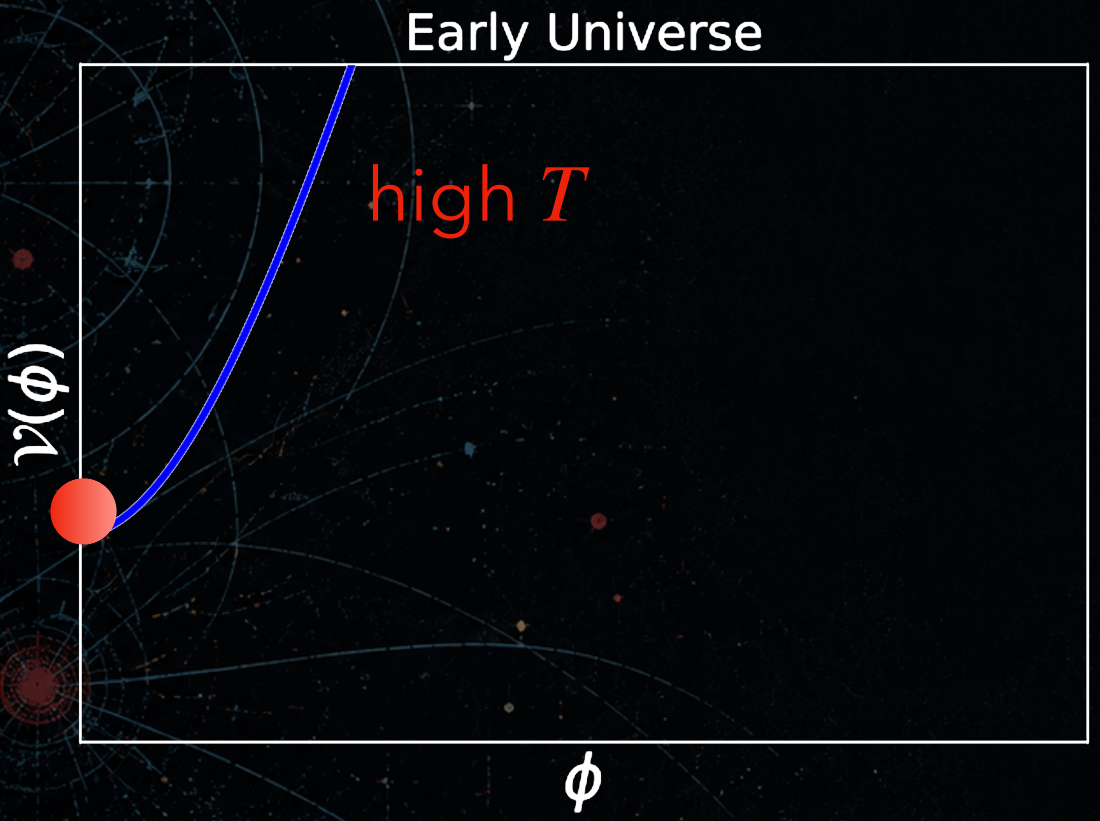
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A First-Order ElectroWeak Phase Transition

Schematically:

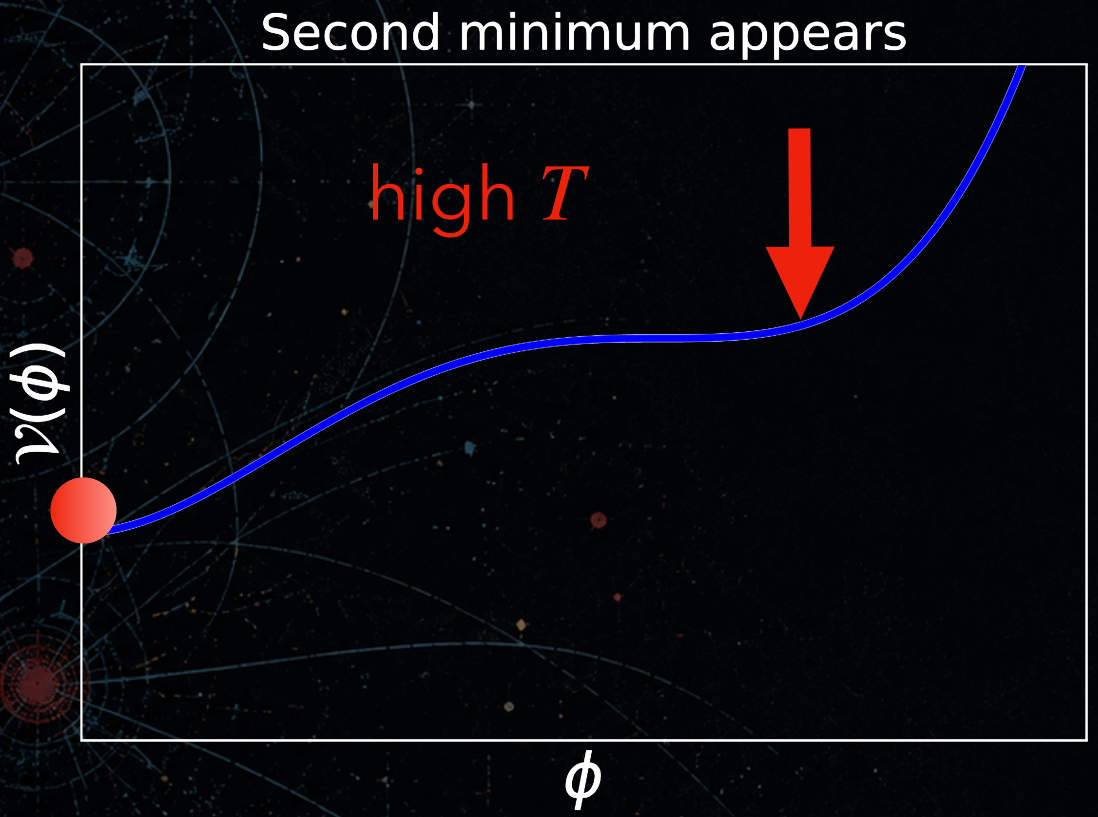


Higgs Potential evolution,

as the universe cools down.

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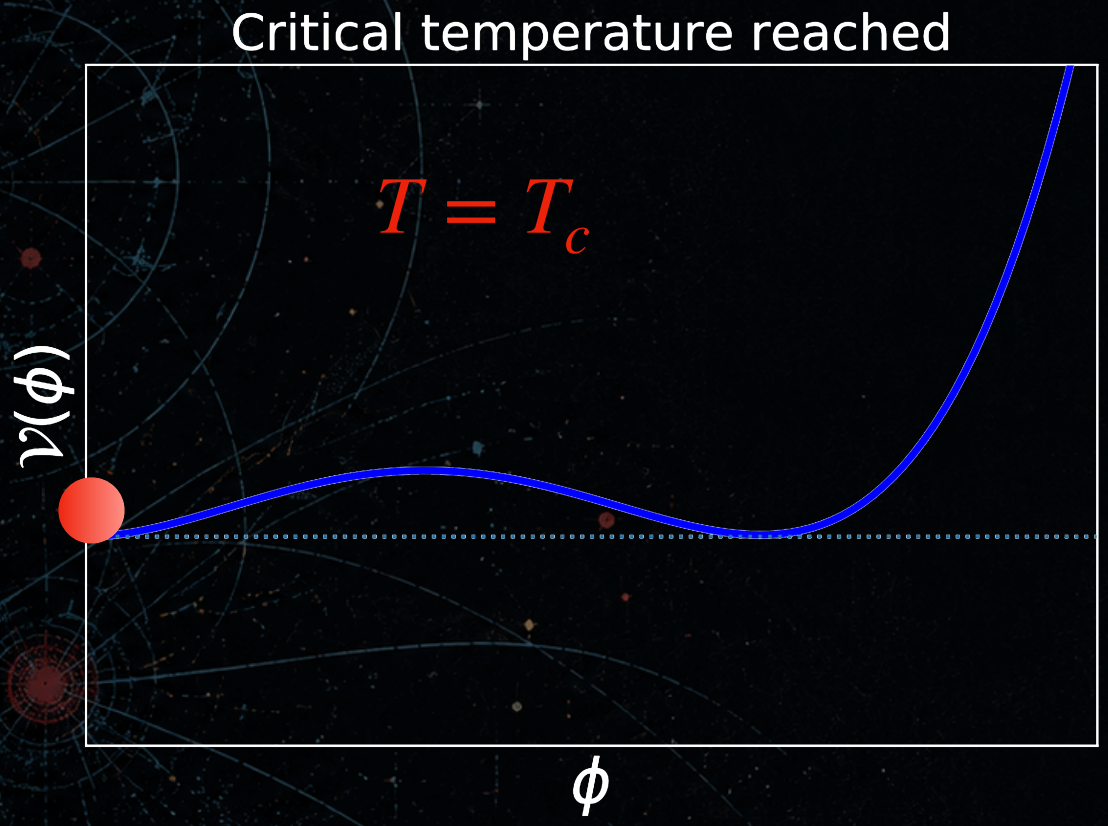


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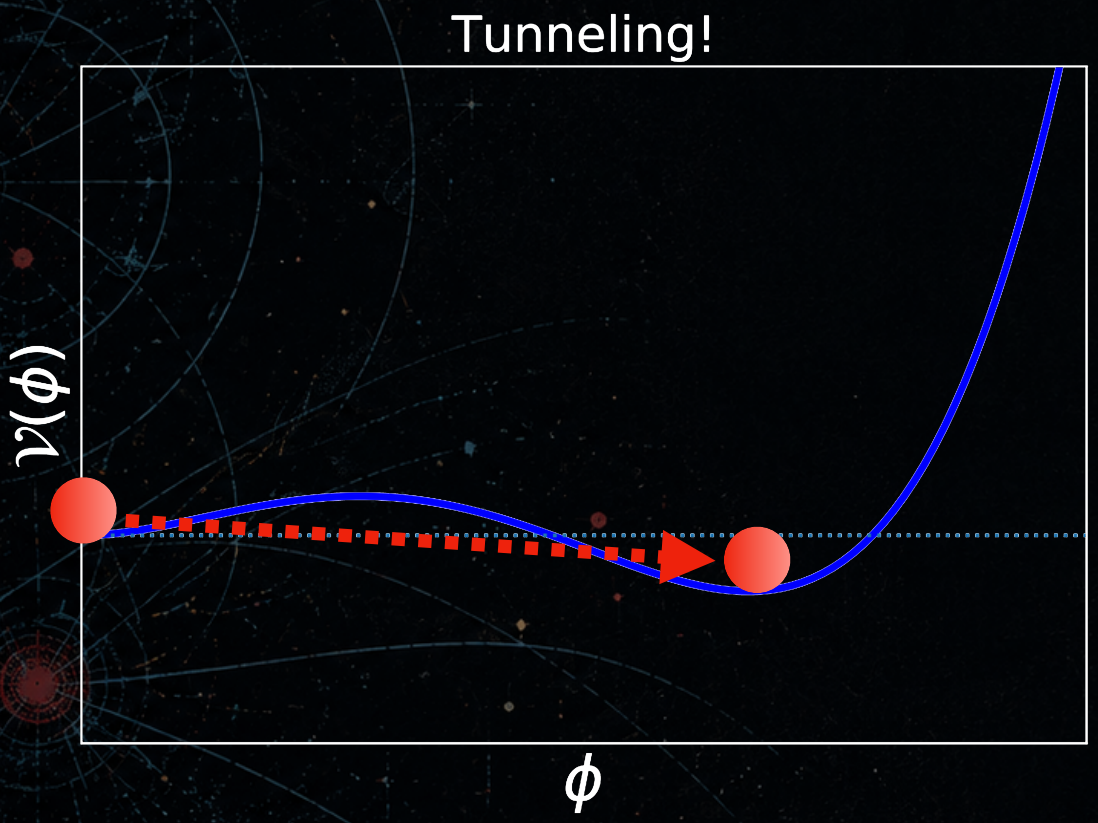


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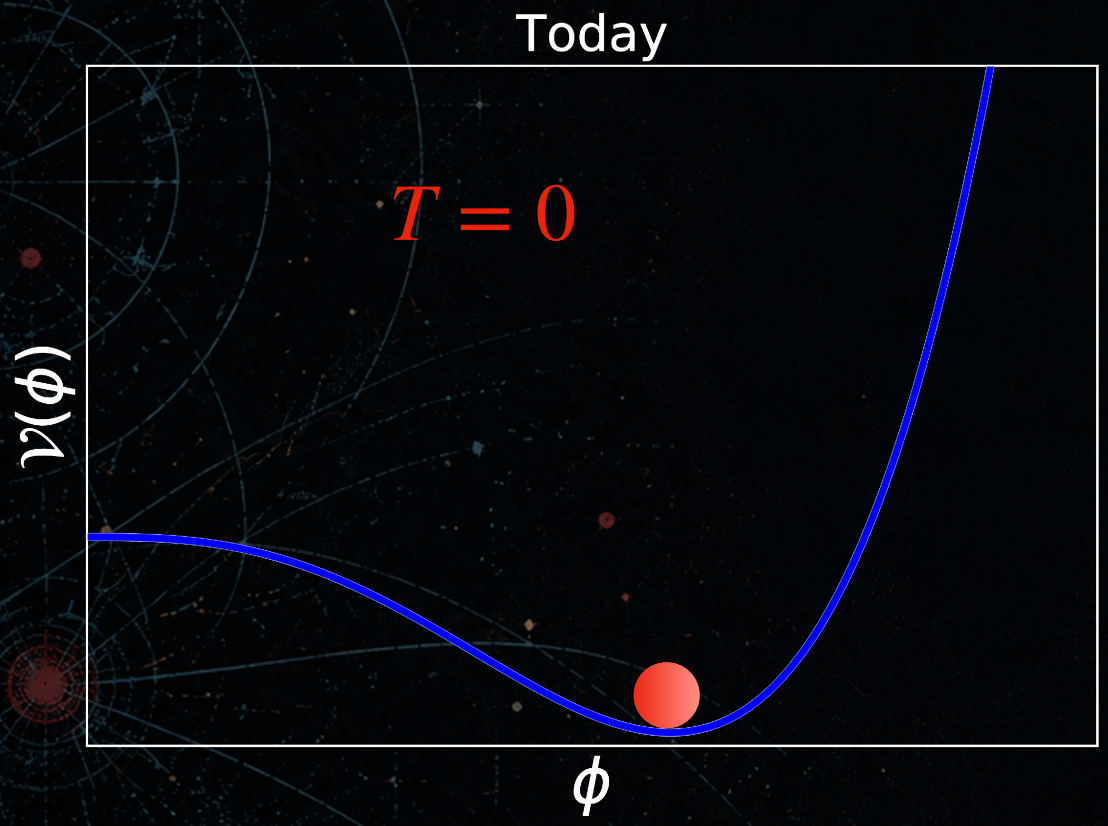


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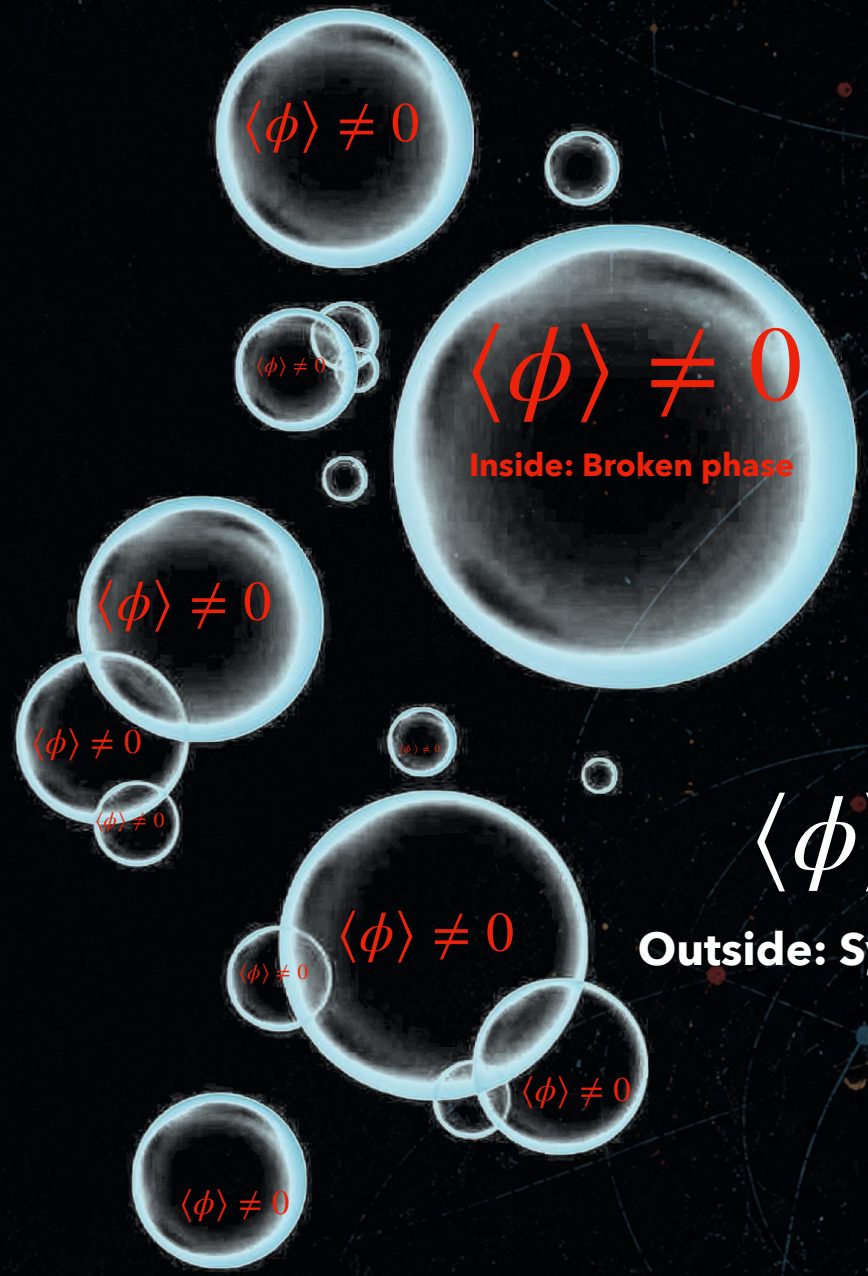
Higgs Potential evolution,

as the universe cools down.

$\langle \phi \rangle \neq 0 \Rightarrow$ **Symmetry breaking**

Some time *after* the critical temperature T_c:

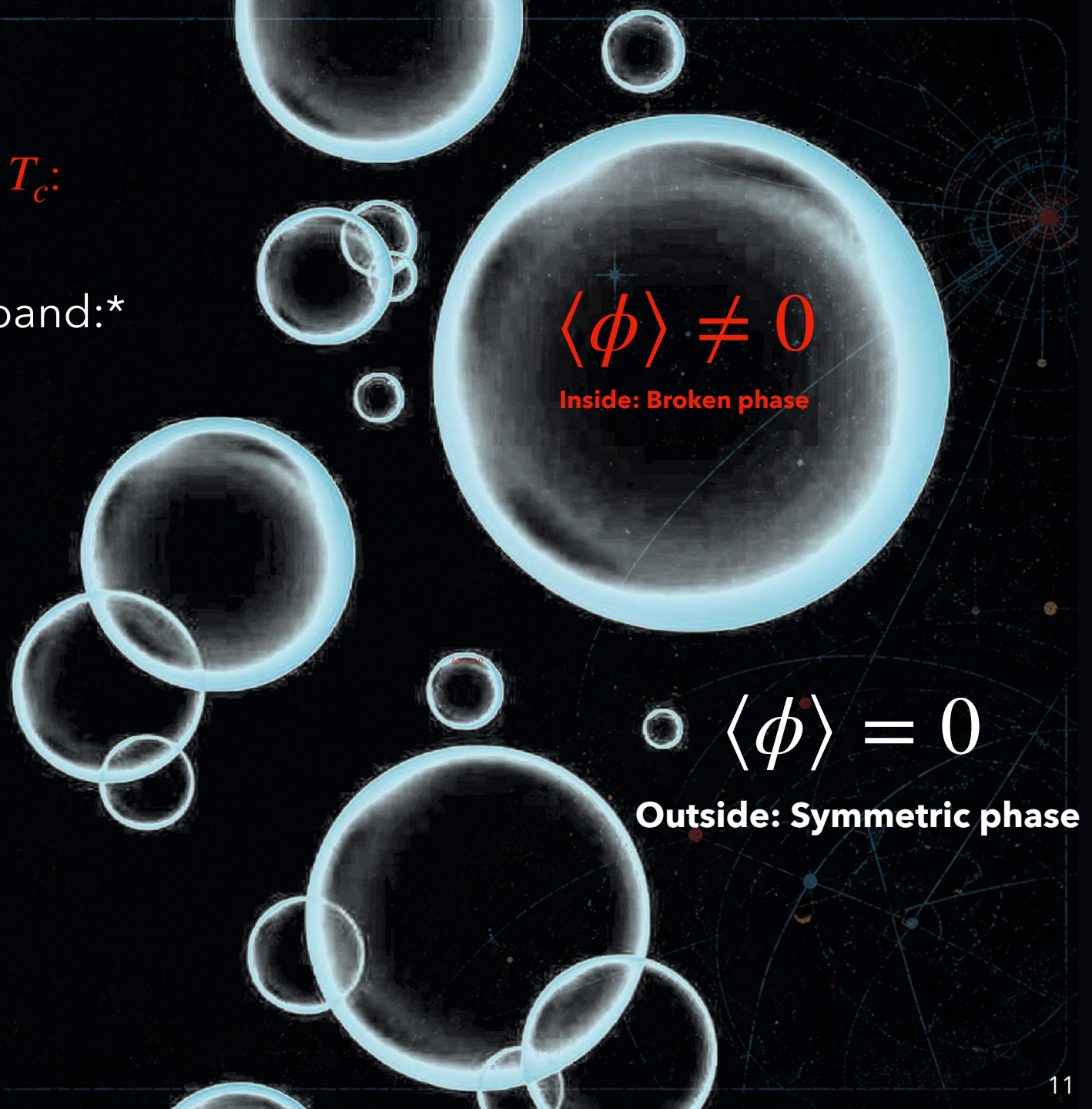
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**Note*: Gravitational Waves can form during this period!

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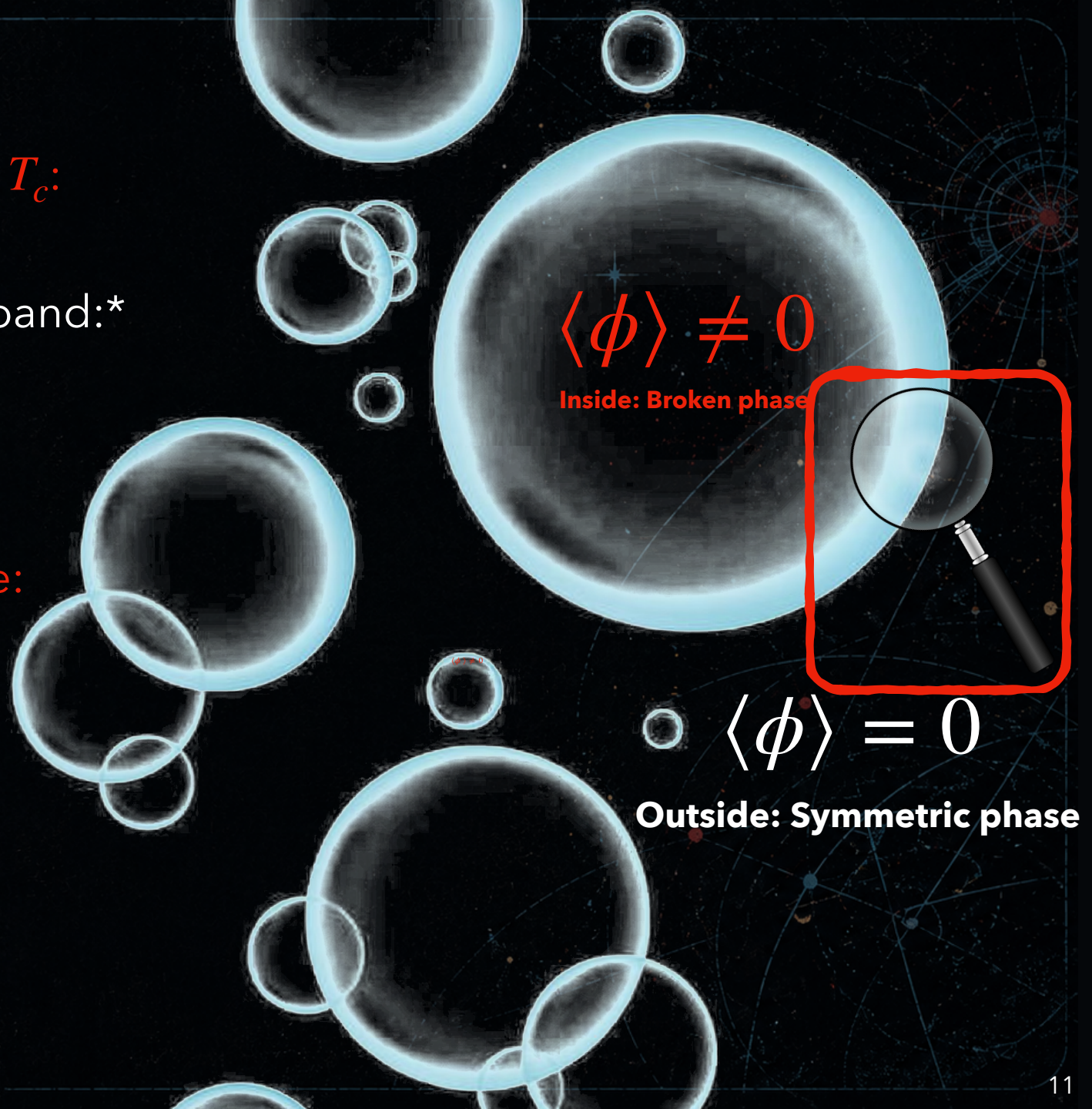
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Zoom in at surface of expanding bubble:

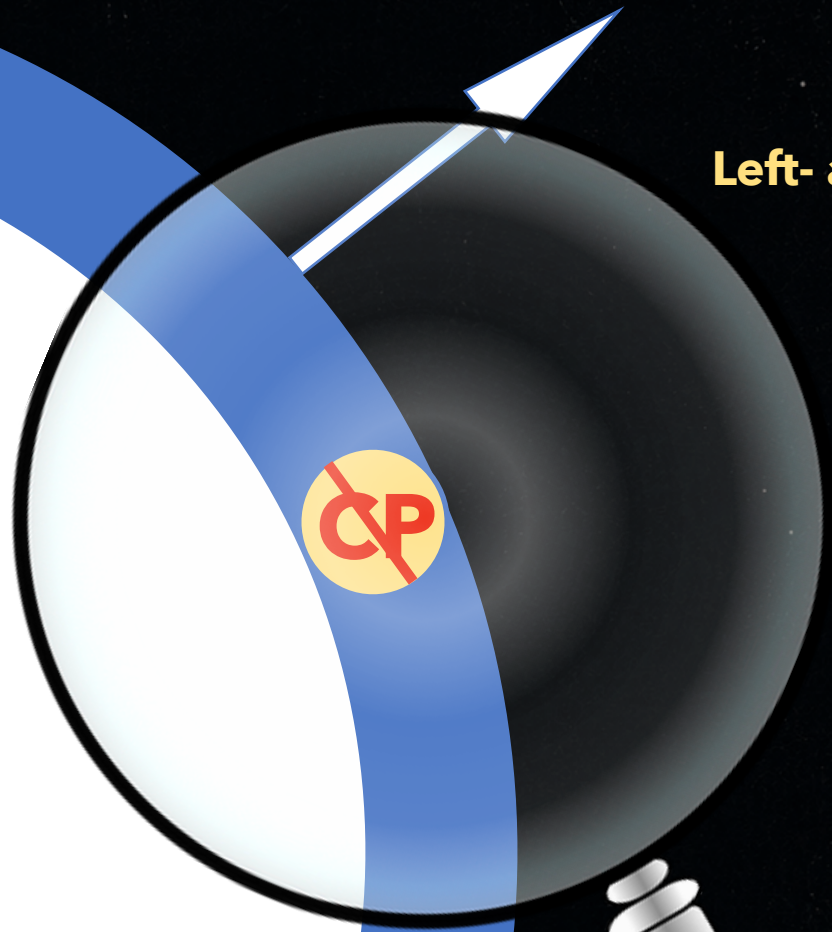
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ElectroWeak Baryogenesis

$$\langle \phi \rangle \neq 0$$

Inside: Broken phase



Left- and right-handed fermions

$$\psi_L + \psi_R$$

$$\langle \phi \rangle = 0$$

Outside: Symmetric phase

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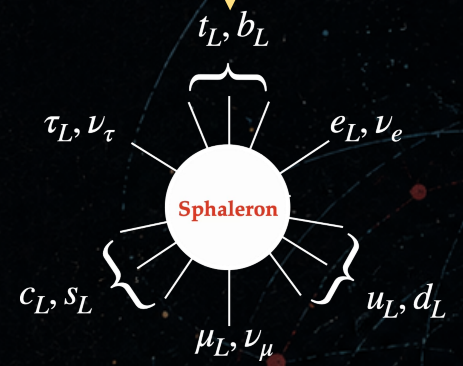
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[**AP**, Sakurai, Plätzer, arXiv:1910.4761]

M

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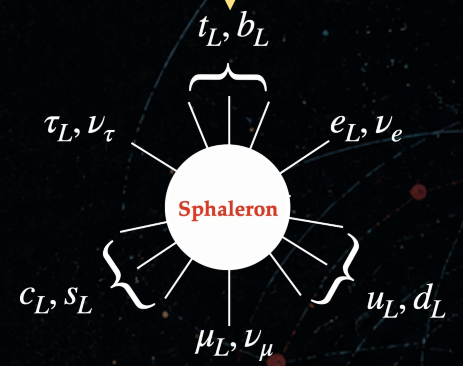
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CP

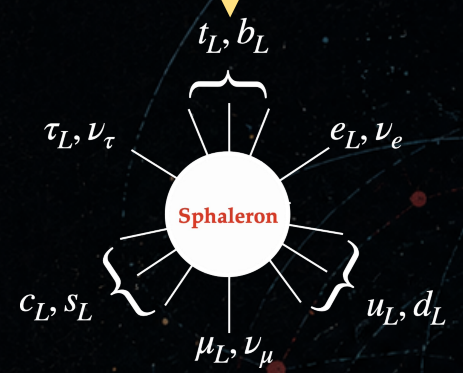
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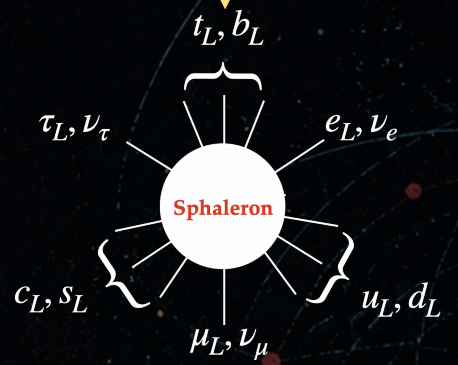
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Important Reminder:

A first-order EWPT *requires* BSM phenomena!

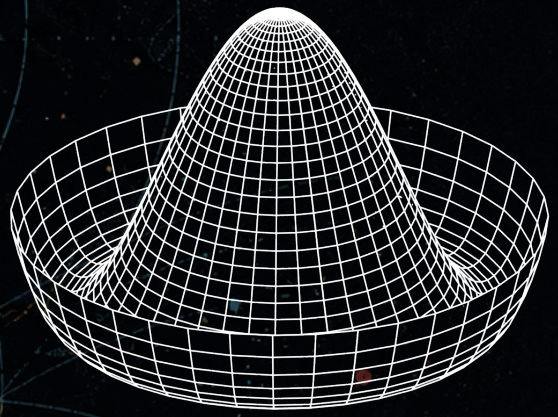
e.g. *Extended scalar sectors*!

SECTION 03

Probing the Scalar Sector: Multi-Higgs Boson Production

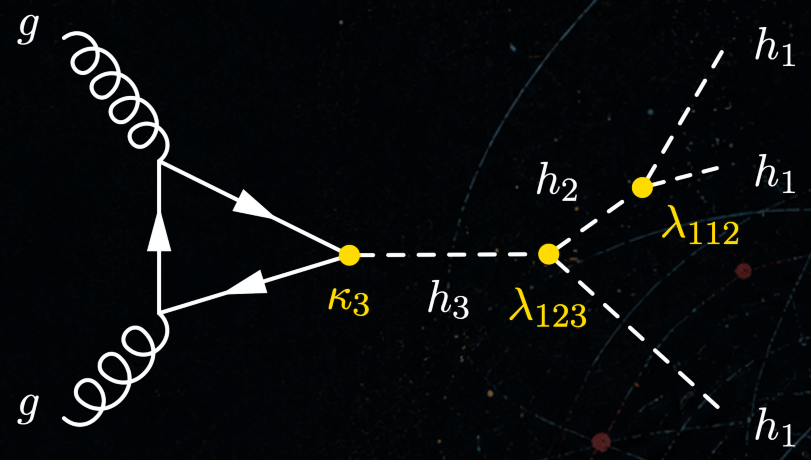
Multi-Higgs Boson Production:

A. Can probe the Higgs Potential:



Does it indeed have the expected shape?

B. Search for extended Scalar Sectors:



⇒ and test electroweak baryogenesis!

SECTION 03

A. Measuring the Higgs Potential

Measuring The Higgs Boson's Potential: Mass

$$\mathscr{V}(\langle\phi\rangle + h) = \frac{1}{2}m_h^2 h^2 + \kappa_3 \frac{m_h^2}{2v} h^3 + \kappa_4 \frac{m_h^2}{8v^2} h^4$$

Verify SM predictions:

$$\kappa_{3,4} \stackrel{?}{=} 1$$

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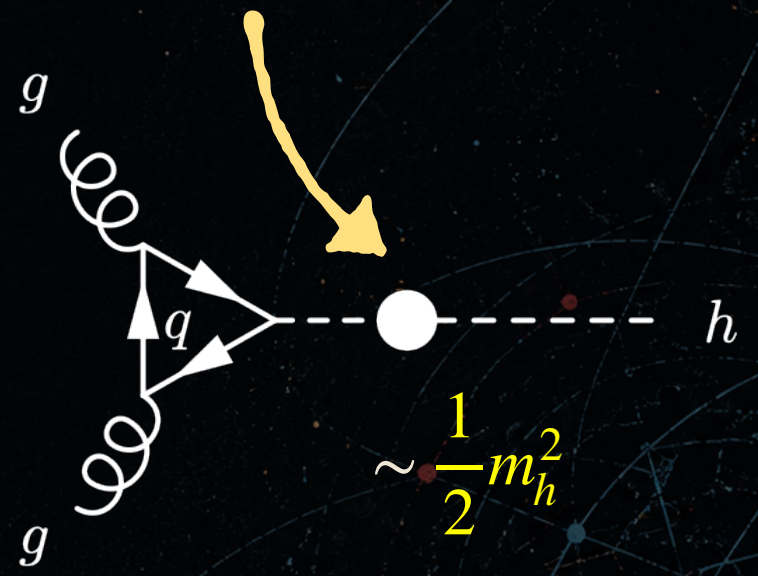
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Step 1: Single Higgs boson production:

Mass measurement:



Measuring The Higgs Boson's Potential: Mass

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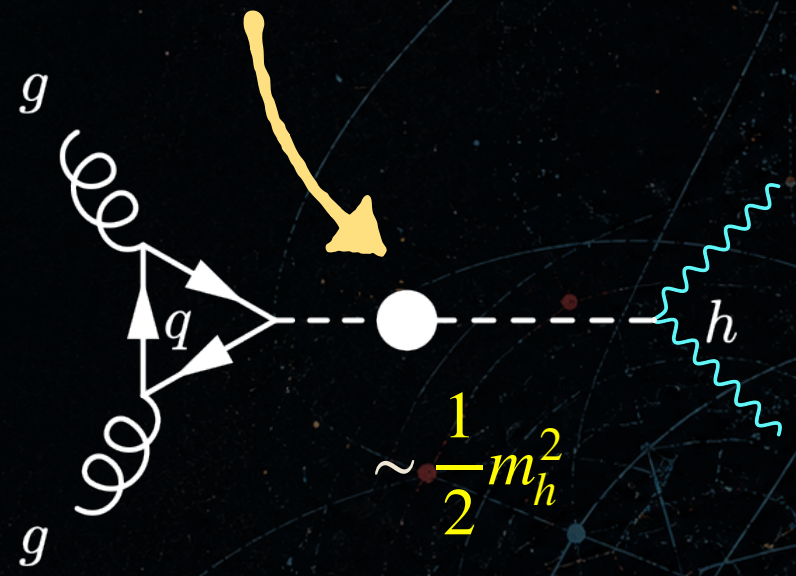
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decay mode	branching ratio
$h \to b\bar{b}$	58.2 %
$h \to WW^*$	21.4 %
$h \to \tau^+\tau^-$	6.27 %
$h \to ZZ^*$	2.62 %
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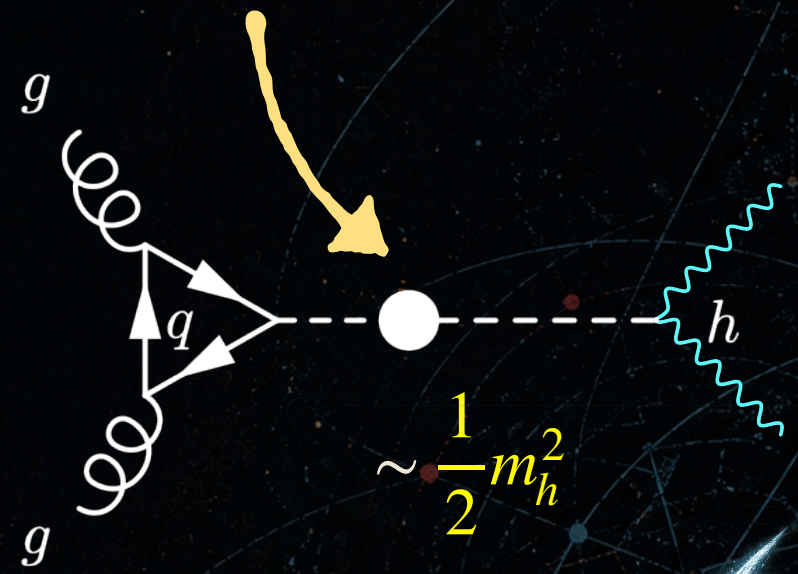
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$$m_h \simeq 125.1 \pm 0.1 \text{ GeV}$$

[CMS: PRD 111 (2025) 092014, ATLAS: Phys. Lett. B 847 (2023) 138315]

Measuring The Higgs Boson's Potential: Trilinear

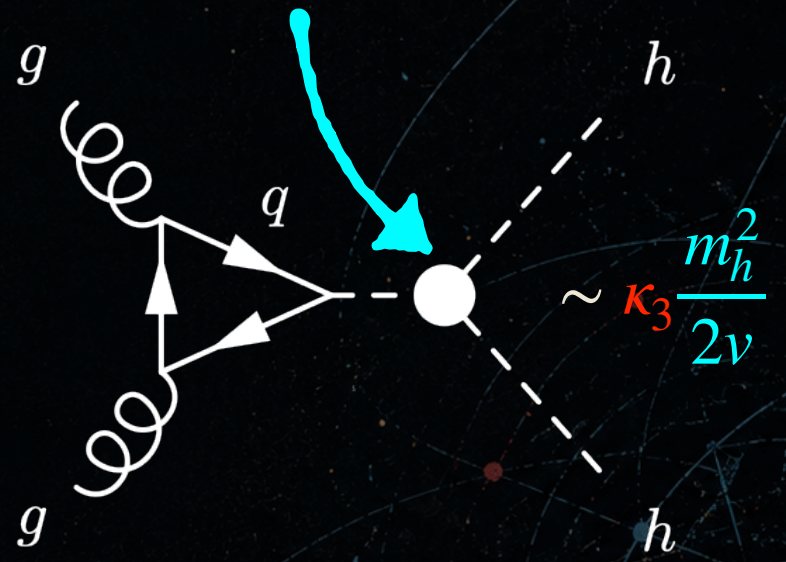
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Step 2: Double Higgs boson production:

Trilinear coupling measurement:



Measuring The Higgs Boson's Potential: Trilinear

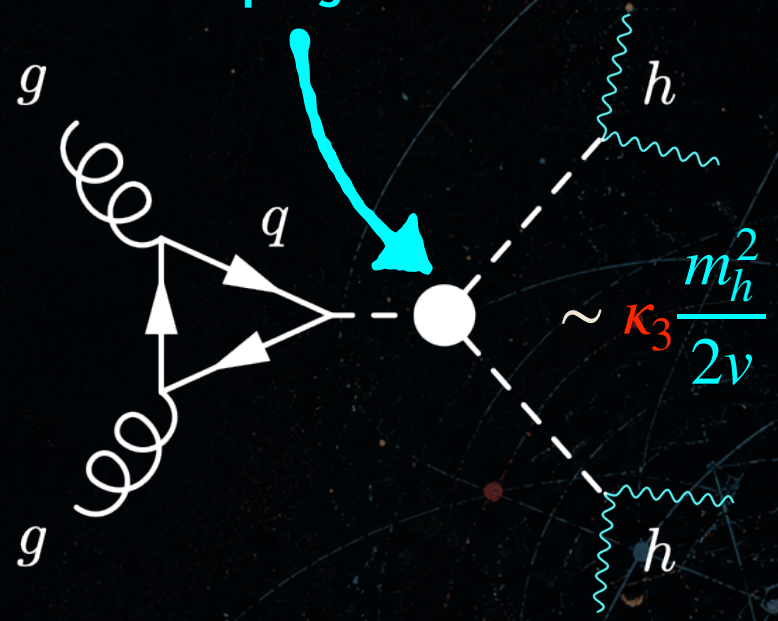
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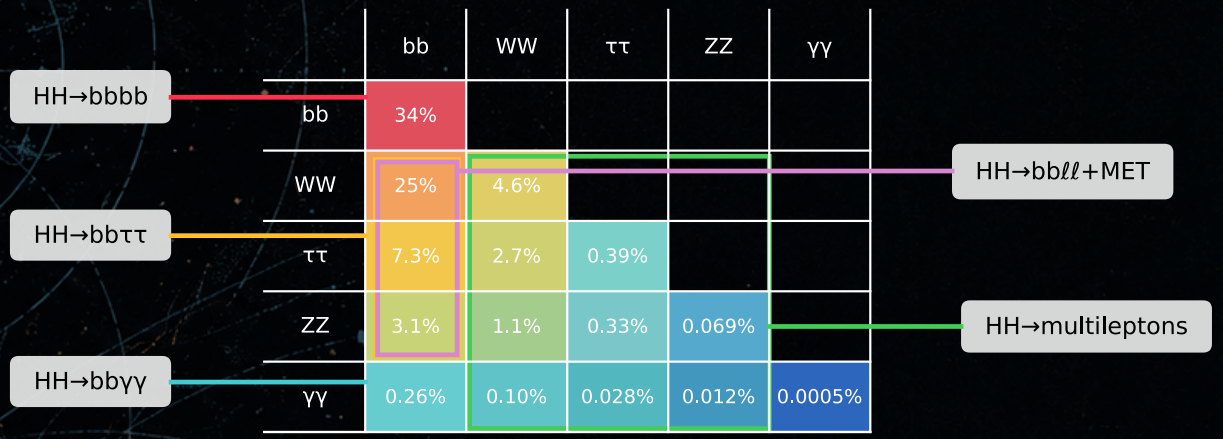
Trilinear coupling measurement:



Measuring The Higgs Boson's Potential: Trilinear

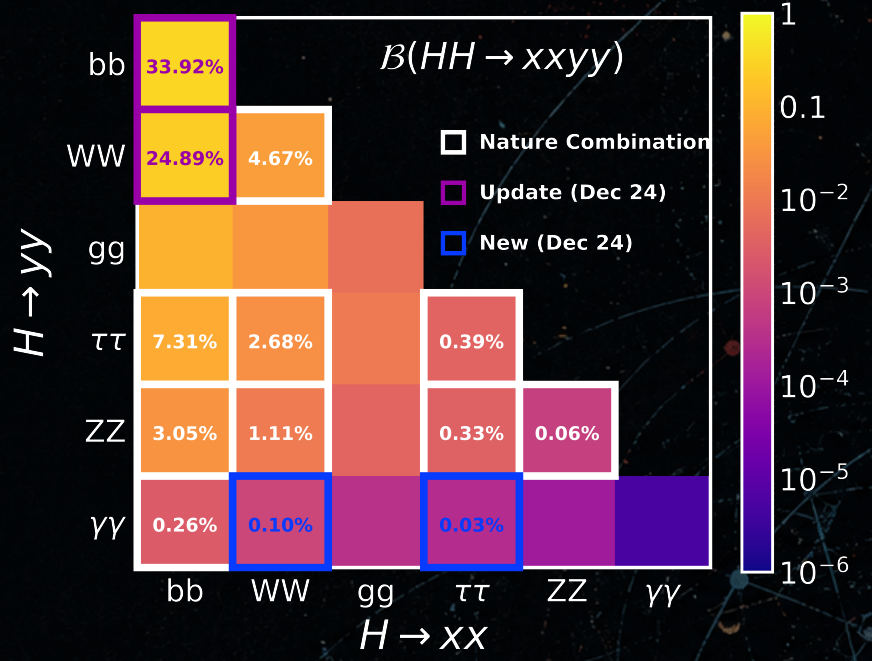
$pp \to hh \to (xx)(yy)$ final states @ LHC, Branching Ratios:

ATLAS:



[A. Sidley, HIGGS 2025]

CMS:

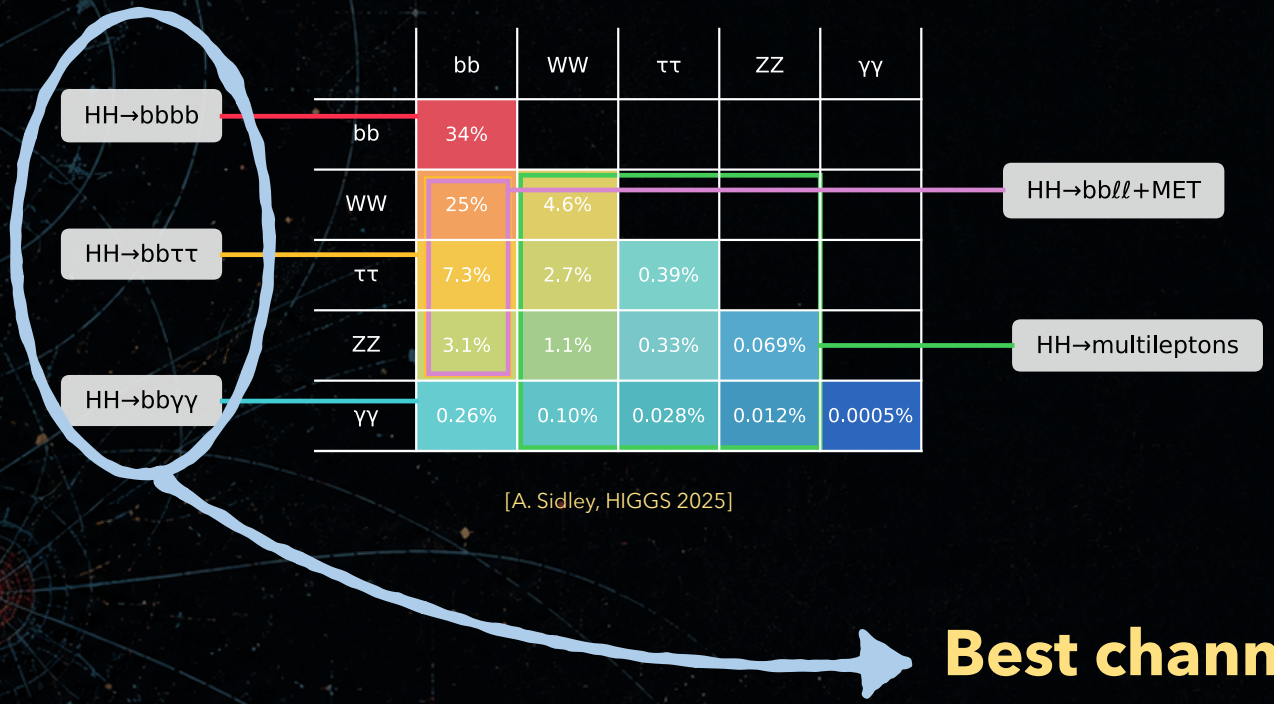


[N. De Filippis, HIGGS 2025]

Measuring The Higgs Boson's Potential: Trilinear

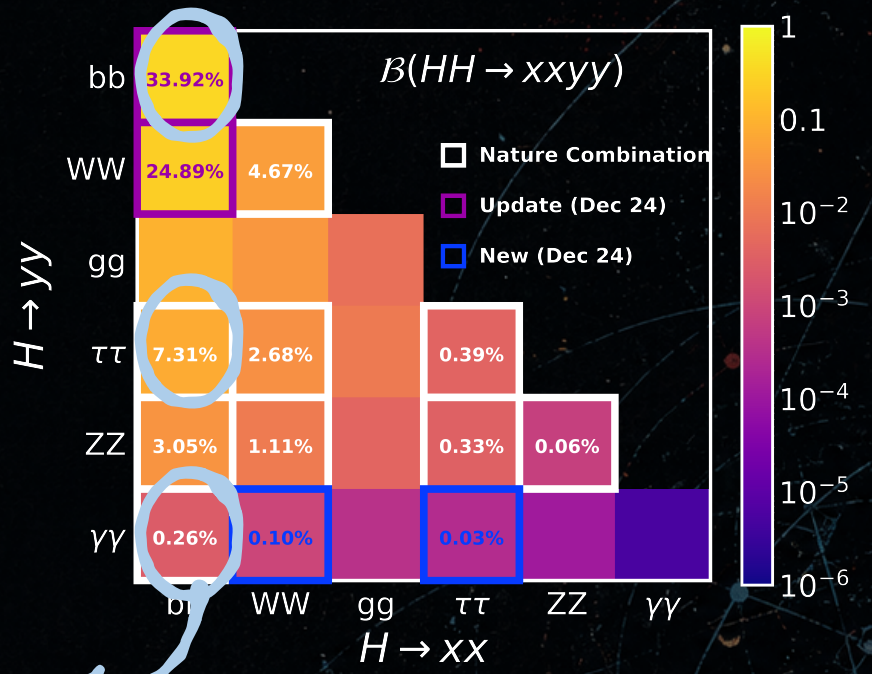
$pp \to hh \to (xx)(yy)$ final states @ LHC, Branching Ratios:

ATLAS:



[A. Sidley, HIGGS 2025]

CMS:

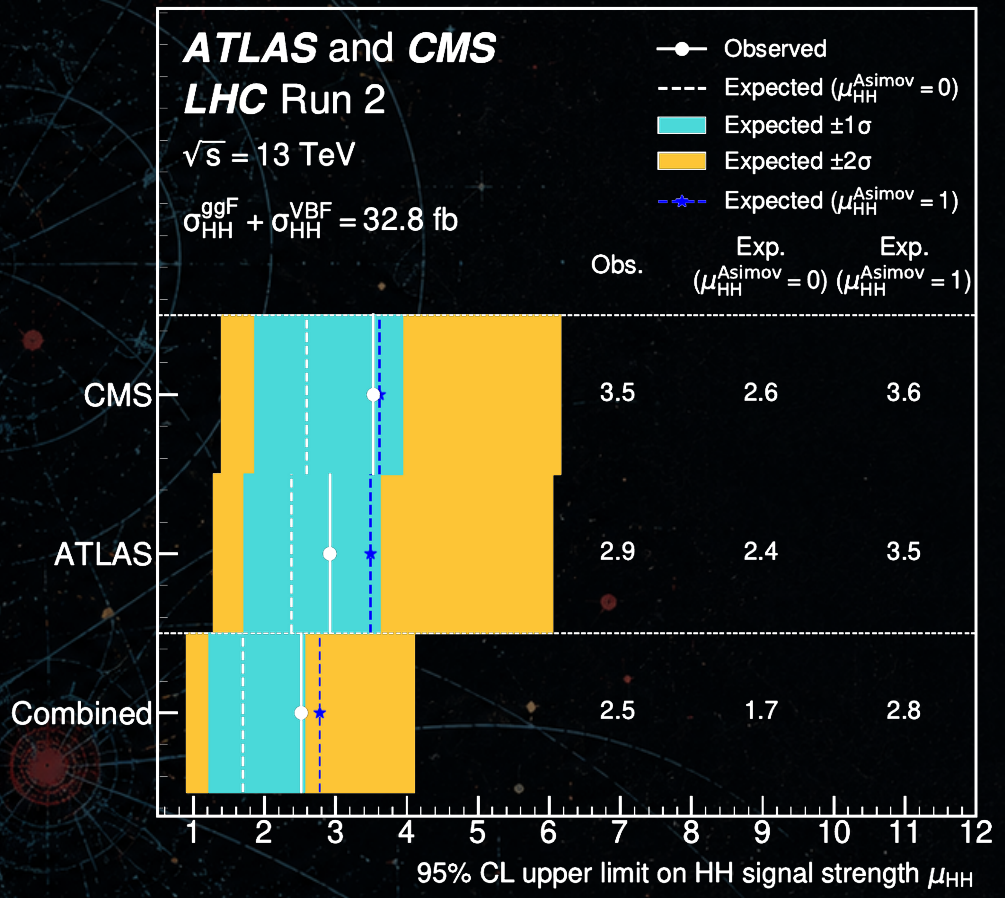


[N. De Filippis, HIGGS 2025]

Best channels!

Trilinear @ LHC (Today!)

ATLAS + CMS Combo (13 TeV): [arXiv:2602.23991]

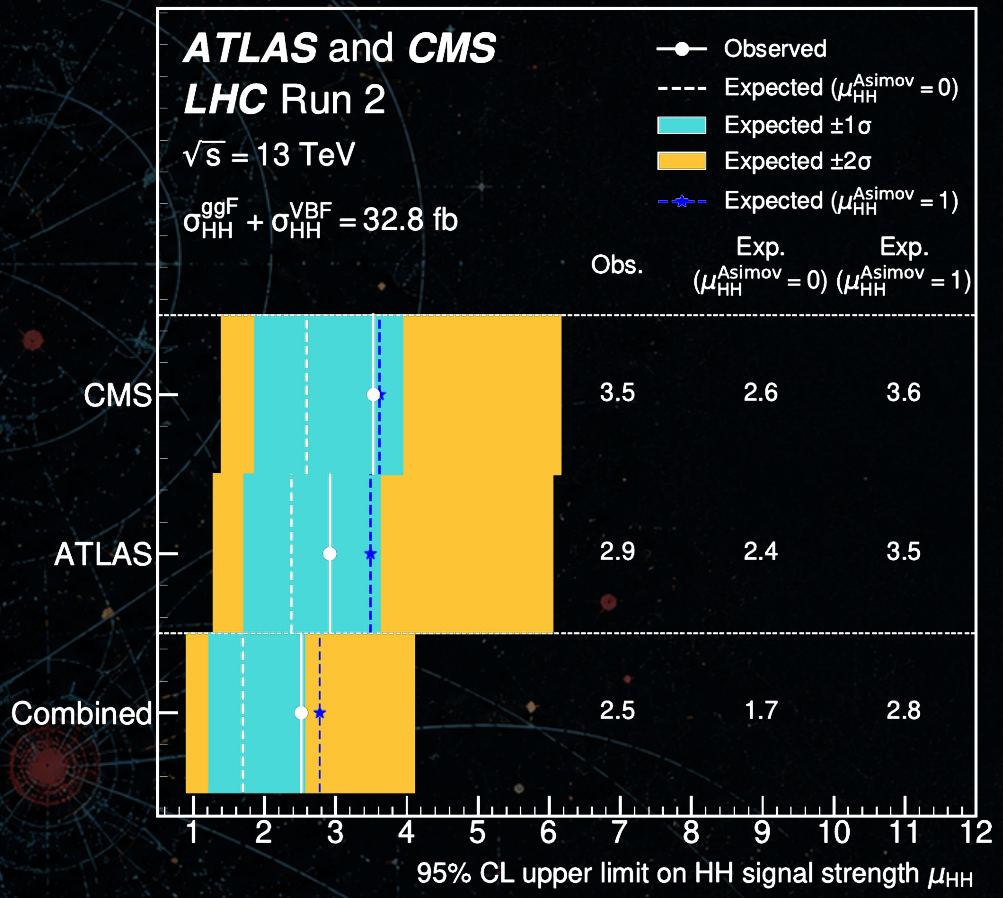


$\mu_{HH} = \sigma_{\text{obs}}/\sigma_{\text{SM}}$

	Channel	Observed signal strength μ_{HH}	Observed κ_3 95% CL interval
ATLAS	$bbbb$	< 5.4	$[-3.4, 11]$
	$bb\tau\tau$	< 5.8	$[-3.3, 9.1]$
	$bb\gamma\gamma$	< 4.1	$[-1.6, 7.0]$
	$bb\ell\ell + E_{\text{T}}^{\text{miss}}$	< 9.6	$[-6.5, 13]$
	Multilepton	< 17	$[-6.4, 12]$
	Combined	< 2.9	$[-1.3, 7.2]$
CMS	$bbbb$	< 7.0	$[-5.0, 12]$
	$bb\tau\tau$	< 3.5	$[-1.9, 8.9]$
	$bb\gamma\gamma$	< 8.7	$[-3.5, 8.0]$
	$bbWW$	< 15	$[-6.1, 13]$
	Multilepton	< 22	$[-5.8, 10]$
	Combined	< 3.5	$[-1.4, 6.6]$
ATLAS + CMS combined		< 2.5	$[-0.71, 6.1]$

Trilinear @ LHC (Today!)

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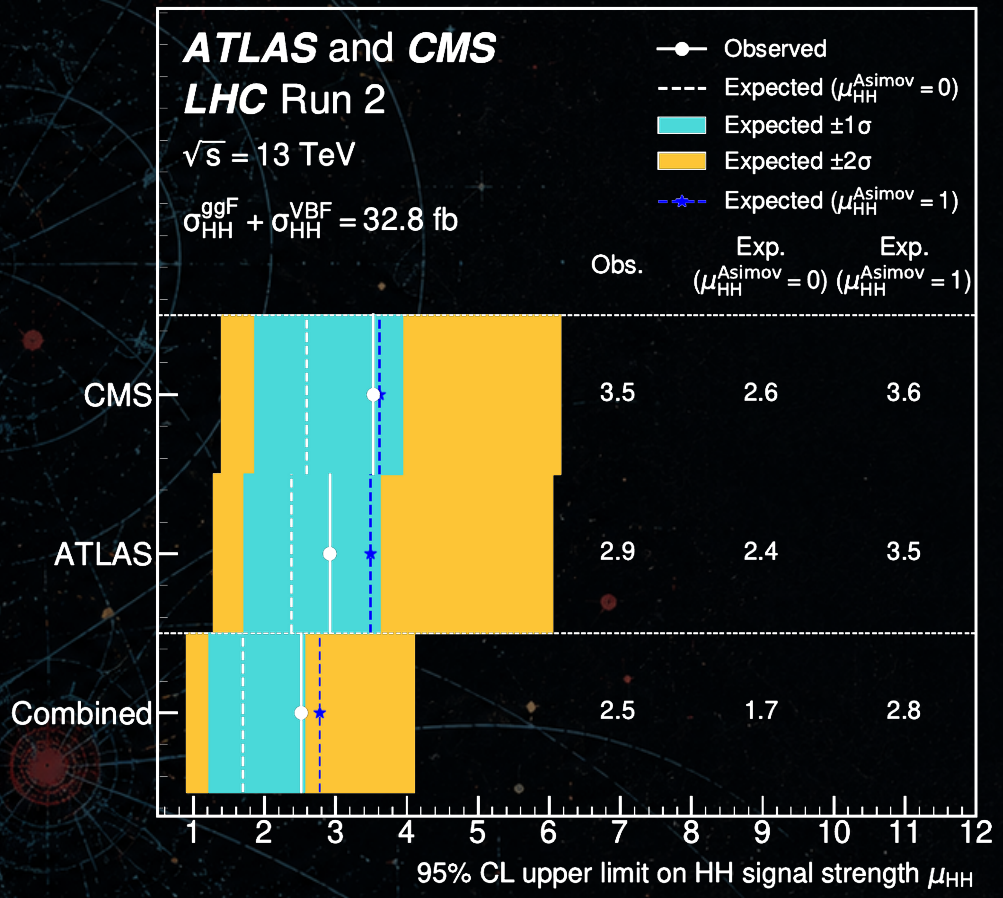


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$\Rightarrow -0.71 < \kappa_3 < 6.1$



Measuring The Higgs Boson's Potential: Quartic

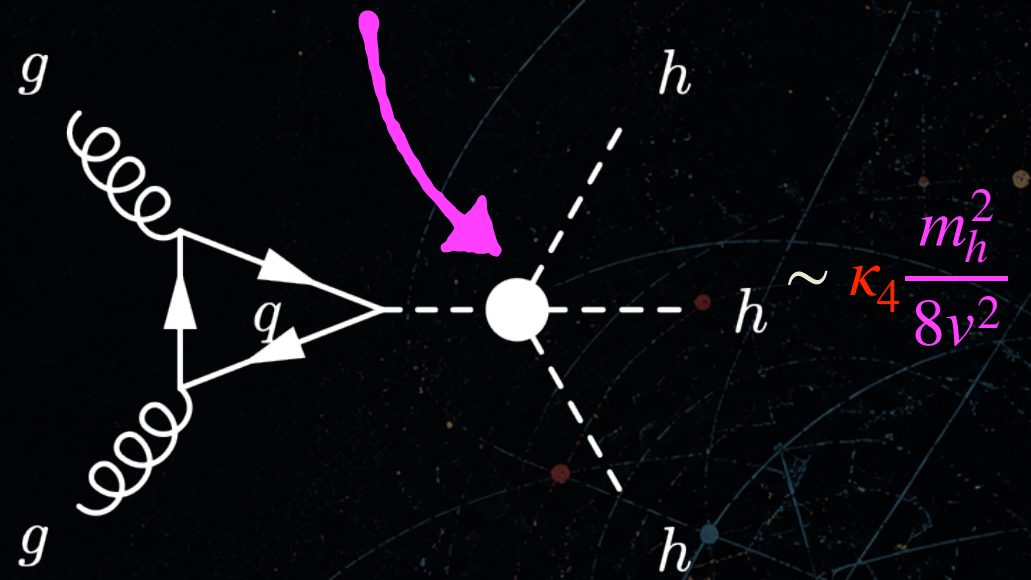
$$\mathscr{V}(\langle\phi\rangle + h) = \frac{1}{2}m_h^2 h^2 + \kappa_3 \frac{m_h^2}{2v} h^3 + \kappa_4 \frac{m_h^2}{8v^2} h^4$$

Verify SM predictions:

$$\kappa_{3,4} \stackrel{?}{=} 1$$

Step 3: Triple Higgs boson production:

Quartic coupling measurement:



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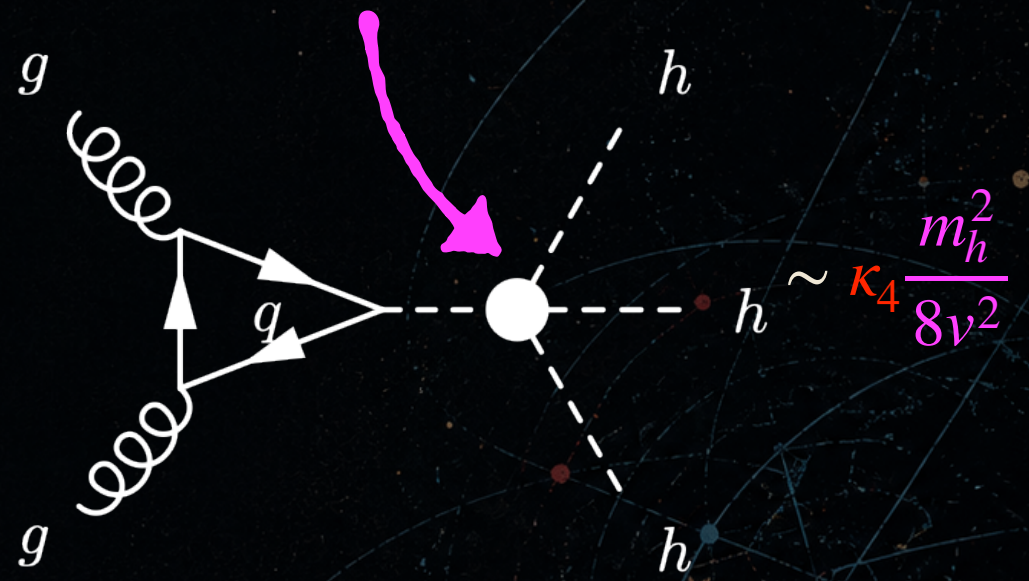
Tiny cross section @ LHC:

SM $\sigma(hhh @ 14\ \text{TeV}) \sim 0.1\ \text{fb}$**!**



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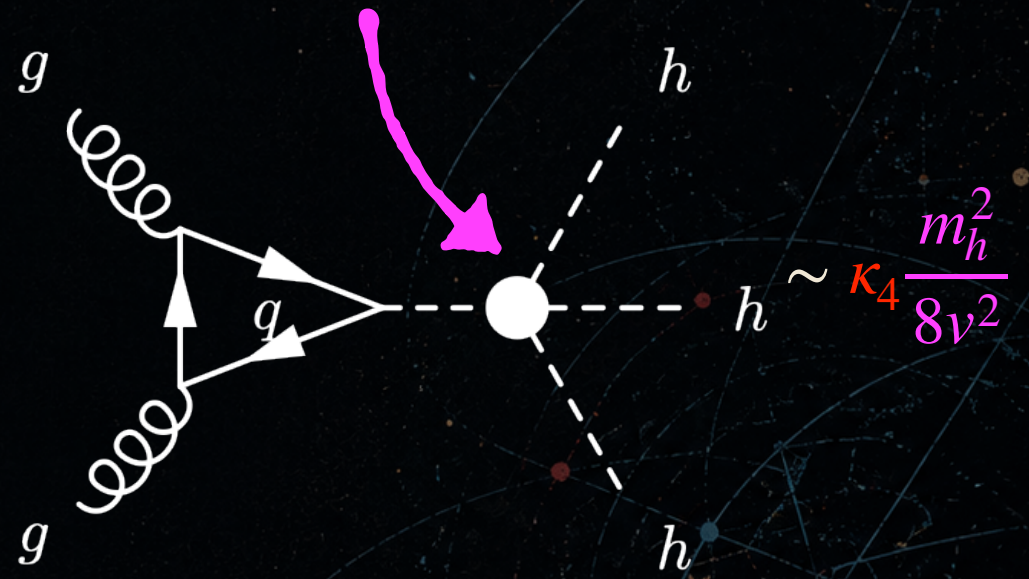
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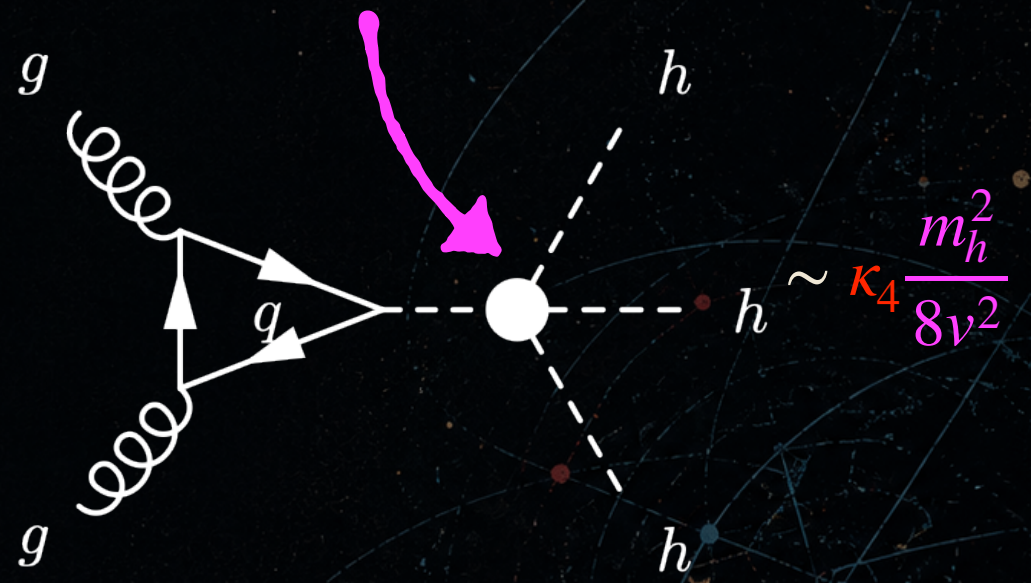
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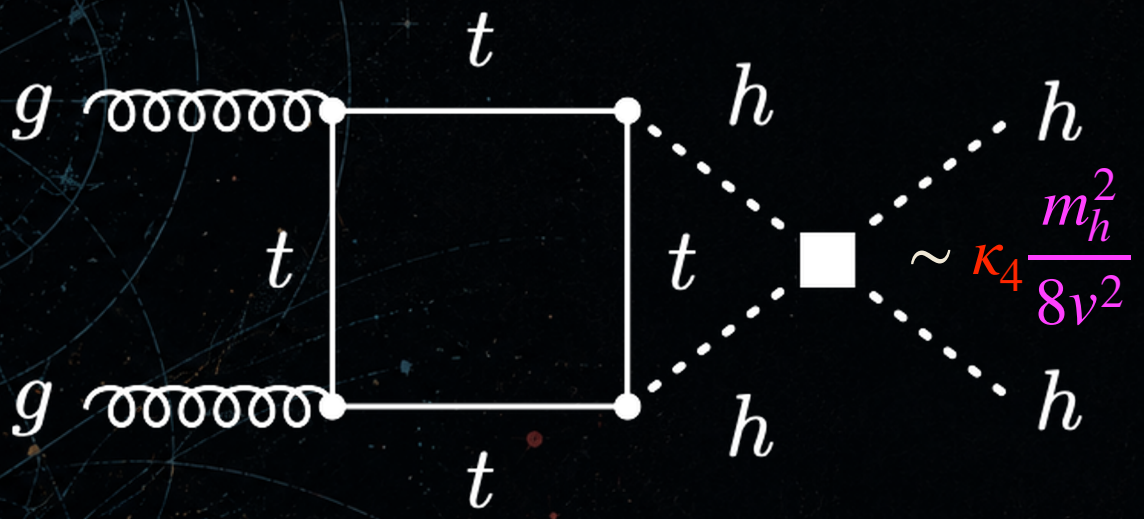
$\kappa_4 \in [-533, 541]$ [CMS, 4b2γ]

$\kappa_4 \in [-190, 190]$ [CMS, 6b]

$\kappa_4 \in [-190, 190]$ [ATLAS, 6b]

Quartic Indirect Constraints through *hh*

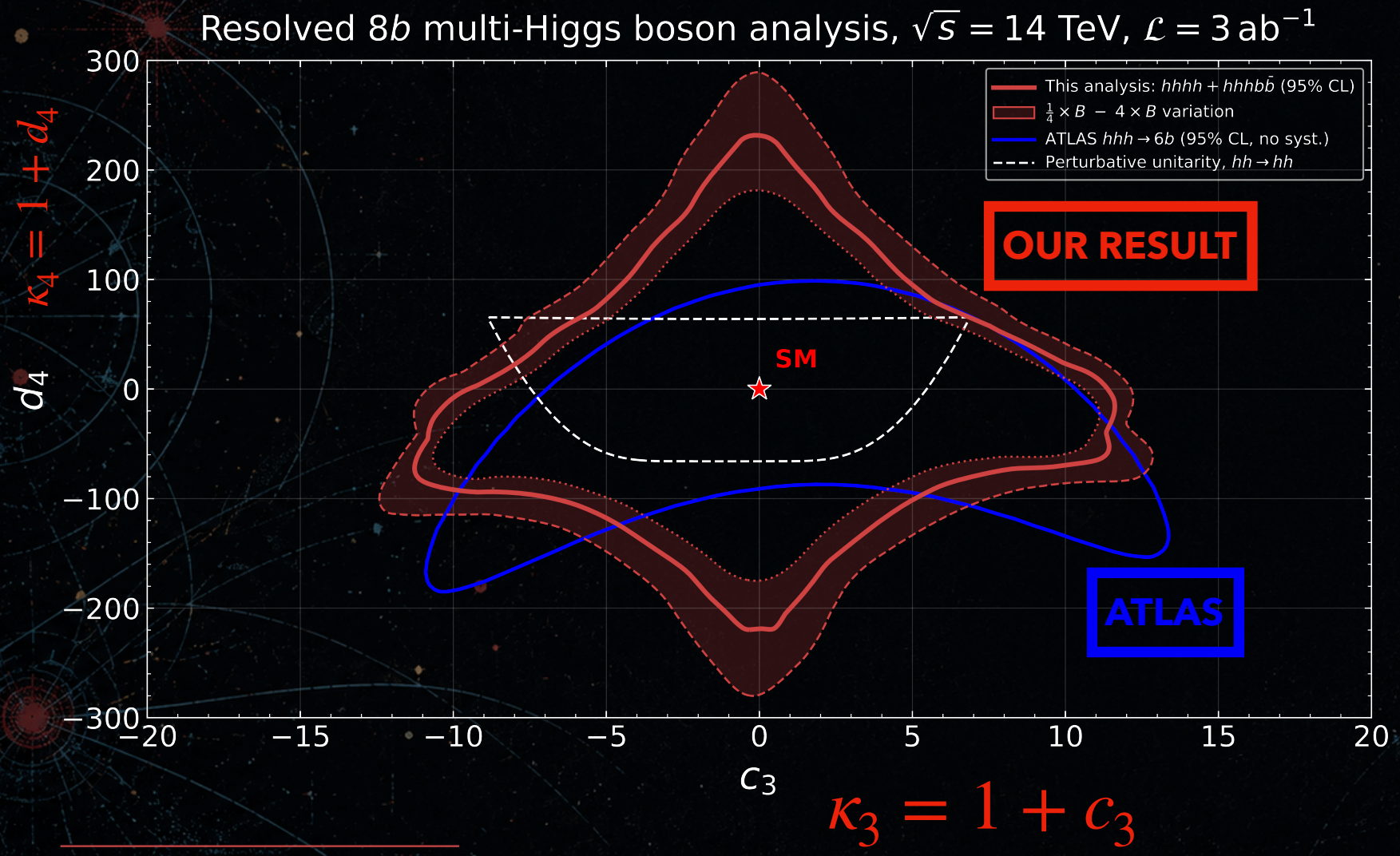
Through electroweak corrections:



[Bizoń, Haisch, Rottoli, **1810.04665**, Li Si, Wang, Zhang, Zhao, **2407.14716**, Heinrich, Jones, Kerner, Stone, Vesten, **2407.04653**]

Quadruple Higgs Boson Production @ High Lumi-LHC ?!

[**AP**, Tetlalmatzi-Xolocotzi, 2608.16998]



Even smaller SM cross section than triple ***h***!

➡ but more diagrams involving κ_4!

➡ **Potentially <u>complementary</u> to** $hhh \to 6b$ jets on (k_3, k_4) plane!

Future Self-Coupling Constraints

Collider:	HL-LHC	FCC-ee	FCC-hh	Muon Coll.
k_3	$\sim 50\,\%$	$\sim 20-30\,\%$	$\sim 5\%$	$5-30\,\%$
k_4	$\mathcal{O}(10-100)$ [ATL-PHYS-PUB-2025-003, ATL-PHYS-PUB-2025-018 CMS-HIG-25-002]	???	$\mathcal{O}(1-10)$ [Fuks, **AP**, Tetlalmatzi-Xolocotzi, **2509.16364**]	$50-100\,\%$ [Chiesa, Maltoni, Mantani, Mele, Piccinini, Zhao, **2003.13628**]

SECTION 03

B. Searching for Extended Scalar Sectors

Extended Scalar Sectors: One Singlet Scalar

A simple extension to SM: add **one** singlet scalar field:

$$\mathcal{V}(\phi, S) = \bullet\,|\phi|^2 + \blacksquare\,|\phi|^4$$

Singlet = *No* SM charges,
⇒ *No direct* $f\bar{f}S$, VVS,... couplings!

$$+ \bullet\, S^2 + \blacktriangle\, S^3 + \blacksquare\, S^4$$



$$+ \blacktriangle\,|\phi|^2 S + \blacksquare\,|\phi|^2 S^2$$ ← "**Portal**" interactions.

$$+ S \times (\text{Hidden Sector}) + \cdots$$ ← **Dark Matter?**

Extended Scalar Sectors: One Singlet Scalar

$$\mathcal{V}(\phi, S) = \bullet\,|\phi|^2 + |\phi|^4 + \bullet\, S^2 + \blacktriangle\, S^3 + S^4 + \blacktriangle\, |\phi|^2 S + |\phi|^2 S^2$$

EWSB $\leftrightarrow$ VEVs:

$\phi \to \langle \phi \rangle + h$

$S \to \langle S \rangle + \chi$

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$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} h \\ \chi \end{pmatrix}$$

Extended Scalar Sectors: One Singlet Scalar

$$\mathcal{V}(\phi, S) = \color{green}{\bullet}\,|\phi|^2 + |\phi|^4 + \color{magenta}{\bullet}\,S^2 + \blacktriangle\, S^3 + S^4 + \blacktriangle\,|\phi|^2 S + |\phi|^2 S^2$$

EWSB ↔ VEVs:

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θ small:

h_1: "SM-like" Higgs boson,

h_2: **New** scalar resonance!

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θ small:

h_1: "SM-like" Higgs boson,

h_2: **New** scalar resonance!

Primary targets for collider studies:

Extended Scalar Sectors: Two Singlet Scalars

Another extension: add **two** singlet scalar fields + $\mathbb{Z}_2$ symmetries = TRSM:

$$\mathcal{V}(\phi, S, X) = \color{green}{\bullet}\,|\phi|^2 + \color{blue}{\blacksquare}\,|\phi|^4 + \color{red}{\bullet}\,S^2 + \color{dodgerblue}{\blacksquare}\,S^4 + \color{magenta}{\bullet}\,X^2 + \color{peru}{\blacksquare}\,X^4 + \color{greenyellow}{\blacksquare}\,S^2 X^2 + \color{indigo}{\blacksquare}\,|\phi|^2 S^2 + \color{teal}{\blacksquare}\,|\phi|^2 X^2$$

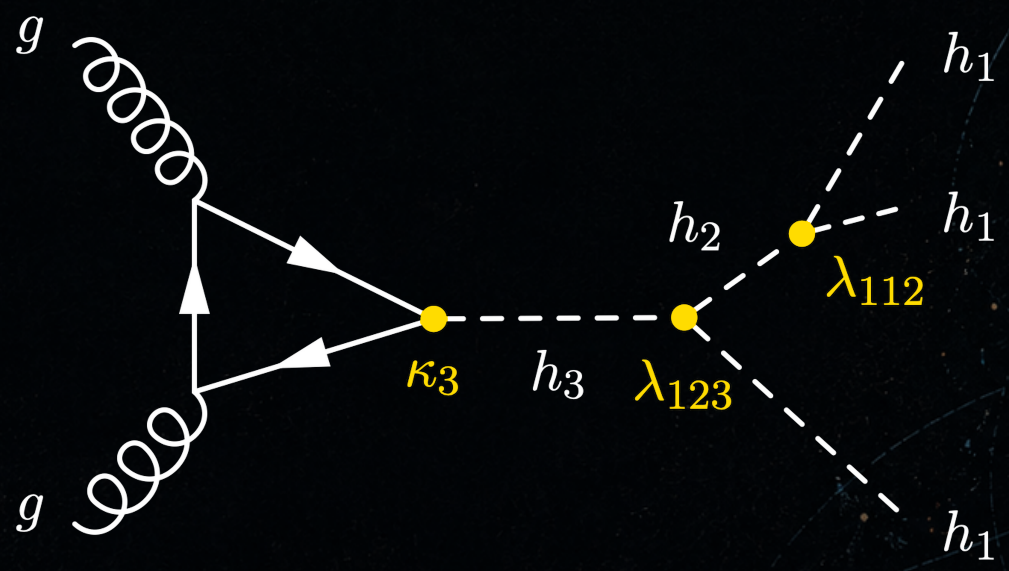
Extended Scalar Sectors: Multi-Scalar Pheno

TRSM ➡ scalar resonances h_1, h_2, h_3 ⇒ richer phenomenology:

$pp \to h_{2,3} \to f\bar{f}, VV, h_1h_1,$ **plus *enhanced*** $pp \to h_1h_1h_1$**:**

double-resonant enhancement!

[Robens, Stefaniak, Wittbrodt, 1908.08554,
AP, Robens, Tetlalmatzi-Xolocotzi, 2101.00037,
Karkout, **AP**, Postma, Tetlalmatzi-Xolocotzi, van de Vis, du Pree, 2404.12425
AP, Tetlalmatzi-Xolocotzi, 2501.14866]



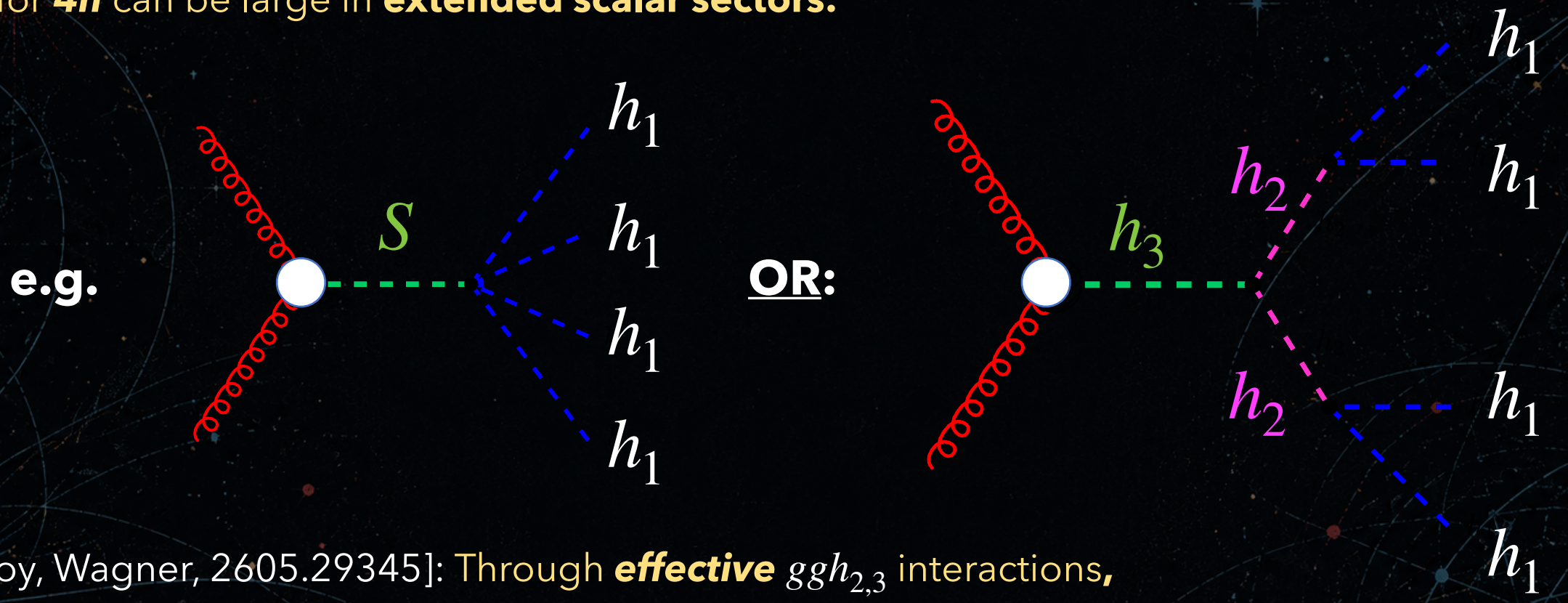
Requires:
$m_3 > m_2 + m_1$, $m_2 > 2m_1$
and
VEVs for X & S nonzero!

* Investigated experimentally in [ATLAS, 2411.02040]!

Quadruple Higgs Boson Production in Extended Scalar Sectors

[**AP**, Tetlalmatzi-Xolocotzi, 2608.16998]

Rate for ***4h*** can be large in **extended scalar sectors:**



[e.g. Roy, Wagner, 2605.29345]: Through ***effective*** $ggh_{2,3}$ interactions**,**

& "**alignment limit**" ($\theta \to 0$): suppresses gauge boson decays + maintains large scalar couplings.

SECTION 04

Conclusions

Conclusions & Outlook

The Higgs potential (i.e. the self-couplings) is one of the *least* understood SM ingredients.

Observed matter-antimatter asymmetry may be generated during electroweak phase transition → if SFO-EWPT ⇒ electroweak baryogenesis!

→ Requires **BSM**! This could be an extended scalar sector.

Multi-Higgs boson production → **Directly** probe self-couplings & extended scalar sectors (that *could* catalyze a SFO-EWPT) .

Extremely rare & challenging (fun!) signals at colliders.

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Thanks!

Questions?

Appendices

A Note on a "Strong" Transition

Suppression of sphaleron rate inside the bubble,

⇒ Matter-antimatter asymmetry swept in broken phase & "**frozen in**".

Rate $\sim \exp[-\langle\phi(T_C)\rangle/T_C \times \ldots]$

[T_C is the critical temperature]

⇒ Require $\langle\phi(T_C)\rangle/T_C \geq 1 \longrightarrow$ a "***Strong***" first-order EWPT **(SFO-EWPT)**.



WG4 *Notice*: $gg \to hh$ @ HL-LHC ⟶ Report 5 on Higher-Order Corrections Out!

Major CERN Report 5 contribution out, LHCHWG-INT-2026-001:

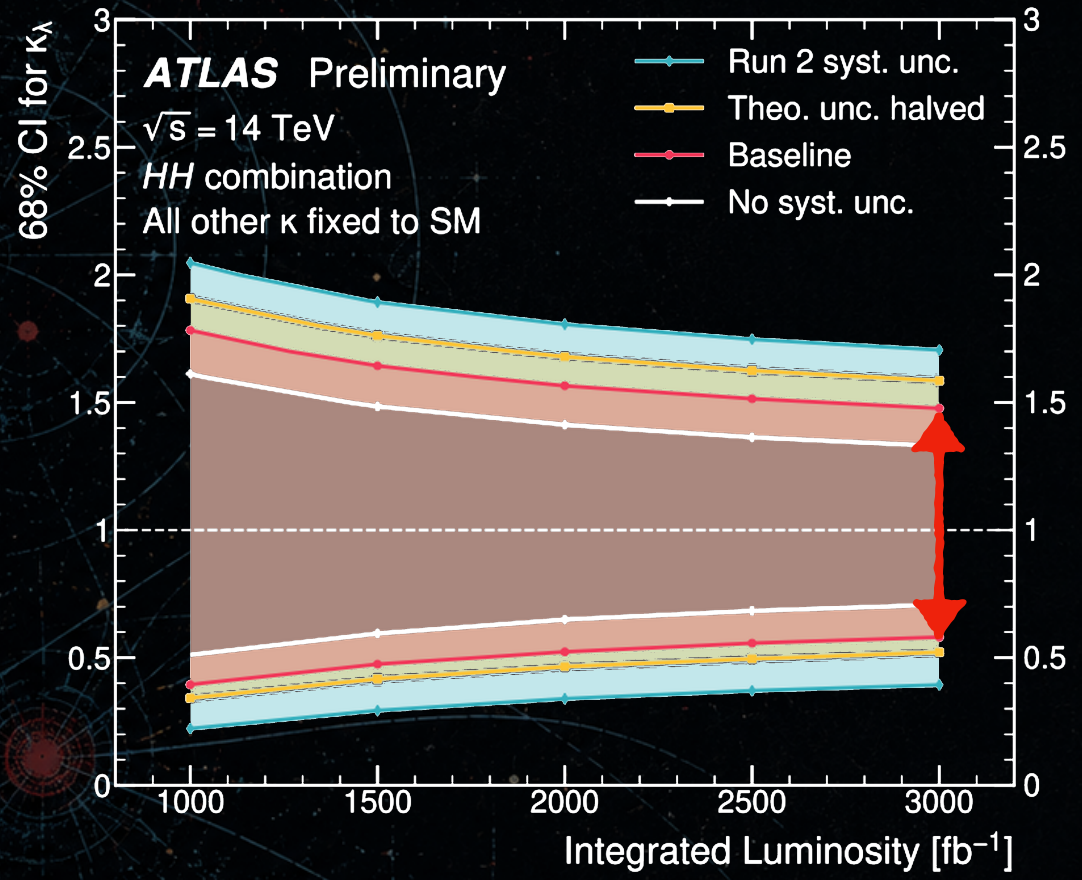
- NLO corrections with top quark mass effects,
- Approximate NNLO/N^{3}LO corrections,
- Electroweak corrections,
- Top mass scheme uncertainty investigations (inclusive),
- Inclusive $gg \to hh$ cross section recommendations.

$$\sigma_{\text{rec.}}(\sqrt{s}) = \sigma_{\text{NNLO}_{\text{FTapprox}}}(\sqrt{s}) \times K_3(\sqrt{s}) \times K_{\text{EW}}(\sqrt{s})$$

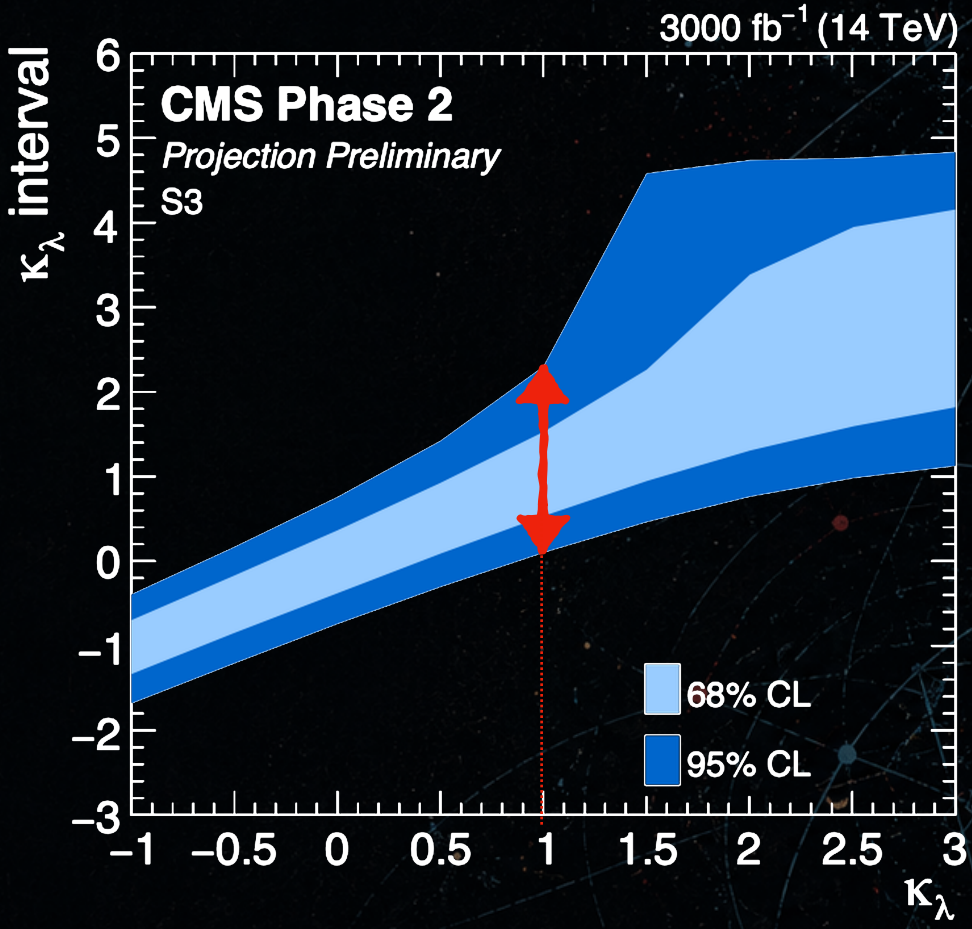
$\sqrt{s}$ [TeV]	13	13.6	14
$\sigma_{\text{rec.}}$ [fb]	30.40	33.62	35.81
± PDF unc. [%]	1.96	1.92	1.89
± α_s unc. [%]	1.51	1.49	1.47
± PDF + α_s unc. [%]	2.47	2.43	2.39
QCD scale unc. [%]	$^{+2.11}_{-4.98}$	$^{+2.07}_{-4.85}$	$^{+2.01}_{-4.78}$
m_t scheme unc. [%]	$^{+5}_{-16}$	$^{+5}_{-16}$	$^{+5}_{-16}$
± TH_{num} [%]	0.14	0.19	0.21
Scale + TH_{num} + m_t scheme unc. [%]	$^{+7.25}_{-21.12}$	$^{+7.26}_{-21.04}$	$^{+7.22}_{-20.99}$

Trilinear @ HL-LHC

ATLAS: $\kappa_3 = 1^{+0.48}_{-0.42}$ [ATL-PHYS-PUB-2025-006]



CMS: $\kappa_3 = 1^{+0.50}_{-0.47}$ [CMS NOTE -2025/006]



$\kappa_\lambda \equiv \kappa_3$

(Modern) History of Triple Higgs Boson Pheno

"Interesting" final states + at least 100 events @ 100 TeV @ 30 ab^{-1}

First studies (mostly) at 100 TeV pp!

$hhh \to$ final state	BR (%)	σ (ab)	$N_{30\text{ab}^{-1}}$
$(b\bar{b})(b\bar{b})(b\bar{b})$	19.21	1110.338	33310
$(b\bar{b})(b\bar{b})(WW_{1\ell})$	7.204	416.41	12492
$(b\bar{b})(b\bar{b})(\tau\bar{\tau})$	6.312	364.853	10945
$(b\bar{b})(\tau\bar{\tau})(WW_{1\ell})$	1.578	91.22	2736
$(b\bar{b})(b\bar{b})(WW_{2\ell})$	0.976	56.417	1692
$(b\bar{b})(WW_{1\ell})(WW_{1\ell})$	0.901	52.055	1561
$(b\bar{b})(\tau\bar{\tau})(\tau\bar{\tau})$	0.691	39.963	1198
$(b\bar{b})(b\bar{b})(ZZ_{2\ell})$	0.331	19.131	573
$(b\bar{b})(WW_{2\ell})(WW_{1\ell})$	0.244	14.105	423
$(b\bar{b})(b\bar{b})(\gamma\gamma)$	0.228	13.162	394
$(b\bar{b})(\tau\bar{\tau})(WW_{2\ell})$	0.214	12.359	370
$(\tau\bar{\tau})(WW_{1\ell})(WW_{1\ell})$	0.099	5.702	171
$(\tau\bar{\tau})(\tau\bar{\tau})(WW_{1\ell})$	0.086	4.996	149
$(b\bar{b})(ZZ_{2\ell})(WW_{1\ell})$	0.083	4.783	143
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First studies (mostly) at 100 TeV pp!

[**AP**, Sakurai, **1508.06524**, Chen, Yan, Zhao, Zhao, Zhong, **1510.04013**, Fuks, Kim, Lee, **1510.07697, ...**]

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$(b\bar{b})(\tau\bar{\tau})(WW_{2\ell})$	0.214	12.359	370
$(\tau\bar{\tau})(WW_{1\ell})(WW_{1\ell})$	0.099	5.702	171
$(\tau\bar{\tau})(\tau\bar{\tau})(WW_{1\ell})$	0.086	4.996	149
$(b\bar{b})(ZZ_{2\ell})(WW_{1\ell})$	0.083	4.783	143
$(b\bar{b})(\tau\bar{\tau})(ZZ_{2\ell})$	0.073	4.191	125

First studies (mostly) at 100 TeV pp!

[Fuks, Kim, Lee, **1510.07697**, Fuks, Kim, Lee, **1704.04298**]

[**AP**, Sakurai, **1508.06524**, Chen, Yan, Zhao, Zhao, Zhong, **1510.04013**, Fuks, Kim, Lee, **1510.07697, ...**]

(Modern) History of Triple Higgs Boson Pheno

"Interesting" final states + at least 100 events @ 100 TeV @ 30 ab^{-1}

$hhh \to$ final state	BR (%)	σ (ab)	$N_{30\text{ab}^{-1}}$
$(b\bar{b})(b\bar{b})(b\bar{b})$	19.21	1110.338	33310
$(b\bar{b})(b\bar{b})(WW_{1\ell})$	7.204	416.41	12492
$(b\bar{b})(b\bar{b})(\tau\bar{\tau})$	6.312	364.853	10945
$(b\bar{b})(\tau\bar{\tau})(WW_{1\ell})$	1.578	91.22	2736
$(b\bar{b})(b\bar{b})(WW_{2\ell})$	0.976	56.417	1692
$(b\bar{b})(WW_{1\ell})(WW_{1\ell})$	0.901	52.055	1561
$(b\bar{b})(\tau\bar{\tau})(\tau\bar{\tau})$	0.691	39.963	1198
$(b\bar{b})(b\bar{b})(ZZ_{2\ell})$	0.331	19.131	573
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[Fuks, Kim, Lee, **1510.07697**, Fuks, Kim, Lee, **1704.04298**]

[Kilian, Sun, Yan, Zhao, Zhao, **1702.03554**]

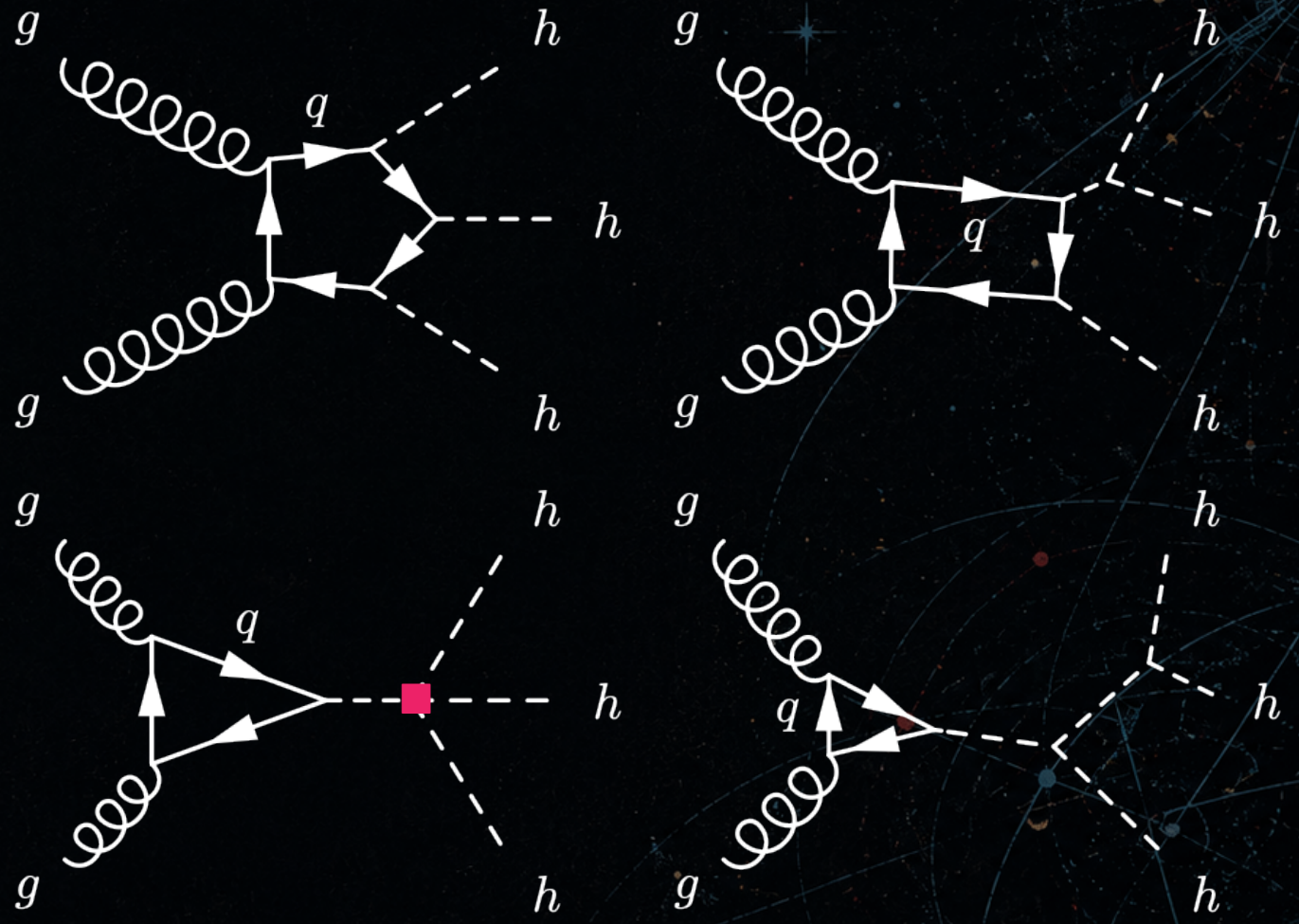
[Stylianou, Weiglein, **2312.04646**, Dong, Sun, Goo, Zhang, Li, Wang, **2504.04037**]

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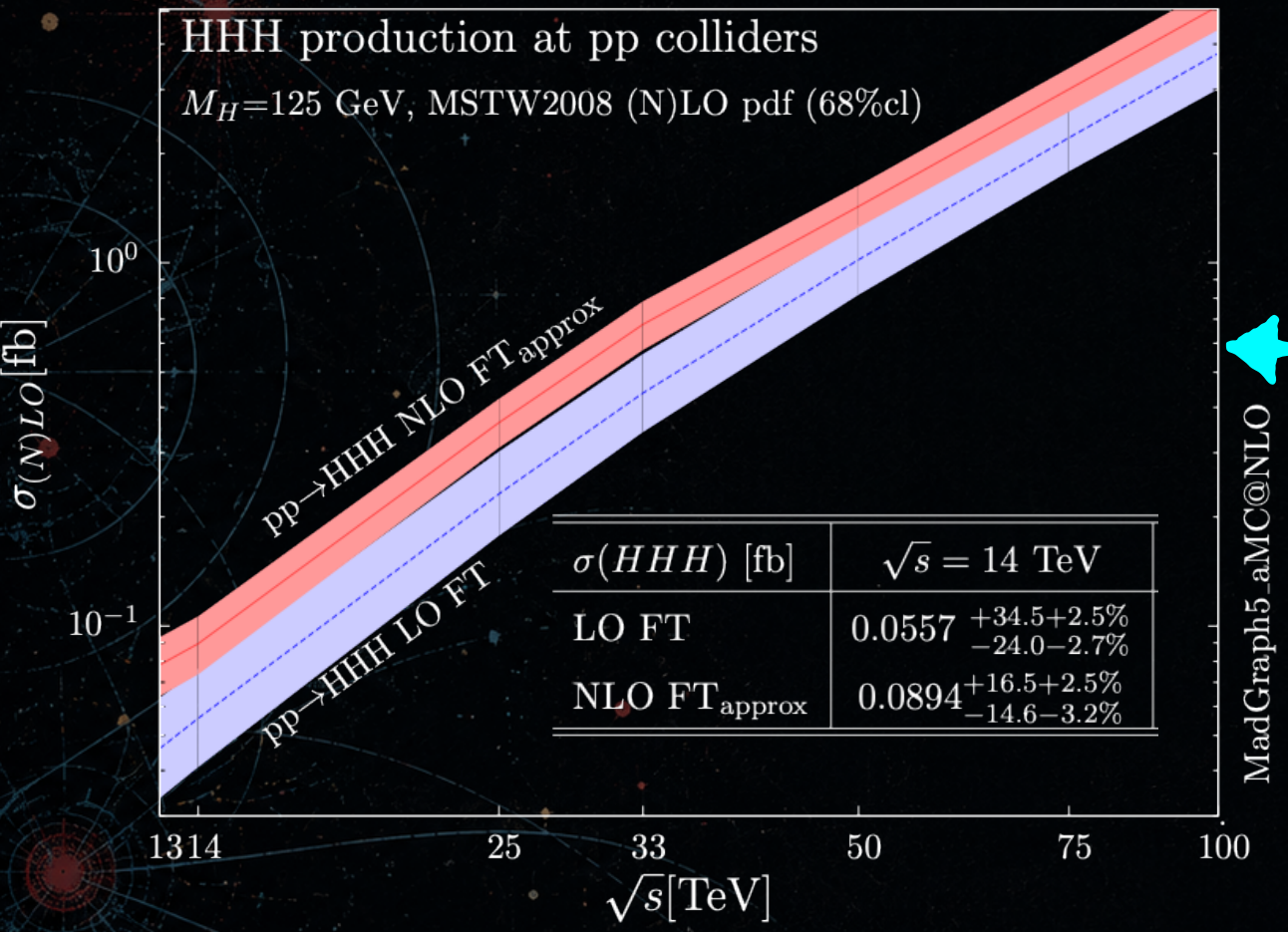
The $pp \to hhh$ Cross Section

At **LO** in SM: Loop-induced!

$$\sigma^{\text{LO}}_{\text{SM}}(14\ \text{TeV}) \sim 0.05\ \text{fb}$$

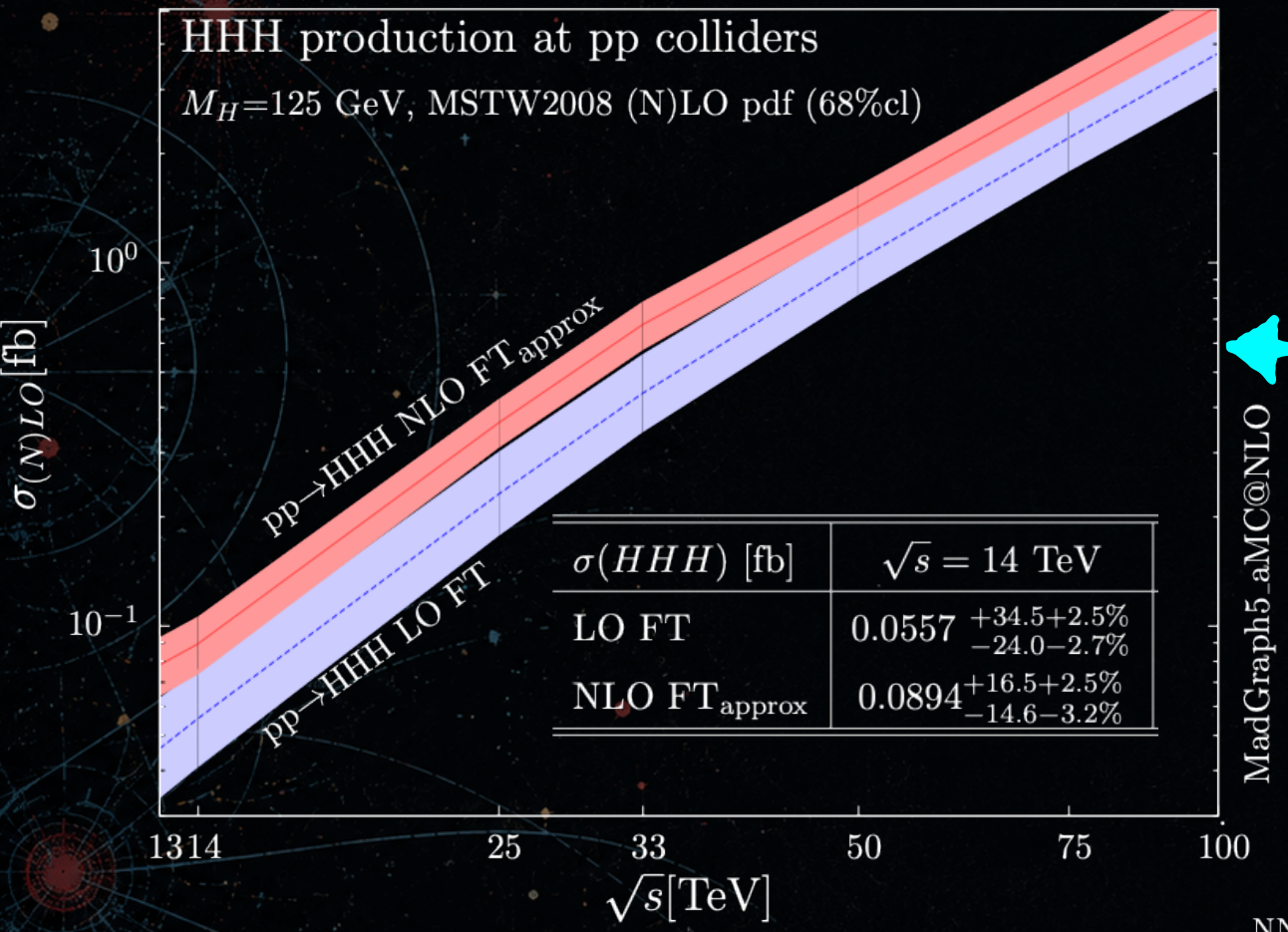


$pp \to hhh$ @ Higher Orders



NLO: [Maltoni, Vryonidou, Zaro, **1408.6542**] → $m_t \to \infty$ plus full real emission (= FT$_{approx}$).

$pp \to hhh$ @ Higher Orders



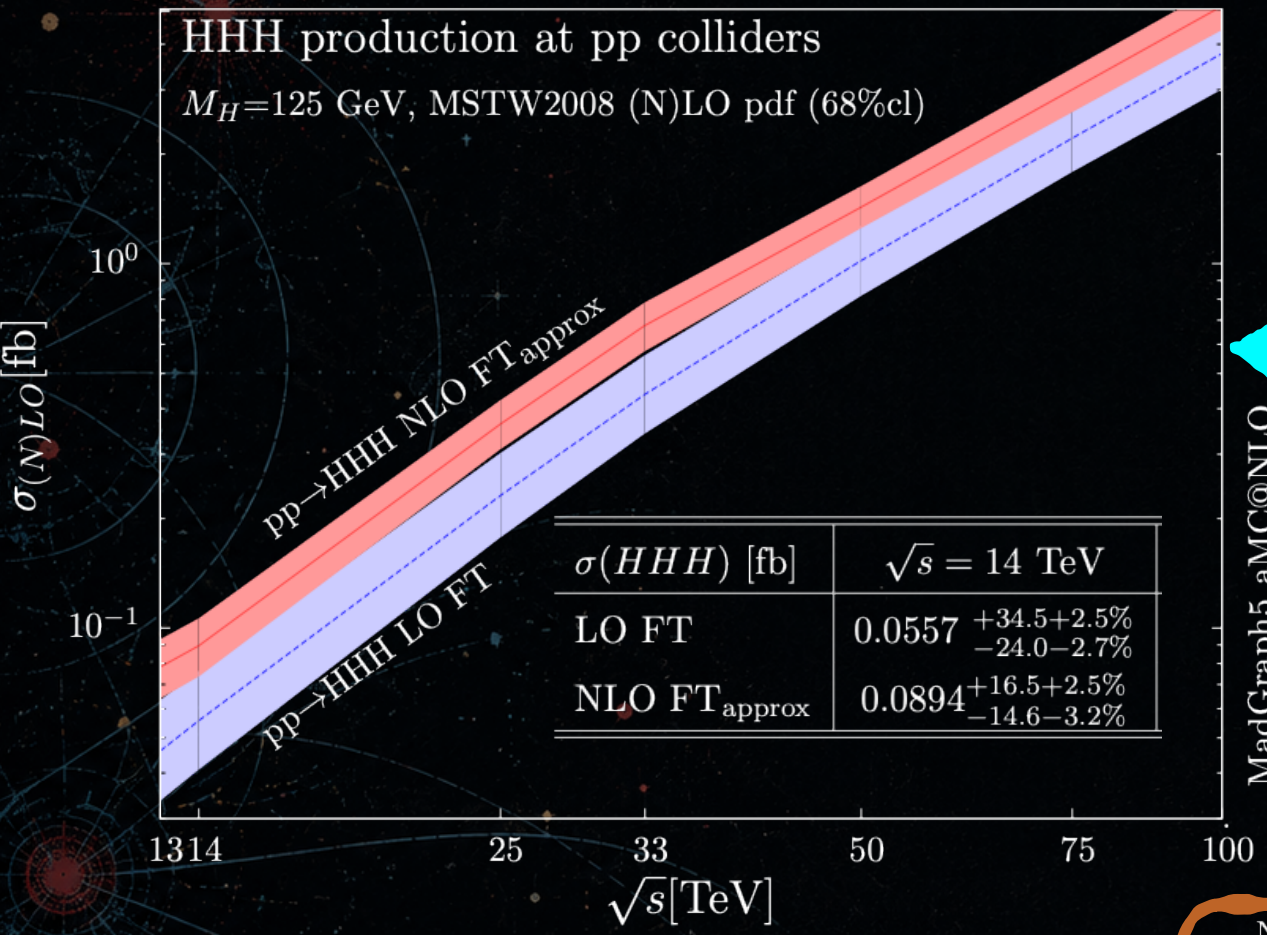
$\sigma(HHH)$ [fb]	$\sqrt{s} = 14$ TeV
LO FT	$0.0557\ ^{+34.5+2.5\%}_{-24.0-2.7\%}$
NLO FT$_{approx}$	$0.0894^{+16.5+2.5\%}_{-14.6-3.2\%}$

NLO: [Maltoni, Vryonidou, Zaro, **1408.6542**] → $m_t \to \infty$ plus full real emission (= FT$_{approx}$).

NNLO: [De Florian, Fabre, Mazzitelli, **1912.02760**] → Full NNLO in $m_t \to \infty$, "Born-Improved" ⇒ ~10% increase wrt. NLO.

$$\sigma^{\rm NNLO}_{\rm Best} = 0.103^{+5\%}_{-8\%} \pm 15\%\,{\rm fb}, \qquad K^{\rm NNLO}_{\rm Best} = 1.70, \qquad (14\ {\rm TeV})$$

$pp \to hhh$ @ Higher Orders



NLO: [Maltoni, Vryonidou, Zaro, **1408.6542**] → $m_t \to \infty$ plus full real emission (= FT$_{approx}$).

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Attack of the Anomalous Couplings

Anomalous couplings can enhance ***hhh***! e.g. *inspired* by an Effective Field Theory.

$$\mathscr{V}(h) \supset \underbrace{(1+c_3)}_{\kappa_3}\frac{m_h^2}{2v}h^3 + \underbrace{(1+d_4)}_{\kappa_4}\frac{m_h^2}{8v^2}h^4$$

⇒ ***hhh*** cross section as ratio to SM (highest-to-lowest coefficient):

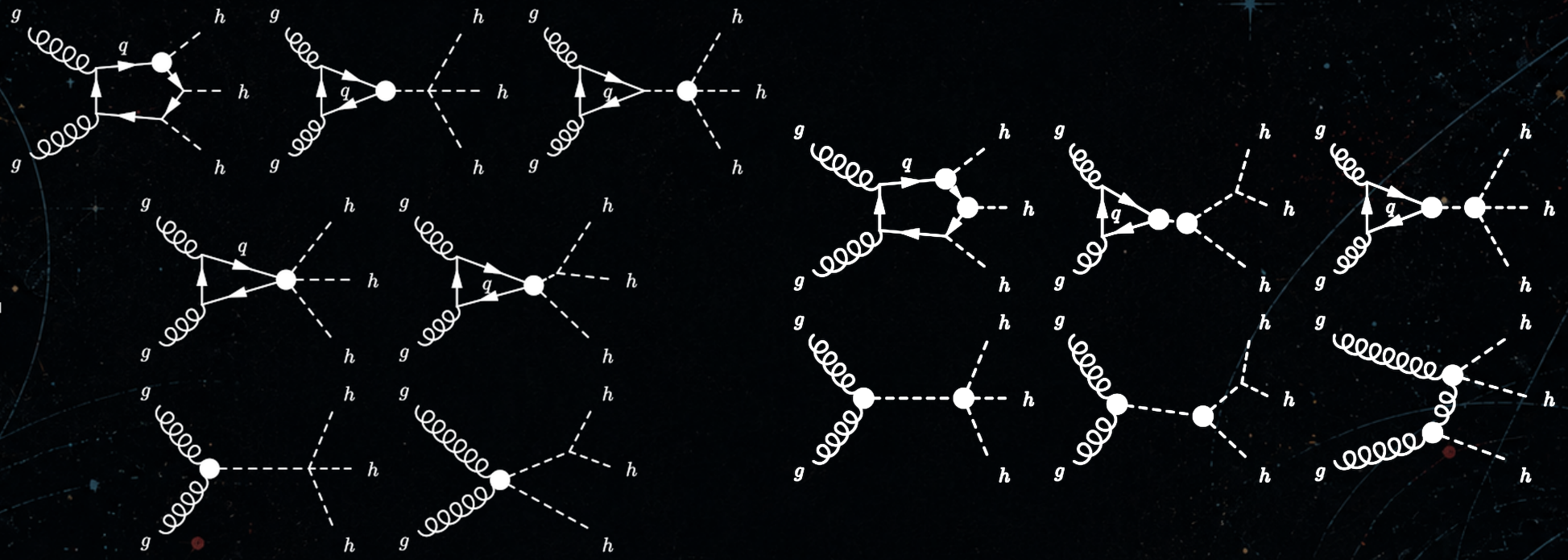
$$\sigma/\sigma_{\rm SM}(13.6\ \text{TeV}) - 1 \approx 0.922c_3^2 - 0.828c_3 - 0.277c_3^3 - 0.178c_3d_4 - 0.0993d_4 + 0.0510c_3^2d_4 - 0.0393c_3^4 + 0.0174d_4^2$$

⇒ Not very sensitive to the quartic!

Return of the Anomalous Couplings

Additional anomalous interactions can also be considered, e.g. $t\bar{t}h^2$, $t\bar{t}h^3$, ggh^2 …:

SM +

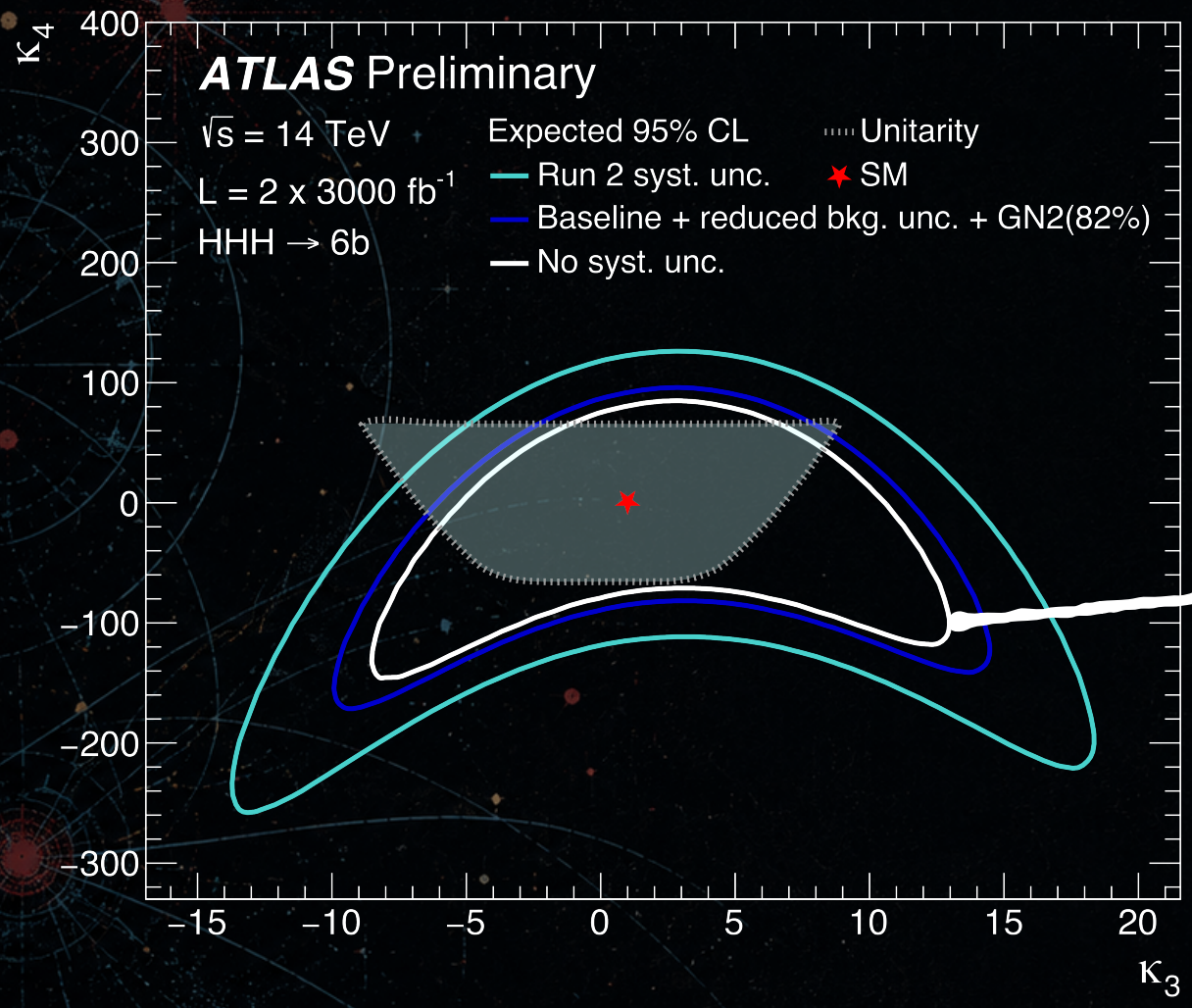


MadGraph5_aMC model at: https://gitlab.com/apapaefs/multihiggs_loop_sm

[**AP**, Tetlalmatzi-Xolocotzi, 2312.13562]

Includes description of interference effects.

Quartic @ HL-LHC



ATLAS HL-LHC extrapolation of **6 b-jet** final state (**6000 fb^{-1}**) with reduced *or* no systematics. [ATL-PHYS-PUB-2025-003]

Best-case scenario:
$\kappa_4 \in [-65, 76]$

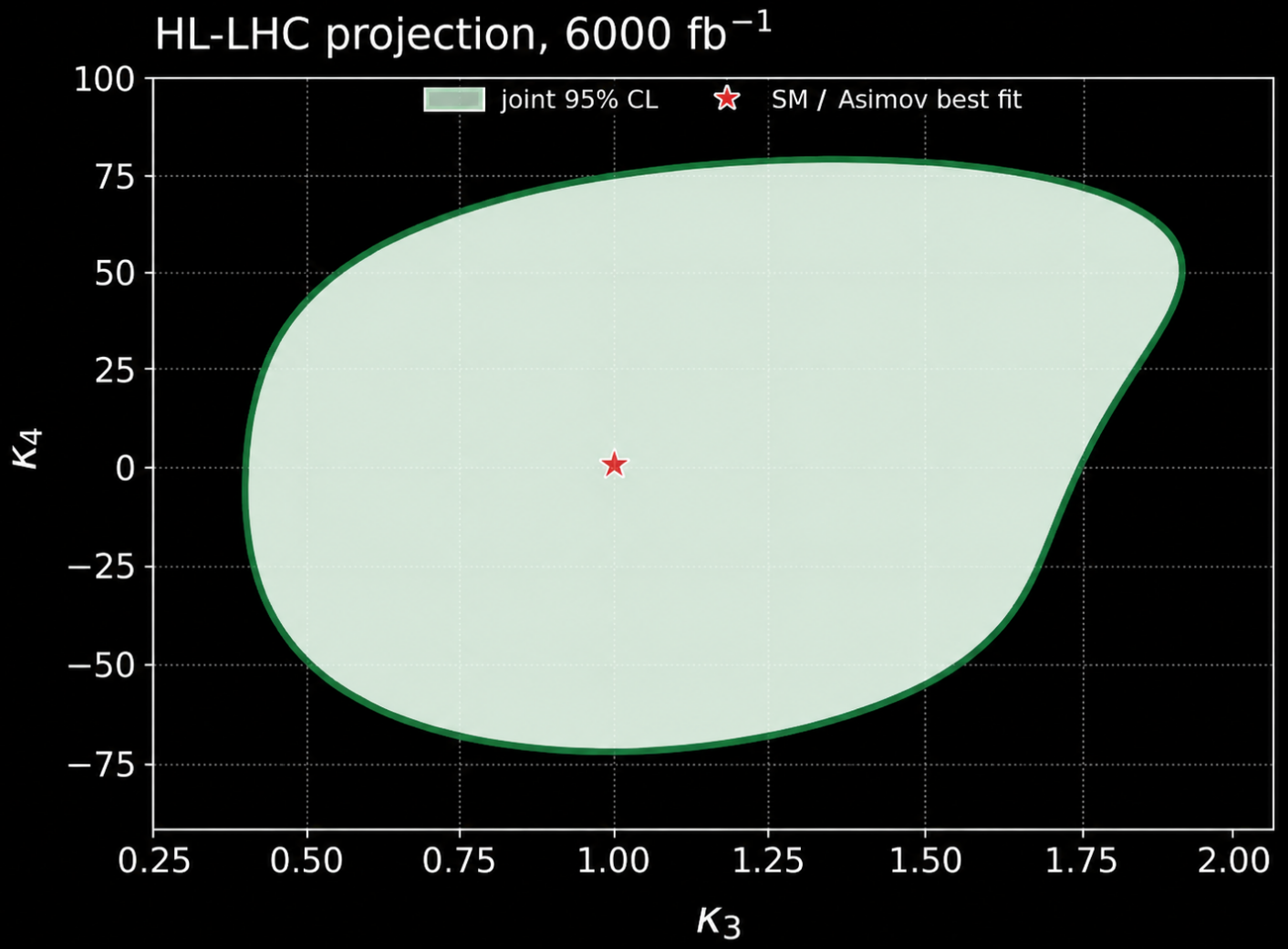
[**Shaded region:** $hh \to hh$ **perturbative unitarity**]

Definitive HL-LHC Self-Coupling Constraints (A Theorist's Dark Fantasy)

Ingredients:

- $gg \to hh$ ATLAS+CMS HL-LHC projection [2504.00672, Fig. 2 left]
  - with κ_4 dependence in $gg \to hh$ (rescaled for $k_4 \neq 1$) [Bizoń, Haisch, Rottoli, Gillis, Moser, Windischhofer, 2402.03463],
- $gg \to hhh$ ATLAS+CMS HL-LHC signal strength projection [ATL-PHYS-PUB-2025-003, Fig. 4 right],
- Signal strengths + κ_3, κ_4 self-coupling dependence in single Higgs: $pp \to h, Vh, \text{VBF}, t\bar{t}h$ [Haisch, Sankar, Zanderighi, 2505.20463],
- $gg \to hhhh$ full Poisson likelihood over κ_3, κ_4 [**AP**, Tetlalmatzi-Xolocotzi, 2608.16998].

Definitive HL-LHC Self-Coupling Constraints (A Theorist's Dark Fantasy)



@ 95% C.L.:

joint two-param. fit:

$\kappa_3 \in [0.4, 1.9]$ $\kappa_4 \in [-71, 79]$

one-param., the other **fixed** to SM:

$\kappa_3 \in [0.5, 1.6]$ $\kappa_4 \in [-63, 67]$

A challenge:
Can we beat these?

Quadruple Higgs Boson Production (!?)

Main background: $gg \to 8b$ jets ➡ ***Extremely*** challenging to generate events ➡ Sherpa LHE + Herwig 7.

Simple **smearing** for jets applied (see e.g. **[Fuks, AP, Tetalmatzi-Xolocotzi, 2509.16364]**).

Pair 8 b-jets into four Higgs candidates and minimize:

$$\chi^2_8(P) = \sum_{k=1}^{4} \left(m_{h,k}(P) - m_{h,k}^{\text{target}} \right)^2 \quad \text{with:} \quad (m_{h,1}^{\text{target}}, m_{h,2}^{\text{target}}, m_{h,3}^{\text{target}}, m_{h,4}^{\text{target}}) = (120,115,110,105)\ \text{GeV}\,.$$

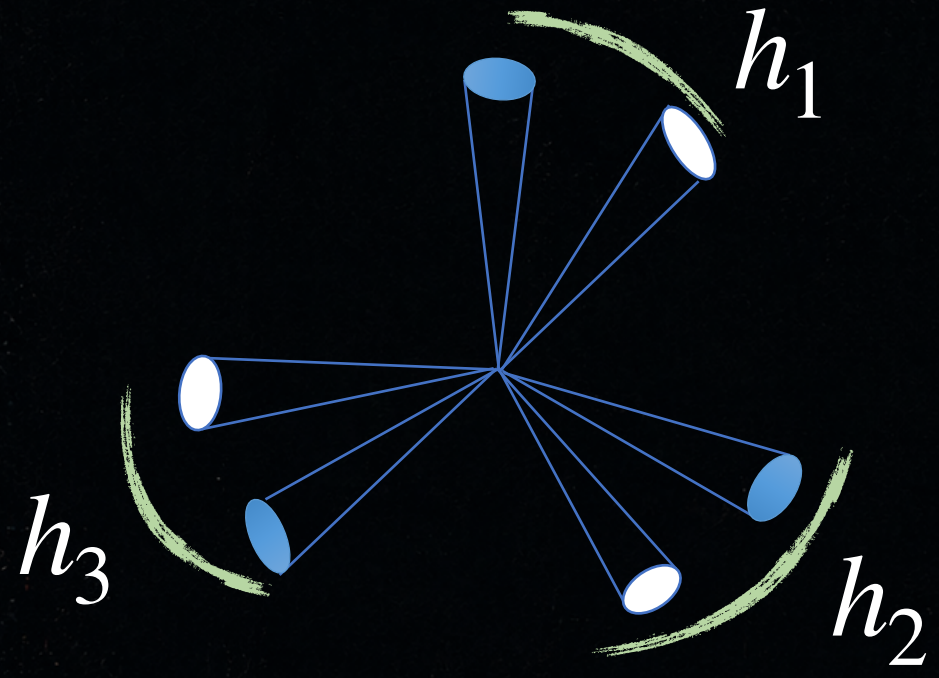
XGBoost analysis on a complete set of observables.

Poisson limited, with *very* low numbers of events! ➡ $B \sim \mathcal{O}(0.1)$ expected events for $gg \to 8b$ jets**!**

Charging Up Multi-Higgs Boson Production

PRELIMINARY

6 b-jets: if charge is unknown → **15** combinations of 6 b-jets into pairs.



$hh \to 4b$ jets: **3** combos

$hhhh \to 8b$ jets: **105 (!)**

Jet-charge tagging reduces the amount of combinations: e.g. CMS DP-2025/07 → **70**% accuracy on b-jets (= **30%** mis-id).

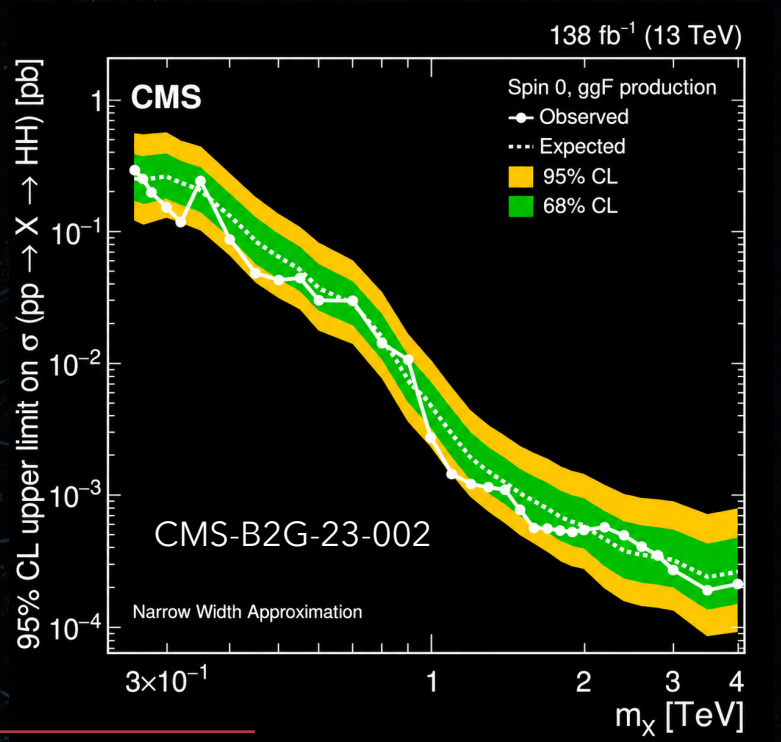
[see e.g. talk by M. Stamenkovic @ HIGGS 2025]

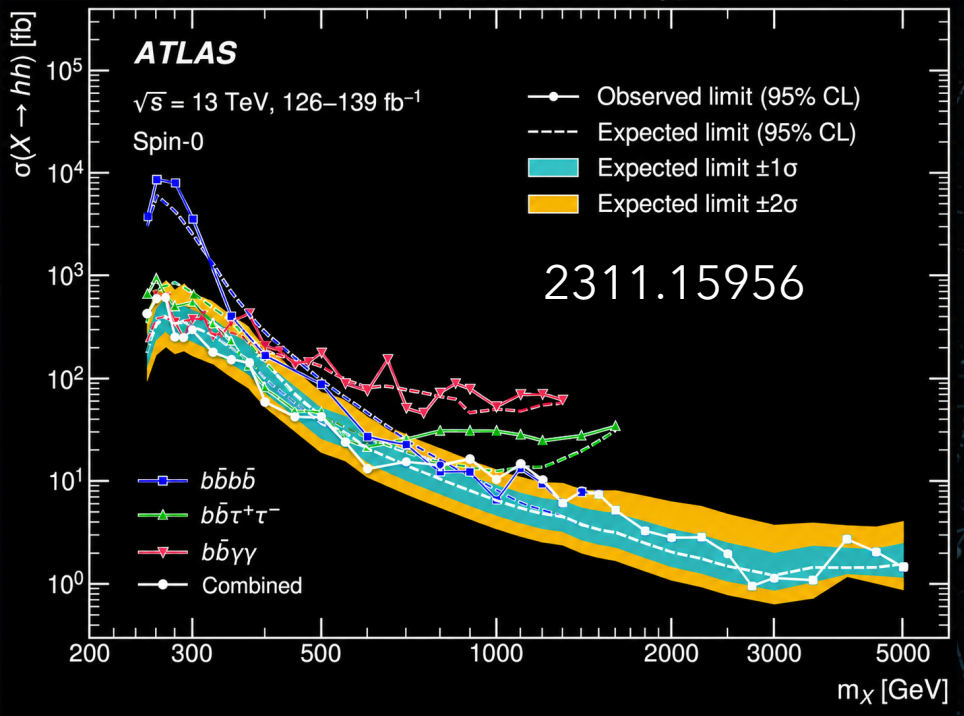
Extended Scalar Sectors: One Singlet Scalar

- Pheno studies, including with **SFO-EWPT** $\longrightarrow h_2 \to ZZ, WW$ and $h_2 \to h_1 h_1$ ***complementary***!

[Kotwal, Ramsey-Musolf, No, Winslow, 1605.06123, **AP**, White, 2010.00597, 2108.11394 , … Many more!]

- Sample of experimental results from $h_2 \to h_1 h_1$:



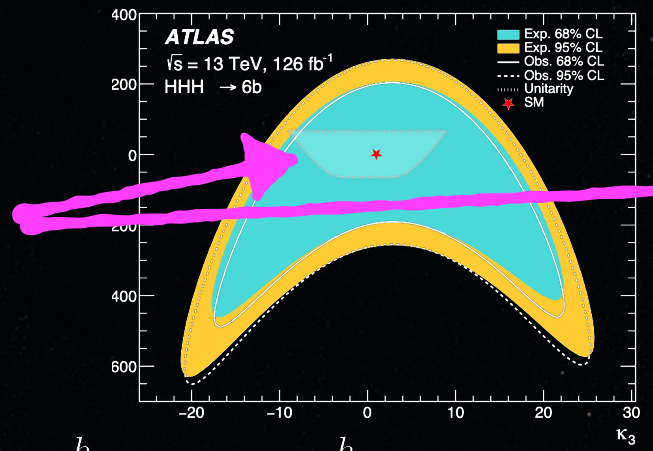


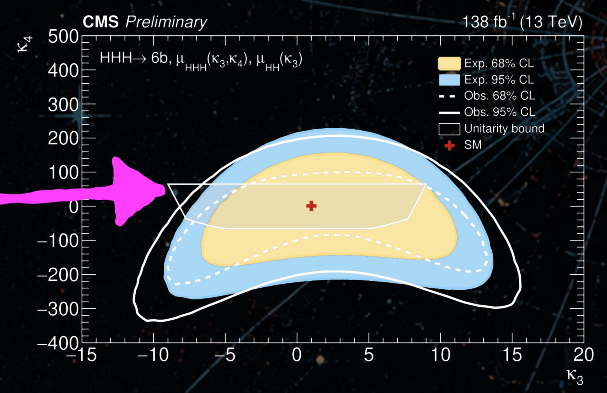
[ASIDE]

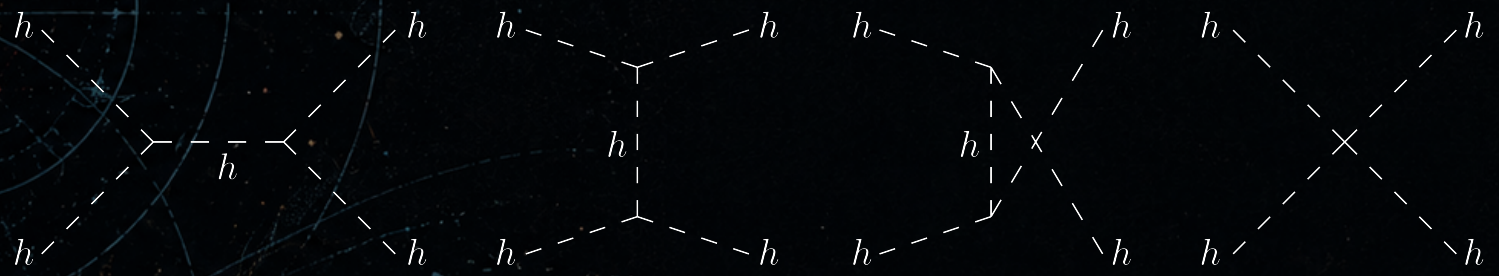
Comments on Perturbative Unitarity

Perturbative Unitarity Constraints

- Experiments show limits on (κ_3, κ_4) like:
- Including **perturbative unitarity** constraints
- From tree-level $hh \to hh$:







$$\mathscr{M} = -\lambda_3^2\left(\frac{1}{s-m_h^2}+\frac{1}{t-m_h^2}+\frac{1}{u-m_h^2}\right)-\lambda_4$$

Via matrix element $\mathscr{M} \sim \langle f\,|\,T\,|\,i\rangle \to T$, interacting part the ***S***-matrix: $S = 1 + iT$, requiring $S^\dagger S = 1$, i.e. unitarity.

All theories must satisfy this: *If* violated ➡ tree level is ***not*** a good approximation for $\mathscr{M}$!

➡ **Higher-order contributions to $\mathscr{M}$ can't be neglected** ➡ **perturbation theory has broken down!**

[Jacob, Wick, Annals Phys. 7, 404 (1959), Di Luzio, Kamenik, Nardecchia, 1604.05746 Di Luzio, Gröber, Spannowsky, 1704.02311, Liu, Lyu, Zhu, 1803.04359, Chang, Luty, 1902.05556, Stylianou, Weiglein, 2312.04646,Fuks, AP, Tetlalmatzi-Xolocotzi, 2509.16364]

See [Logan, 2207.01064] for a **pedagogical** introduction.

More Perturbative Unitarity Constraints

What about constraints from ***other*** scattering processes? e.g. longitudinal gauge boson scattering at high energy:

$$W_L W_L \to W_L W_L hh\,,\ W_L W_L \to W_L^4,\ W_L W_L \to h^4, \ldots$$

For these, ∃ breakdown energy scale Λ, e.g.: [Mahmud, Tobioka, 2406.03522, 2501.16439]

$$\left|c_3 - \frac{1}{5}d_4\right| < \left(\frac{12.3\ \text{TeV}}{\Lambda}\right)^2, \qquad |c_3| < \left(\frac{18.9\ \text{TeV}}{\Lambda}\right)^2, \qquad \left|c_3 - \frac{1}{3}d_4\right| < \left(\frac{20.9\ \text{TeV}}{\Lambda}\right)^2, \ldots$$

[**quintic** interaction set to zero]

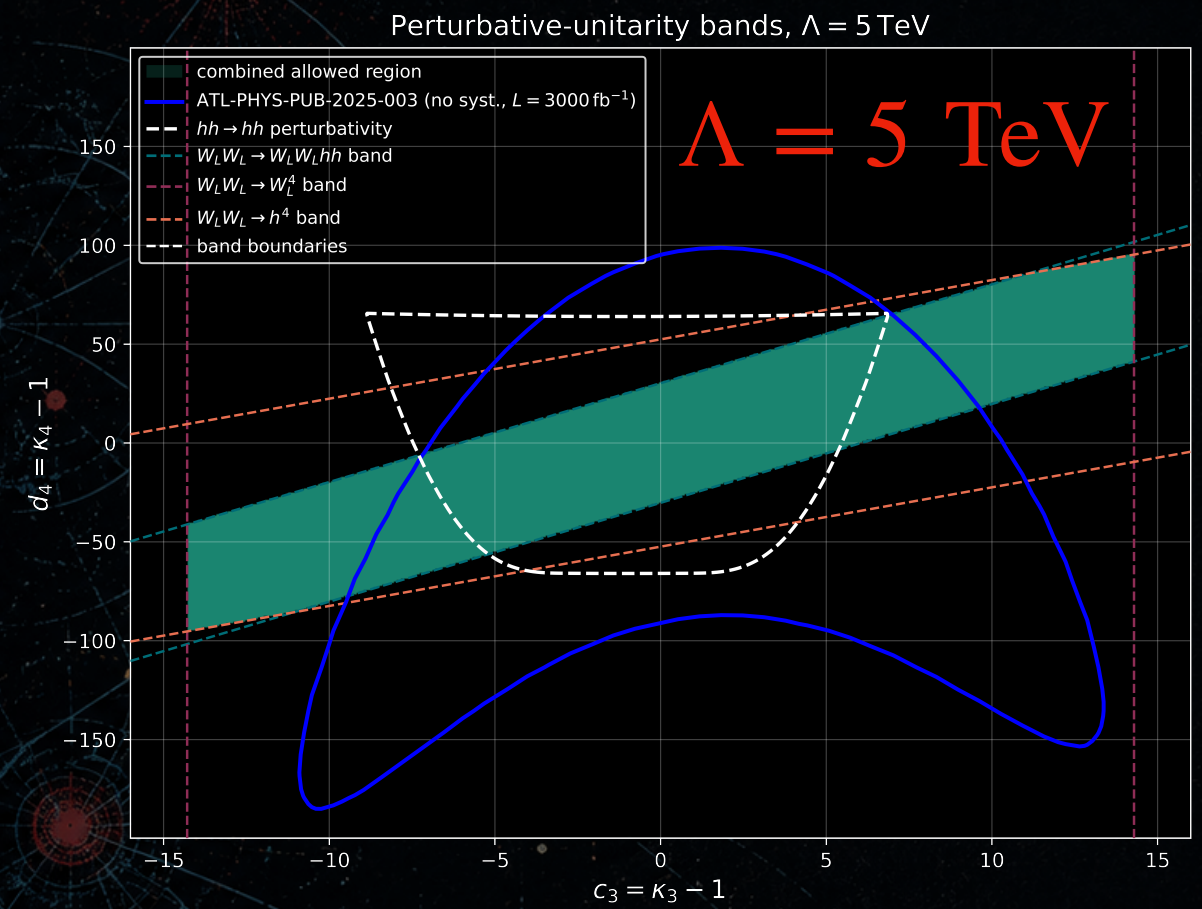
[In order of most constraining processes.]

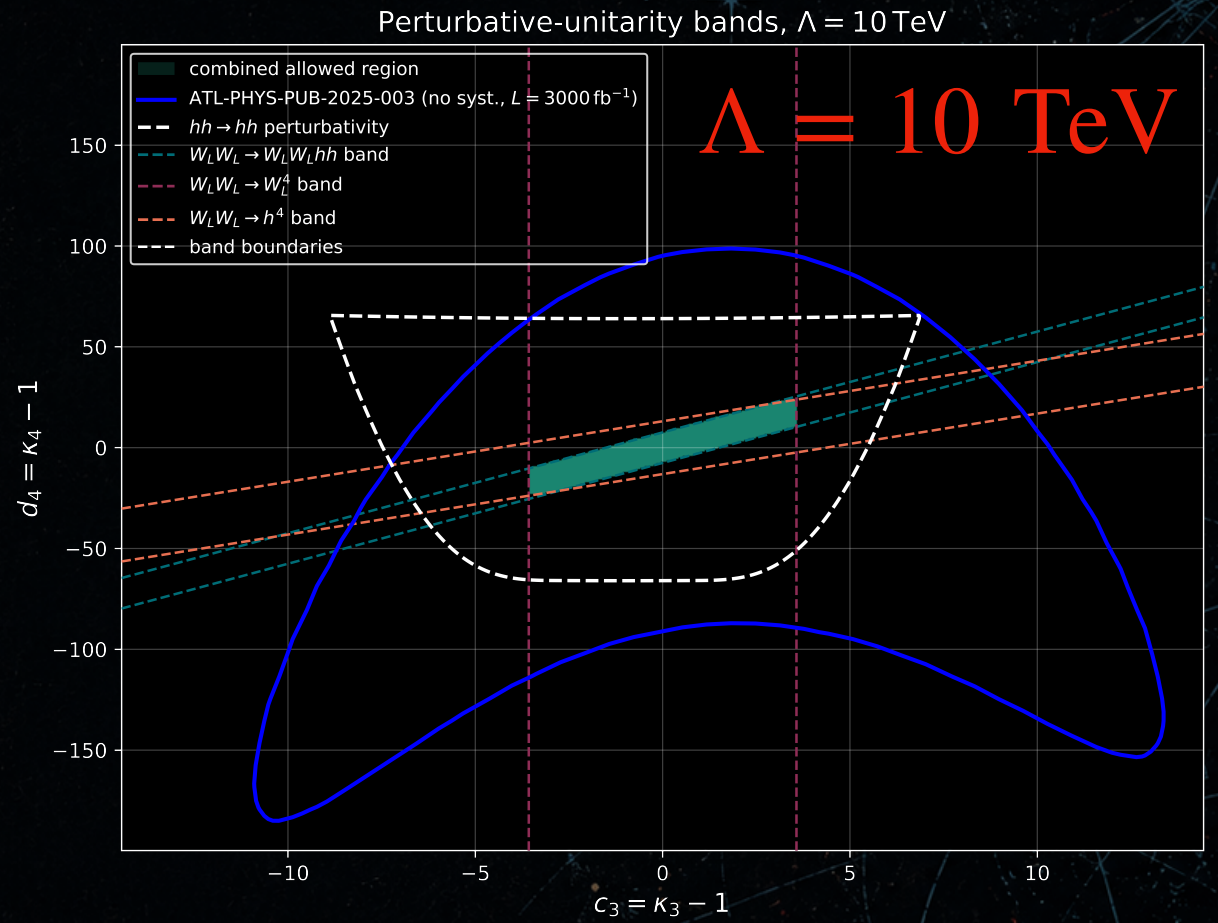
[*Note*: The $\Lambda \to \infty$ line is a **leading-power** null direction: sub-leading terms become important.]

Note that: for $hh \to hh$ the result is valid at ***all*** energy scales.

More Perturbative Unitarity Constraints

Using: $W_L W_L \to W_L W_L hh\,,\; W_L W_L \to W_L^4\,,\; W_L W_L \to h^4$ [Mahmud, Tobioka, 2406.03522, 2501.16439]





Statements on perturbative unitarity should be associated with a breakdown scale Λ!

[*Note*: The $\Lambda \to \infty$ line is a **leading-power** null direction: sub-leading terms become important.]

Perturbative Unitarity Constraints @ NLO?

Calculate perturbative unitarity at **one loop**, in Higgs Effective Field Theory:

Already available for: $W_L W_L \to W_L W_L$, $W_L W_L \to hh$, $hh \to hh$. [Asáin, Espriu, Mescia, 2109.02673l, 2301.13030]

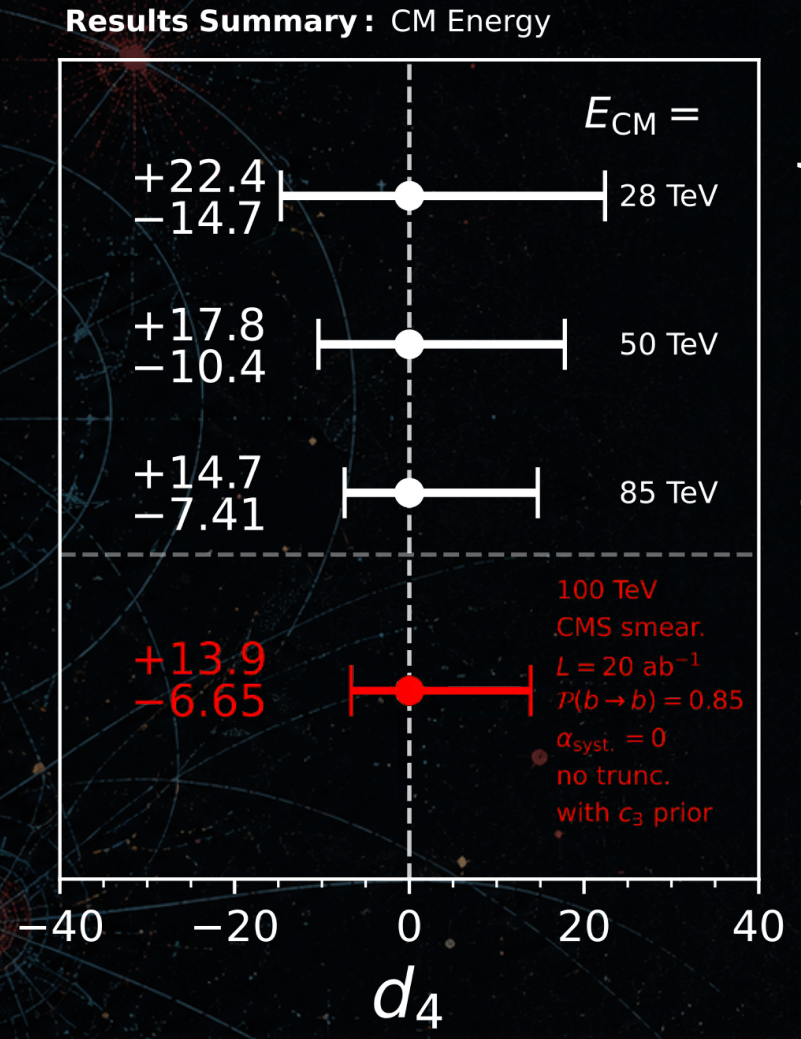
More ***model dependent*** than tree-level:

e.g. @ one loop $hh \to hh$: κ_3, κ_4 ***not*** sufficient: e.g. need derivative operator $\frac{\gamma}{v^4}(\partial_\mu h\, \partial^\mu h)^2$!

Also:

- **Massless** formulas for $hh \to hh$ are insufficient (no κ_3, κ_4 dependence).
- They set $g' = 0$ (neglect e/m) & assume exact custodial symmetry.
- Calculation is in bosonic HEFT (no top quarks).

Quartic @ Hadron Colliders @ Higher Energies



Analysis of **6 b-jet** final state [Fuks, **AP**, Tetalmatzi-Xolocotzi, **2509.16364**]

pp collider **energy** variation
(85 → 50 → 28 TeV)

hhh 100 TeV/20 ab^{-1} +
combination with future trilinear constraint,
assuming: $\delta c_3 = 5\,\%$

Note: $1 + d_4 = \kappa_4$

Trilinear @ Other Colliders

FCC-ee @ 240 GeV/365 GeV:

⇒ not enough for *hhZ* ➡ use loop diagrams:



$\delta(\kappa_3) \sim 28\,\%$

$\delta(\kappa_3) \sim 18\,\%$ (HL-LHC combo)

FCC-hh (compared to others):

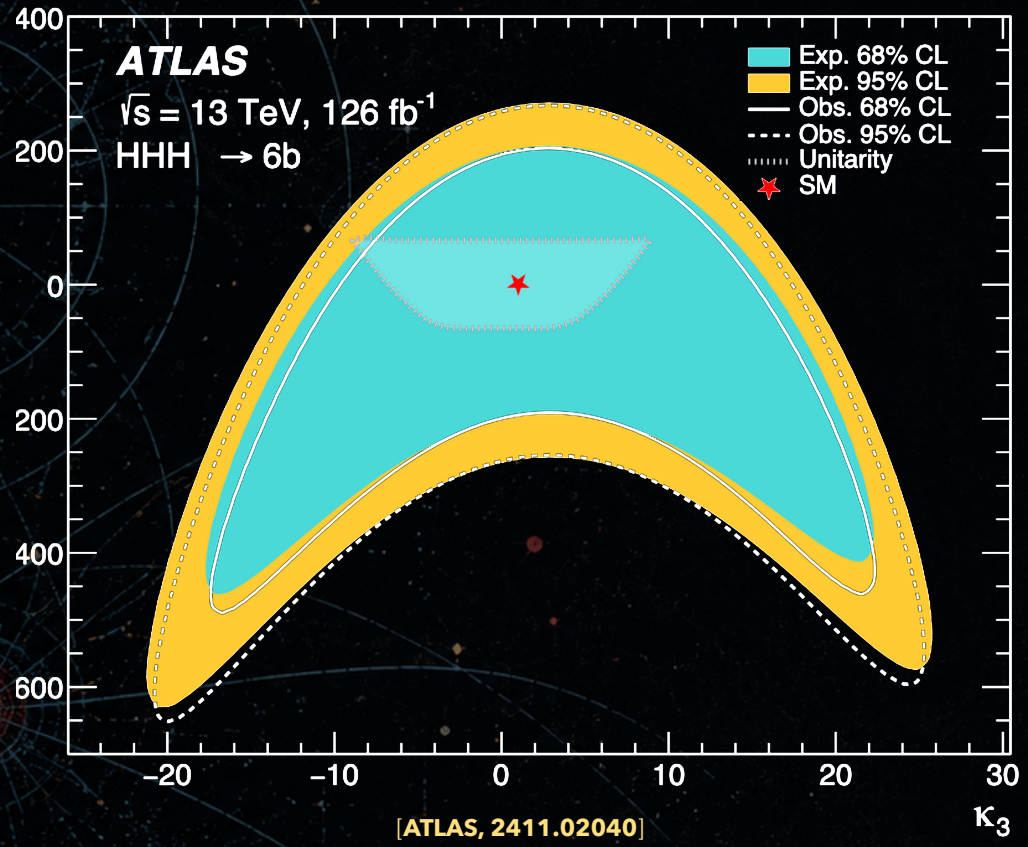


$\delta(\kappa_3) = 3.2\% - 5.4\,\%$ @84 TeV

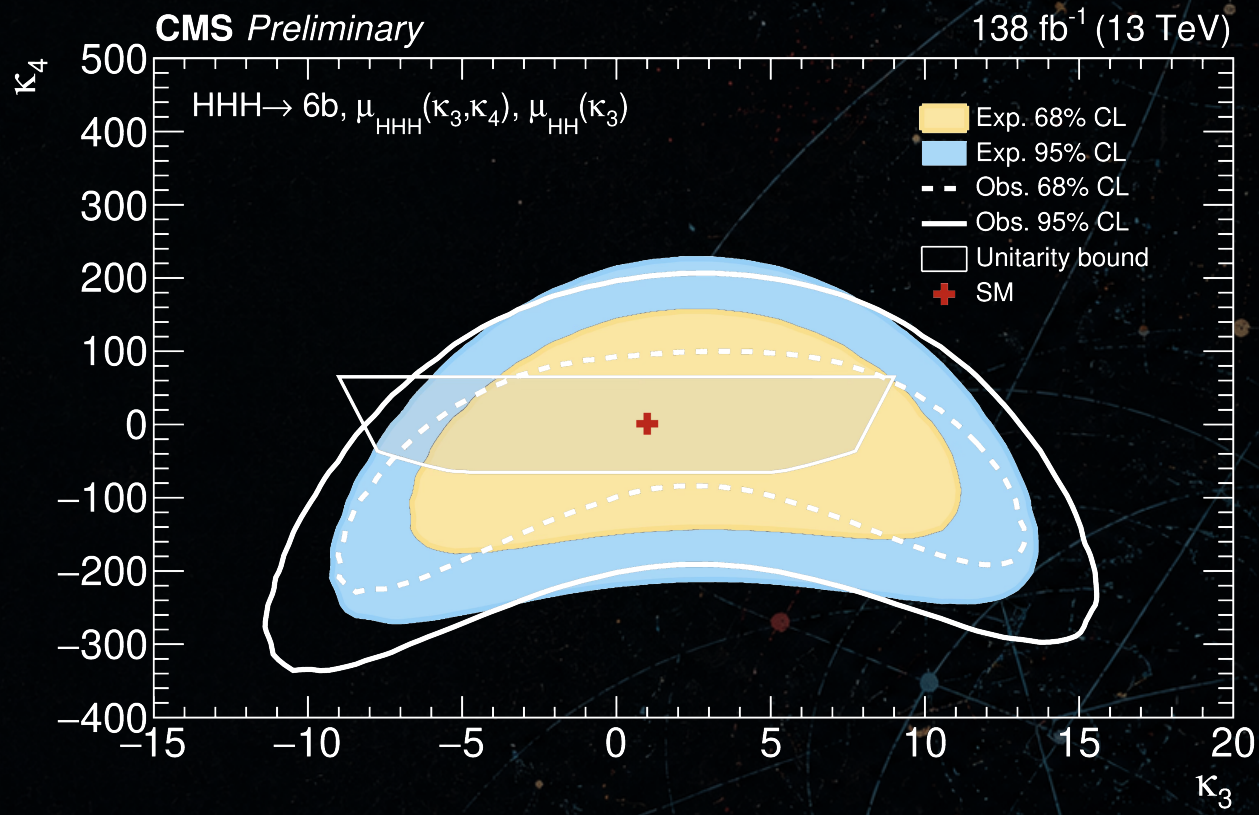
$\delta(\kappa_3) = 2.8\% - 4.8\,\%$ @100 TeV

Quartic @ LHC (Present)

Final state: $pp \to (b\bar{b})(b\bar{b})(b\bar{b})$ ➡ **6 b-jets!**



[ATLAS, 2411.02040]



[CMS-PAS-HIG-24-012 & CMS-PAS-HIG-24-015]

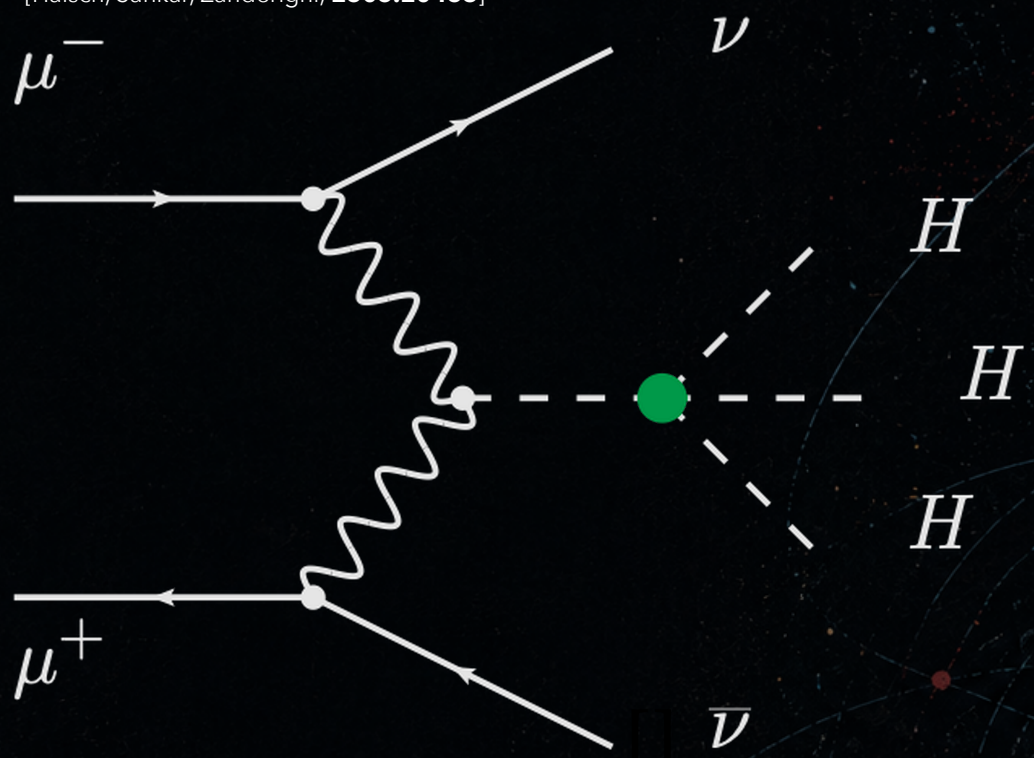
$pp \to (b\bar{b})(b\bar{b})(\gamma\gamma)$

Quartic @ Lepton Colliders

FCC-ee: **insufficient** energy for ***Zhh or Zhhh*** production & constraints for k_4 very weak in ***Zh***.

[Haisch, Sankar, Zanderighi, **2505.20463**]

Muon Collider, through VBF:



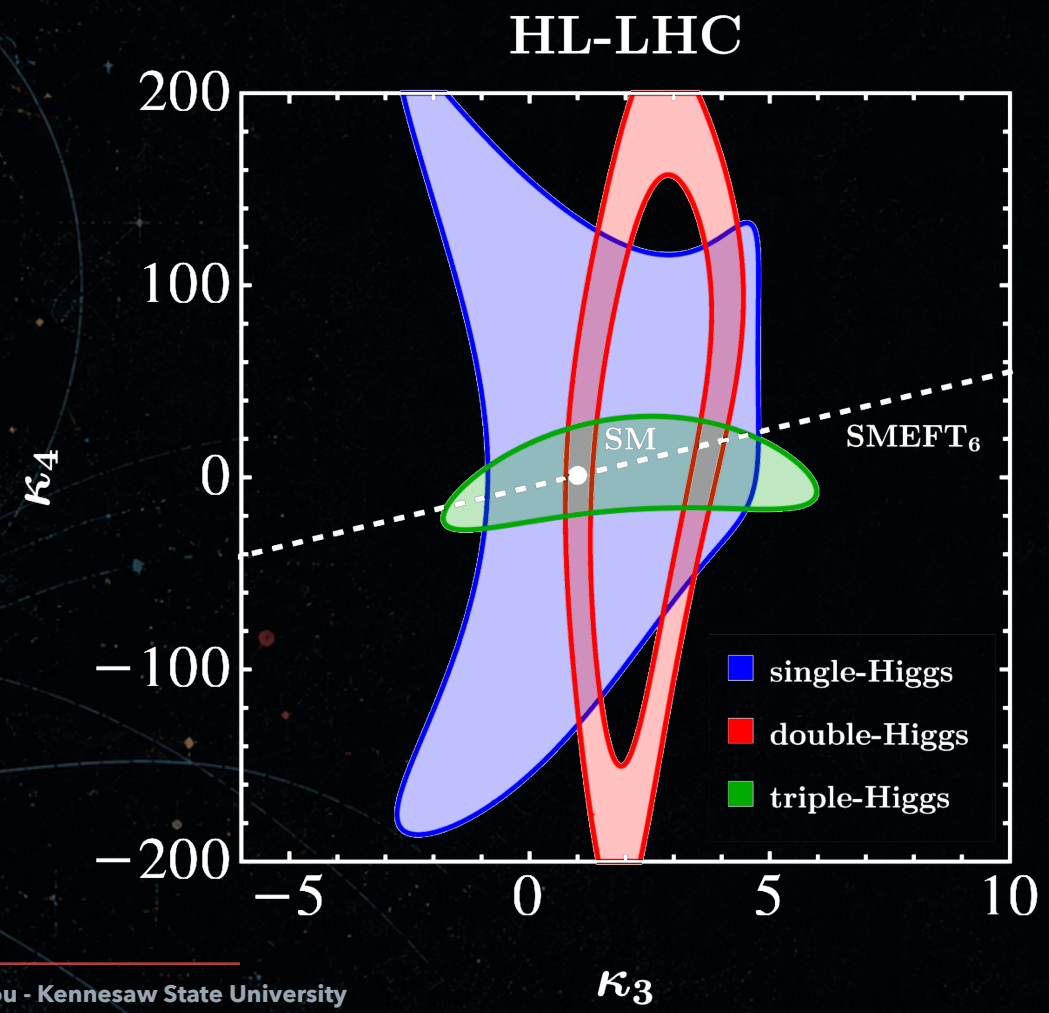
e.g. for $\sqrt{s} = 14$ TeV, 33 ab^{-1}: $\kappa_4 \sim [0.65, 1.55]$

[Chiesa, Maltoni, Mantani, Mele, Piccinini, Zhao, **2003.13628**]

Quartic Indirect Constraints: Single h

- Through **corrections to Higgs wave-function renormalization** from κ_3, κ_4 up to 2 **loops**.

[Haisch, Sankar, Zanderighi, 2505.20463]



@HL-LHC: $\kappa_4 \in [-21, 28]$

But: Indirect constraints somewhat more model-dependent than direct probes!

Quadruple Higgs Boson Production (!?)

[**AP**, Tetlalmatzi-Xolocotzi, 2608.16998]

Cross-section fits (14 TeV):

$$\sigma(gg \to hhh)/\sigma_{\rm SM}(14\ \text{TeV}) - 1 \approx 0.881c_3^2 - 0.872c_3 - 0.261c_3^3 - 0.168c_3d_4 - 0.0934d_4 + 0.0490c_3^2d_4 + 0.0397c_3^4 + 0.0170d_4^2$$

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For $d_4 \sim 70$, enhancement is ~ 200 (=100-200 events @ 3000 fb^{-1})

Quadruple Higgs Boson Production (!?)

[**AP**, Tetlalmatzi-Xolocotzi, 2608.16998]

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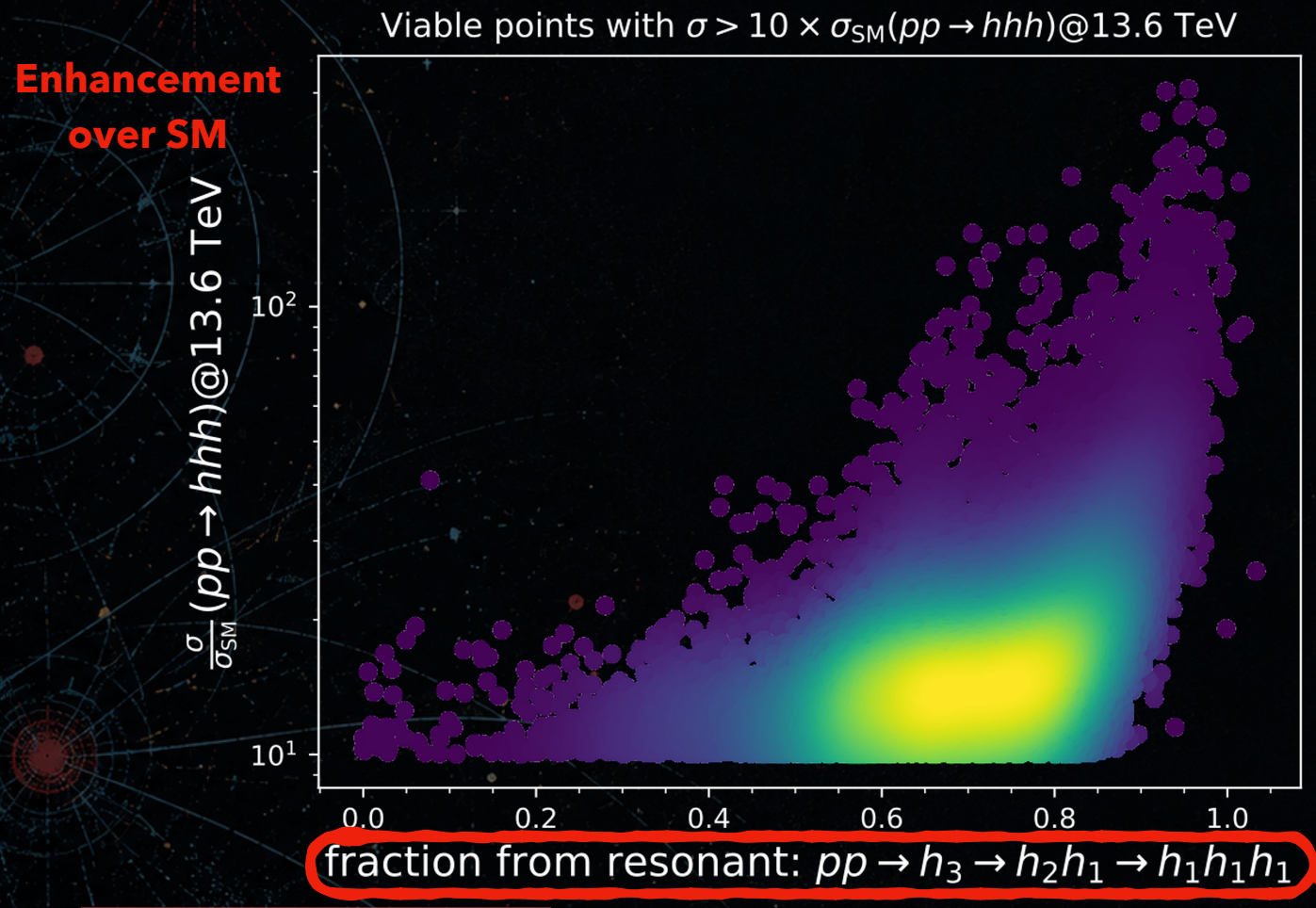
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Extended Scalar Sectors: Multi-scalar Pheno

- Enhancement of ***hhh*** ⇔ Large "fraction" of double-resonant process!

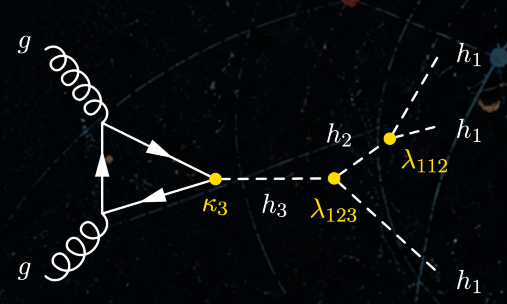


[Karkout, **AP**, Postma, Tetlalmatzi-Xolocotzi, van de Vis, du Pree, **2404.12425**]

TRSM parameter-space scan:

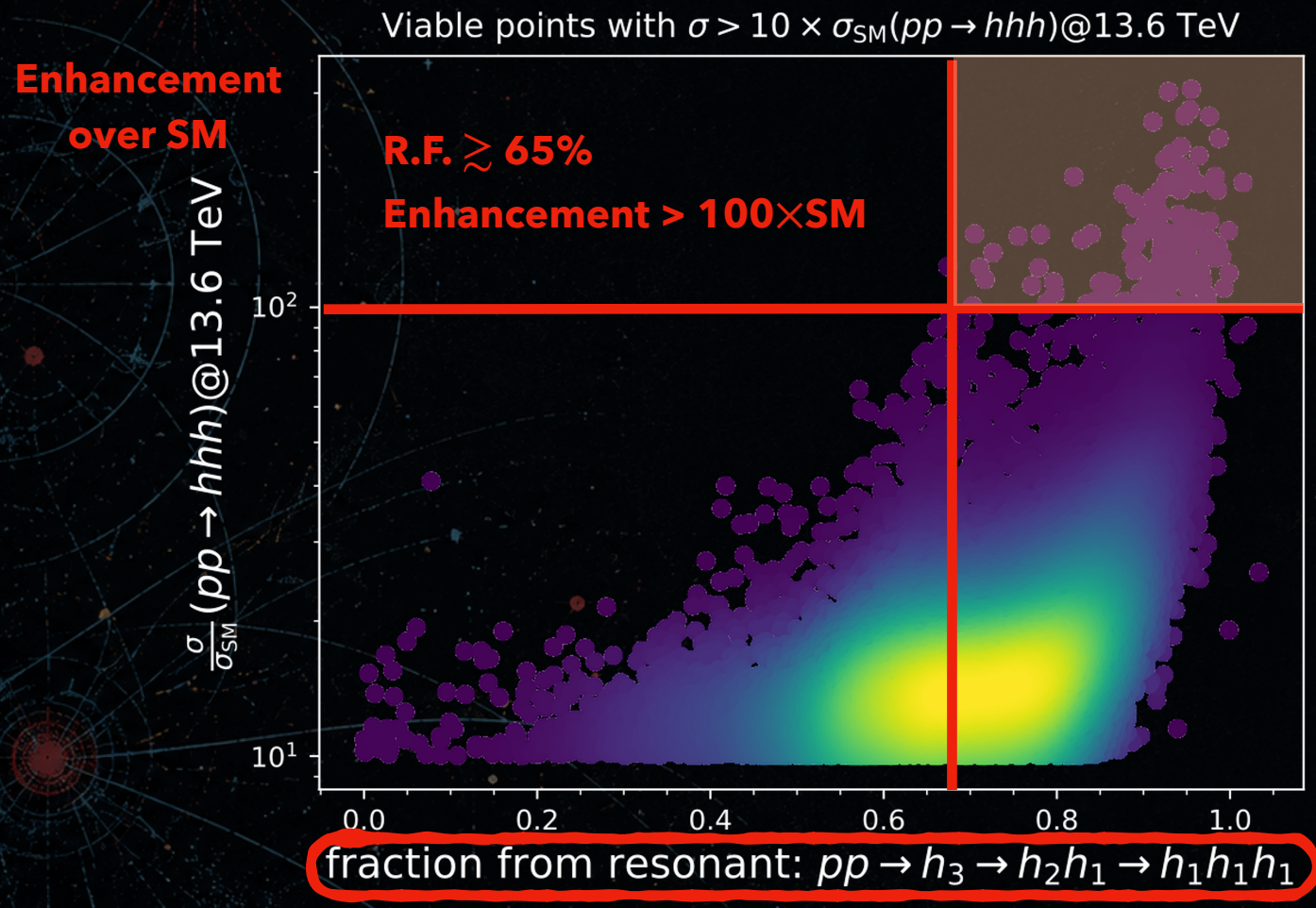
Including boundedness of potential + perturbativity + `HiggsTools` constraints.

Resonant Fraction (R.F.) =
How much of the total cross section comes from… ?



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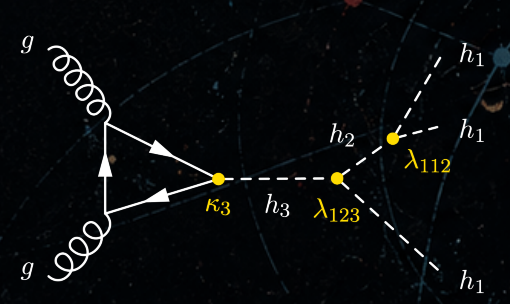


[Karkout, **AP**, Postma, Tetlalmatzi-Xolocotzi, van de Vis, du Pree, **2404.12425**]

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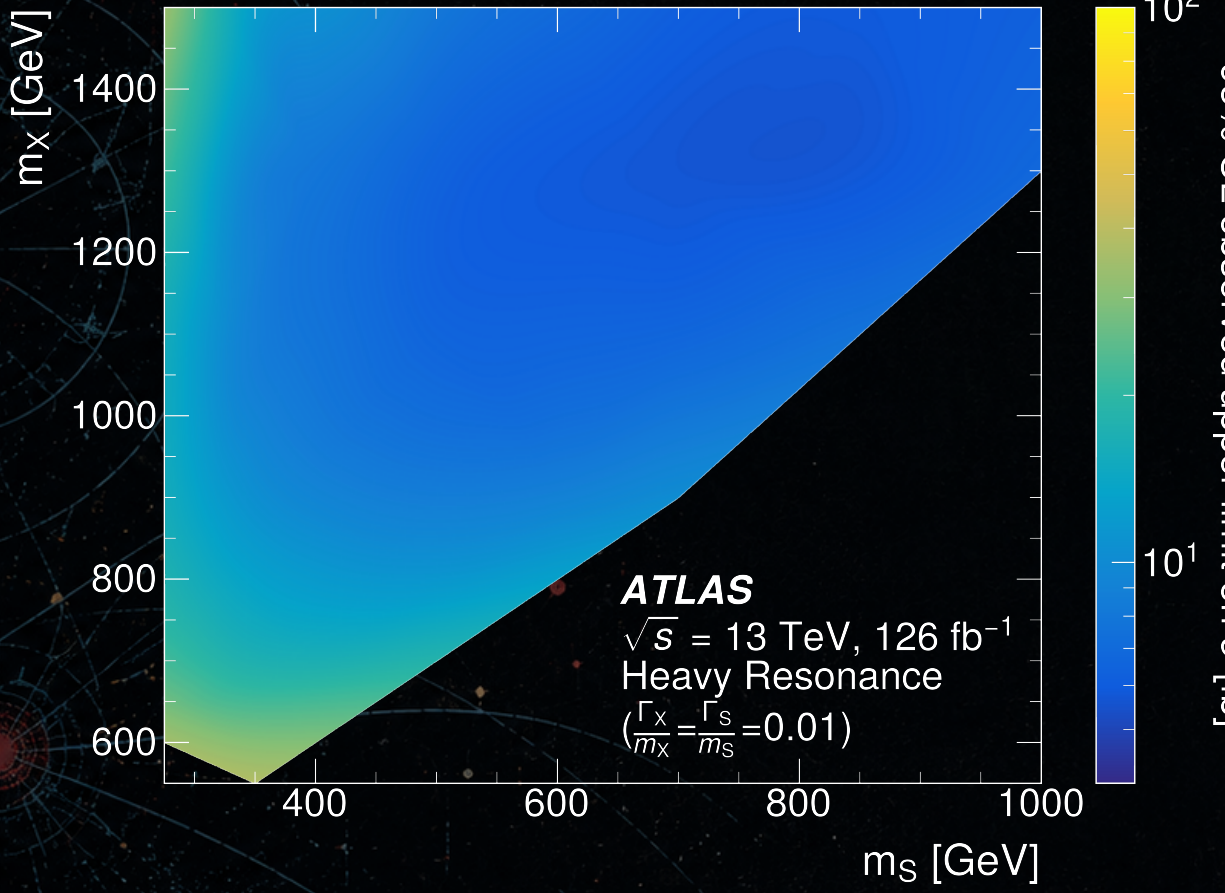
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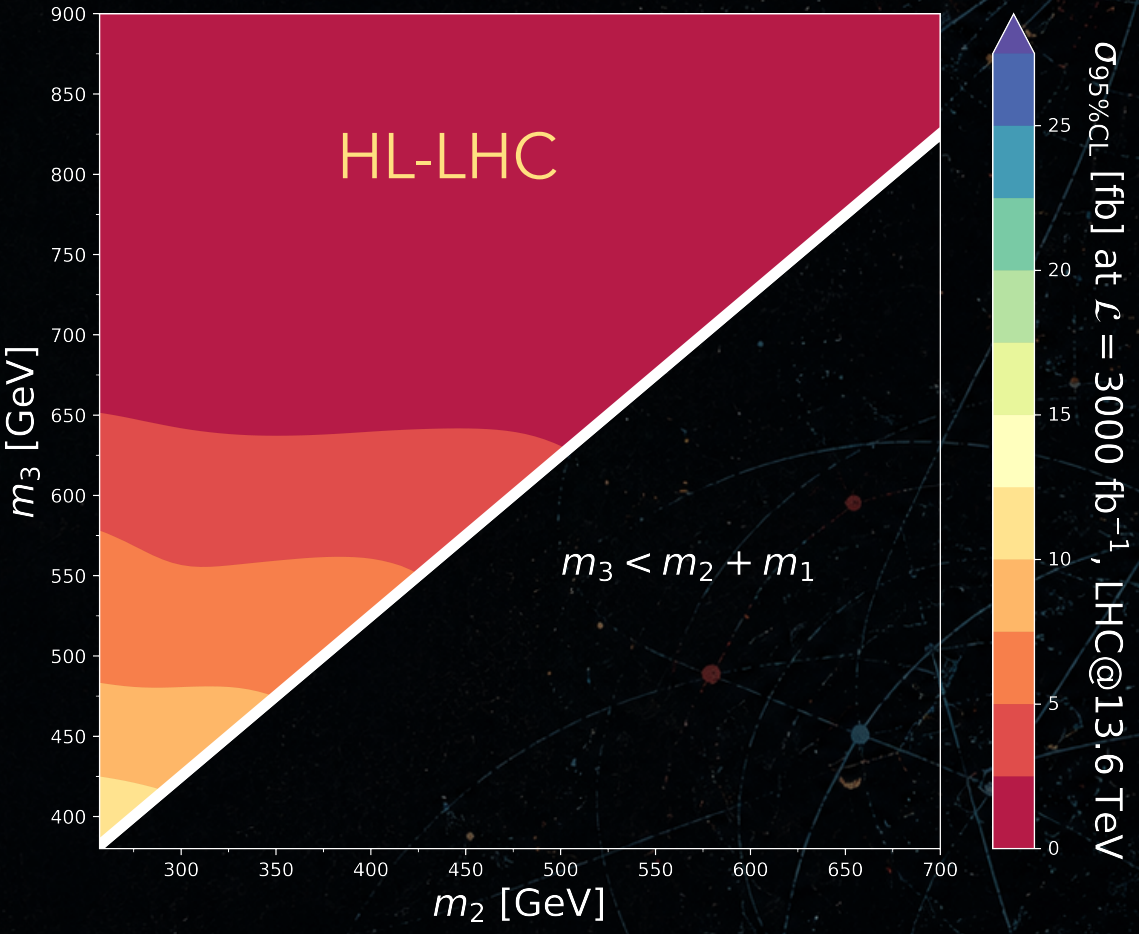
Extended Scalar Sectors: Double-Resonant *hhh*

Already being searched for in ATLAS $pp \to hhh \to 6b$ jets:

[ATLAS, 2411.02040]



[**AP**, Tetlalmatzi-Xolocotzi, 2501.14866]



Quadruple Higgs Boson Production in Extended Scalar Sectors

[**AP**, Tetlalmatzi-Xolocotzi, 2608.16998]

- Expected constraints at **HL-LHC** through the **8 b**-jet final state:

