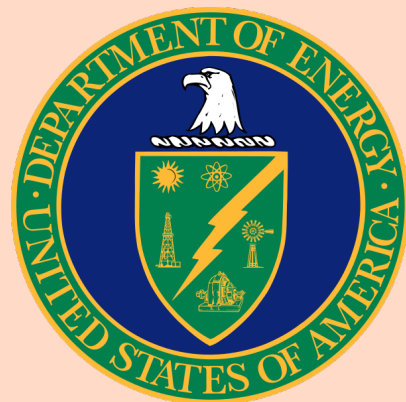


EXPLORING SCALAR SECTORS

at the LHC and Beyond



Andreas Papaefstathiou
Kennesaw State University, GA, USA
@ The University of Kansas [Dec. 1st, 2025]



The Plan:



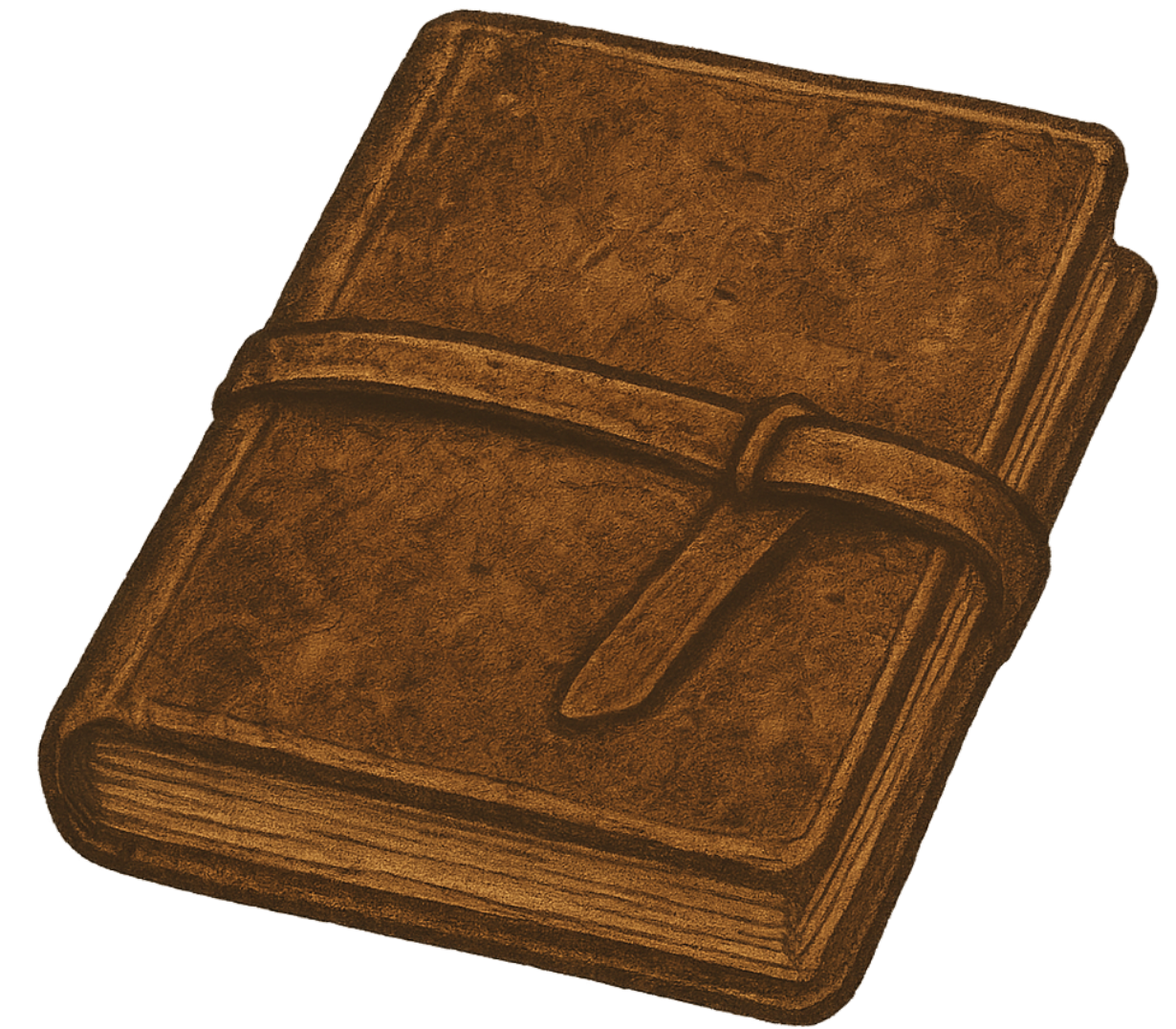
The Plan:

👒 ***PART I: THE STANDARD MODEL SCALAR SECTOR.***

👒 ***PART II: MEASURING THE HIGGS POTENTIAL.***

👒 ***PART III: TRIPLE HIGGS BOSON PRODUCTION.***

👒 ***PART IV: EXTENDED SCALAR SECTORS.***



PART I: THE KINGDOM OF THE SM SCALAR SECTOR



The **Standard Model** Scalar Sector

- 👒 Explicit mass terms in SM $\rightarrow$ violate local gauge invariance ($\rightarrow$ spoils renormalizability).
- 👒 Higgs mechanism $\rightarrow$ spontaneous symmetry breaking $\rightarrow$ **mass** to W/Z bosons & fermions.
- 👒 Results in (SM), where h is the Higgs boson:

$$\mathcal{L} \supset \left[m_W^2 W^{\mu+} W_{\mu}^{-} + \frac{1}{2} m_Z^2 Z^{\mu} Z_{\mu} \right] \left(1 + \frac{h}{v} \right)^2 - m_f \bar{f} f \left(1 + \frac{h}{v} \right)$$

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Experimental Scrutiny: Higgs Couplings

$$\mathcal{L} \supset \left[m_W^2 W^{\mu+} W_{\mu}^{-} \left(1 + 2\kappa_W \frac{h}{v} + \dots \right) + \frac{1}{2} m_Z^2 Z^{\mu} Z_{\mu} \left(1 + 2\kappa_Z \frac{h}{v} + \dots \right) \right] \\ - m_f \bar{f} f \left(1 + \kappa_f \frac{h}{v} \right)$$

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$$- m_f \bar{f} f \left(1 + \kappa_f \frac{h}{v} \right)$$

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🎩 Test experimentally, via coupling modifiers (“kappa” framework) $\rightarrow$ SM: $\kappa_V = \kappa_f = 1$.

🎩 A stringent test of the SM prediction through $h \rightarrow xx$ decays!

Experimental Scrutiny: Higgs Couplings

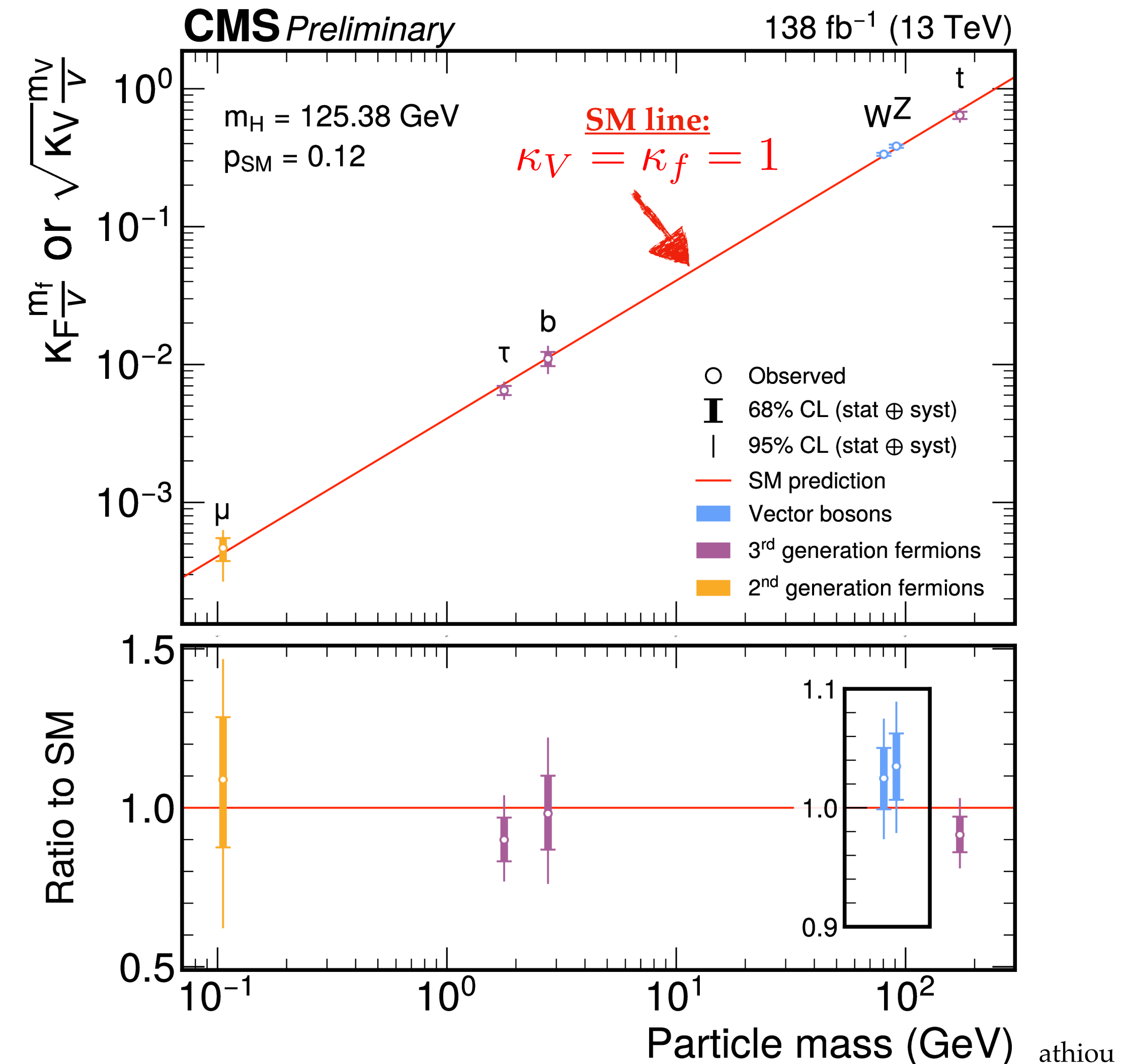
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[CMS PAS-HIG-21-018, April 2025]



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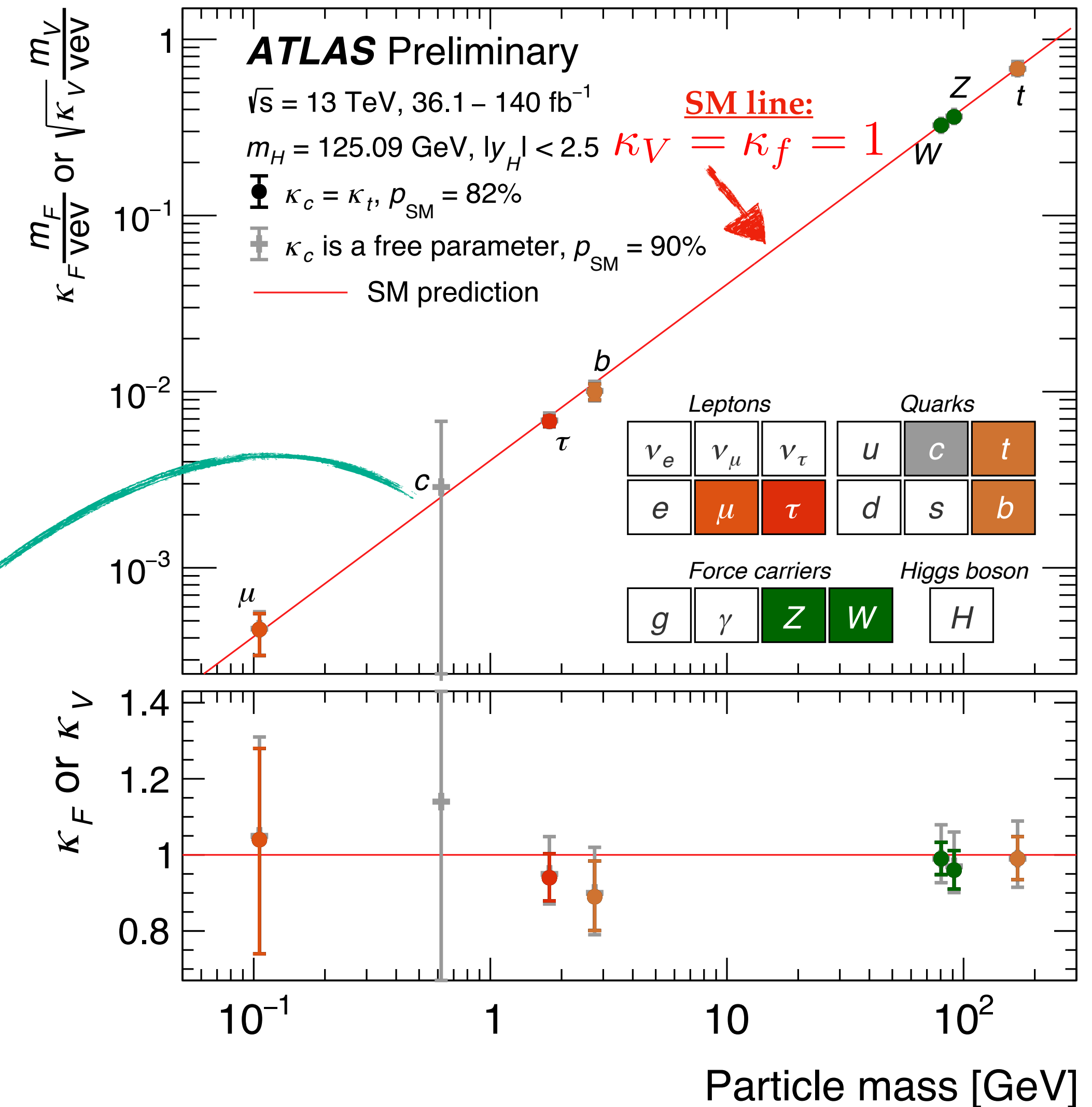
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[ATLAS-CONF-2025-006, July 2025]

c-quark couplings elusive:
small mass + tagging challenges

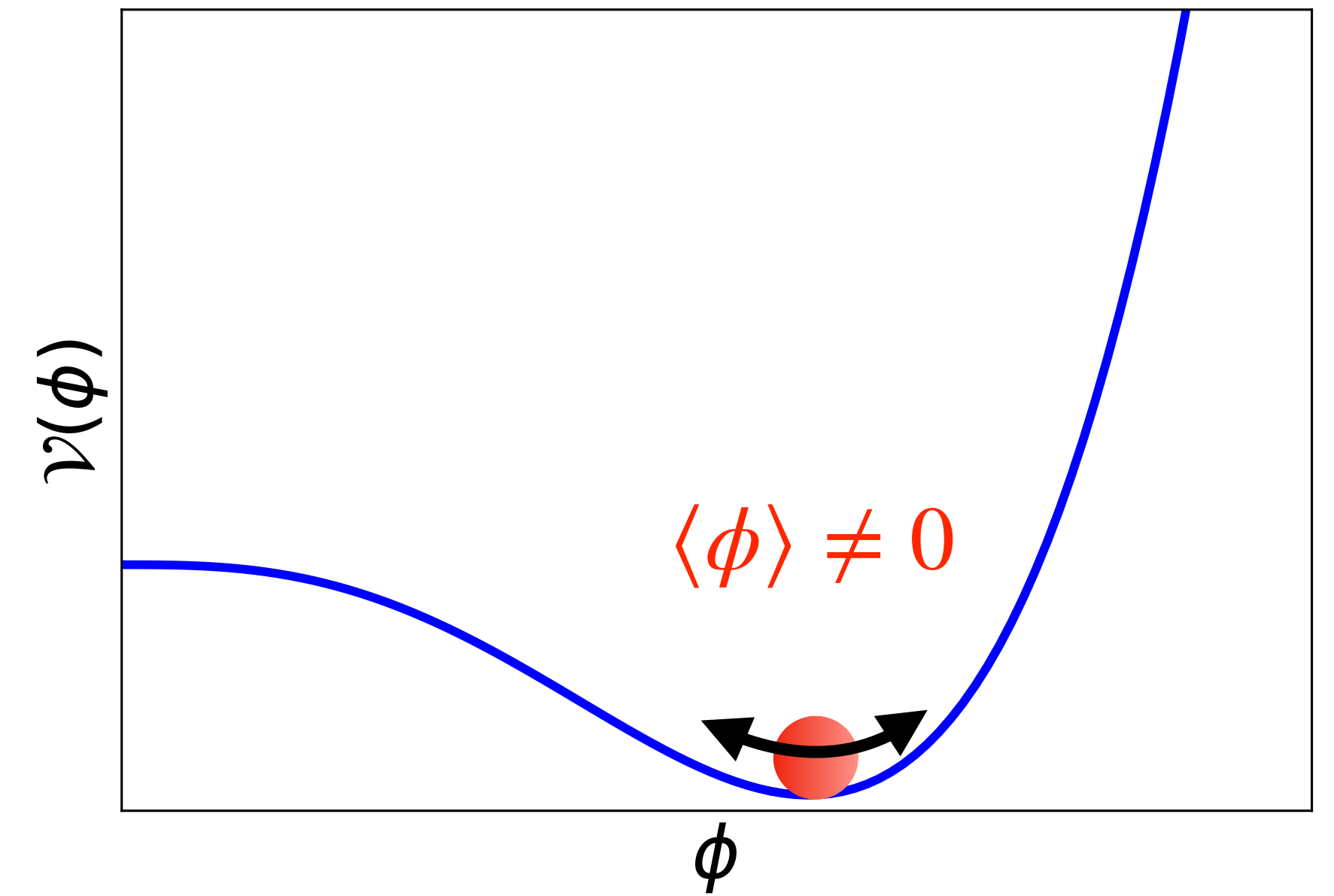


PART II: RAIDERS OF THE HIGGS POTENTIAL



The Higgs Potential

👒 Higgs field ϕ , “sits” in potential (SM): $\mathcal{V}(\phi) = \text{●} |\phi|^2 + \text{■} |\phi|^4$.



The Higgs Potential

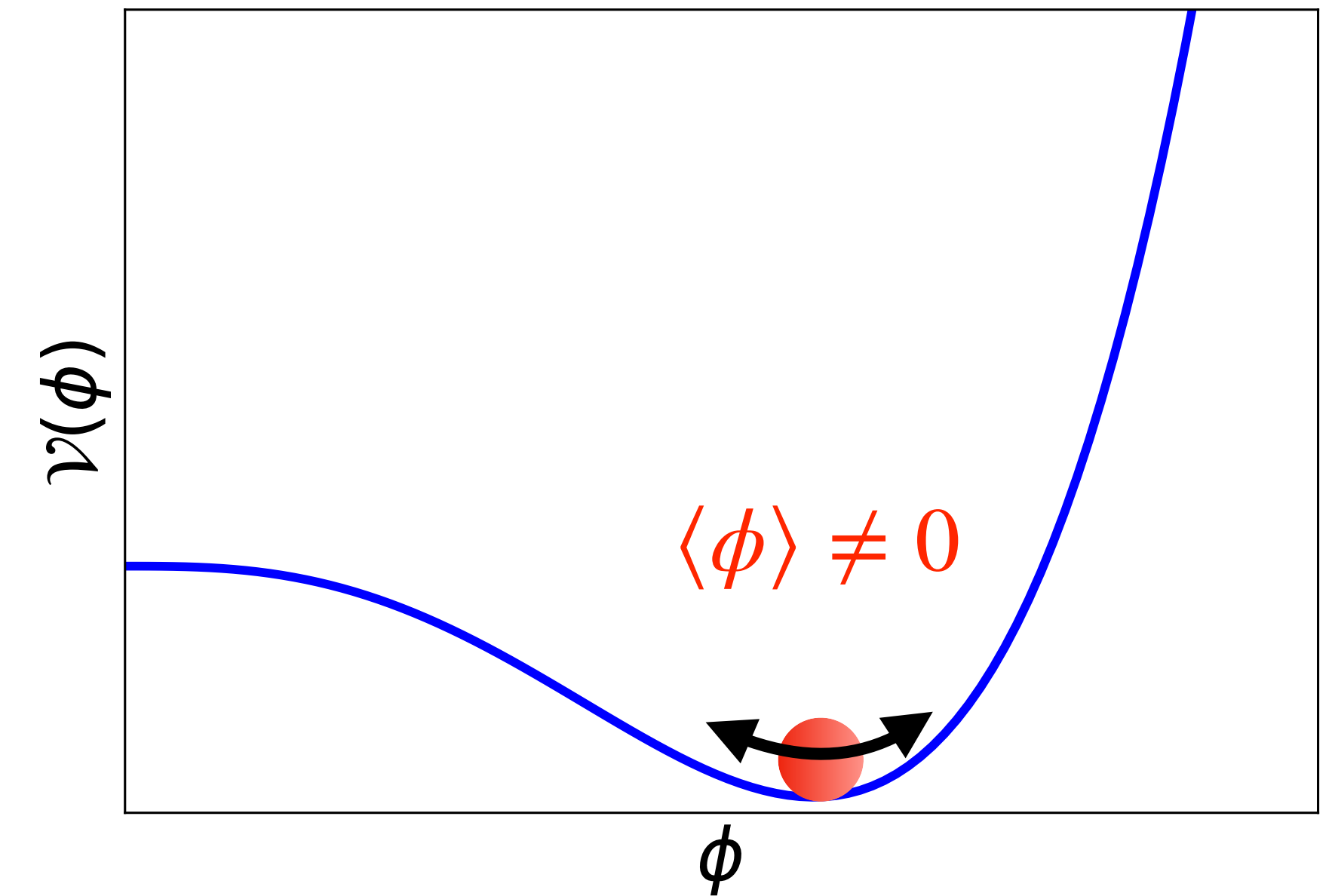
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🎩 **Electroweak Symmetry Breaking:**

$$\mathcal{V}(\langle \phi \rangle + h) = \text{●} h^2 + \text{▲} h^3 + \text{■} h^4$$

where: h = the Higgs boson.

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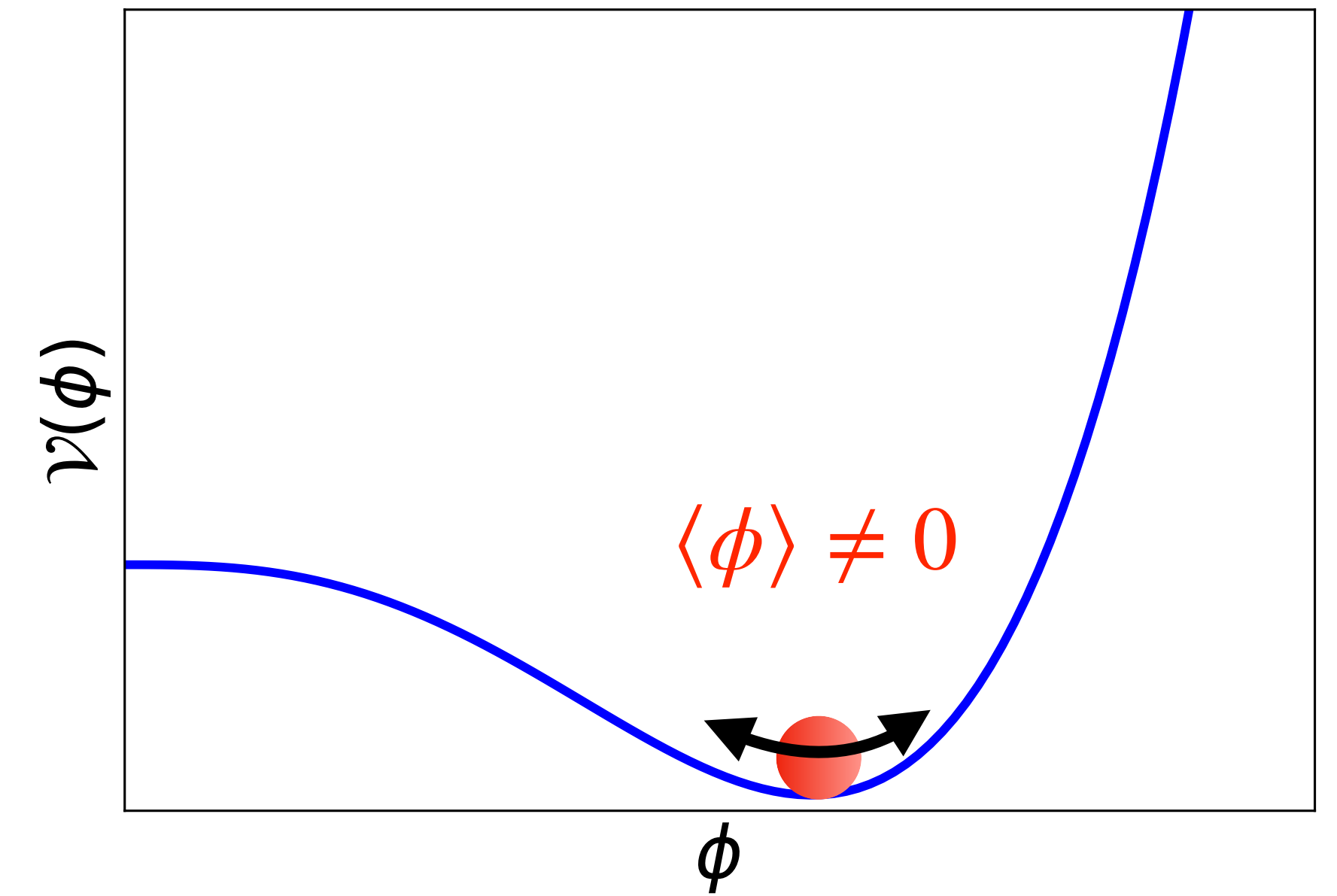
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🎩 SM: predicted via the Higgs boson mass & VEV:

$$\text{●}_{\text{SM}} = \frac{1}{2} m_h^2 ; \quad \underbrace{\text{▲}_{\text{SM}}}_{\lambda_3^{\text{SM}}} = \frac{m_h^2}{2v} ; \quad \underbrace{\text{■}_{\text{SM}}}_{\lambda_4^{\text{SM}}} = \frac{m_h^2}{8v^2}$$

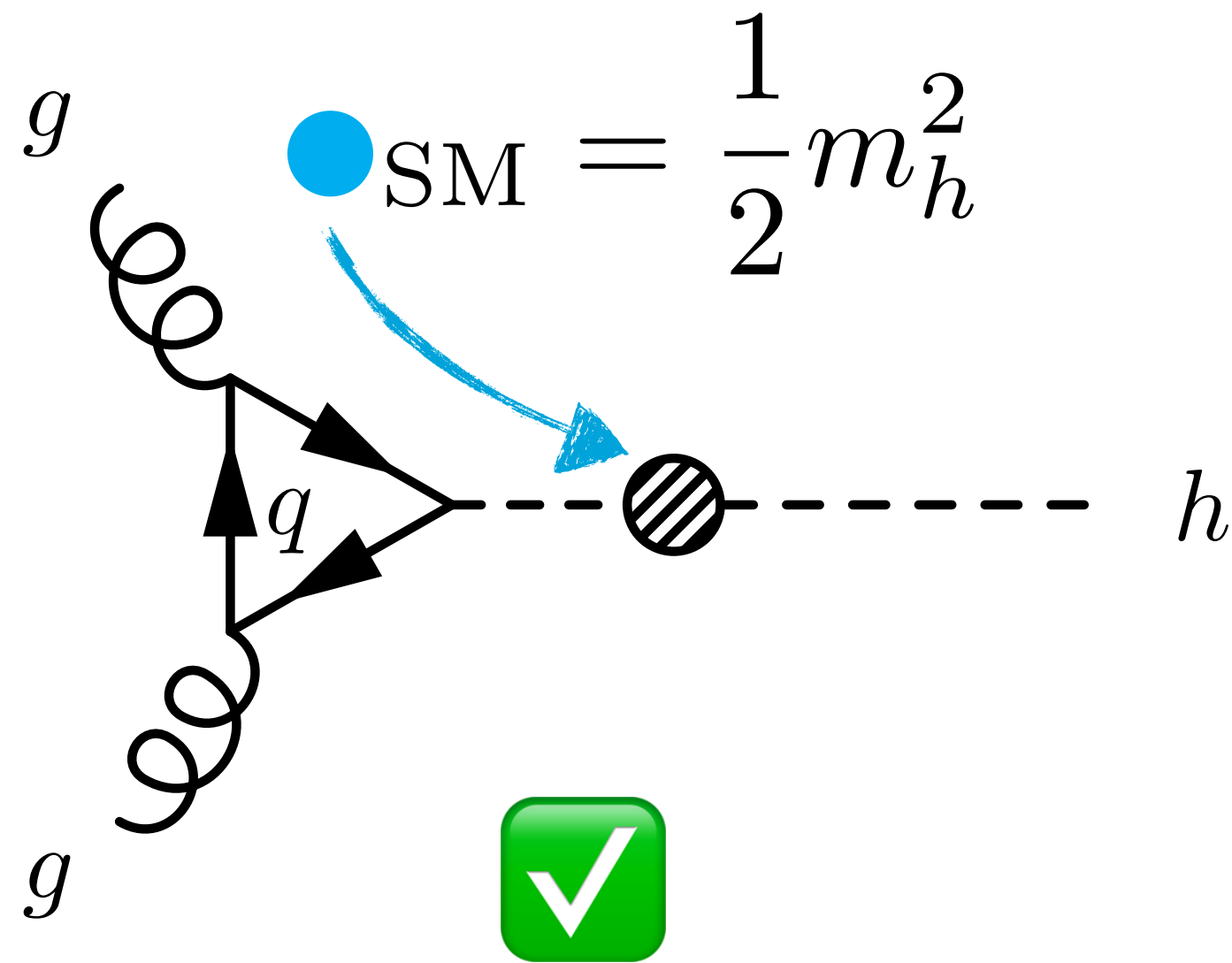


Measuring the Higgs Boson Self-Couplings

👒 **Verify** that $\{\bullet, \blacktriangle, \blacksquare\} \approx \{\bullet, \blacktriangle, \blacksquare\}_{\text{SM}}$ at hadron colliders, by measuring:

Measuring the Higgs Boson Self-Couplings

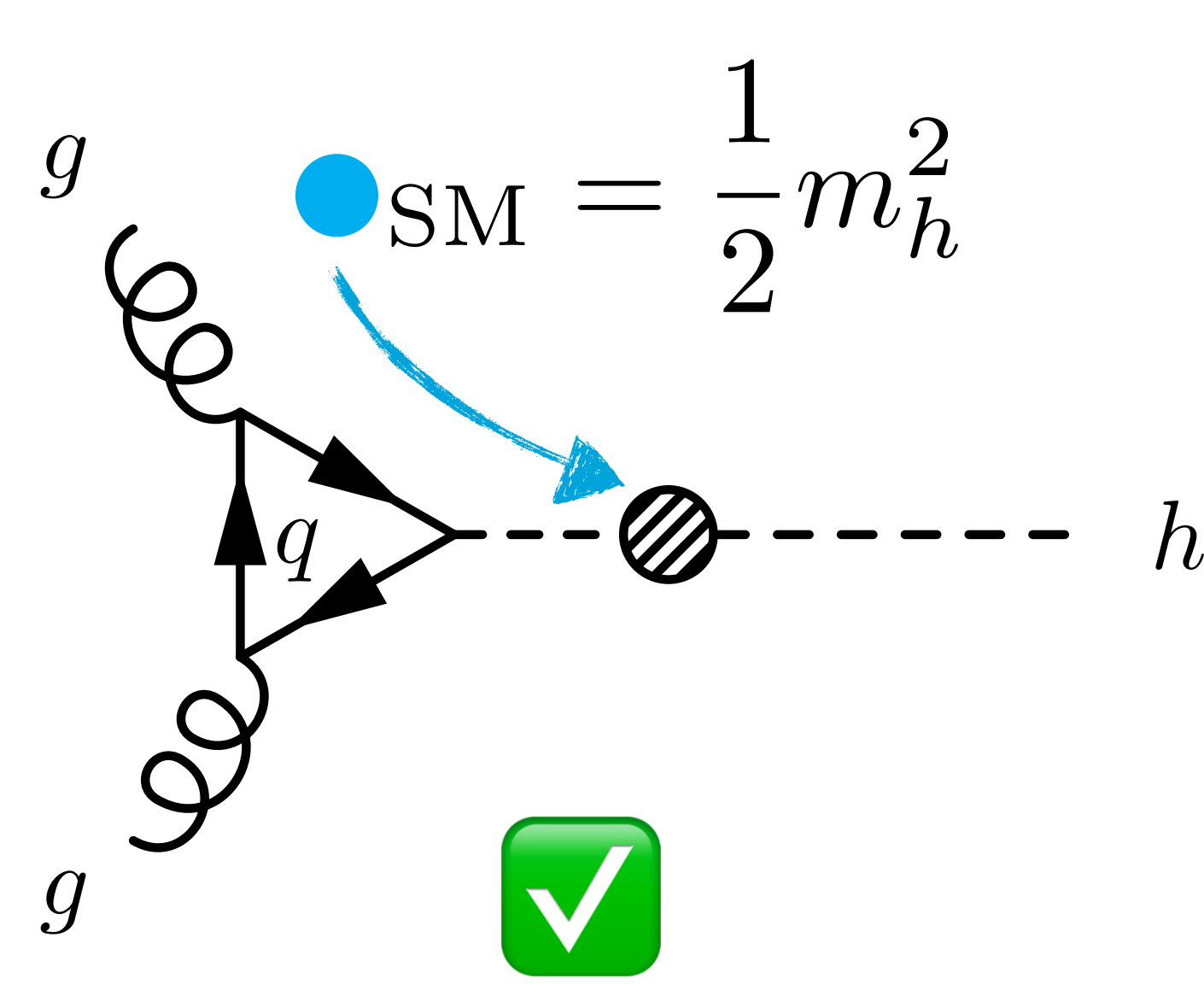
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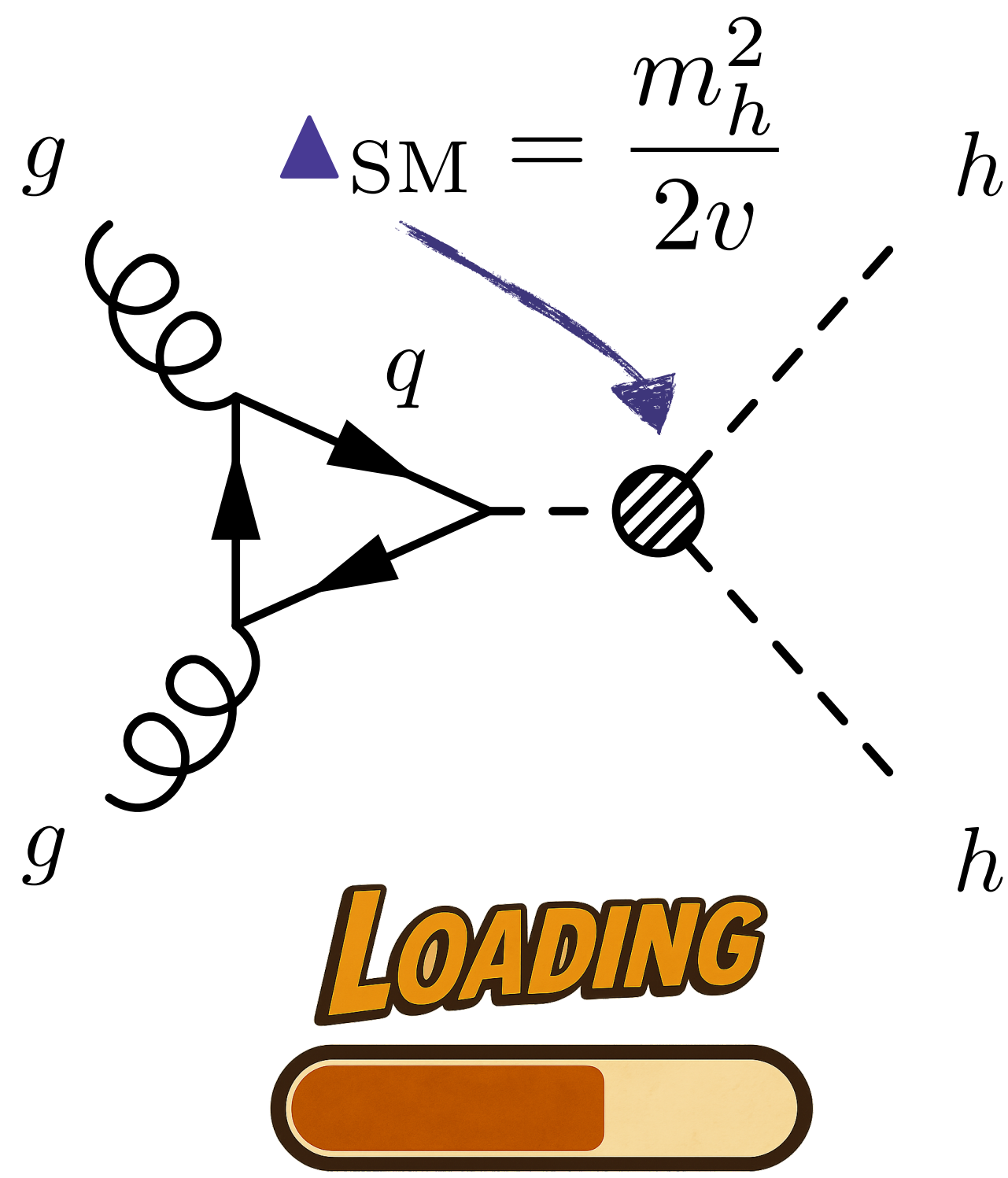
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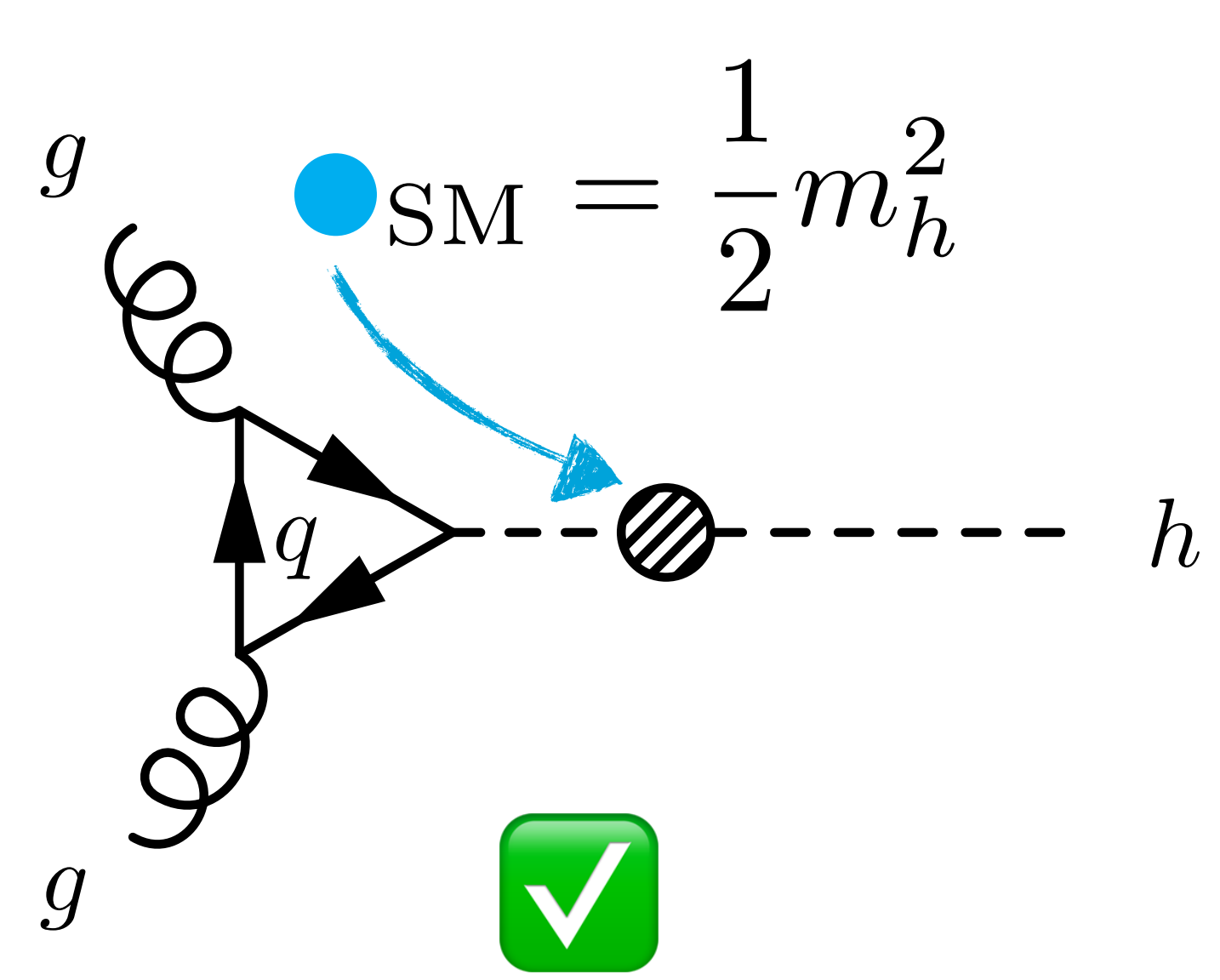


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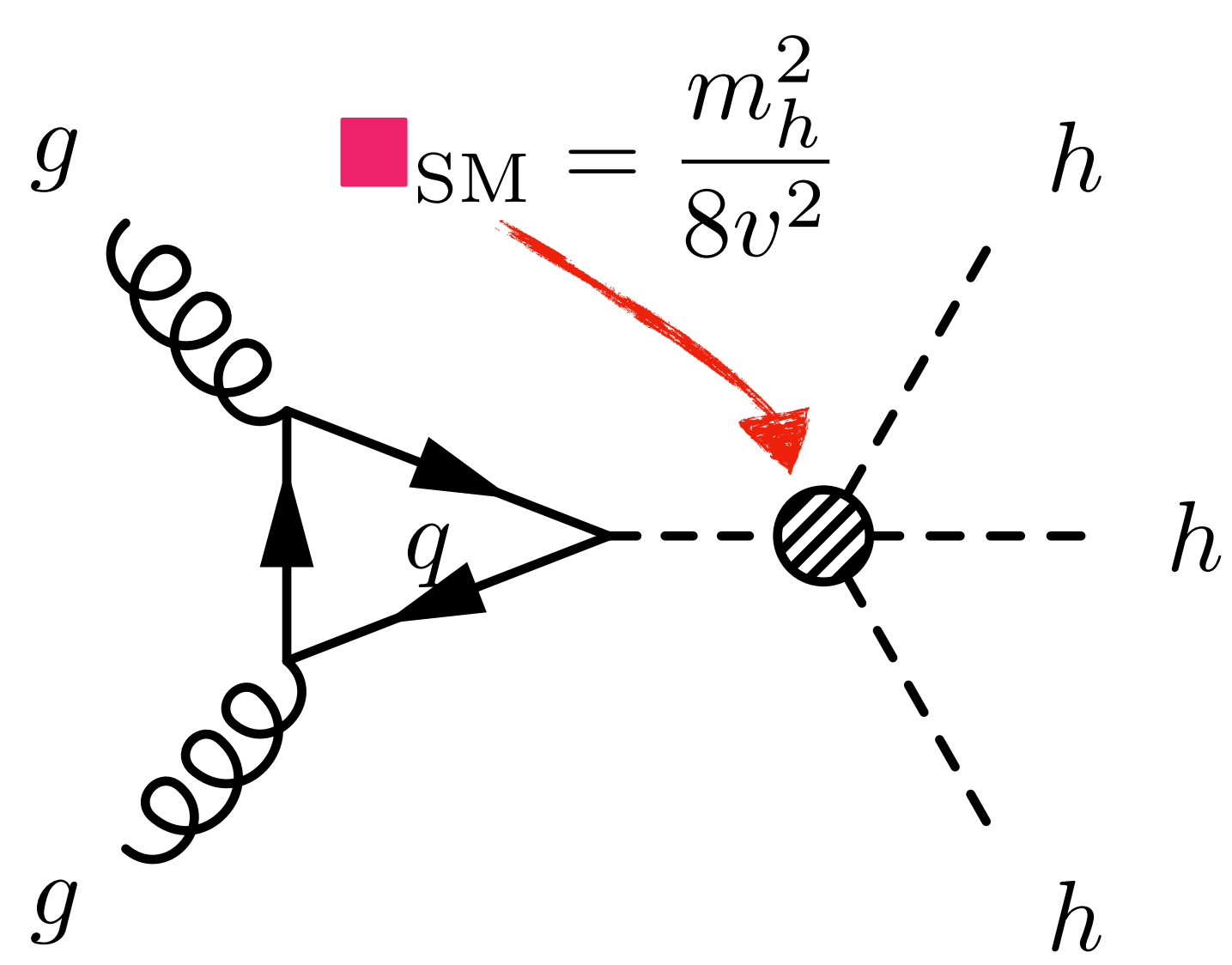
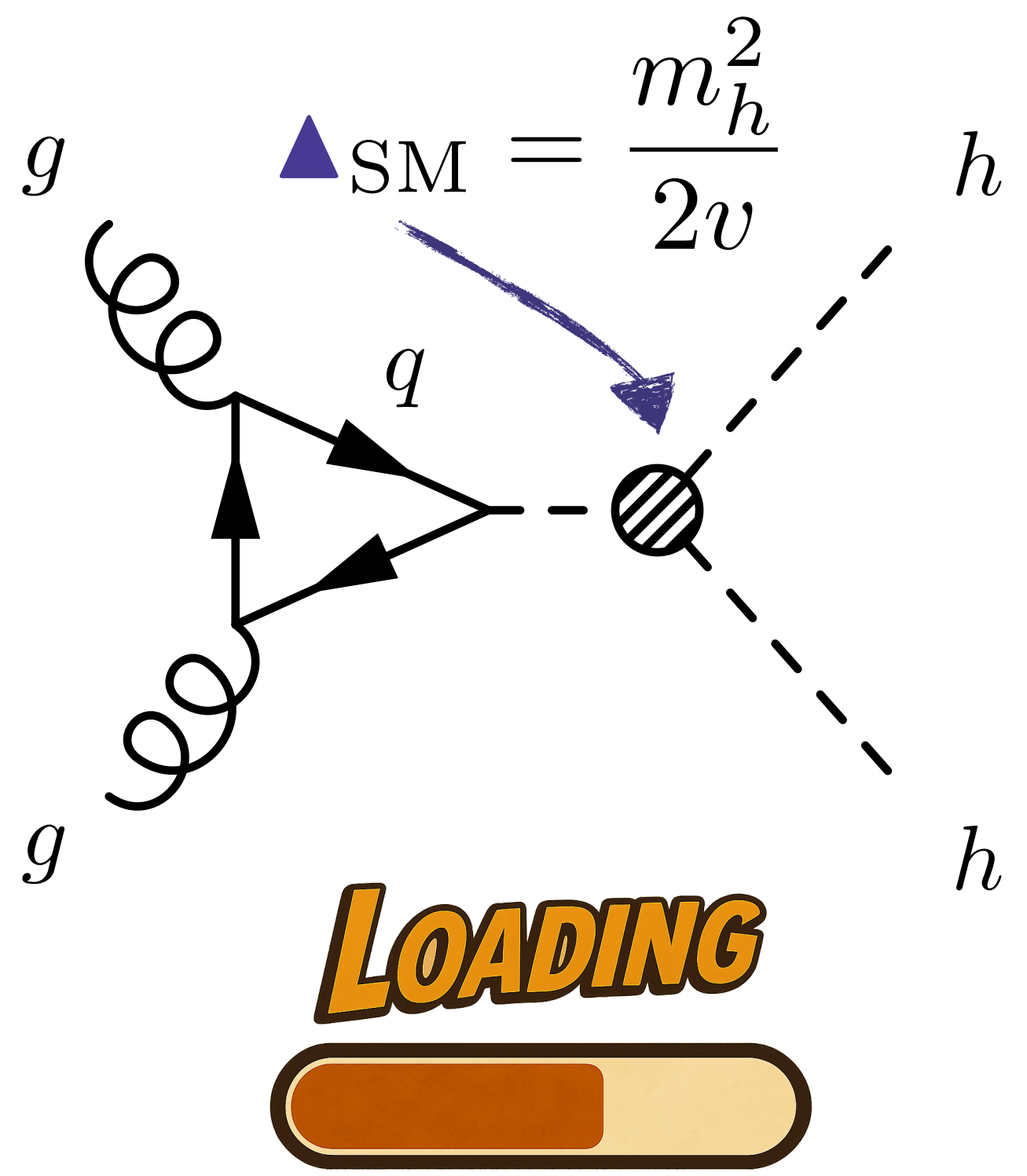


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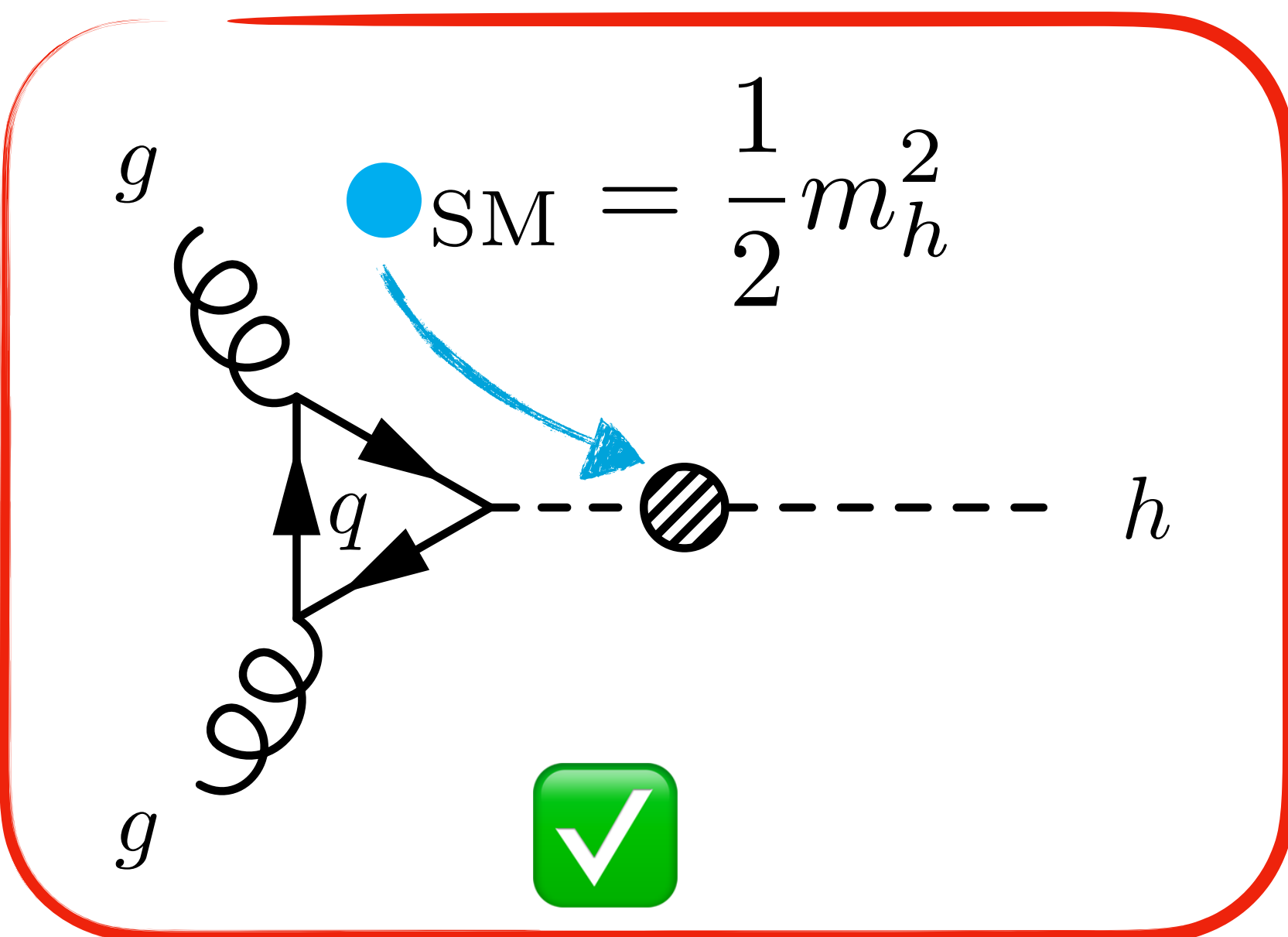


Tiny cross section @ LHC:
SM $\sigma(hhh @ 14 \text{ TeV}) \sim 0.1 \text{ fb!}$

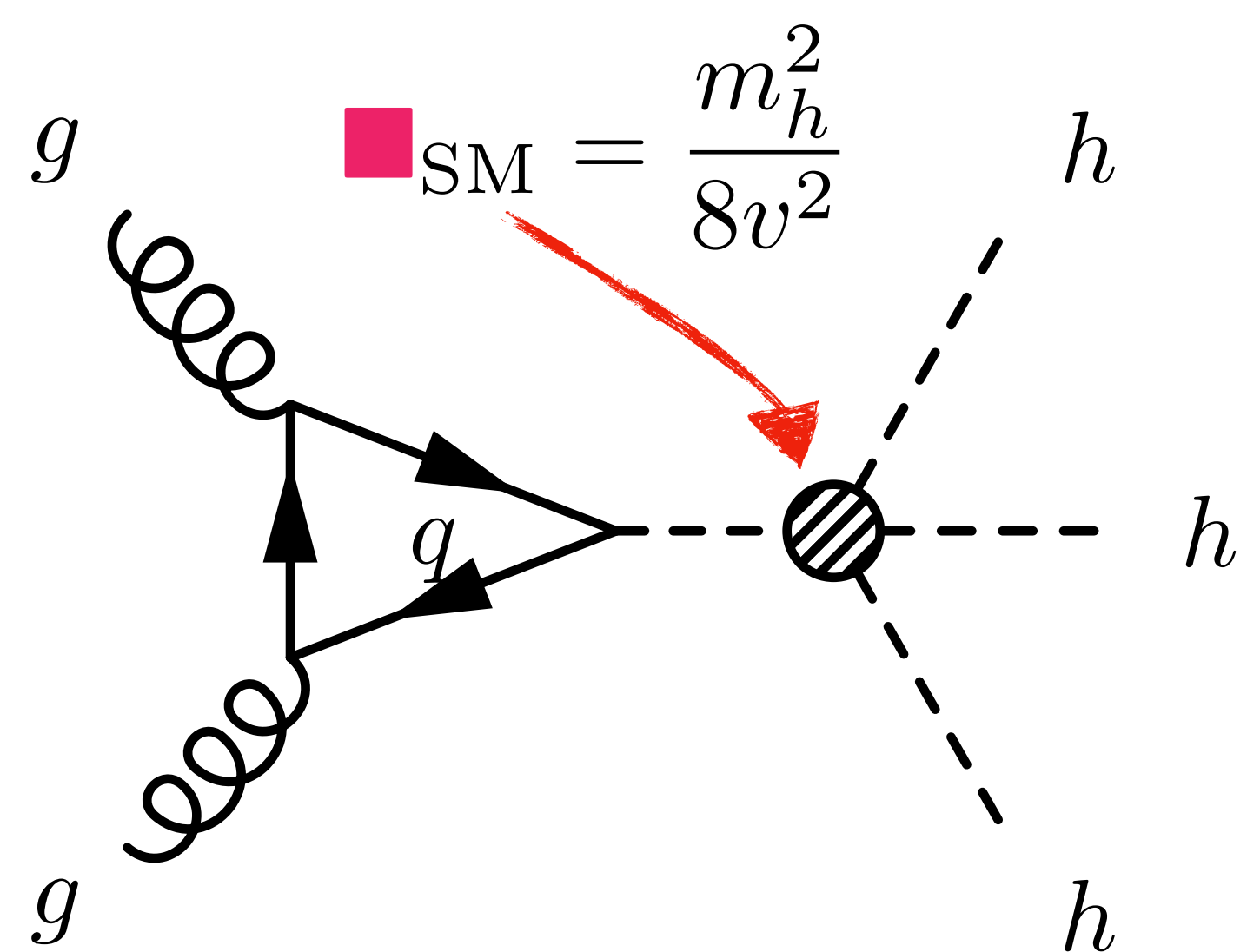
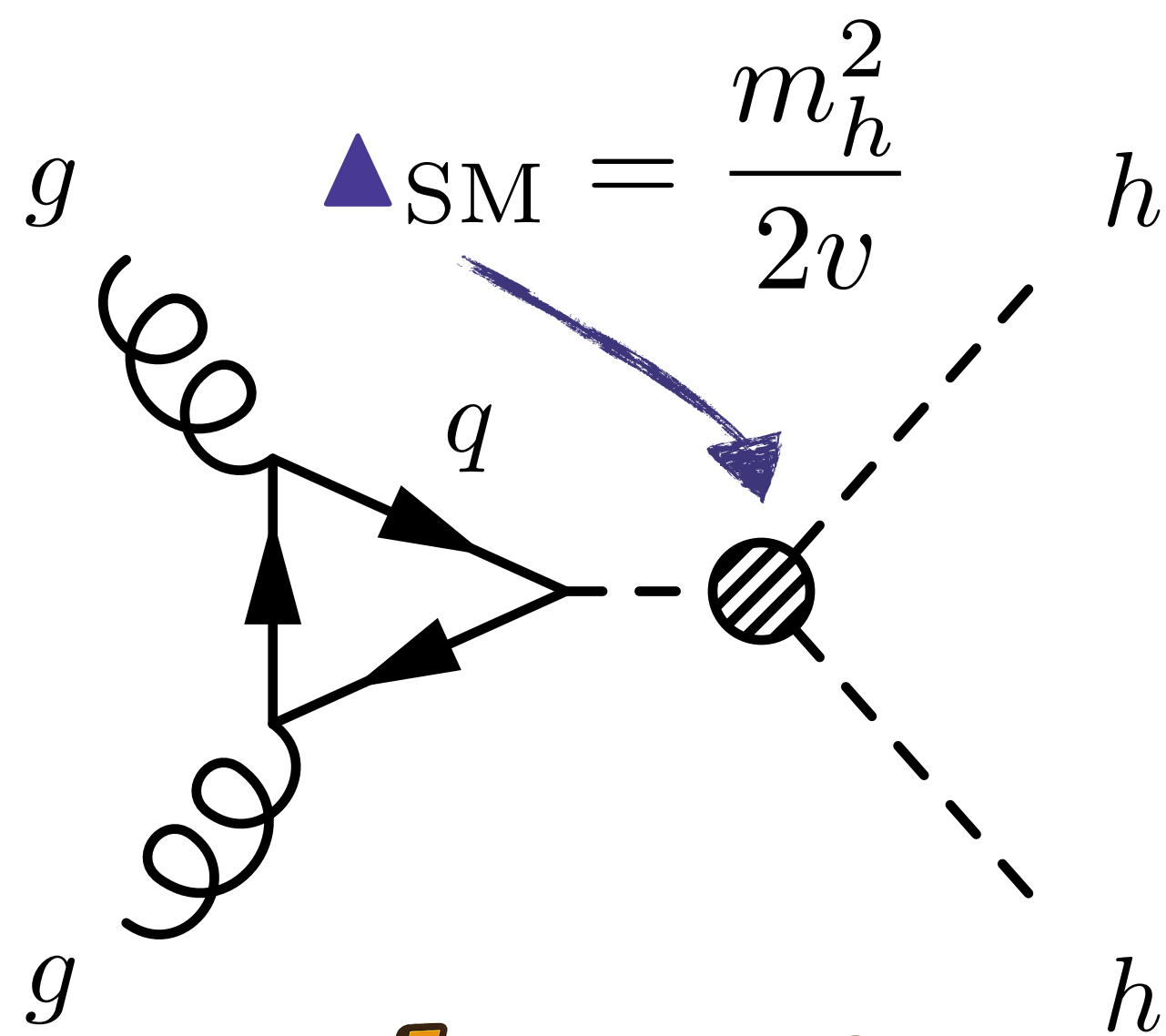


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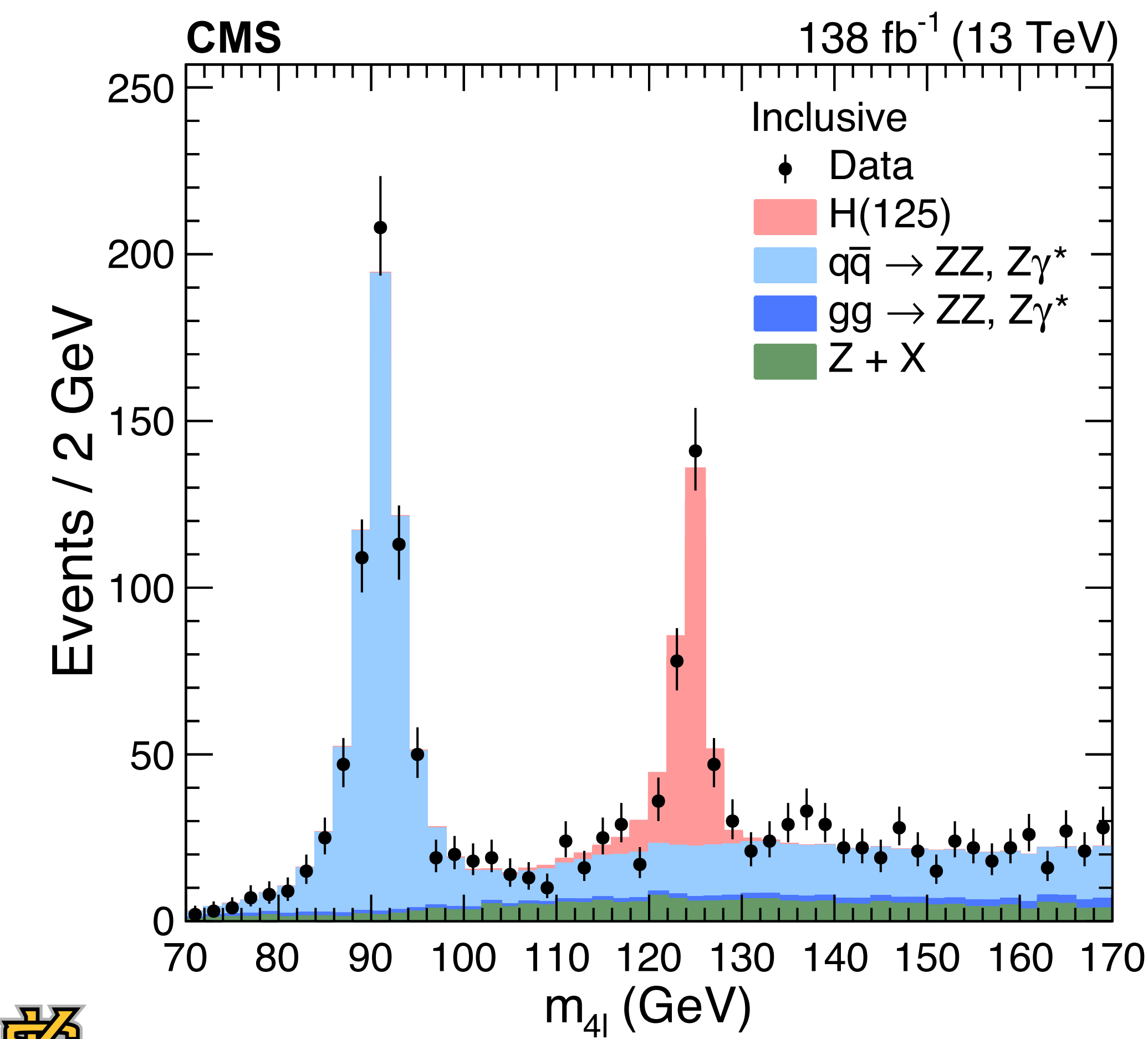


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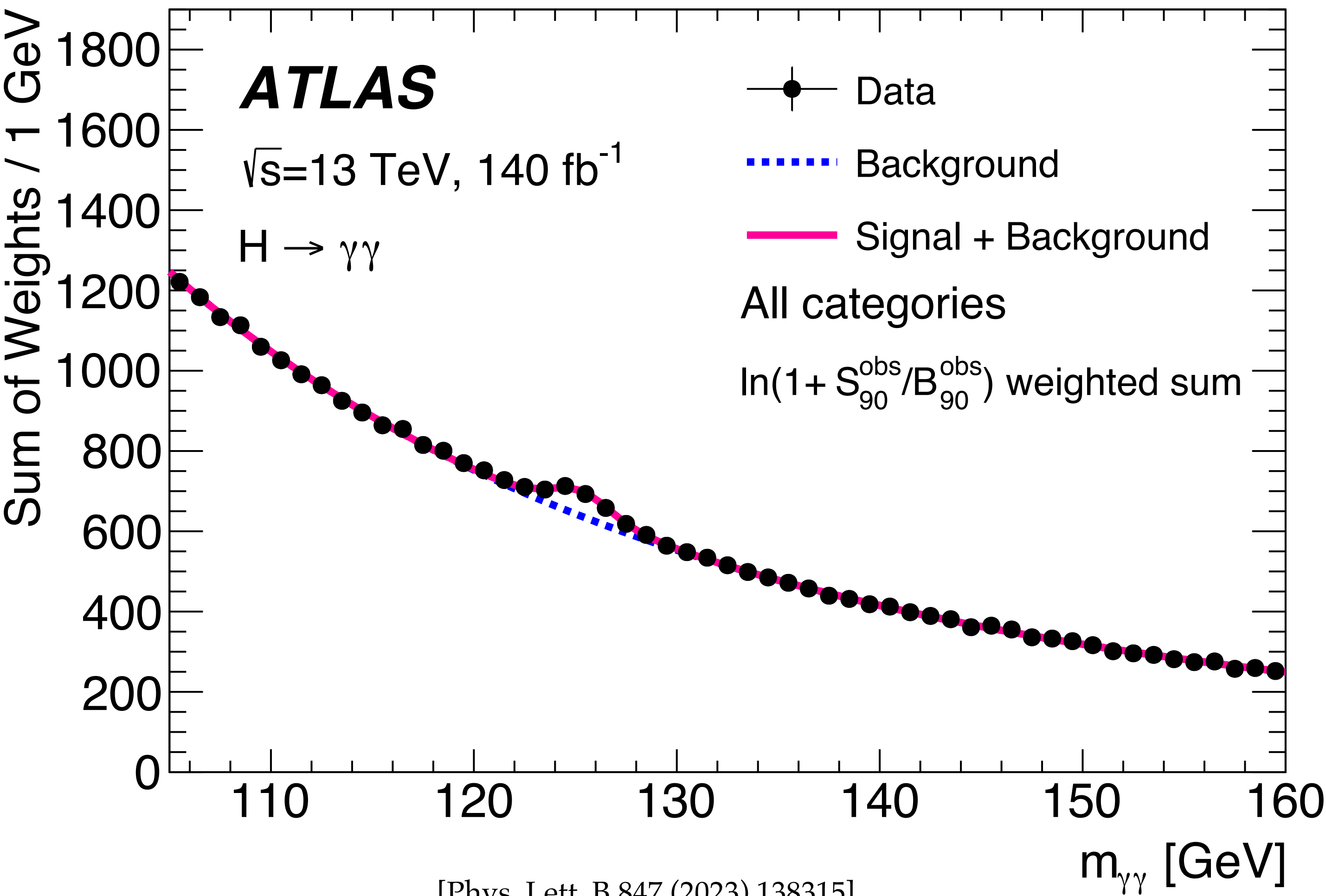


Measuring the Higgs Boson Mass

🎩 Measured in $h \rightarrow ZZ \rightarrow 4\ell$ and $h \rightarrow \gamma\gamma$ (high-res. channels).



[PRD 111 (2025) 092014]



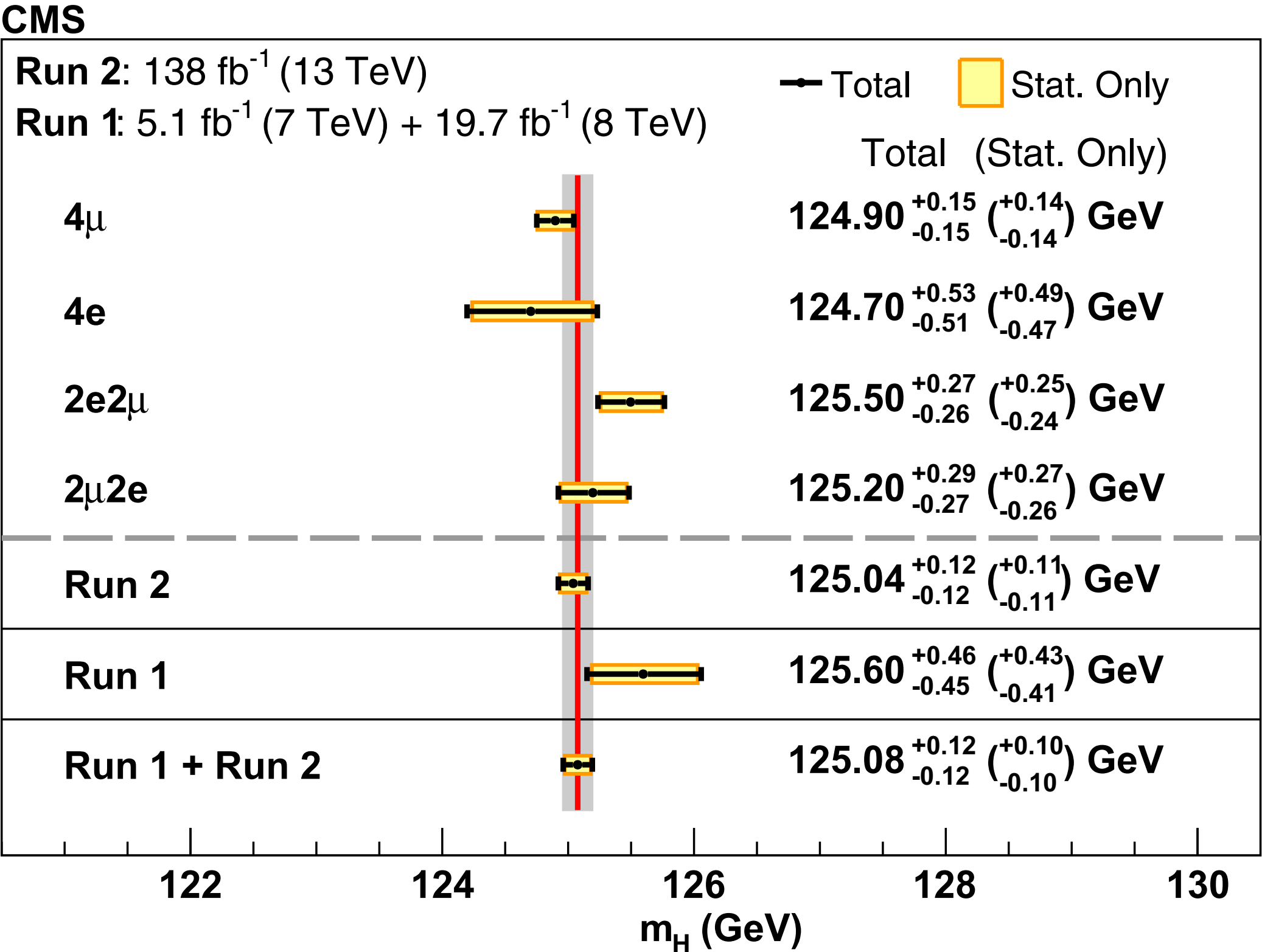
[Phys. Lett. B 847 (2023) 138315]

Andreas Papaefstathiou

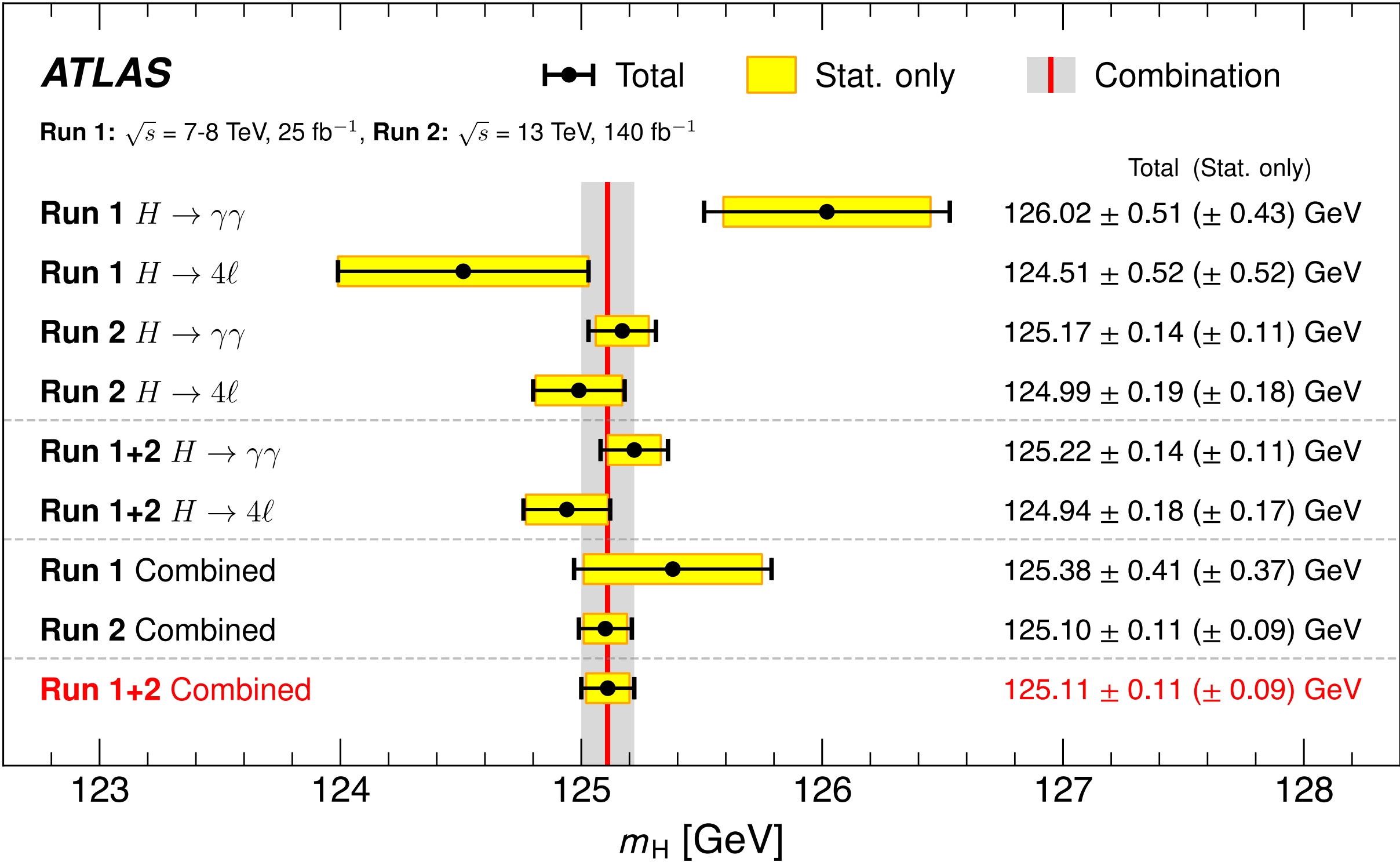
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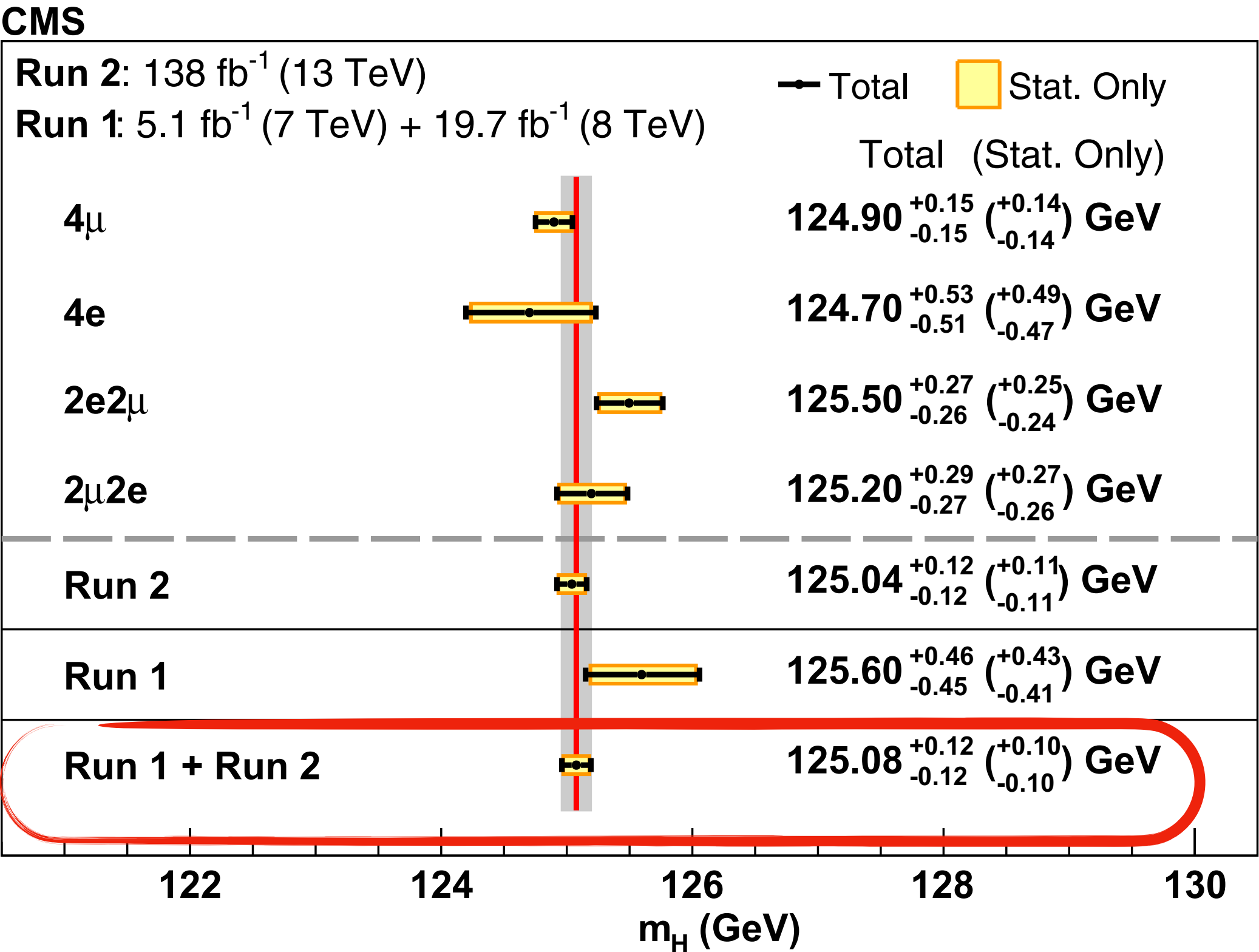


[Phys. Lett. B 847 (2023) 138315]

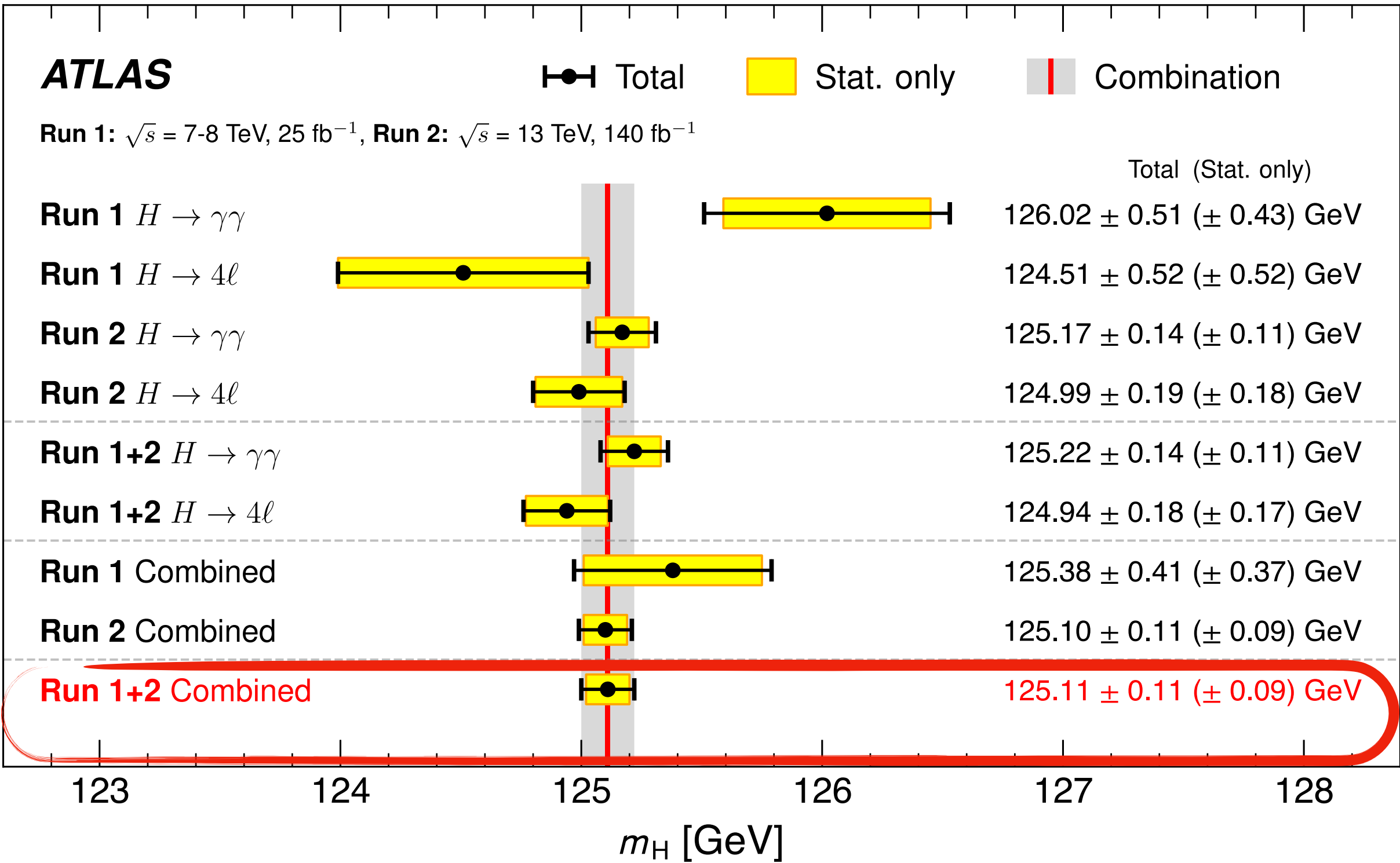
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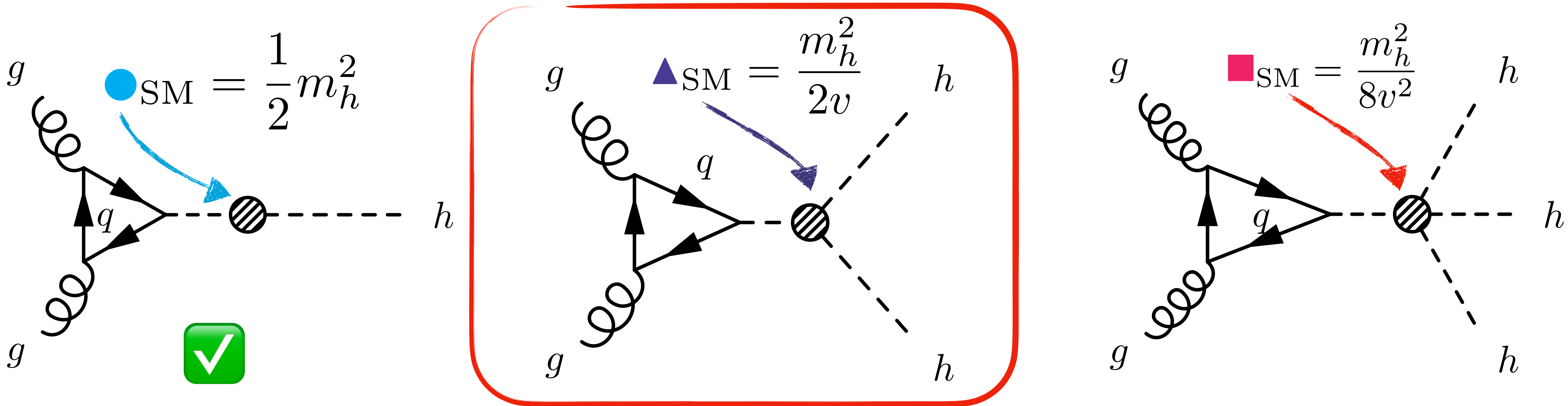
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[Phys. Lett. B 847 (2023) 138315]

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Measuring the Higgs Boson Trilinear Self-Coupling

🎩 $pp \rightarrow hh \rightarrow (xx)(yy)$ final states @ LHC, Branching Ratios:

ATLAS: [A. Sidley, HIGGS 2025]

	bb	WW	$\tau\tau$	ZZ	$\gamma\gamma$
bb	34%				
WW	25%	4.6%			
$\tau\tau$	7.3%	2.7%	0.39%		
ZZ	3.1%	1.1%	0.33%	0.069%	
$\gamma\gamma$	0.26%	0.10%	0.028%	0.012%	0.0005%

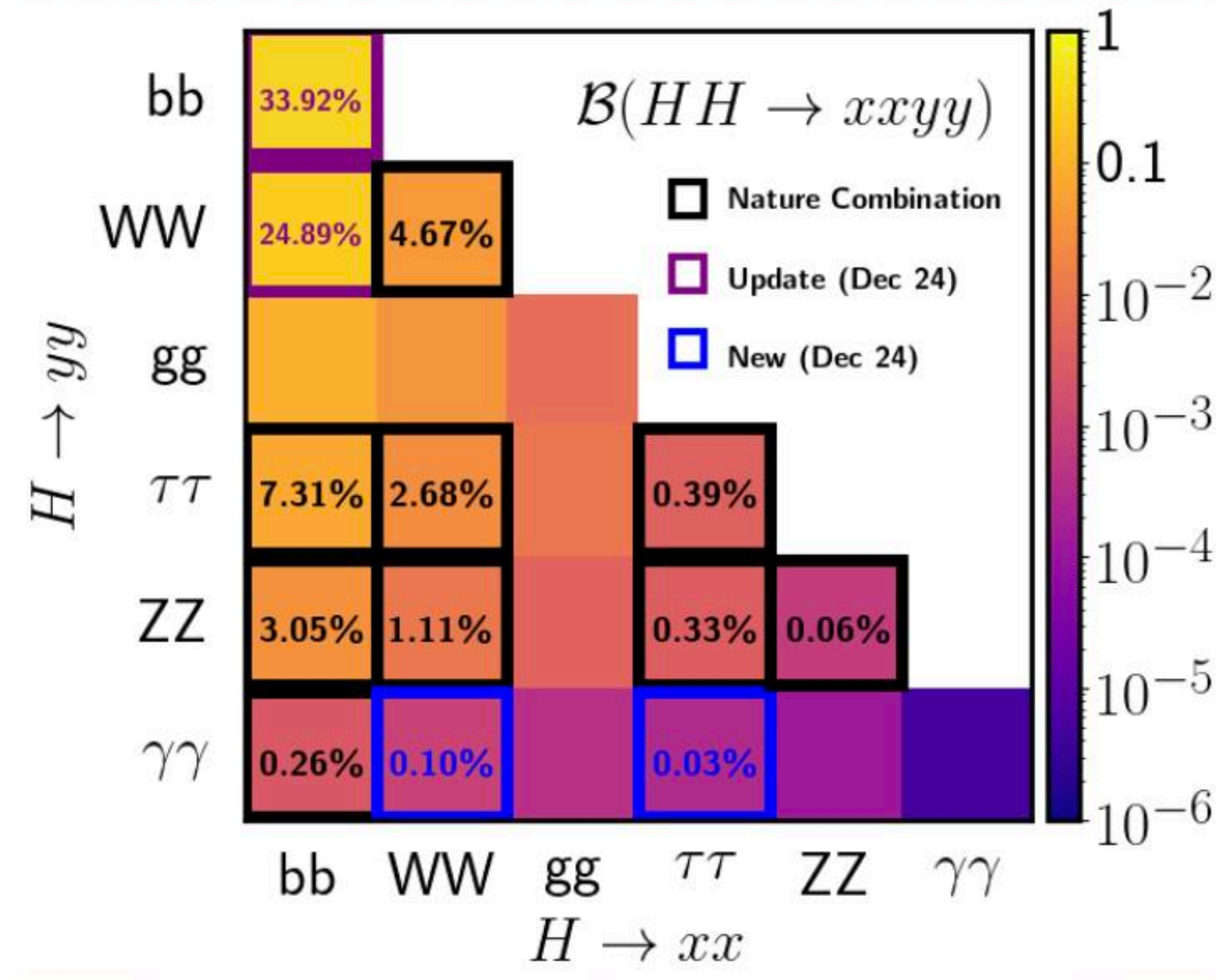
- HH→bbbb
- HH→bbττ
- HH→bbγγ

HH→bbℓℓ+MET

HH→multileptons

➡ Best channels!

CMS: [N. De Filippis, HIGGS 2025]

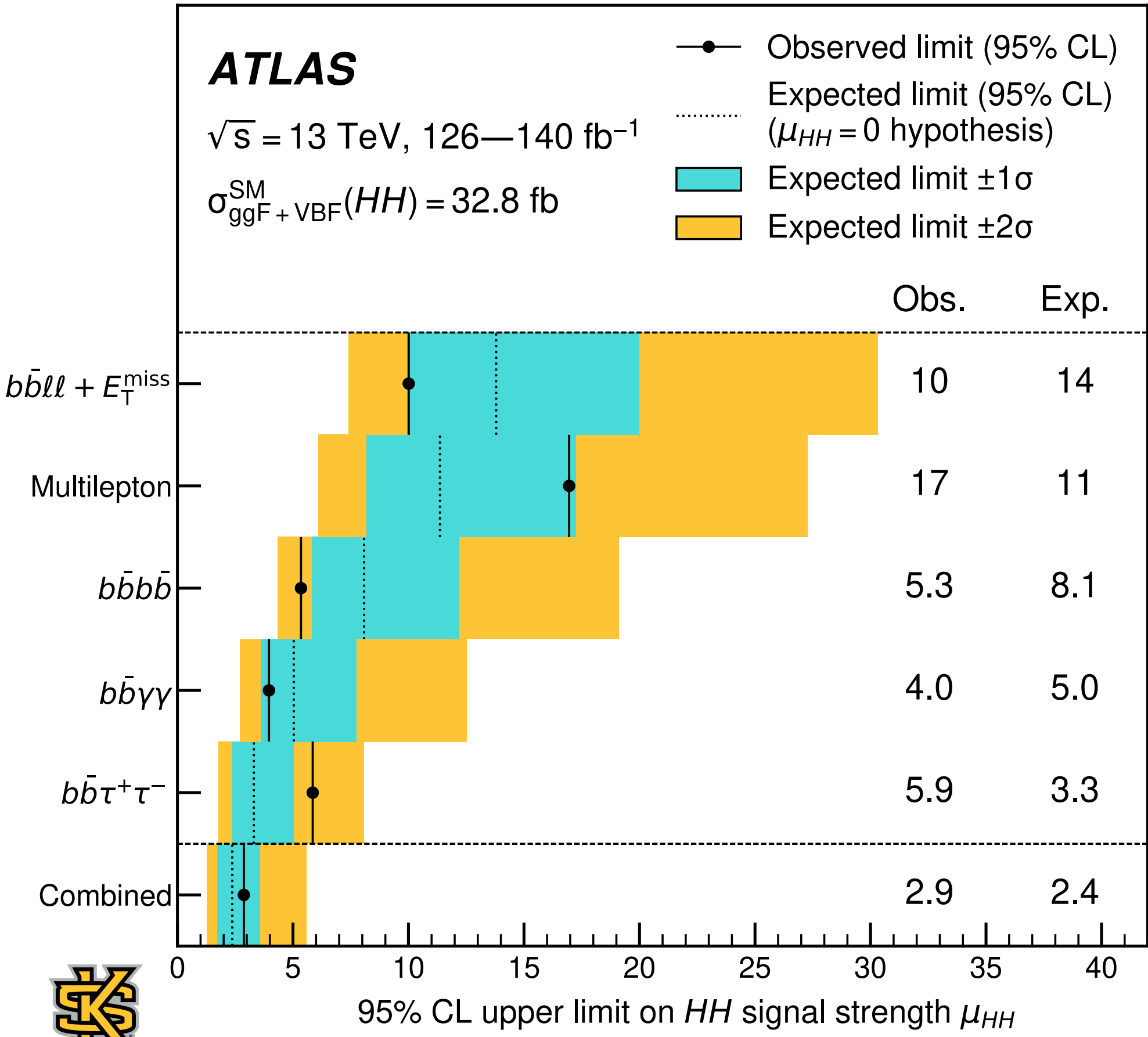


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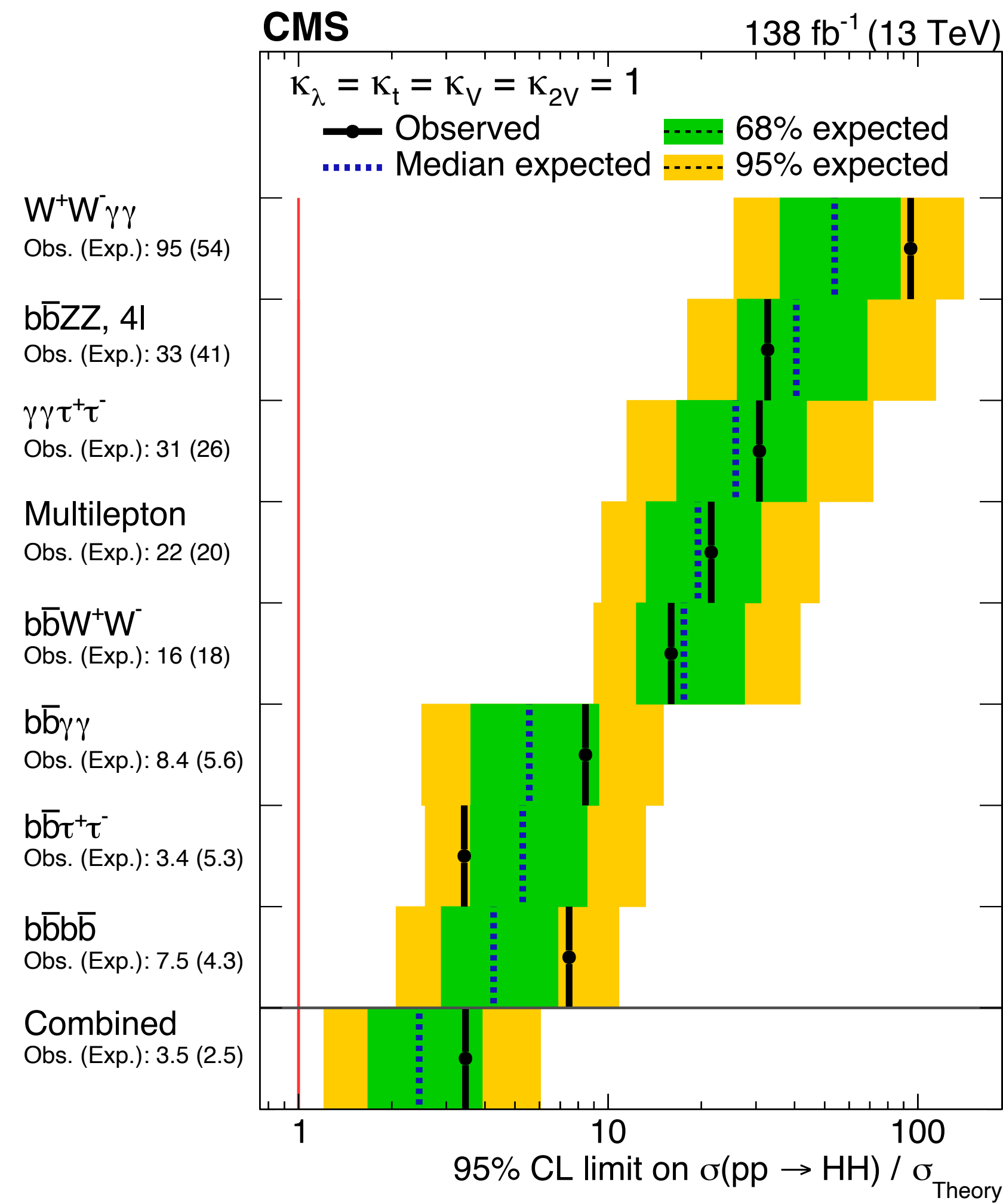
Combinations over several channels (current): **CMS:** [arXiv: 2510.07527]

ATLAS: [Phys. Rev. Lett. 133 (2024) 101801]

$\mu_{hh} = \sigma_{hh}/\sigma_{hh,SM} < 2.9; \quad -1.6 < \lambda_3/\lambda_3^{SM} < 7.2$



$\mu_{hh} = \sigma_{hh}/\sigma_{hh,SM} < 3.5; \quad -1.39 < \lambda_3/\lambda_3^{SM} < 7.02$

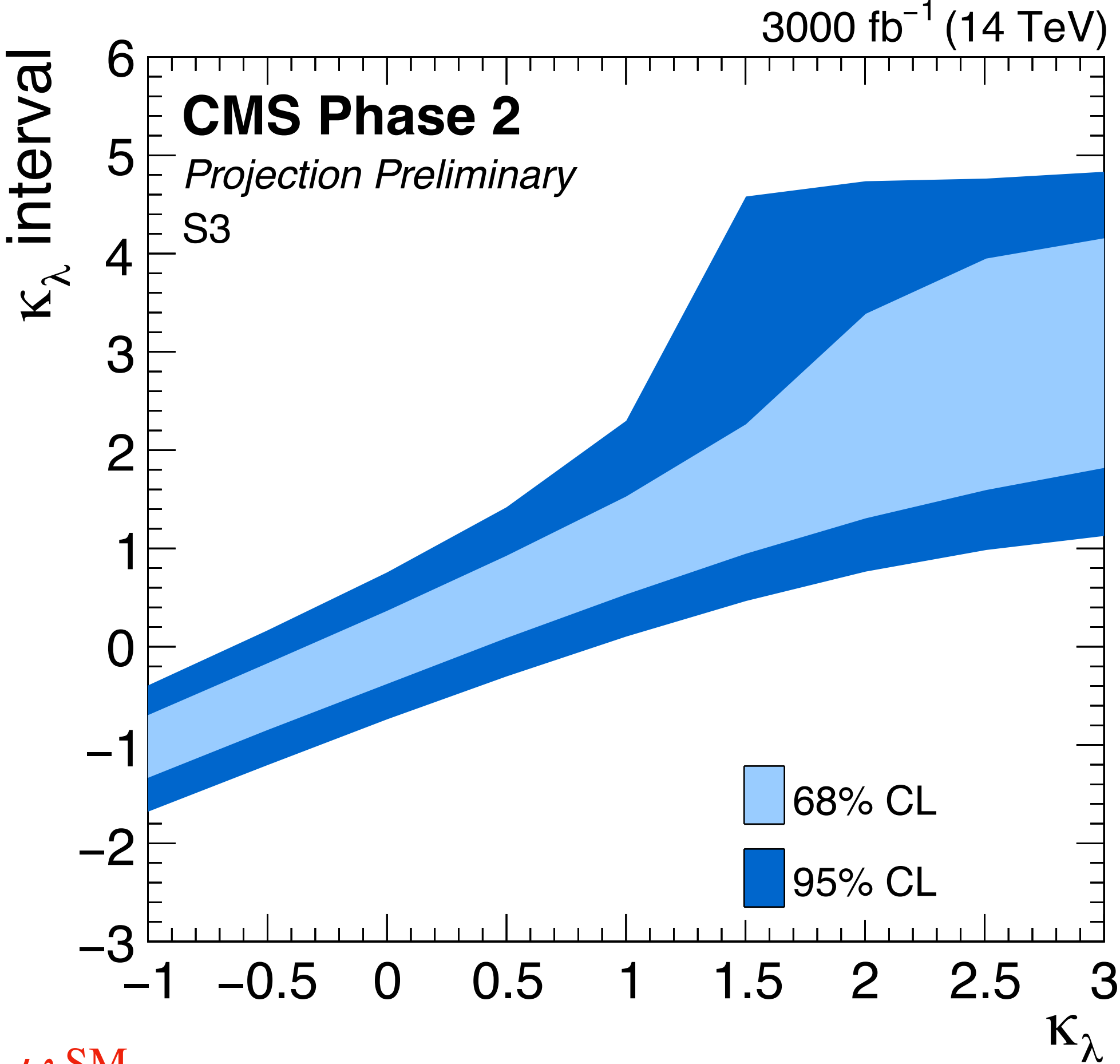
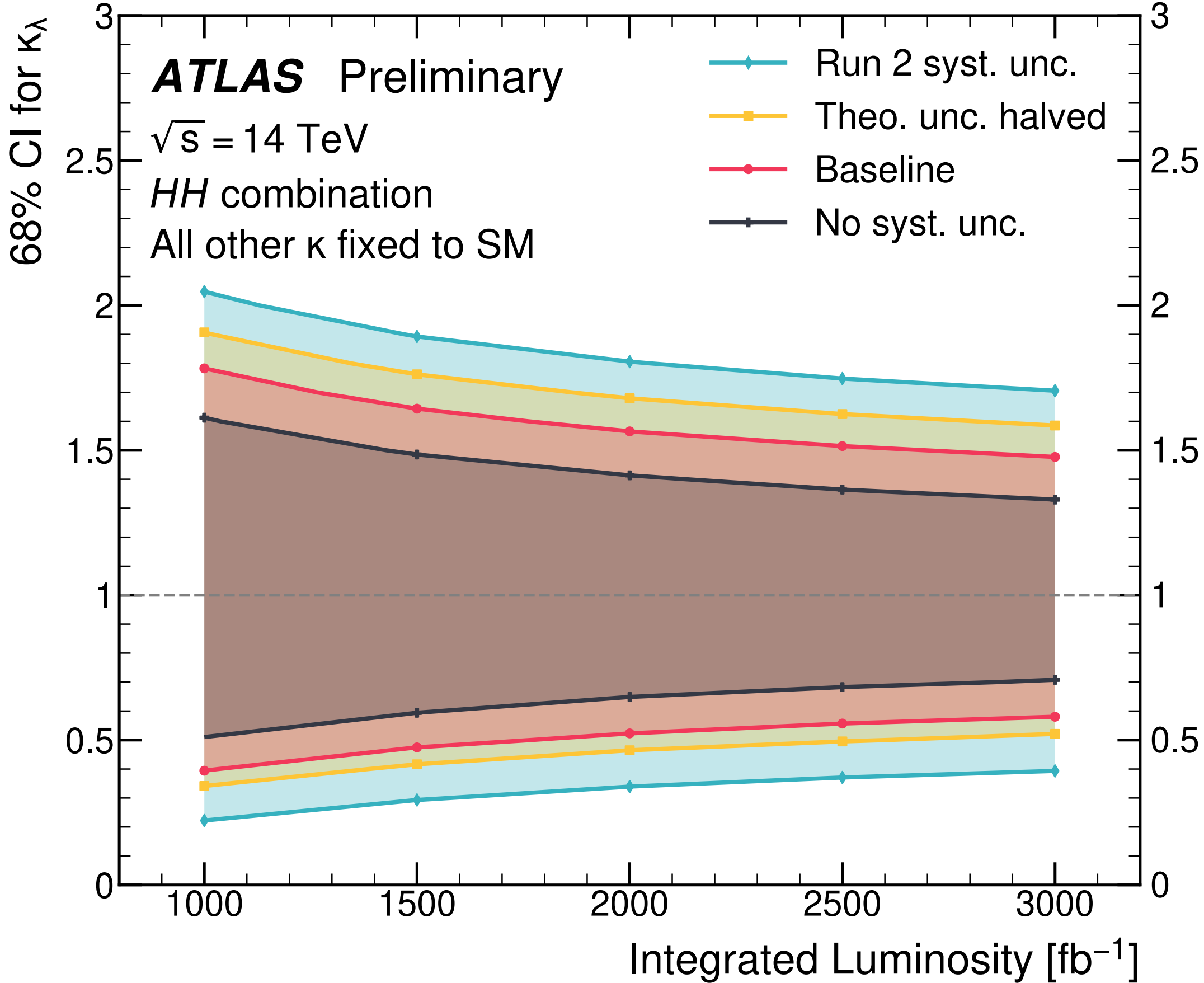


HL-LHC (3000/fb) Projections: Higgs Trilinear

Future prospects:

ATLAS: [ATL-PHYS-PUB-2025-006] $\lambda_3/\lambda_3^{\text{SM}} = 1^{+0.48}_{-0.42}$

CMS: [CMS NOTE -2025/006] $\lambda_3/\lambda_3^{\text{SM}} = 1^{+0.50}_{-0.47}$

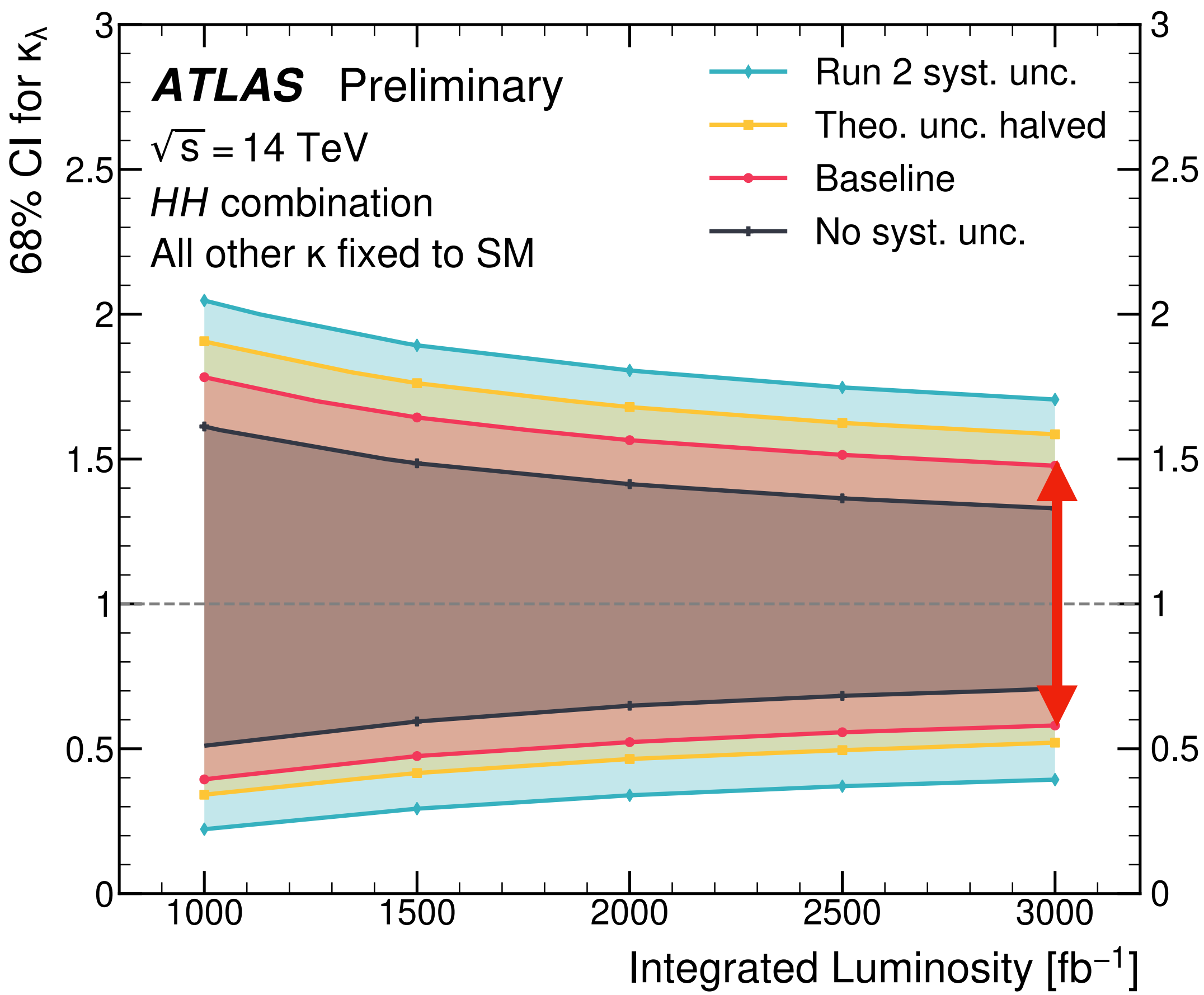


Note: $\kappa_\lambda \equiv \lambda_3/\lambda_3^{\text{SM}}$

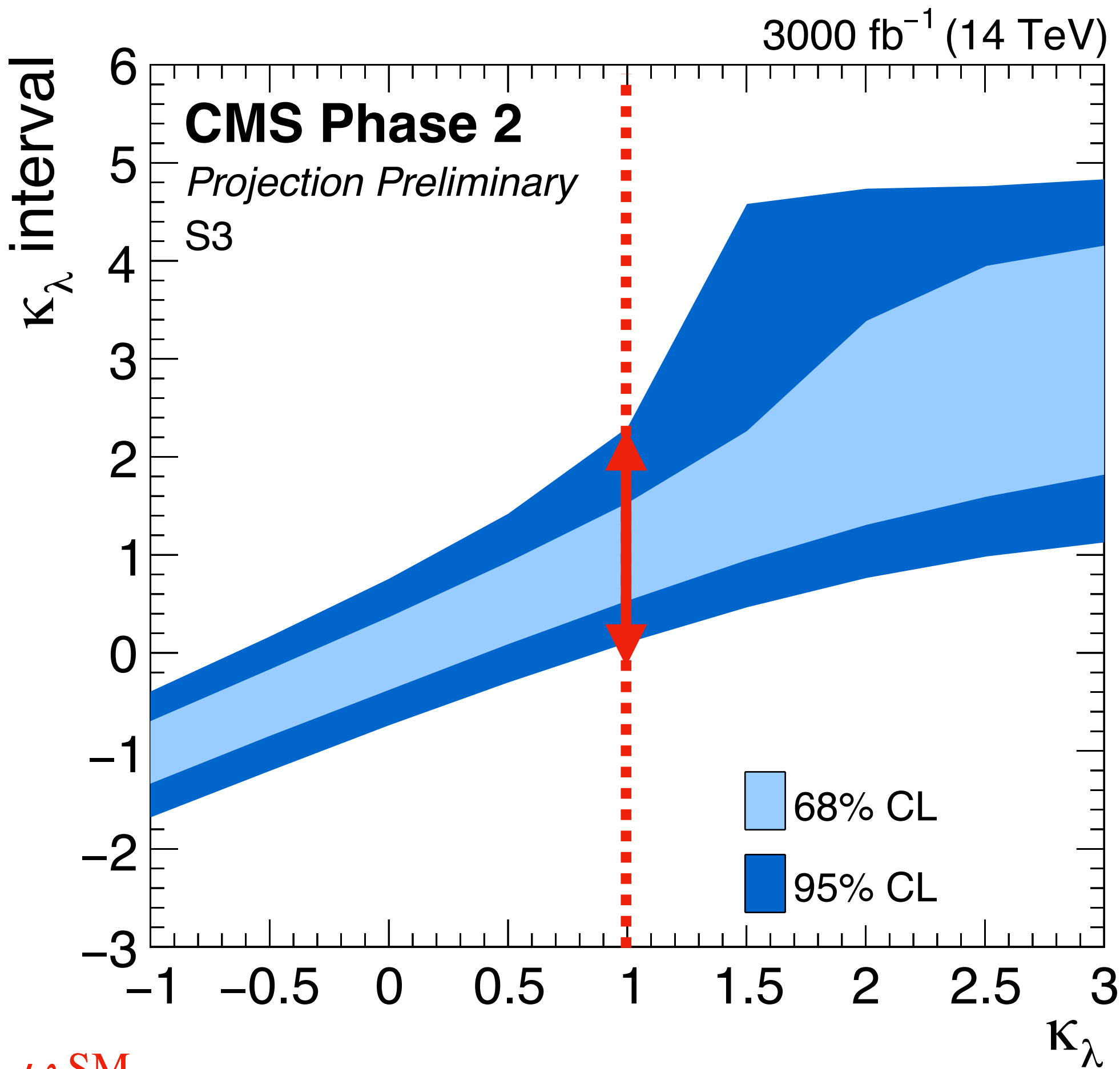
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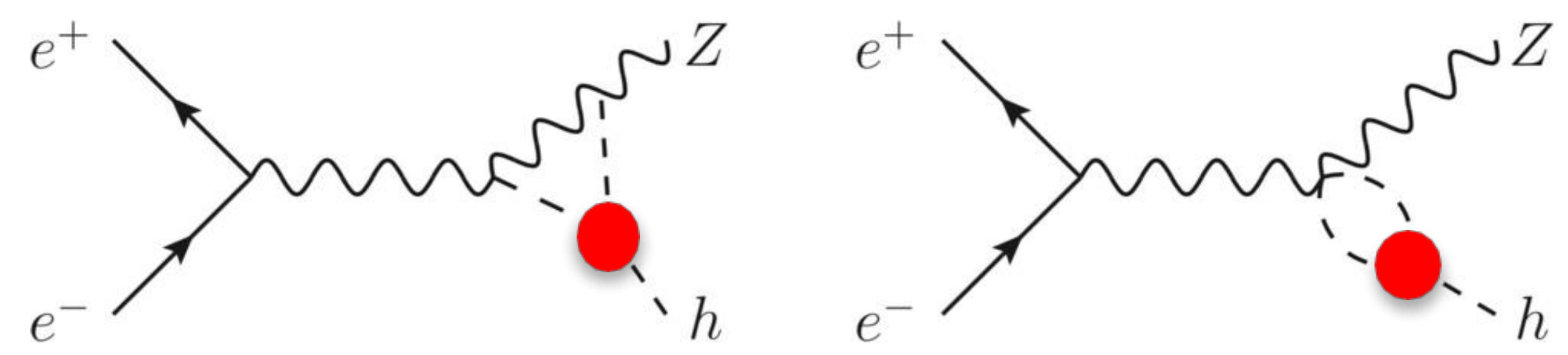
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Future Collider Projections: Higgs Boson Trilinear Self-Coupling

🎩 Prospects: [N. De Filippis, HIGGS 2025, arXiv:2511.03883]

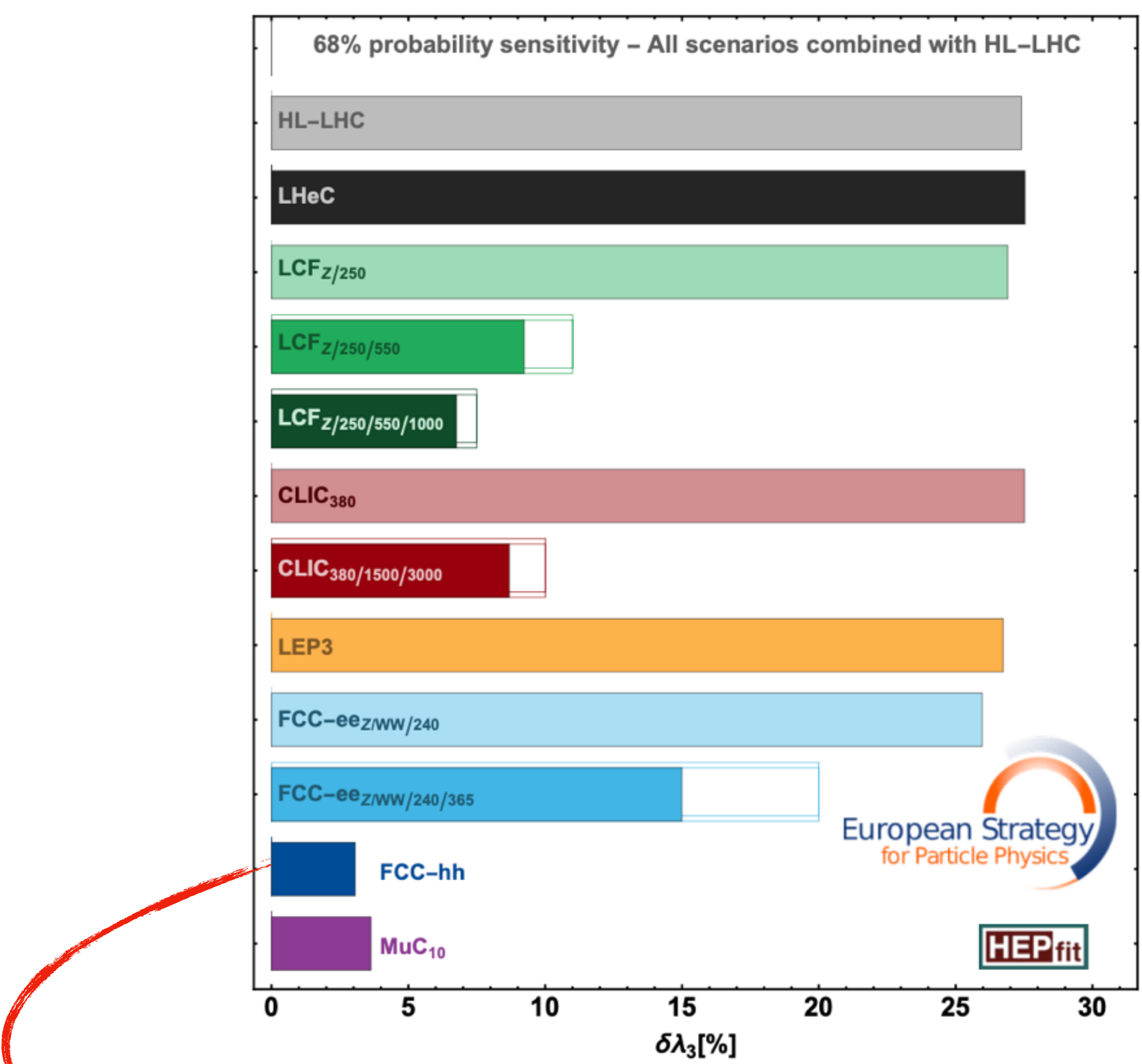
FCC-ee @ 240 GeV/365 GeV:

⇒ can't produce *hhZ*, but use loop diagrams:



$\delta(\lambda_3/\lambda_3^{SM}) \sim 28 \%$
 $\delta(\lambda_3/\lambda_3^{SM}) \sim 18 \%$ (HL-LHC combo)

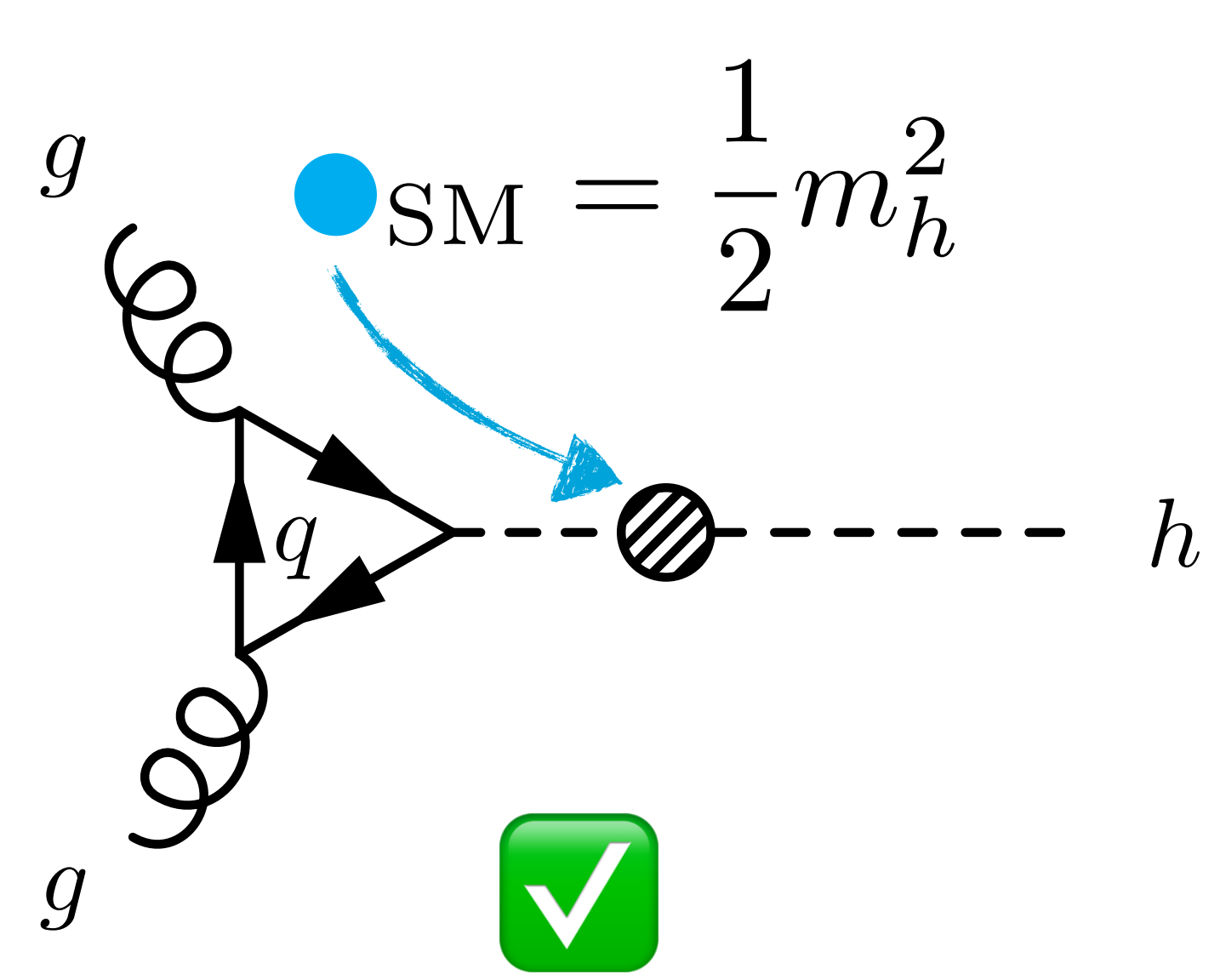
FCC-hh (compared to others):



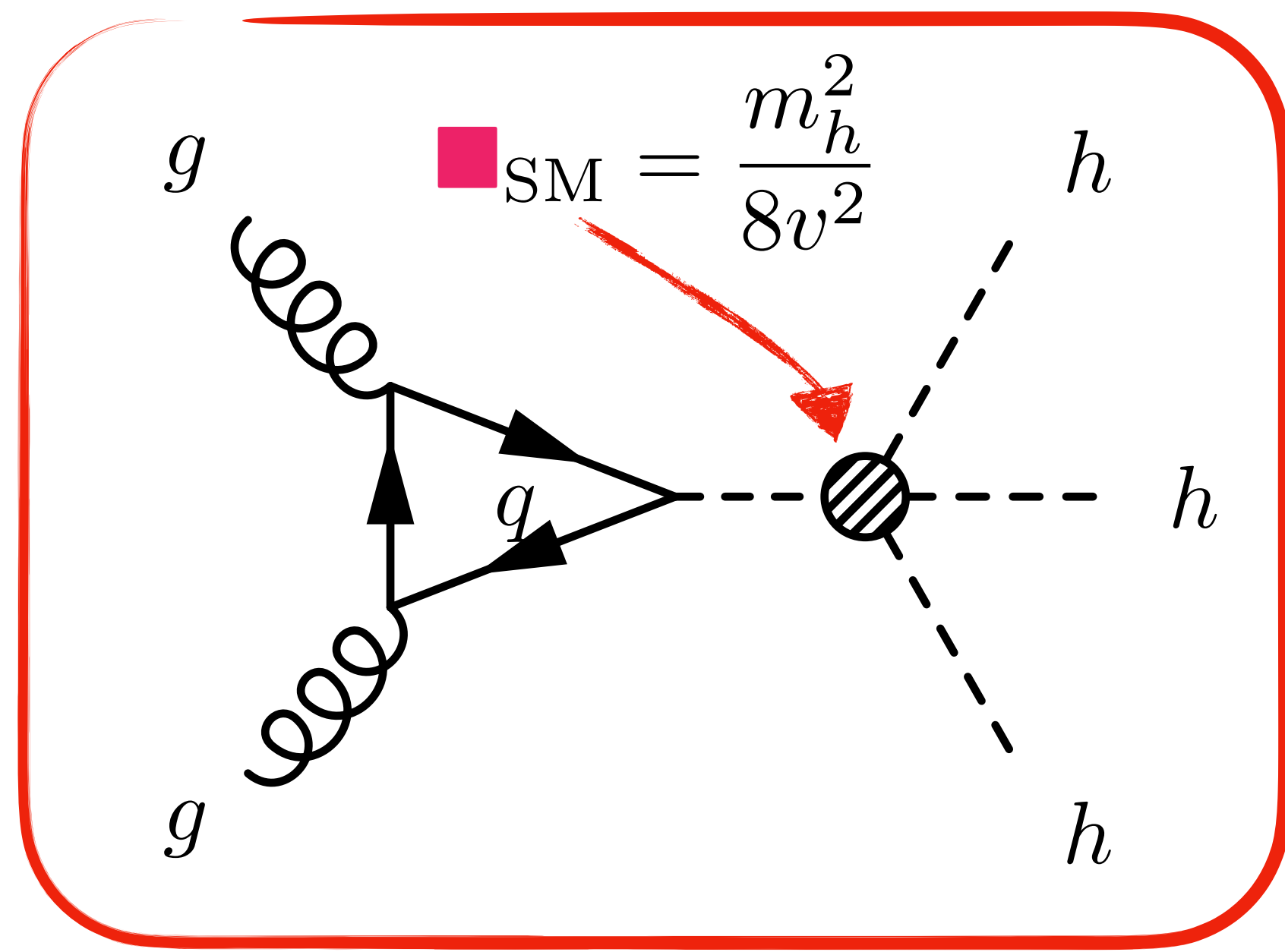
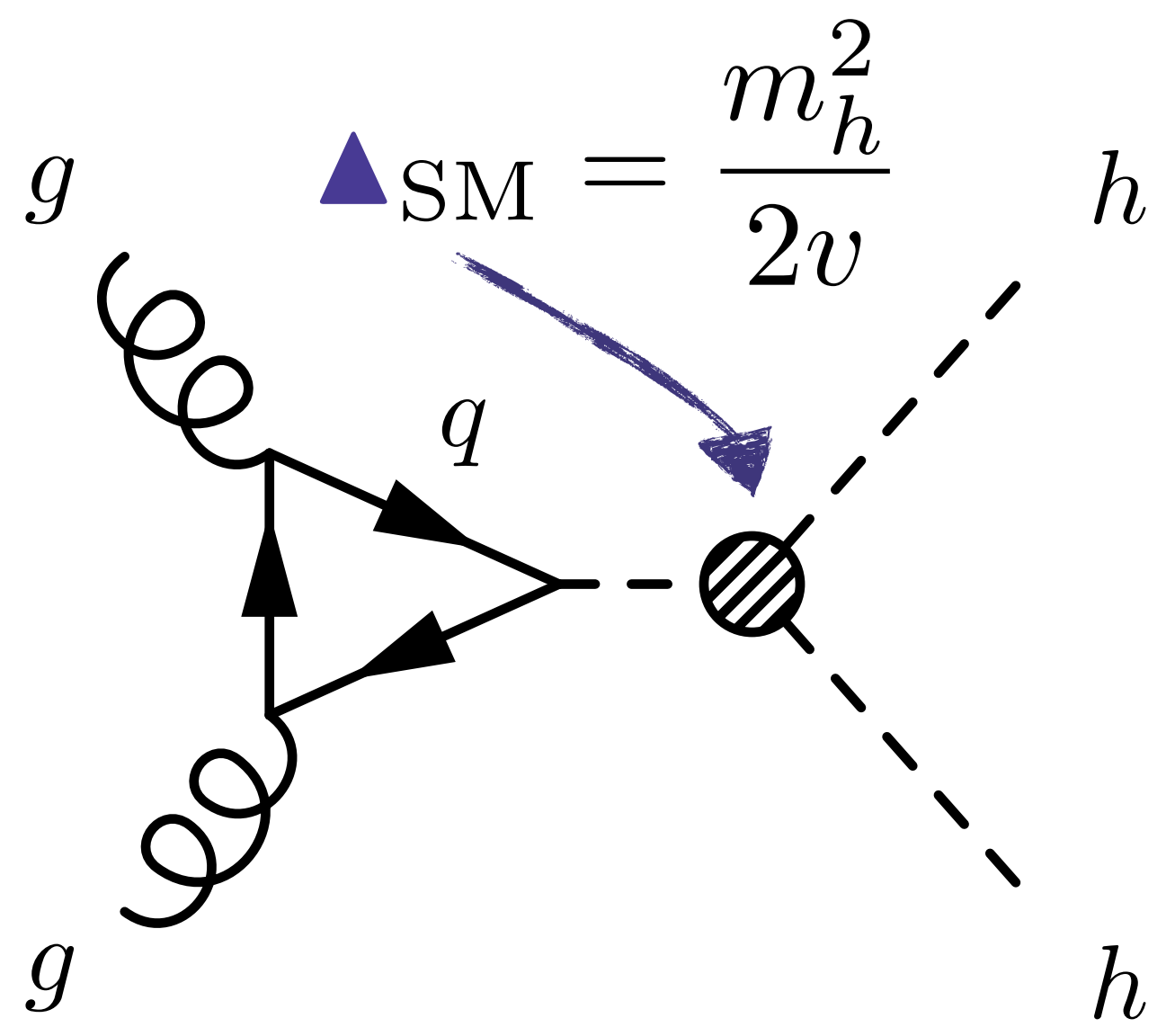
$\delta(\lambda_3/\lambda_3^{SM}) = 3.2\% - 5.4 \%$ @ 84 TeV
 $\delta(\lambda_3/\lambda_3^{SM}) = 2.8\% - 4.8 \%$ @ 100 TeV

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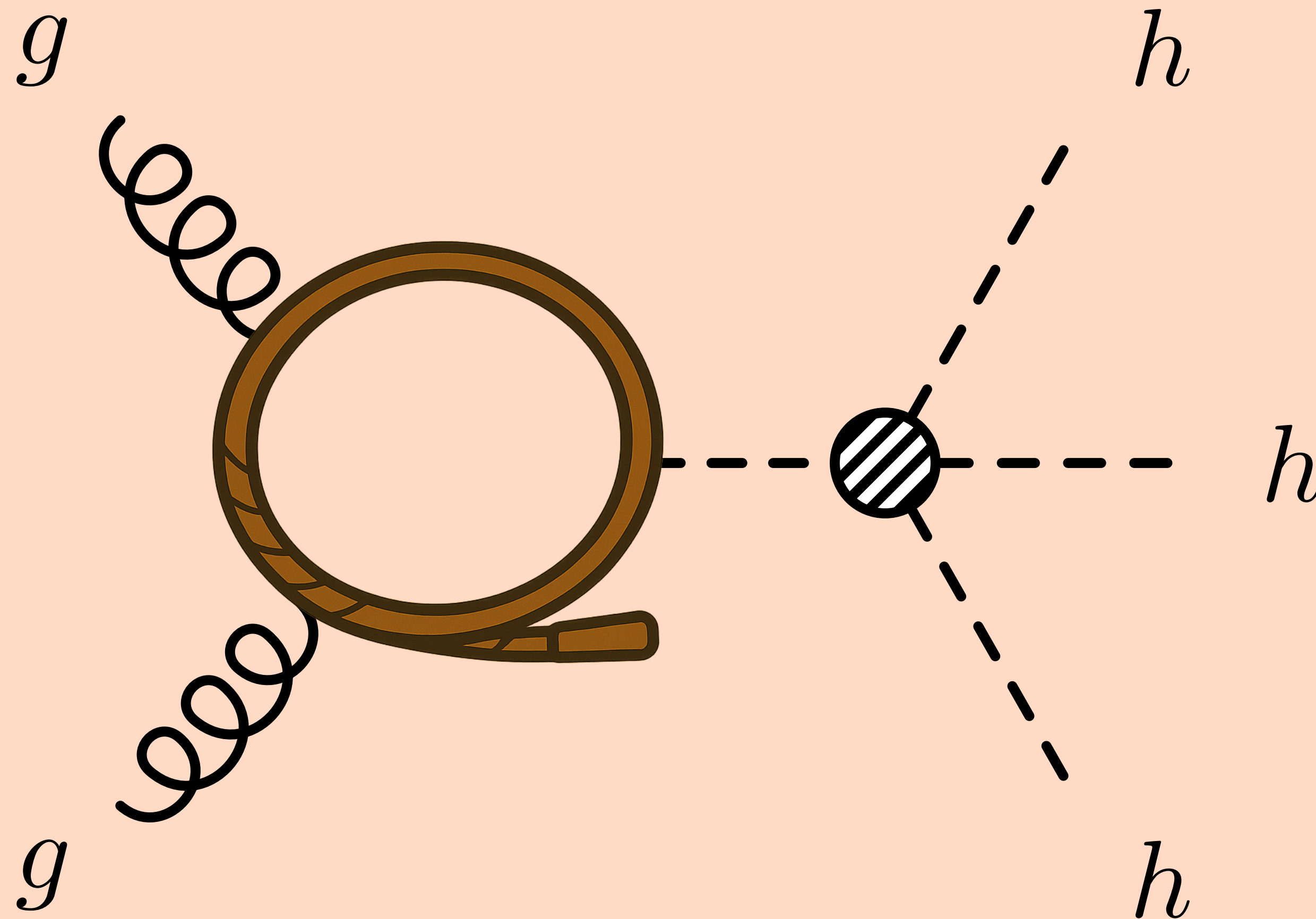
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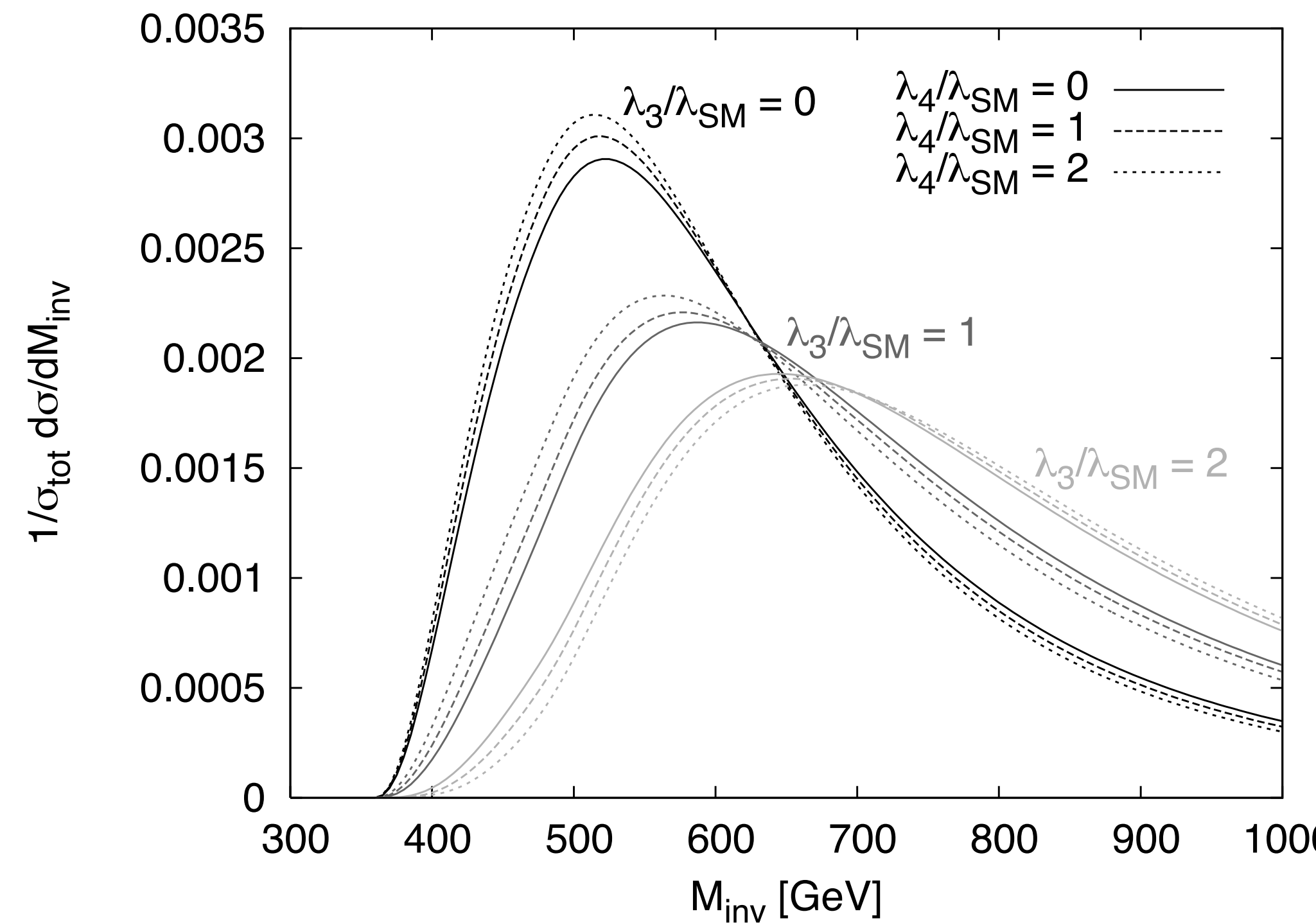
PART III: TRIPLE HIGGS BOSON PRODUCTION



Triple Higgs Boson Production: First Studies

👒 [Plehn, Rauch, hep-ph/0507321]: “VLHC” @ 200 TeV → “*Little Hope! & Measuring quartic requires good knowledge of trilinear*”.

👒 [Binoth, Karg, Kauer, Rückl, hep-ph/0608057] →
“*multi- h rates sensitive to BSM, extreme values of quartic lead to $\mathcal{O}(10 \text{ fb})$ cross sections at LHC*”.



Note: In these studies: no knowledge of h mass!

Post-Higgs Boson Discovery Studies

🎩 Following Higgs boson discovery → (most) pheno studies @ pp@100 TeV:

“Interesting” final states + at least 100 events @ 100 TeV @ 30 ab⁻¹

$hhh \rightarrow$ final state	BR (%)	σ (ab)	$N_{30\text{ab}^{-1}}$
$(b\bar{b})(b\bar{b})(b\bar{b})$	19.21	1110.338	33310
$(b\bar{b})(b\bar{b})(WW_{1\ell})$	7.204	416.41	12492
$(b\bar{b})(b\bar{b})(\tau\bar{\tau})$	6.312	364.853	10945
$(b\bar{b})(\tau\bar{\tau})(WW_{1\ell})$	1.578	91.22	2736
$(b\bar{b})(b\bar{b})(WW_{2\ell})$	0.976	56.417	1692
$(b\bar{b})(WW_{1\ell})(WW_{1\ell})$	0.901	52.055	1561
$(b\bar{b})(\tau\bar{\tau})(\tau\bar{\tau})$	0.691	39.963	1198
$(b\bar{b})(b\bar{b})(ZZ_{2\ell})$	0.331	19.131	573
$(b\bar{b})(WW_{2\ell})(WW_{1\ell})$	0.244	14.105	423
$(b\bar{b})(b\bar{b})(\gamma\gamma)$	0.228	13.162	394
$(b\bar{b})(\tau\bar{\tau})(WW_{2\ell})$	0.214	12.359	370
$(\tau\bar{\tau})(WW_{1\ell})(WW_{1\ell})$	0.099	5.702	171
$(\tau\bar{\tau})(\tau\bar{\tau})(WW_{1\ell})$	0.086	4.996	149
$(b\bar{b})(ZZ_{2\ell})(WW_{1\ell})$	0.083	4.783	143
$(b\bar{b})(\tau\bar{\tau})(ZZ_{2\ell})$	0.073	4.191	125

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[AP, Sakurai, 1508.06524, Chen, Yan, Zhao, Zhao, Zhong, 1510.04013, Fuks, Kim, Lee, 1510.07697, ...]

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🎩 Following Higgs boson discovery → (most) pheno studies @ pp@100 TeV:

“Interesting” final states + at least 100 events @ 100 TeV @ 30 ab⁻¹

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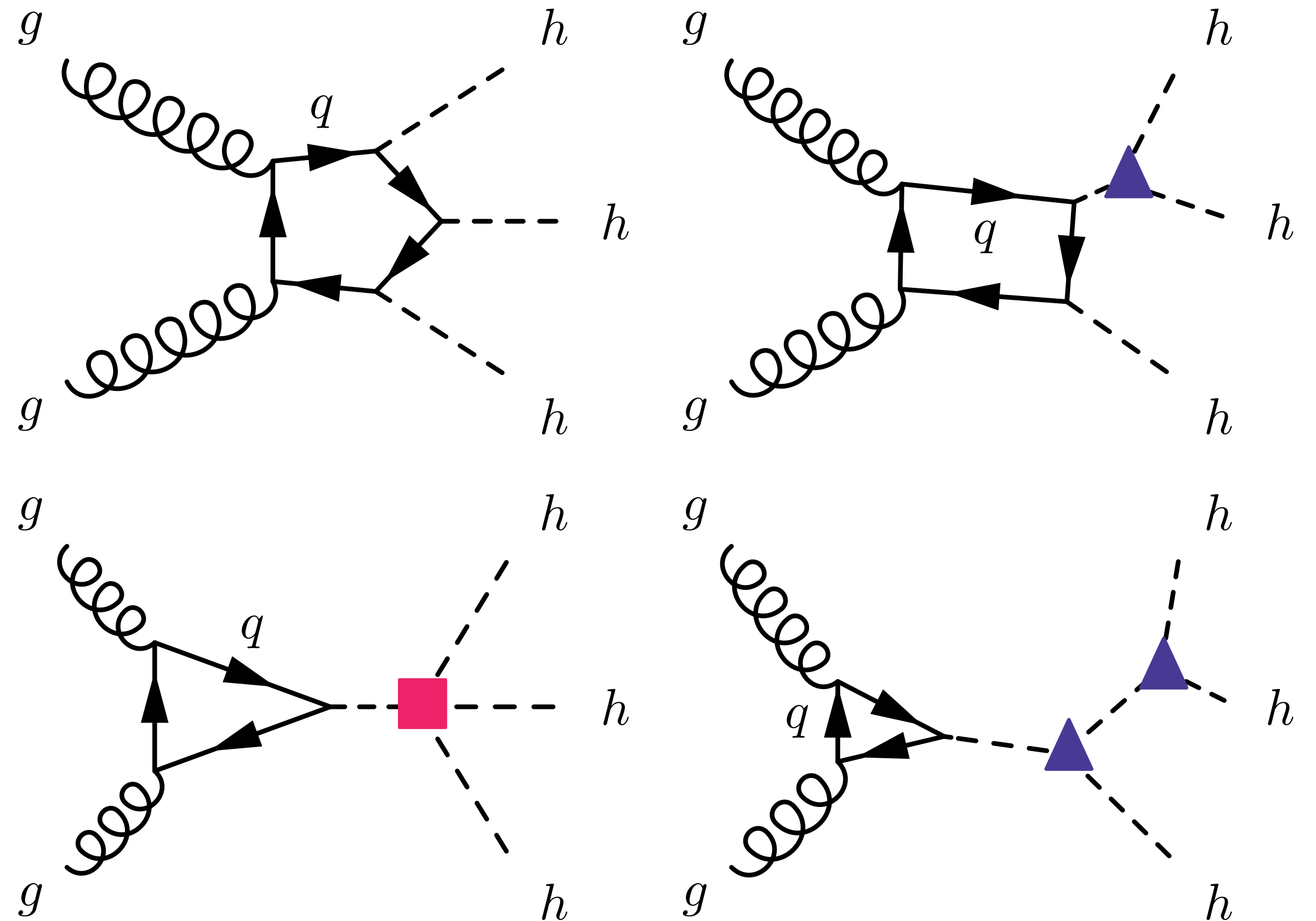
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The $pp \rightarrow hhh$ Cross Section

👒 [Note: VBF hhh $\sigma \sim 10^{-4}$ fb @ 13 TeV $\Rightarrow$ Gluon fusion dominant by far, see: Belyaev, Schaefers, Thomas, 1801.10157].

👒 At **LO** in SM: Loop-induced!

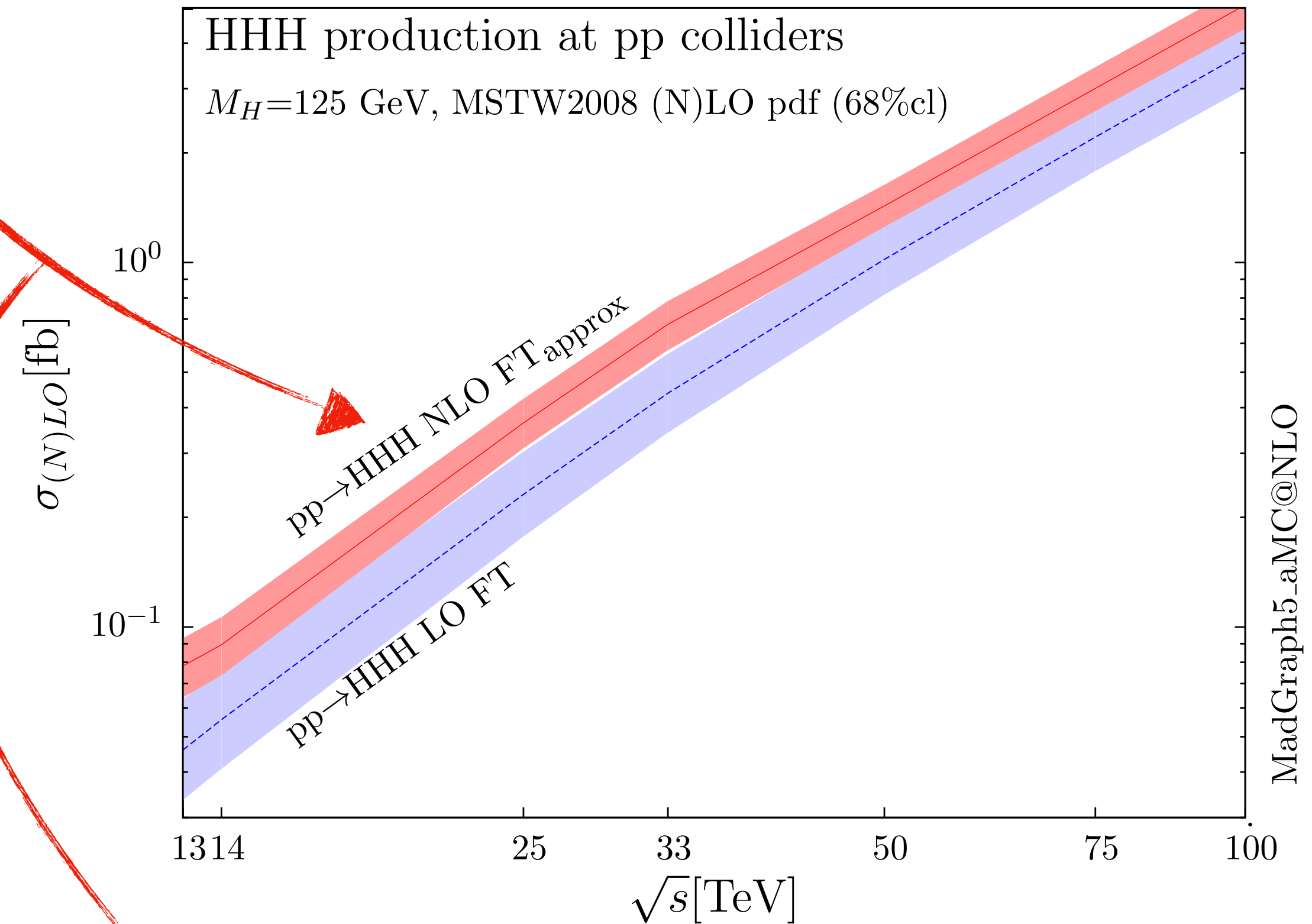
$$\sigma_{\text{SM}}^{\text{LO}}(14 \text{ TeV}) \sim 0.05 \text{ fb}$$



$pp \rightarrow hhh$ @ Higher Orders

🎩 **NLO**: [Maltoni, Vryonidou, Zaro, 1408.6542] $\rightarrow m_t \rightarrow \infty$ plus full real emission (= **FT_{approx}**).

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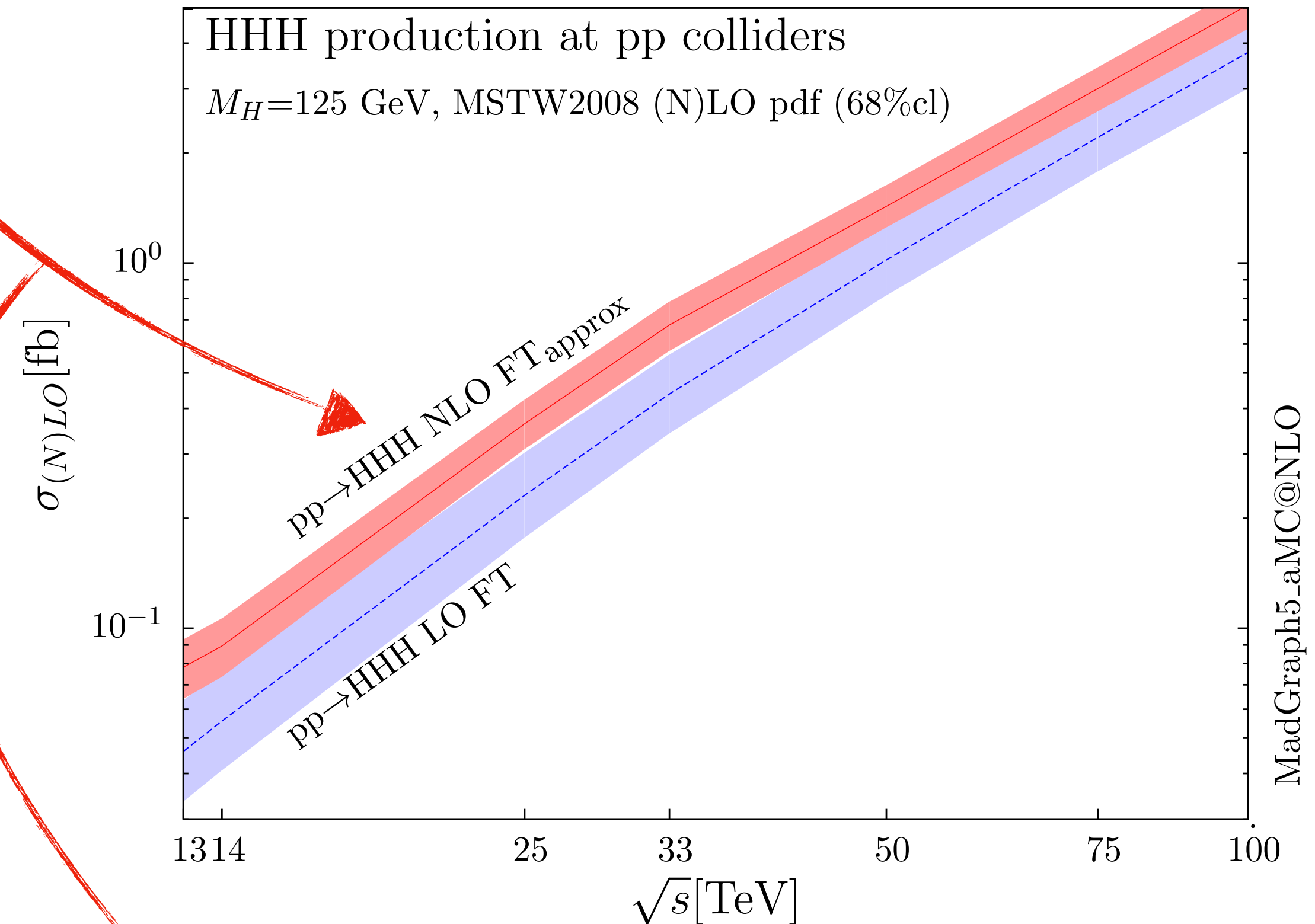
$$\begin{aligned} \sigma_{\text{Best}}^{\text{NNLO}} &= 0.103^{+5\%}_{-8\%} \pm 15\% \text{ fb}, & K_{\text{Best}}^{\text{NNLO}} &= 1.70, & (14 \text{ TeV}) \\ \sigma_{\text{Best}}^{\text{NNLO}} &= 0.501^{+5\%}_{-7\%} \pm 15\% \text{ fb}, & K_{\text{Best}}^{\text{NNLO}} &= 1.70, & (27 \text{ TeV}) \\ \sigma_{\text{Best}}^{\text{NNLO}} &= 5.56^{+5\%}_{-6\%} \pm 20\% \text{ fb}, & K_{\text{Best}}^{\text{NNLO}} &= 1.43, & (100 \text{ TeV}) \end{aligned}$$

$\sigma(HHH)$ [fb]	$\sqrt{s} = 14 \text{ TeV}$	$\sqrt{s} = 33 \text{ TeV}$	$\sqrt{s} = 100 \text{ TeV}$
LO FT	$0.0557^{+34.5+2.5\%}_{-24.0-2.7\%}$	$0.438^{+26.8+1.5\%}_{+20.0-2.0\%}$	$3.78^{+24.1+0.9\%}_{-18.7-1.7\%}$
NLO FT _{approx}	$0.0894^{+16.5+2.5\%}_{-14.6-3.2\%}$	$0.677^{+14.5+1.4\%}_{-13.4-1.7\%}$	$5.09^{+13.5+1.0\%}_{-12.7-1.3\%}$

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Attack of the Anomalous Couplings

👒 Anomalous couplings can enhance *hhh*! e.g. inspired by an Effective Field Theory.

$$\mathcal{L} \supset -\lambda_{\text{SM}} v \underbrace{(1+c_3)}_{\kappa_3 = \lambda_3/\lambda_3^{\text{SM}}} h^3 - \frac{\lambda_{\text{SM}}}{4} \underbrace{(1+d_4)}_{\kappa_4 = \lambda_4/\lambda_4^{\text{SM}}} h^4$$

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⇒ *hhh* cross section as ratio to SM (highest-to-lowest coefficient):

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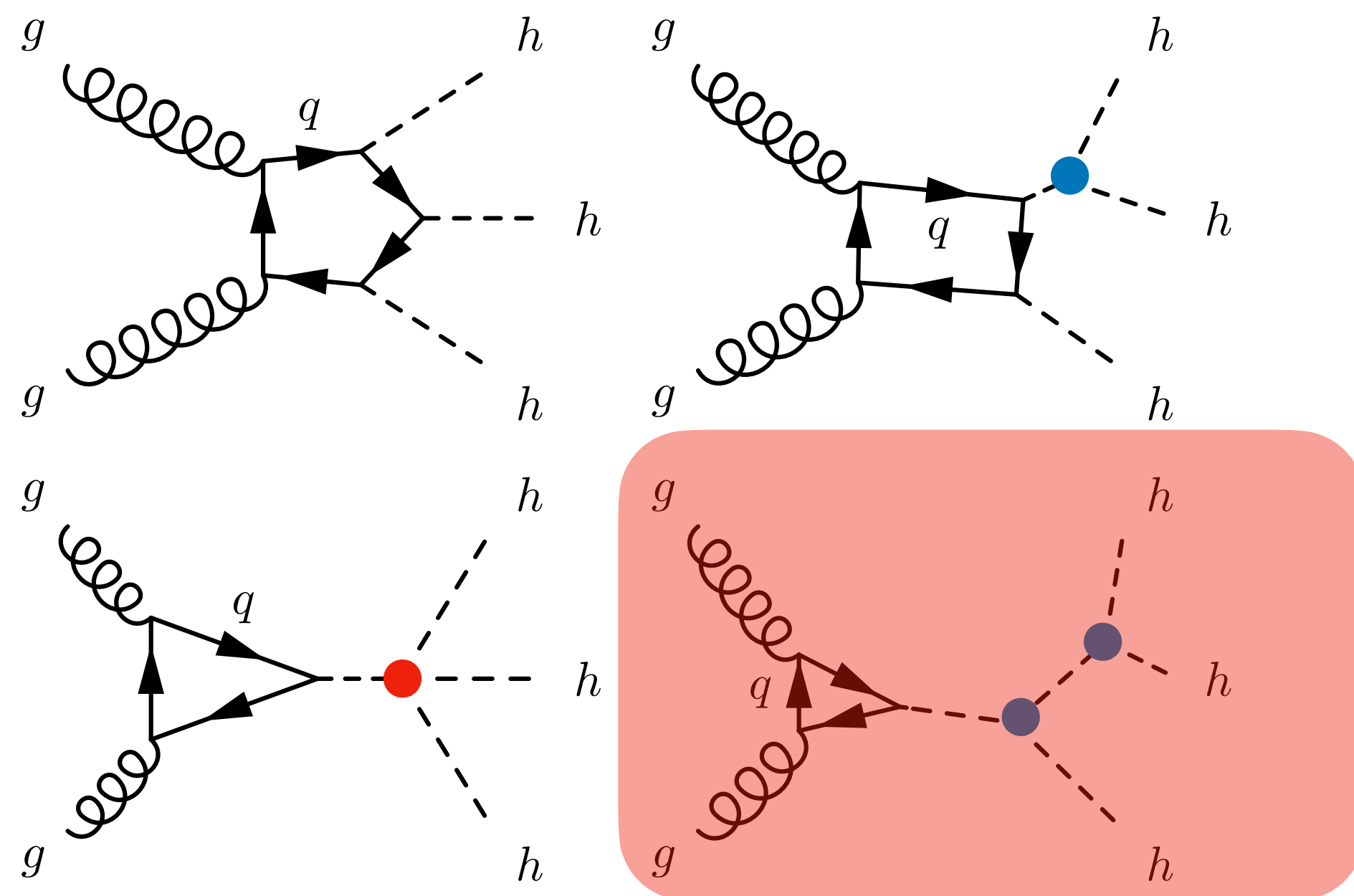
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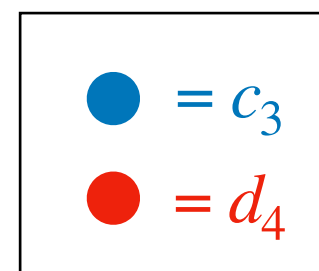
Q: Do all of these terms make sense in an effective field theory?

Matrix Element Truncation

- 🎩 **Q:** Which anomalous contributions to keep @ the **matrix element-squared** level?
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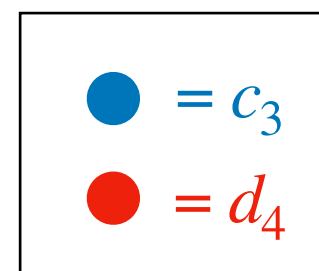
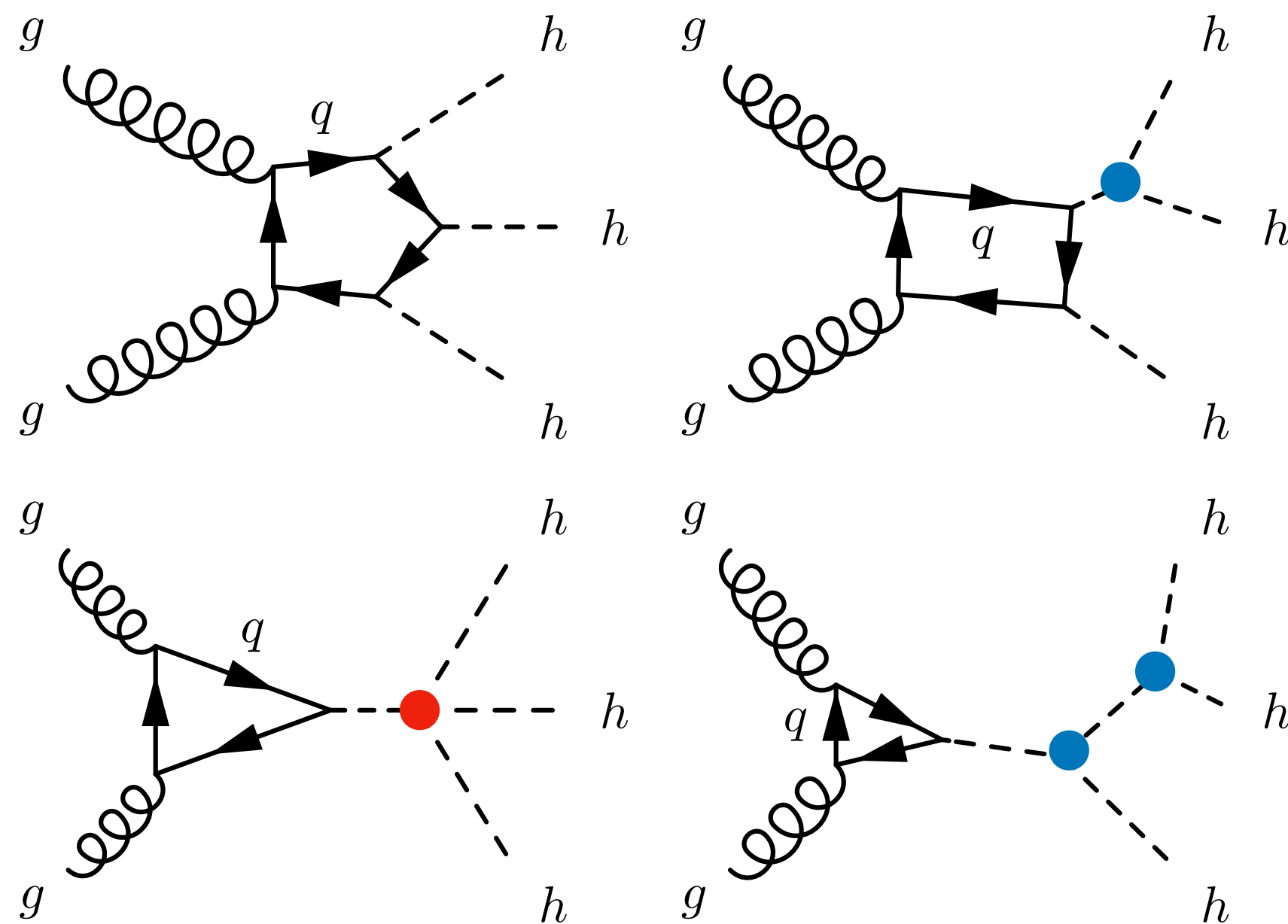


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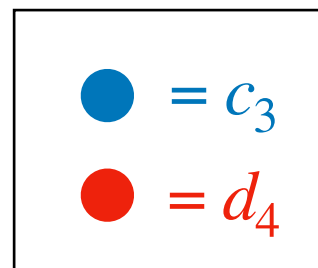
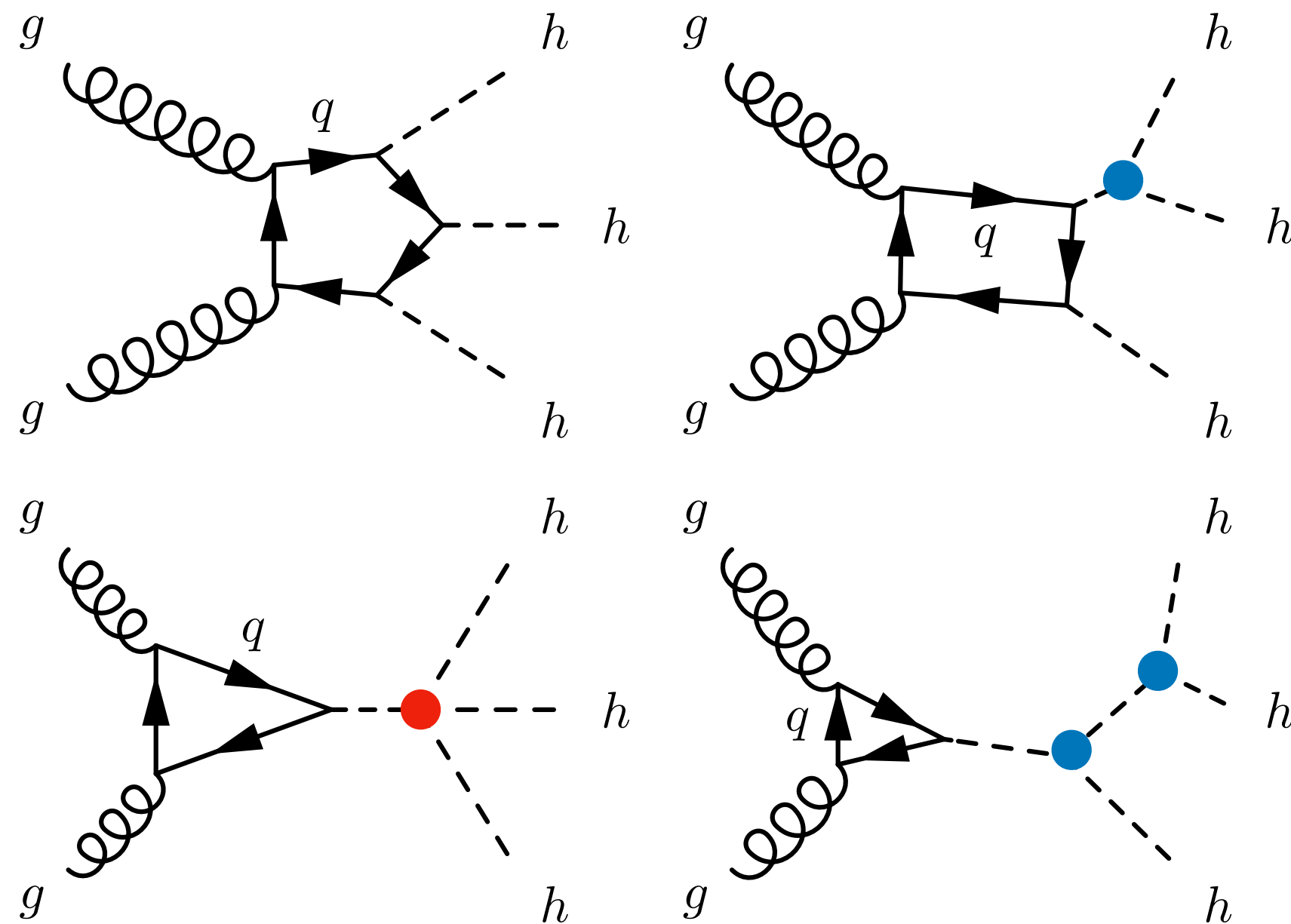


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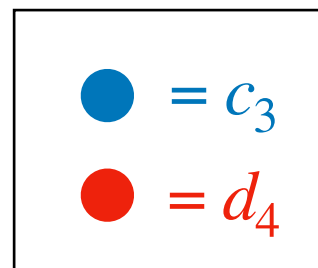
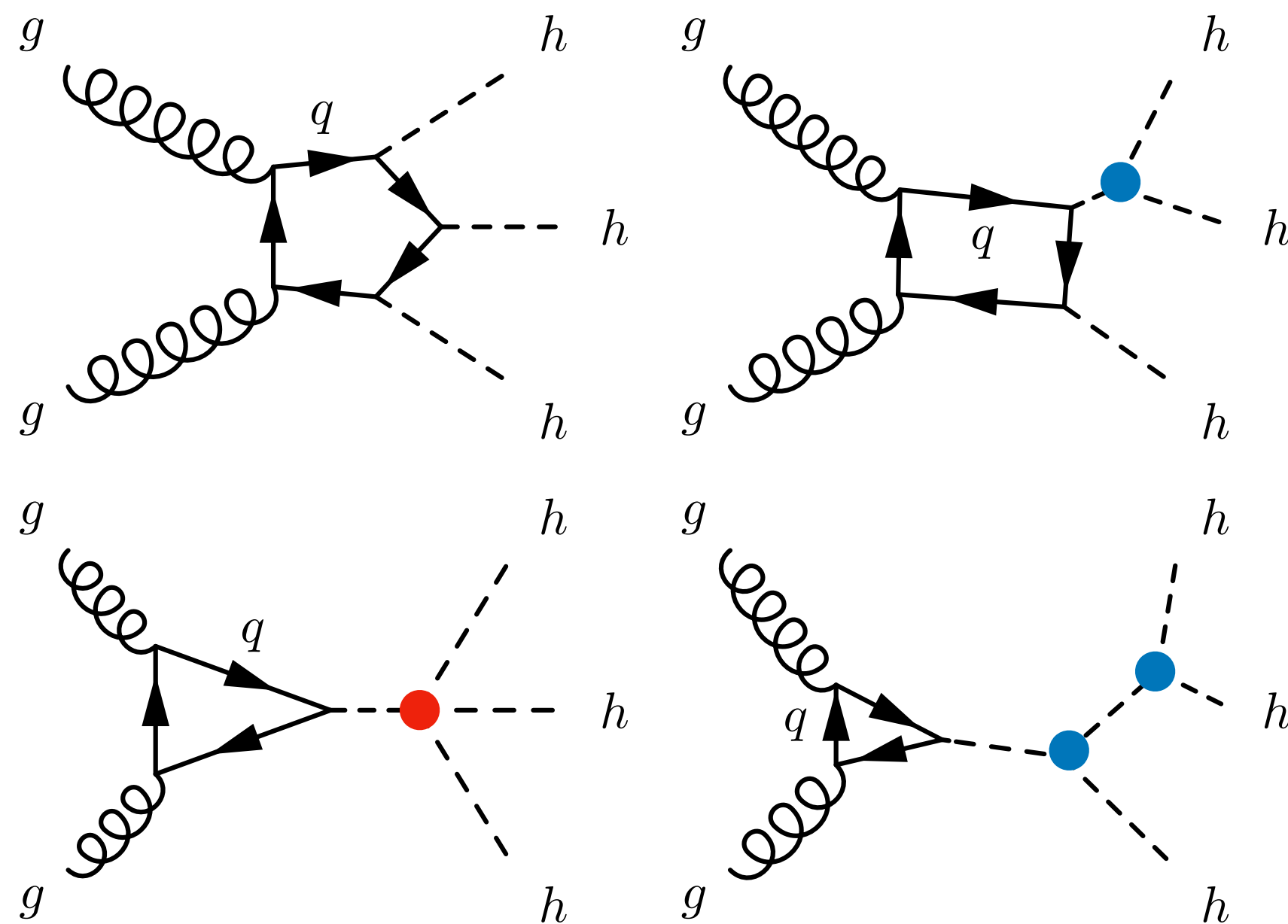
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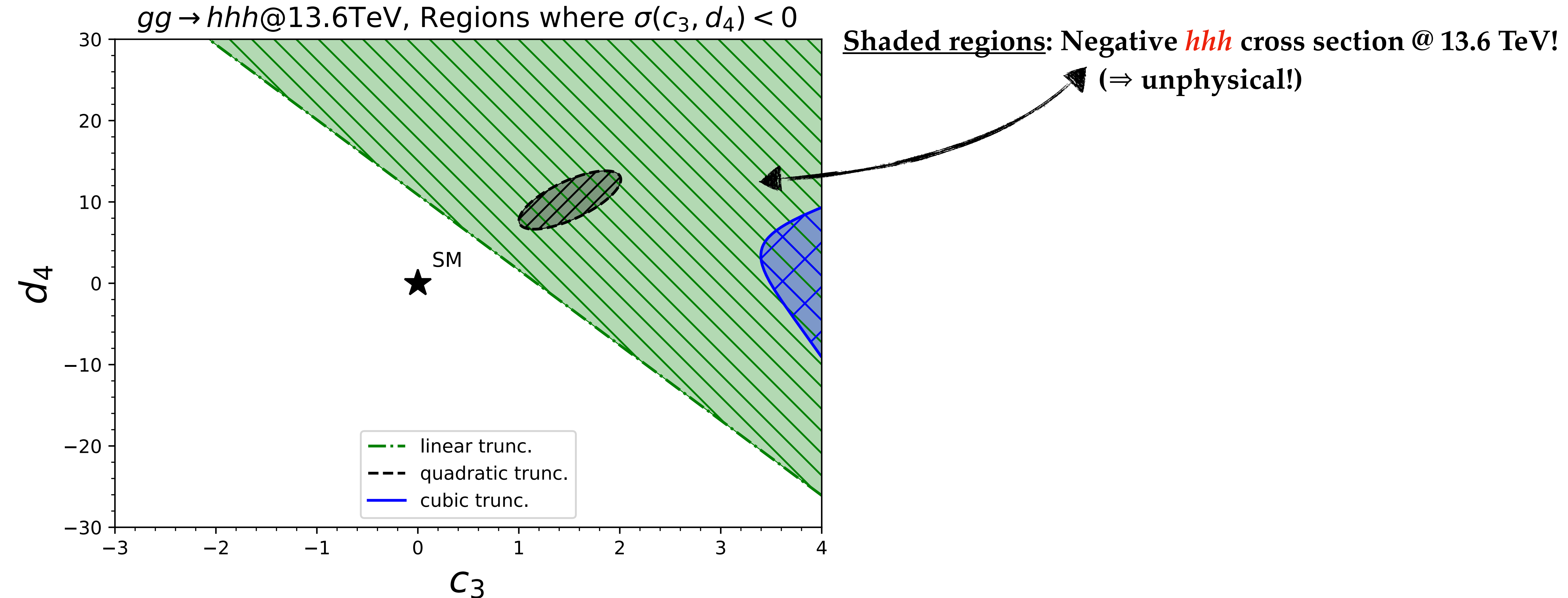
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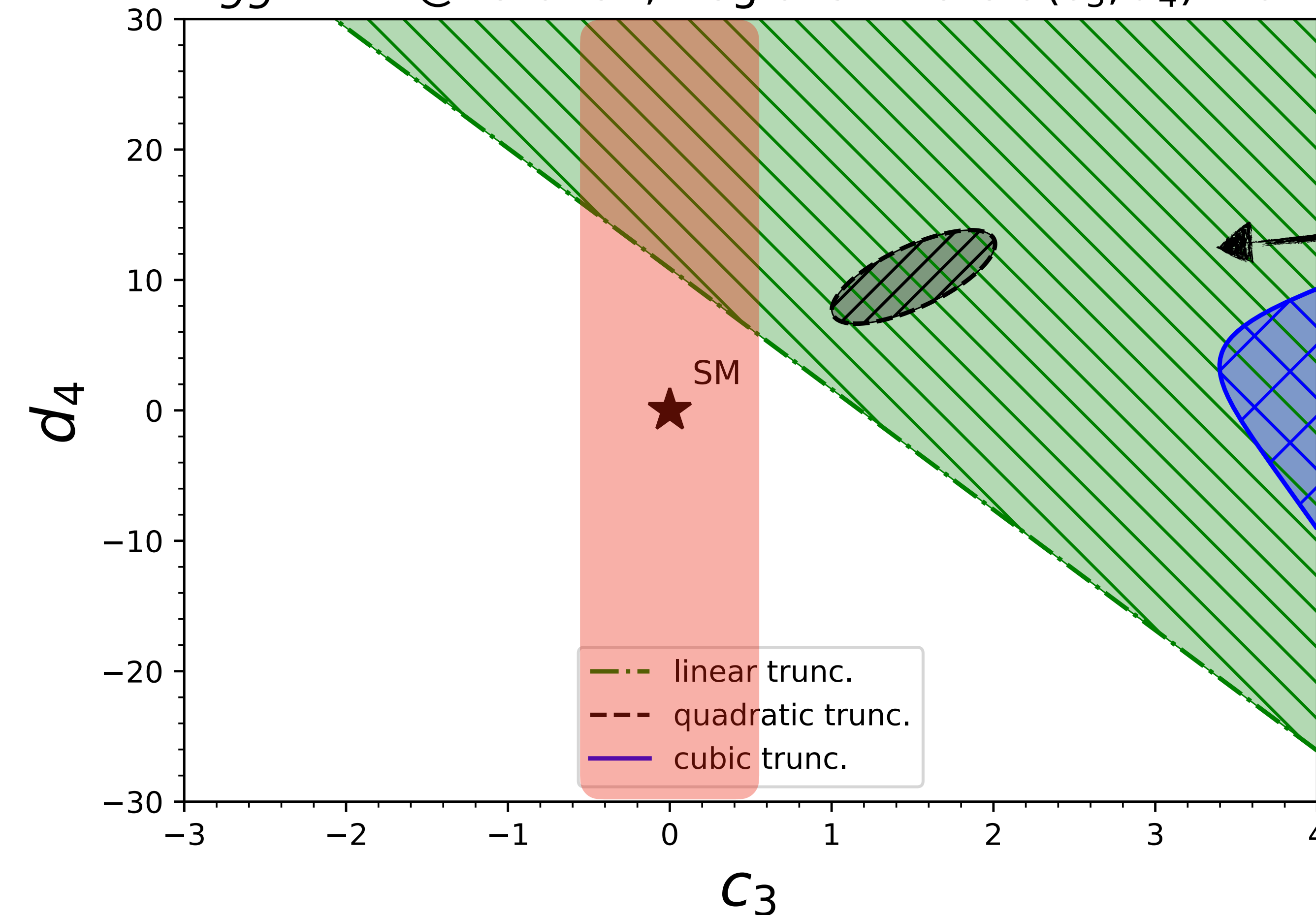


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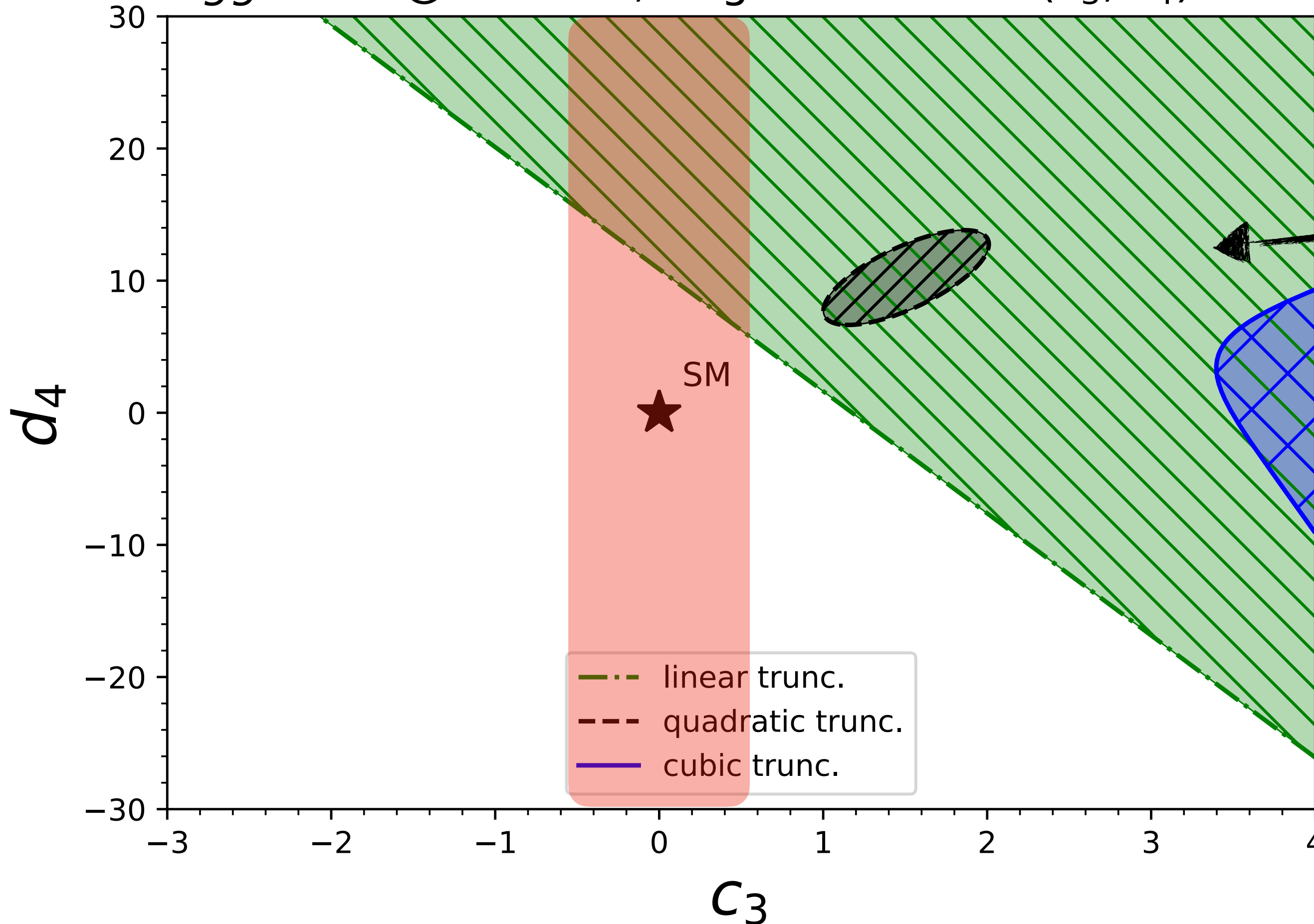
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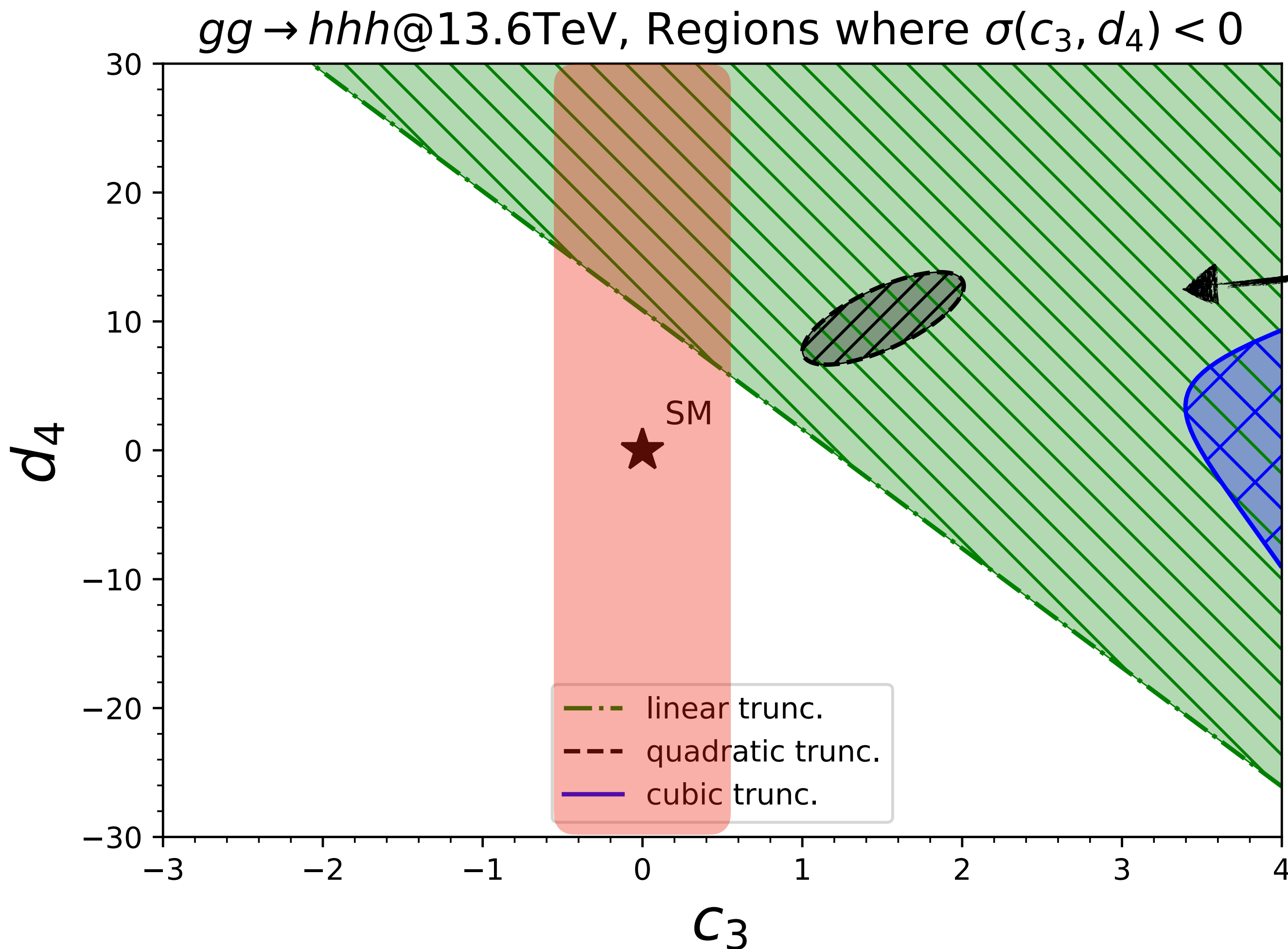
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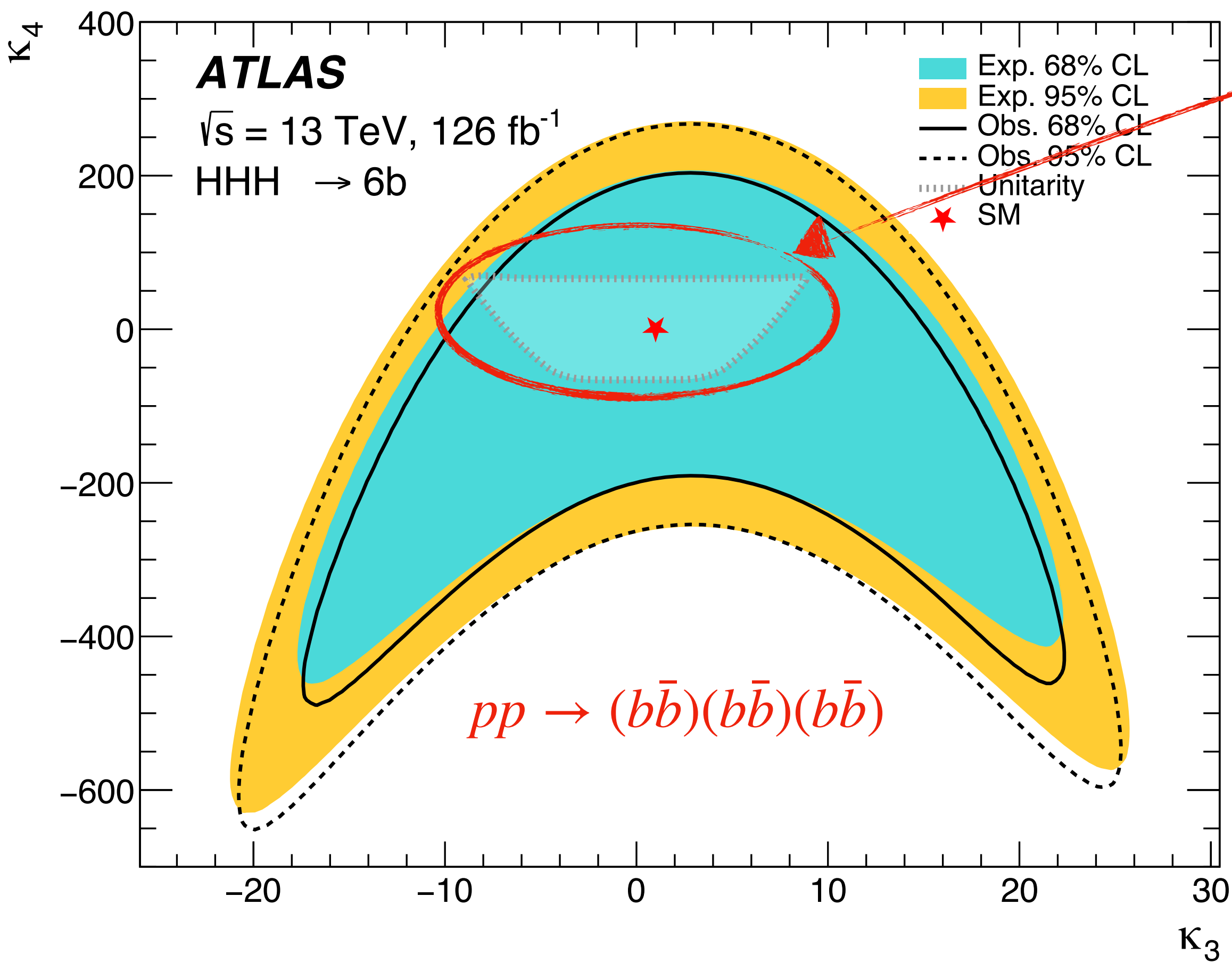
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achievable at High-Luminosity LHC

$\Rightarrow$ When combining with HL-LHC Higgs trilinear constraints, quadratic or cubic truncations could also make sense!

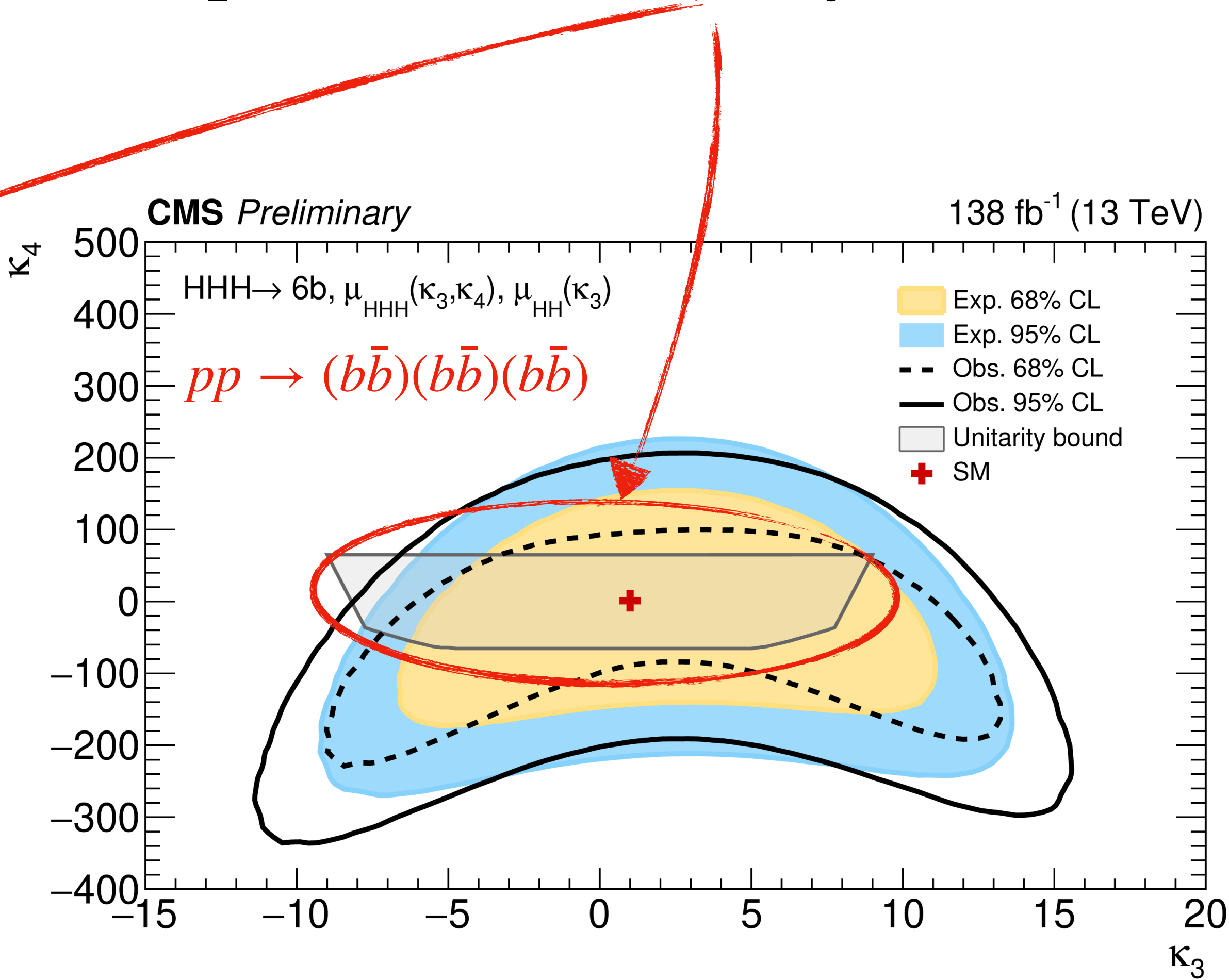
$\Rightarrow$ **Should experiments consider these?**

LHC Searches for Triple Higgs Boson Production

LHC results! But: currently *NOT* competitive vs perturbative unitarity!



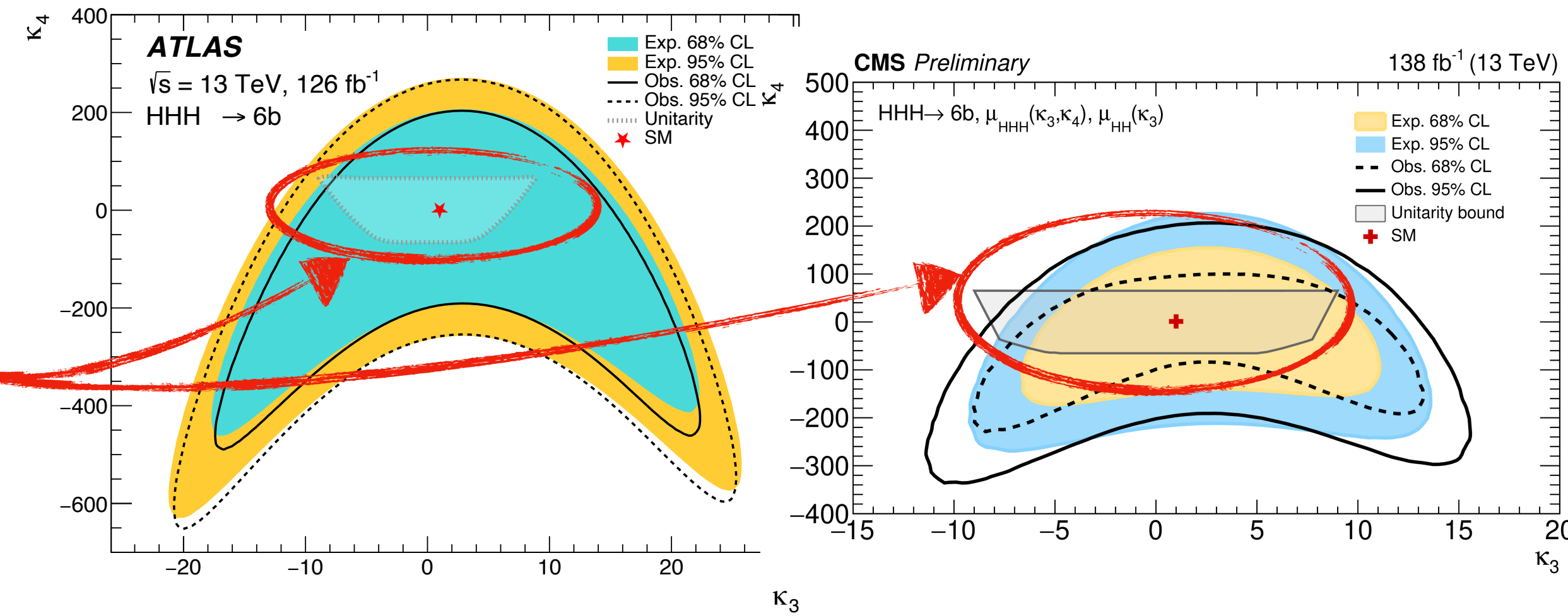
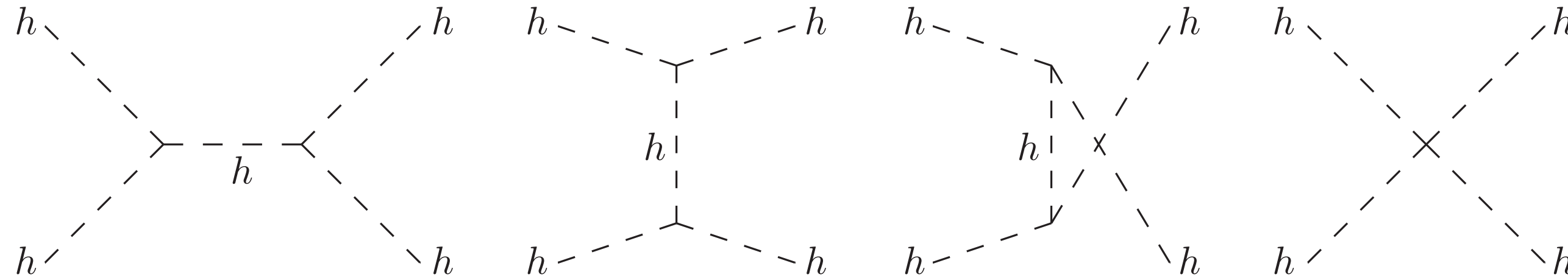
[ATLAS, 2411.02040]



[CMS-PAS-HIG-24-012 & CMS-PAS-HIG-24-015]

Perturbative Unitarity (in 5 easy steps)

- Experiments show limits on (κ_3, κ_4) like these:
- including constraint from **perturbative unitarity**,
- 1. Consider $hh \rightarrow hh$ scattering:



$$\mathcal{M} = -\lambda_3^2 \left(\frac{1}{s - m_h^2} + \frac{1}{t - m_h^2} + \frac{1}{u - m_h^2} \right) - \lambda_4$$

- via the matrix element $\mathcal{M} \sim \langle f | T | i \rangle \rightarrow T$ the interacting part of the S-matrix: $S = 1 + iT$, requiring $S^\dagger S = 1$, i.e. **unitarity**.

- 2. Project into partial waves $a_{fi}^J \propto \int d(\cos \theta) \overbrace{d_{\mu_i \mu_f}^J(\theta)}^{\text{Wigner-}d \text{ function}} \langle f | T | i \rangle$ of total angular momentum J ,

- 3. consider same-helicity state and $J=0$ (higher typically smaller amplitudes) $\rightarrow d_{00}^0 = 1$.

Perturbative Unitarity (in 5 easy steps)

4. If real amplitude, then require: $\left| \text{Re}(a_{ii}^J) \right| \leq \frac{1}{2}$, at all energies.

(in reality: issues arise before saturating this $\Rightarrow$ **bound is conservative**).

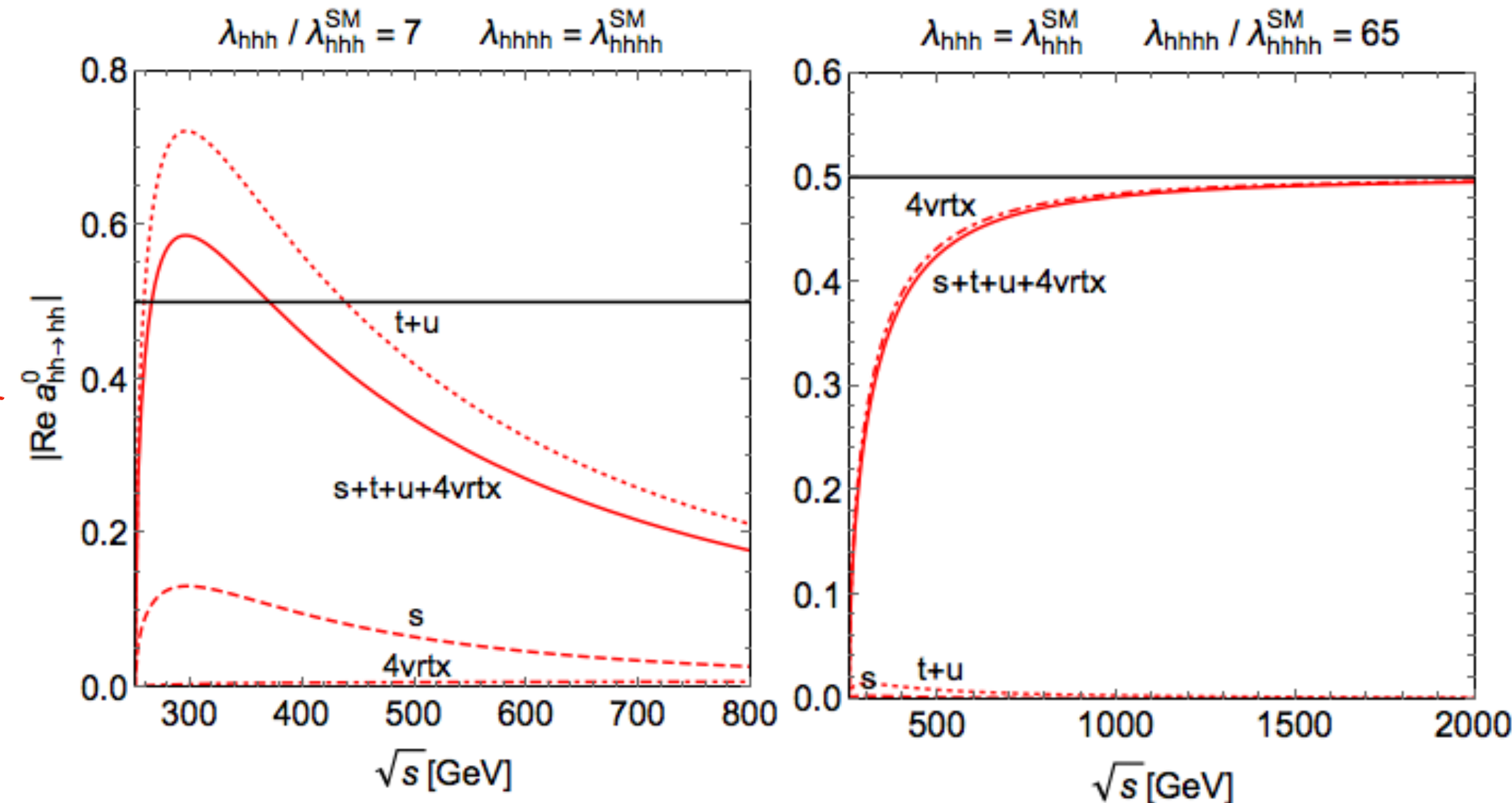
5. Apply to $hh \rightarrow hh$: one finds that κ_3 bound is **maximized at low energy**, κ_4 at **high energy**:

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[Fuks, AP, Tetlalmatzi-Xolocotzi, 2509.16364]

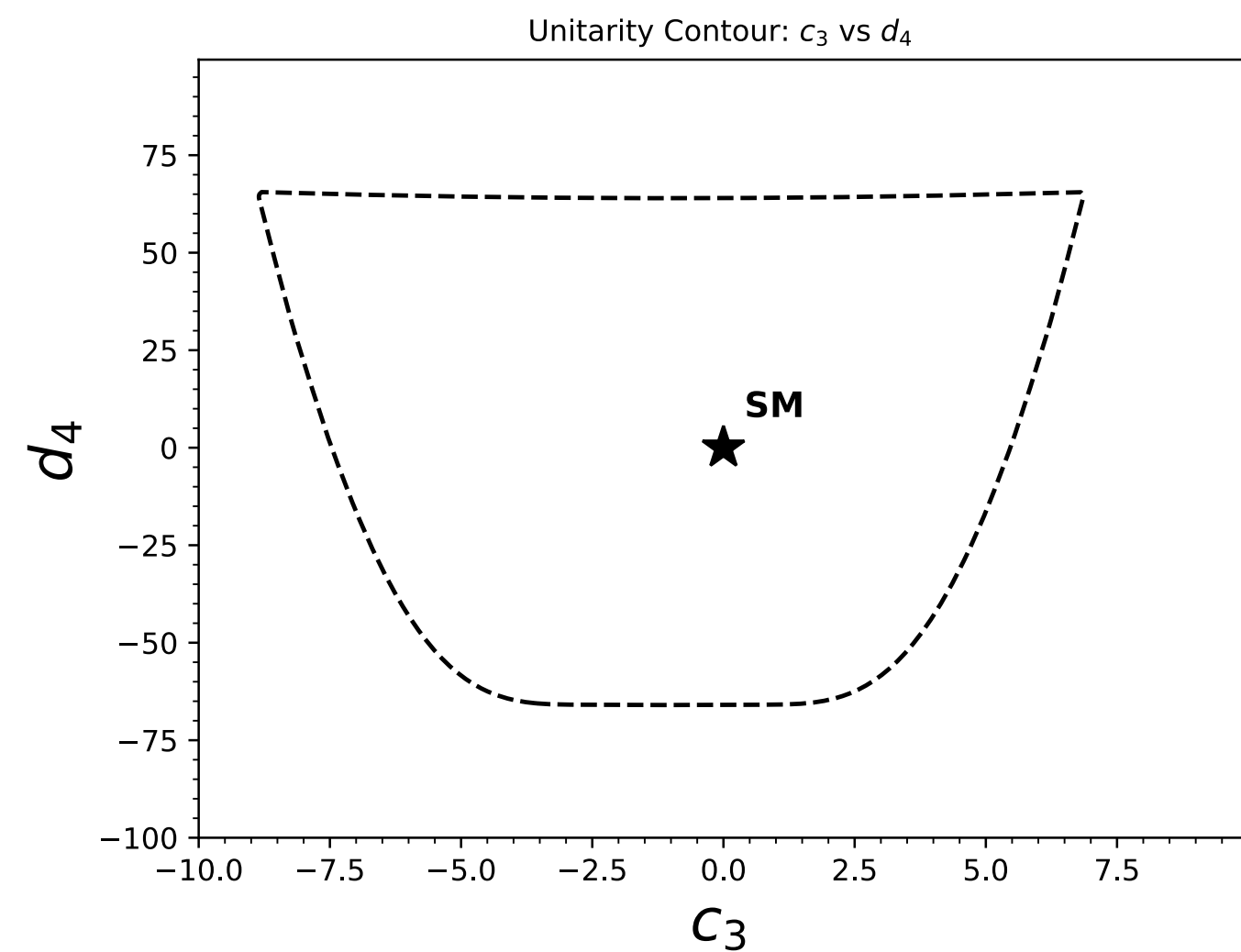
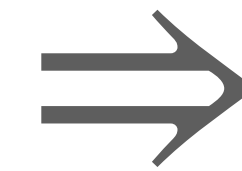
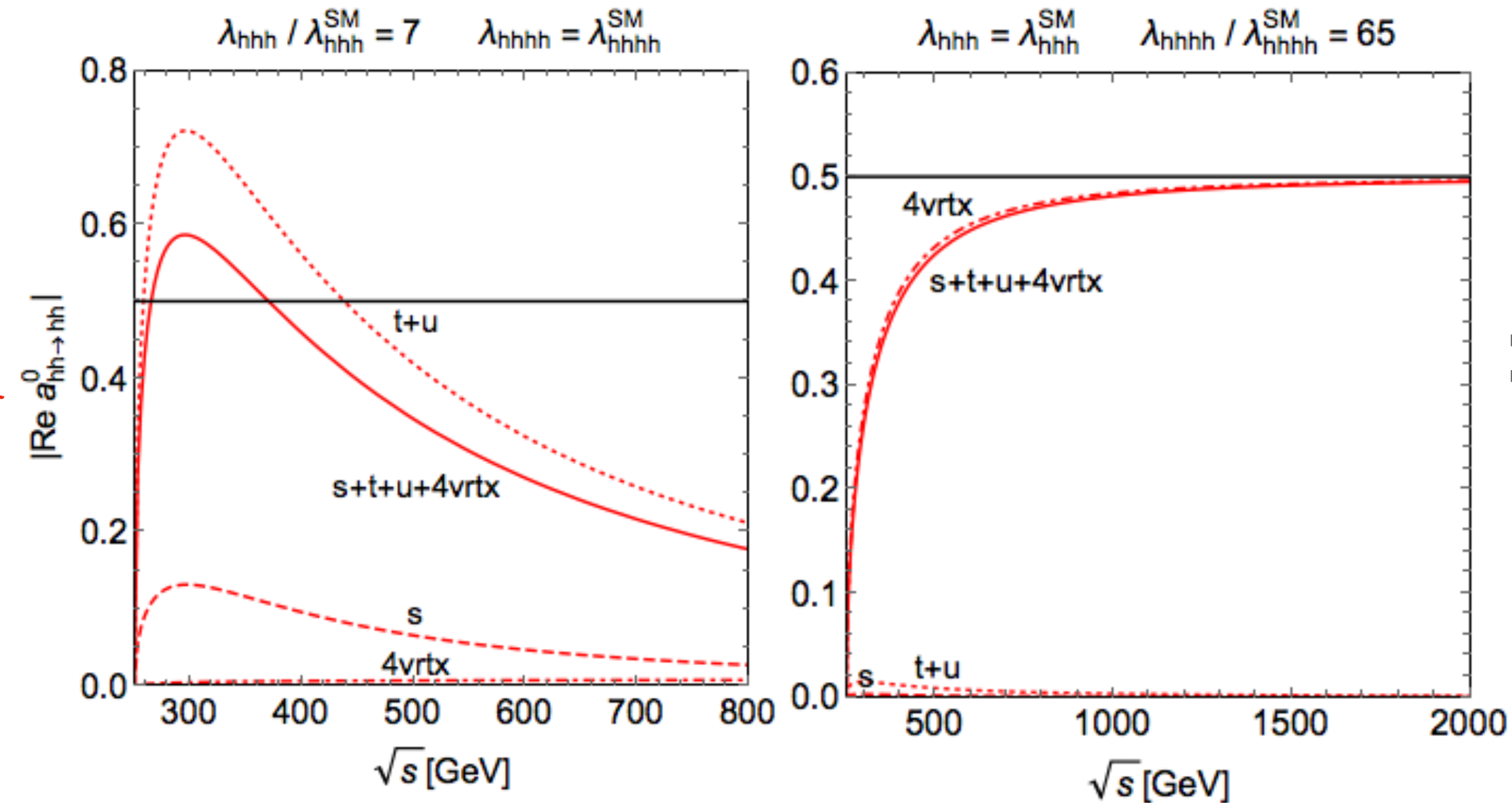
[Jacob, Wick, Annals Phys. 7, 404 (1959),
Di Luzio, Kamenik, Nardecchia, 1604.05746
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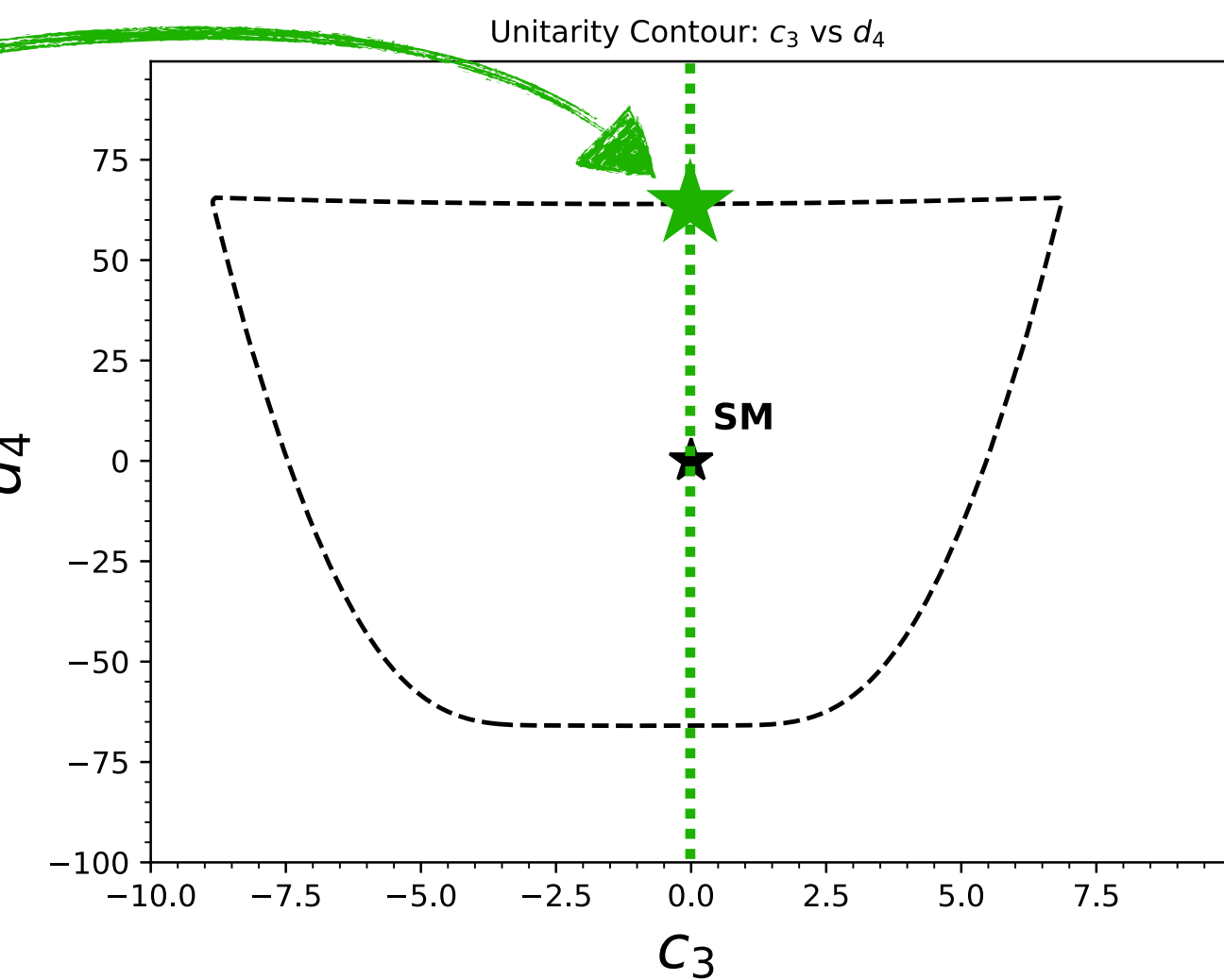
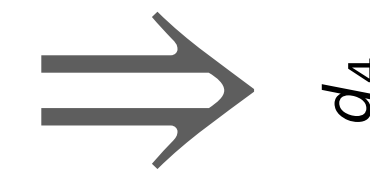
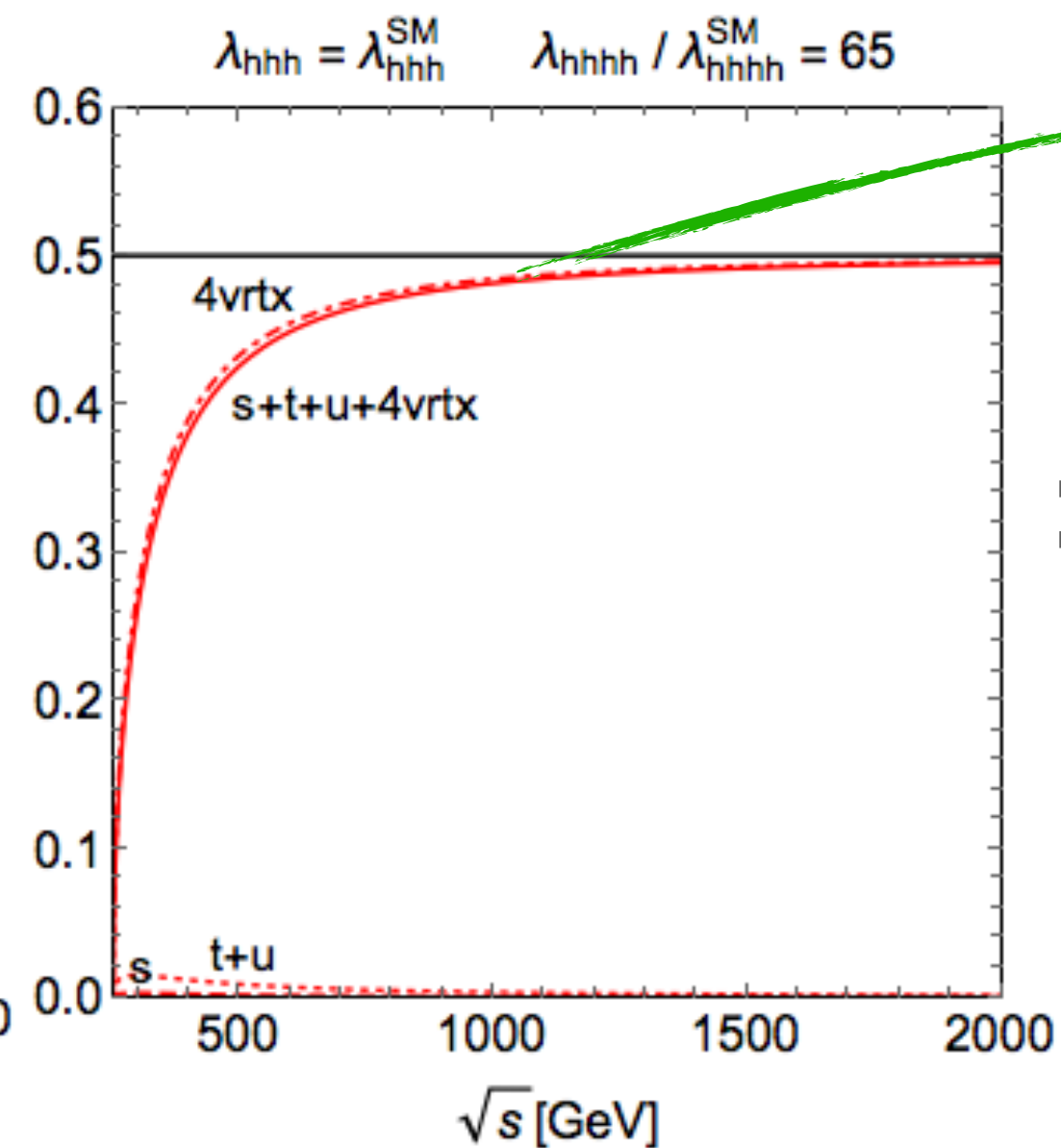
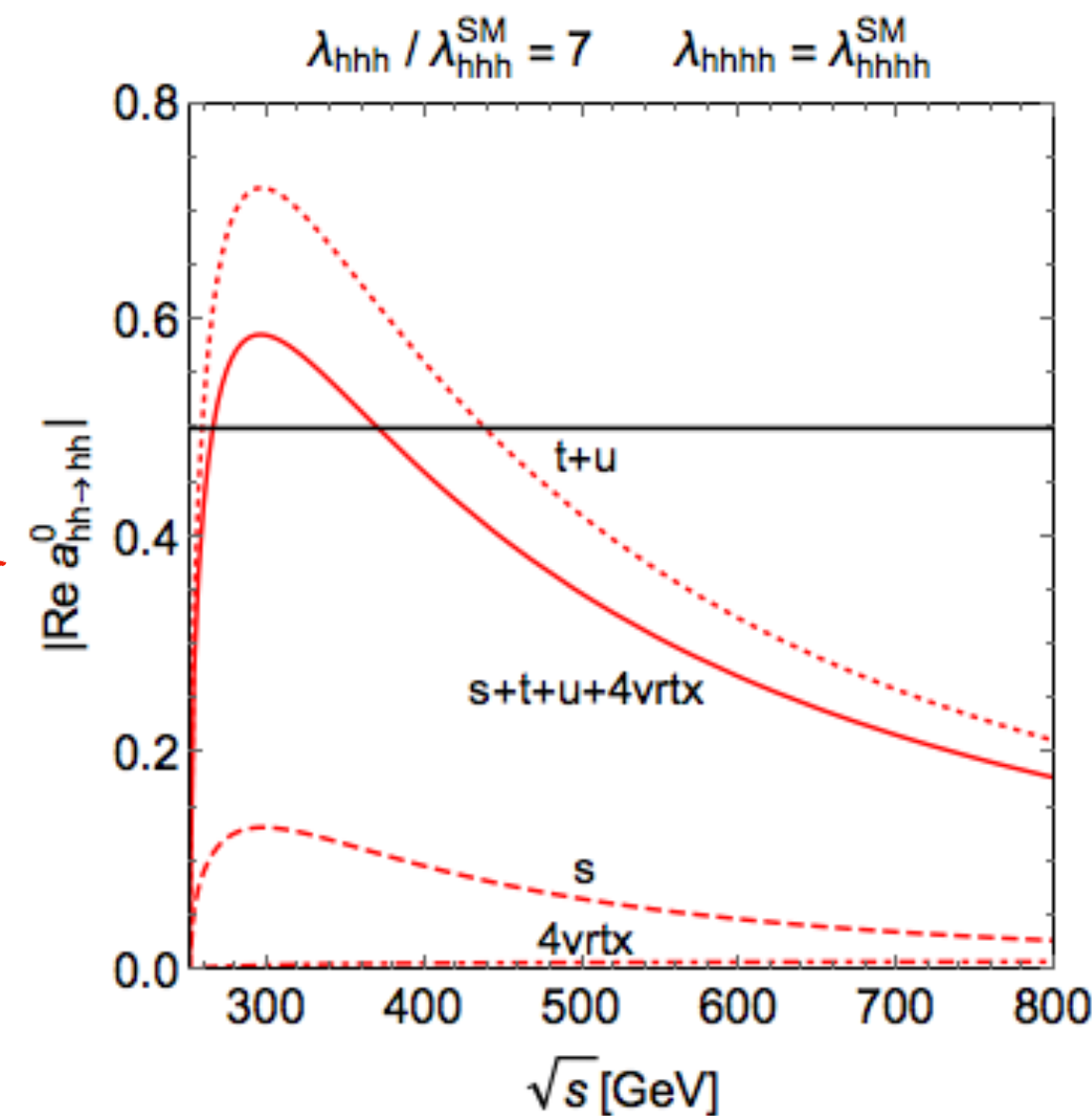
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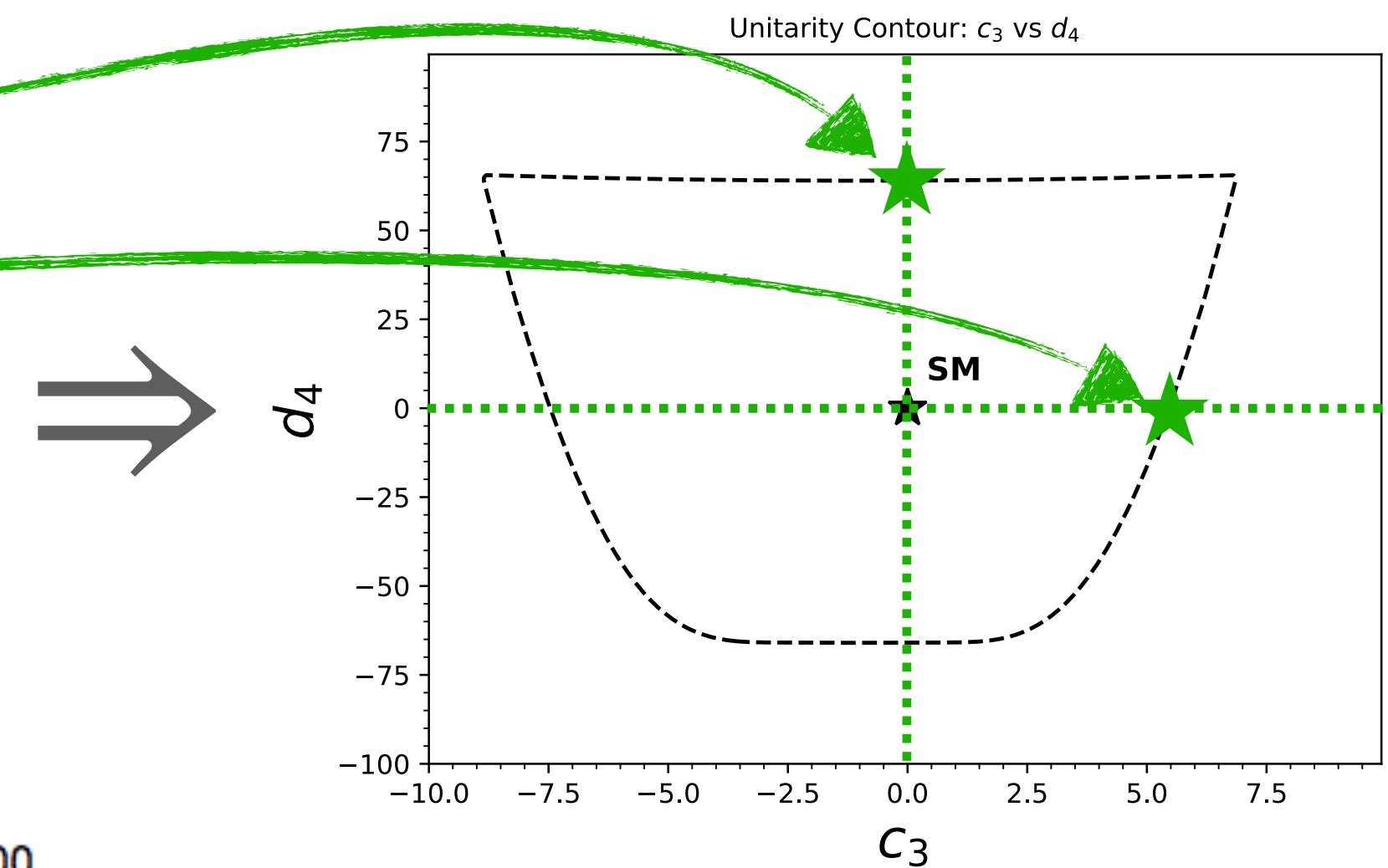
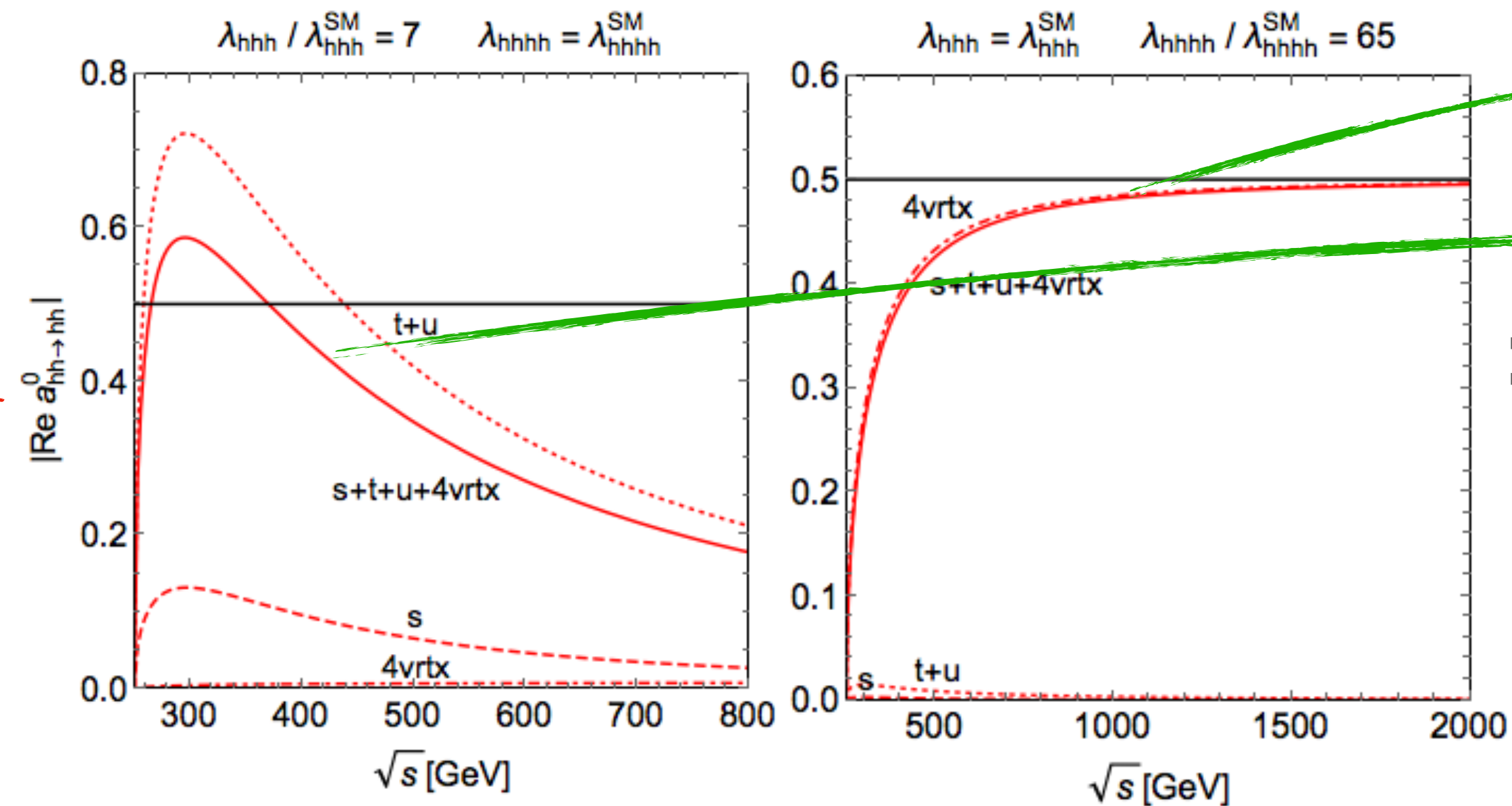
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Return of the Anomalous Couplings

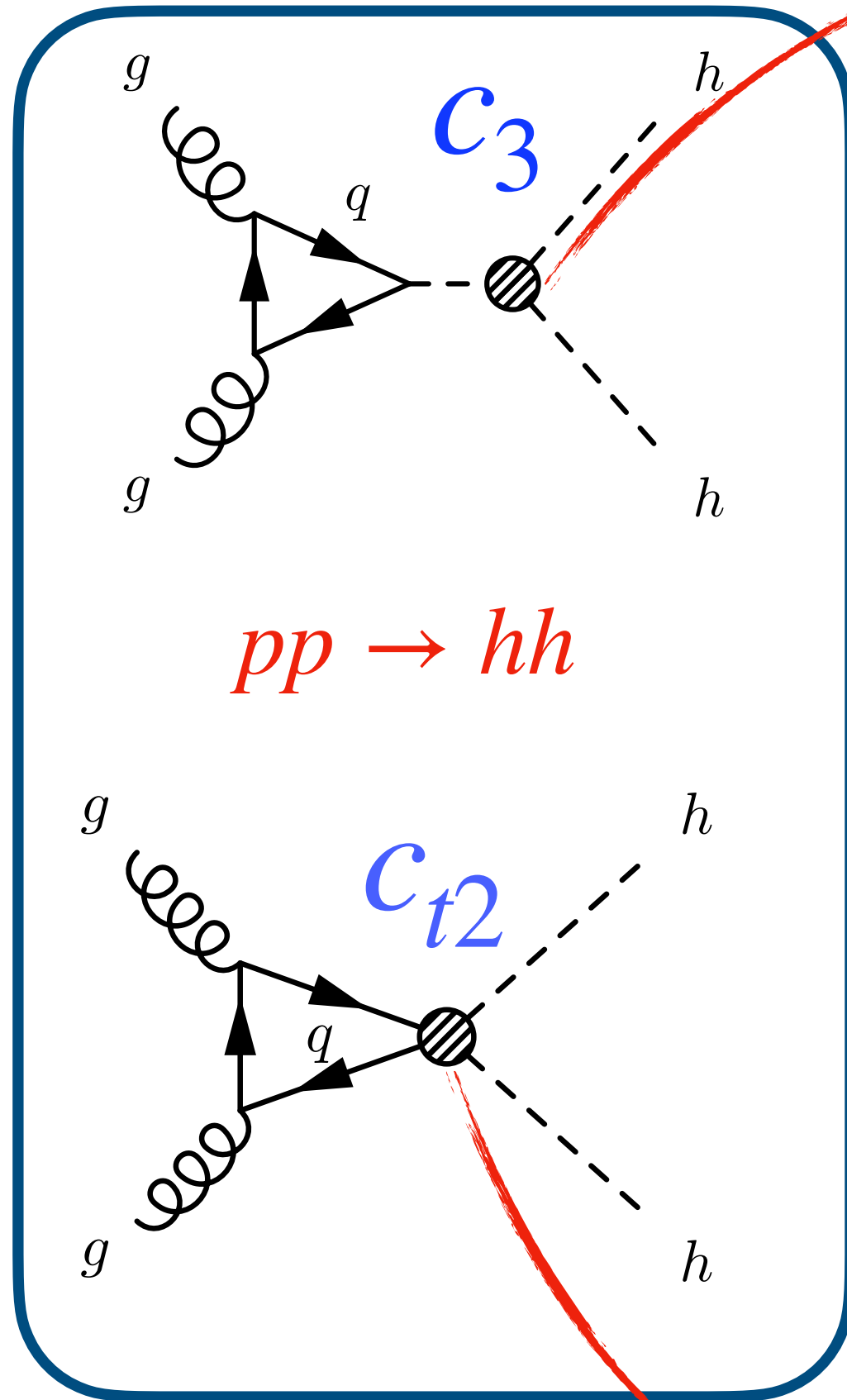
👒 Anomalous couplings can enhance *hhh*! e.g. $\kappa_3 \neq 1, \kappa_4 \neq 1$, but also others!

$$\begin{aligned} \mathcal{L}_{\text{PhenoExp}} \supset & -\lambda_{\text{SM}} v (1+c_3) h^3 - \frac{\lambda_{\text{SM}}}{4} (1+d_4) h^4 \\ & + \frac{\alpha_s}{12\pi} \left(c_{g1} \frac{h}{v} - c_{g2} \frac{h^2}{2v^2} \right) G_{\mu\nu}^a G_a^{\mu\nu} \\ & - \left[\frac{m_t}{v} (1+c_{t1}) \bar{t}_L t_R h + \frac{m_b}{v} (1+c_{b1}) \bar{b}_L b_R h + \text{h.c.} \right] \\ & - \left[\frac{m_t}{v^2} c_{t2} \bar{t}_L t_R h^2 + \frac{m_b}{v^2} c_{b2} \bar{b}_L b_R h^2 + \text{h.c.} \right] \\ & - \left[\frac{m_t}{v^3} \left(\frac{c_{t3}}{2} \right) \bar{t}_L t_R h^3 + \frac{m_b}{v^3} \left(\frac{c_{b3}}{2} \right) \bar{b}_L b_R h^3 + \text{h.c.} \right] \end{aligned}$$

[AP, Tetlalmatzi-Xolocotzi, 2312.13562]

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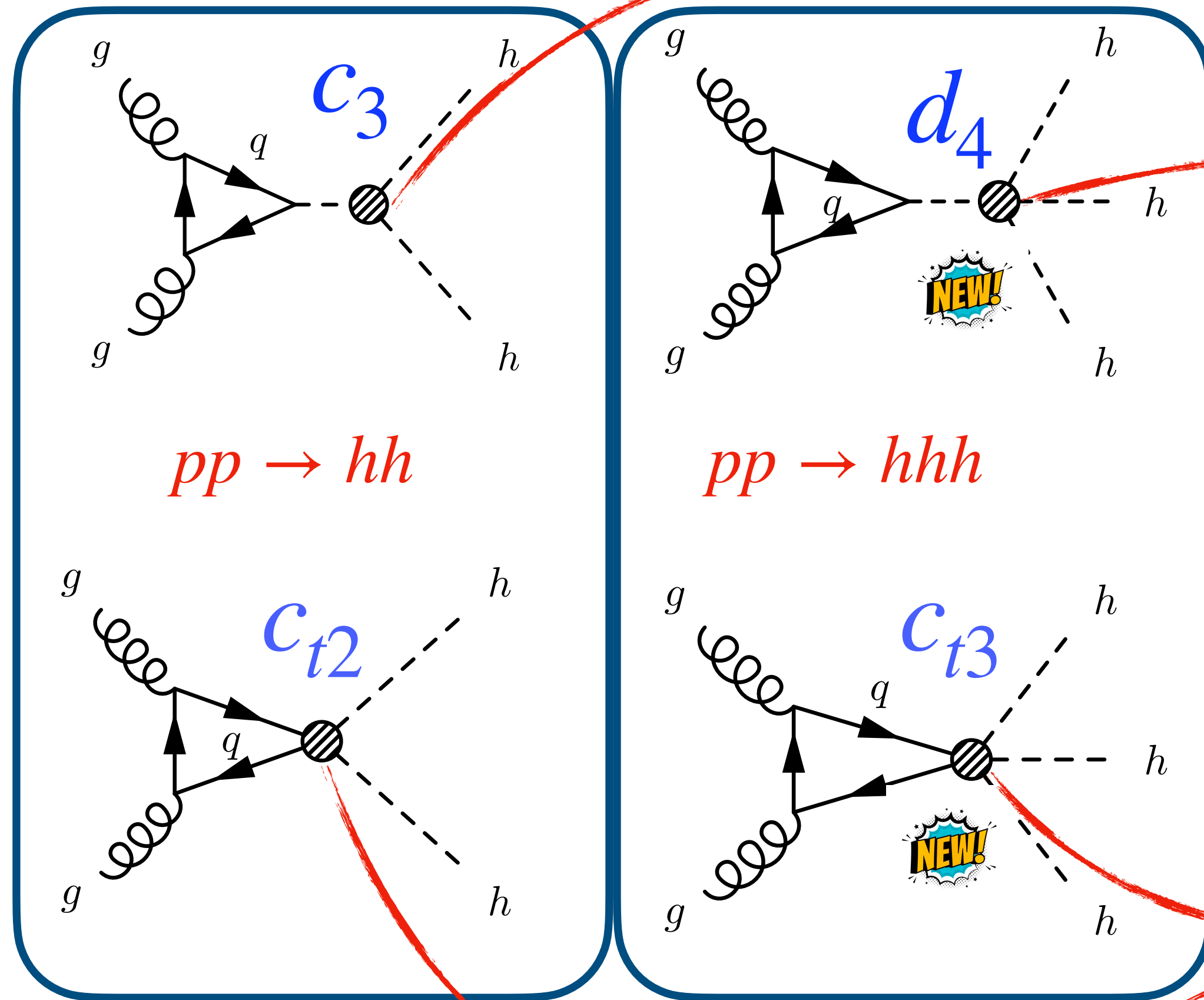
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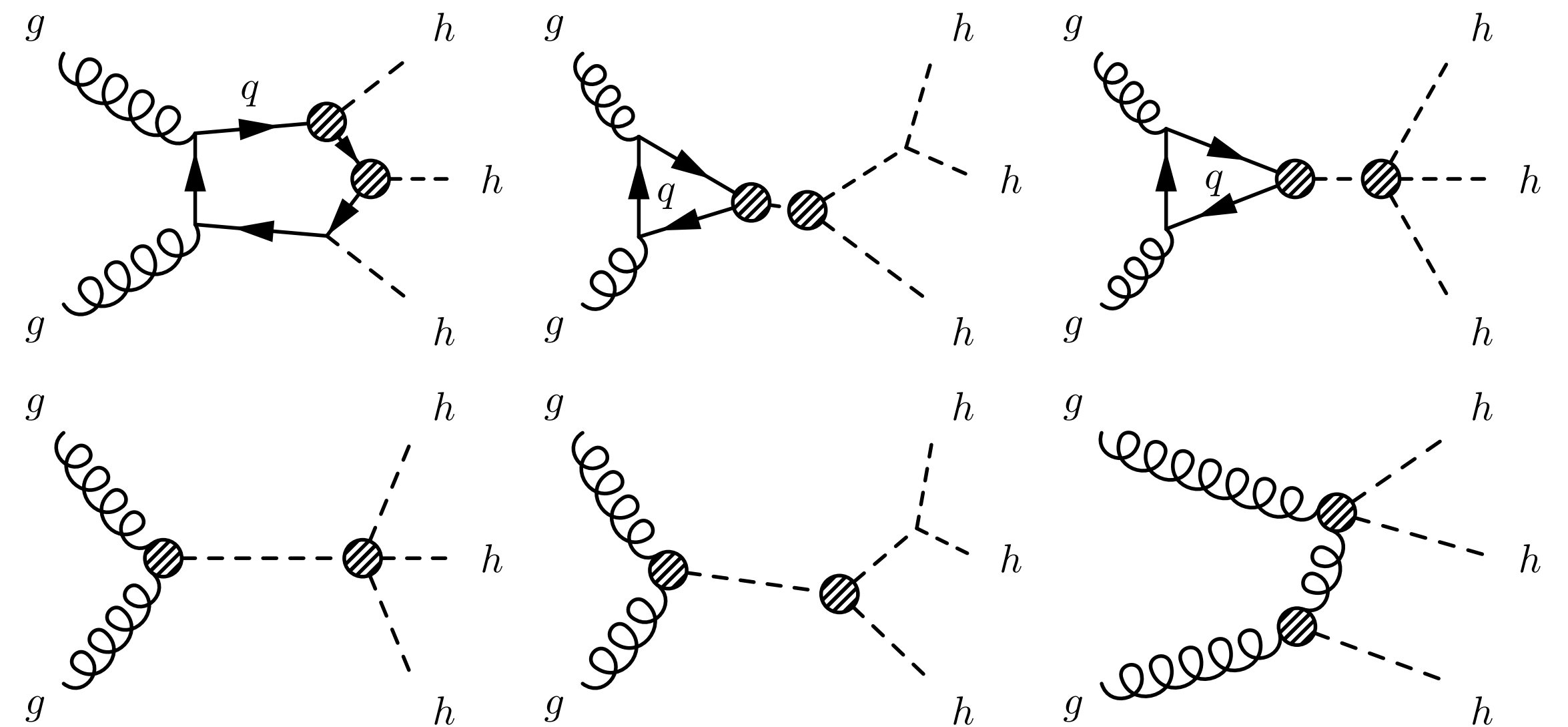
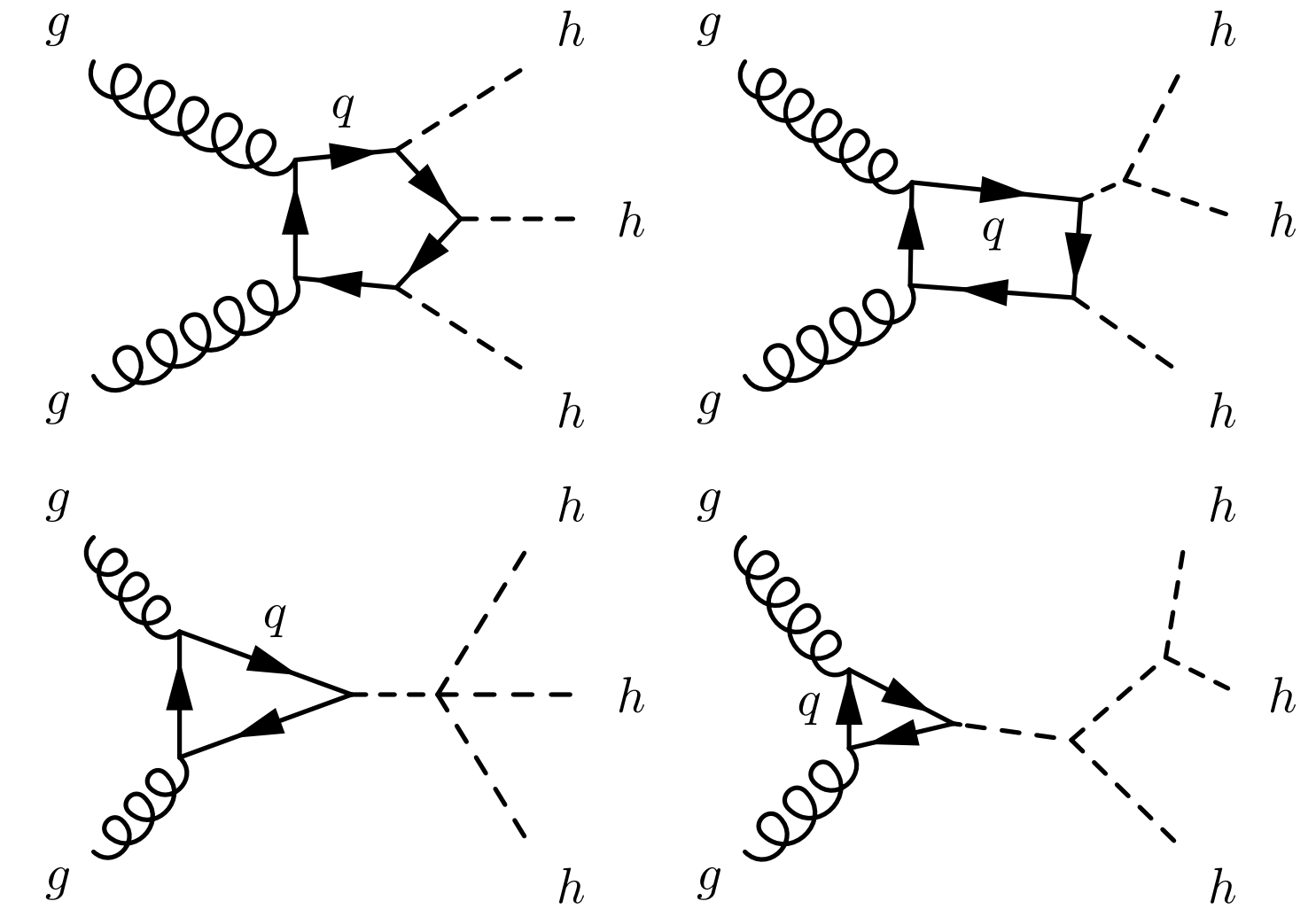
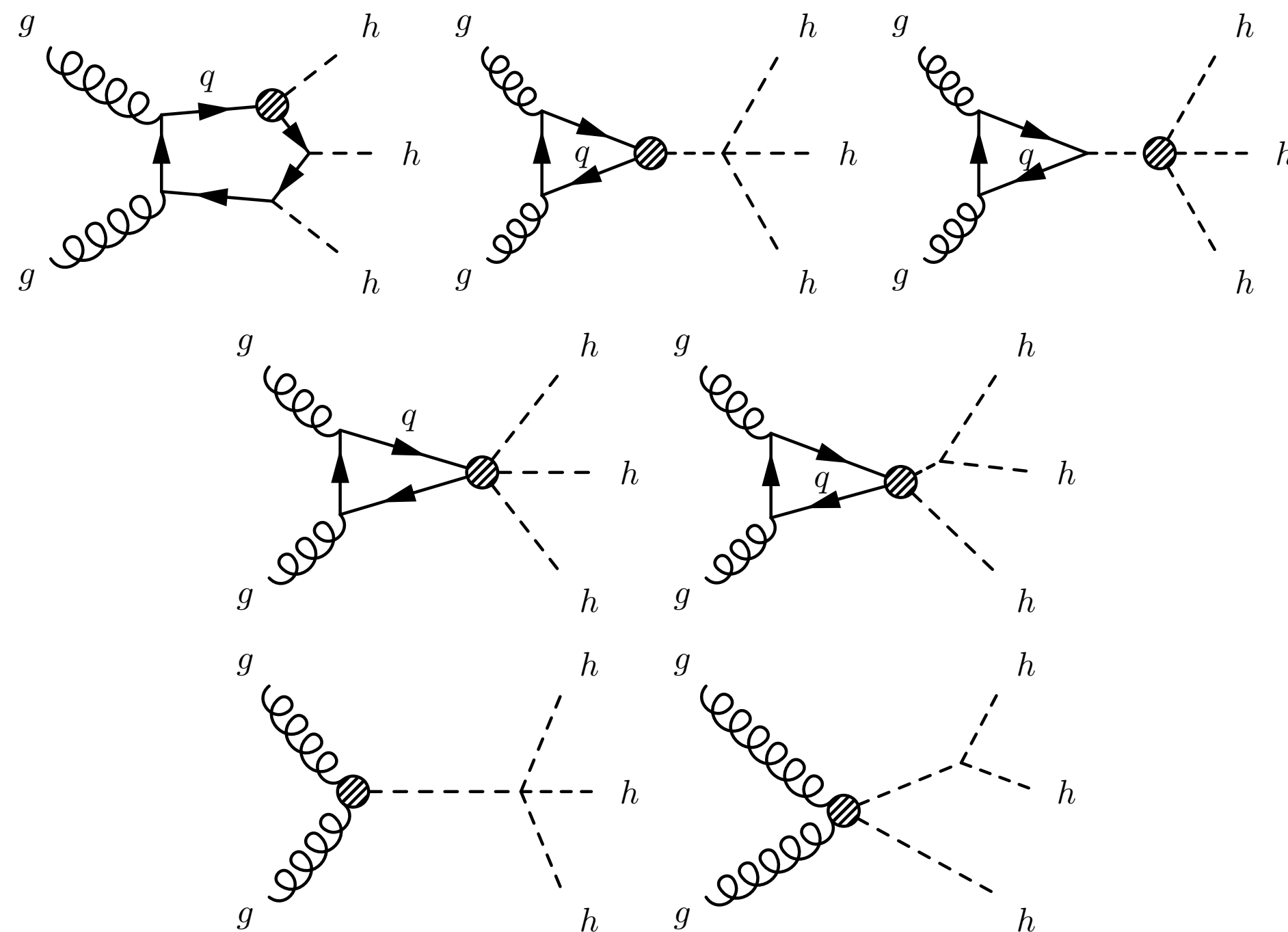
[AP, Tetlalmatzi-Xolocotzi, 2312.13562]

Return of the Anomalous Couplings

MadGraph5_aMC model at:
https://gitlab.com/apapaeefs/multihiggs_loop_sm

***Includes description of interference effects**
& allows for ME-squared truncation!

[AP, Tetlalmatzi-Xolocotzi, 2312.13562]



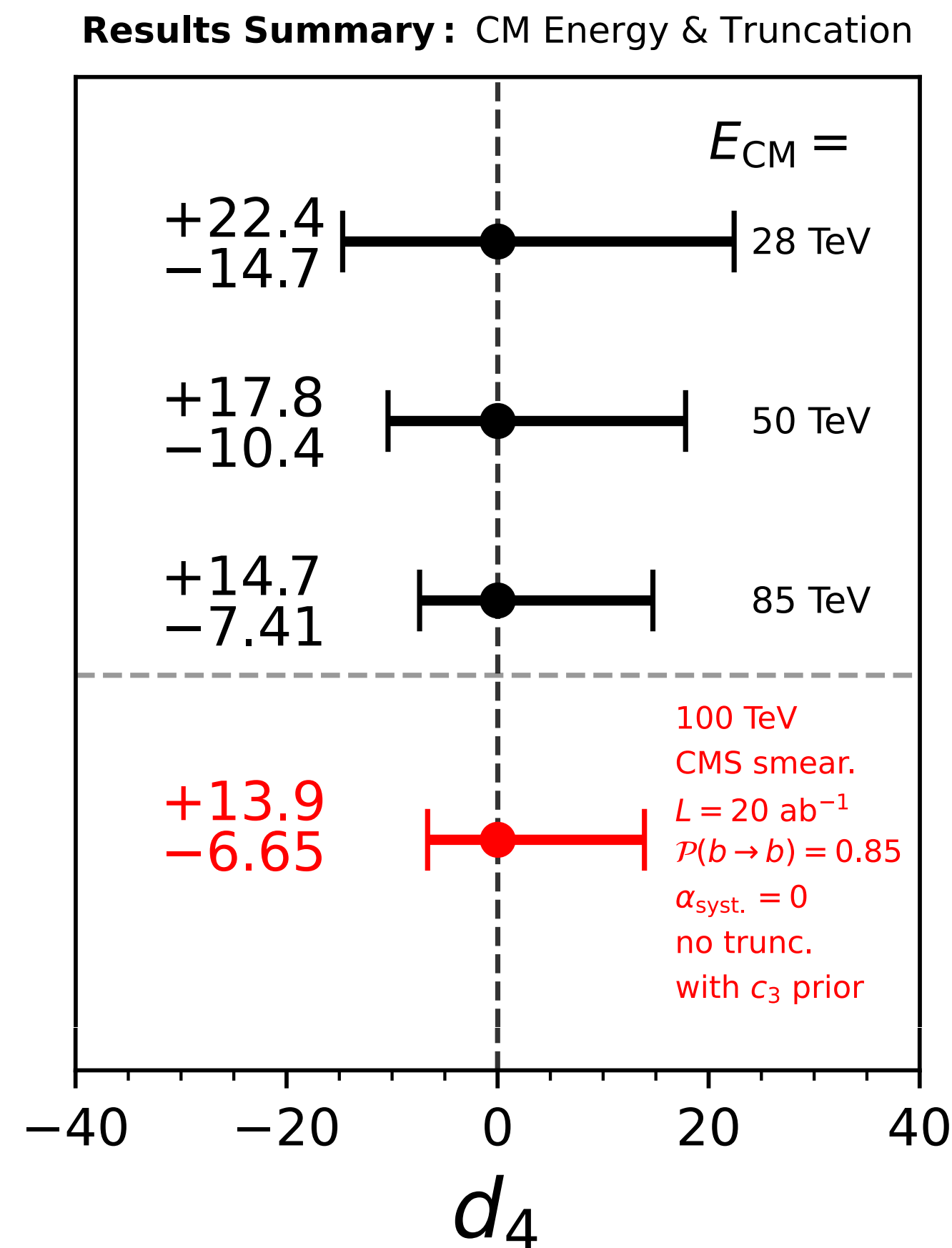
QUARTIC FUTURE PROSPECTS



Future Hadron Colliders

👒 Proton colliders: [Fuks, AP, Tetlalmatzi-Xolocotzi, 2509.16364]

Six b-jet final state $pp \rightarrow hhh \rightarrow (b\bar{b})(b\bar{b})(b\bar{b})$:



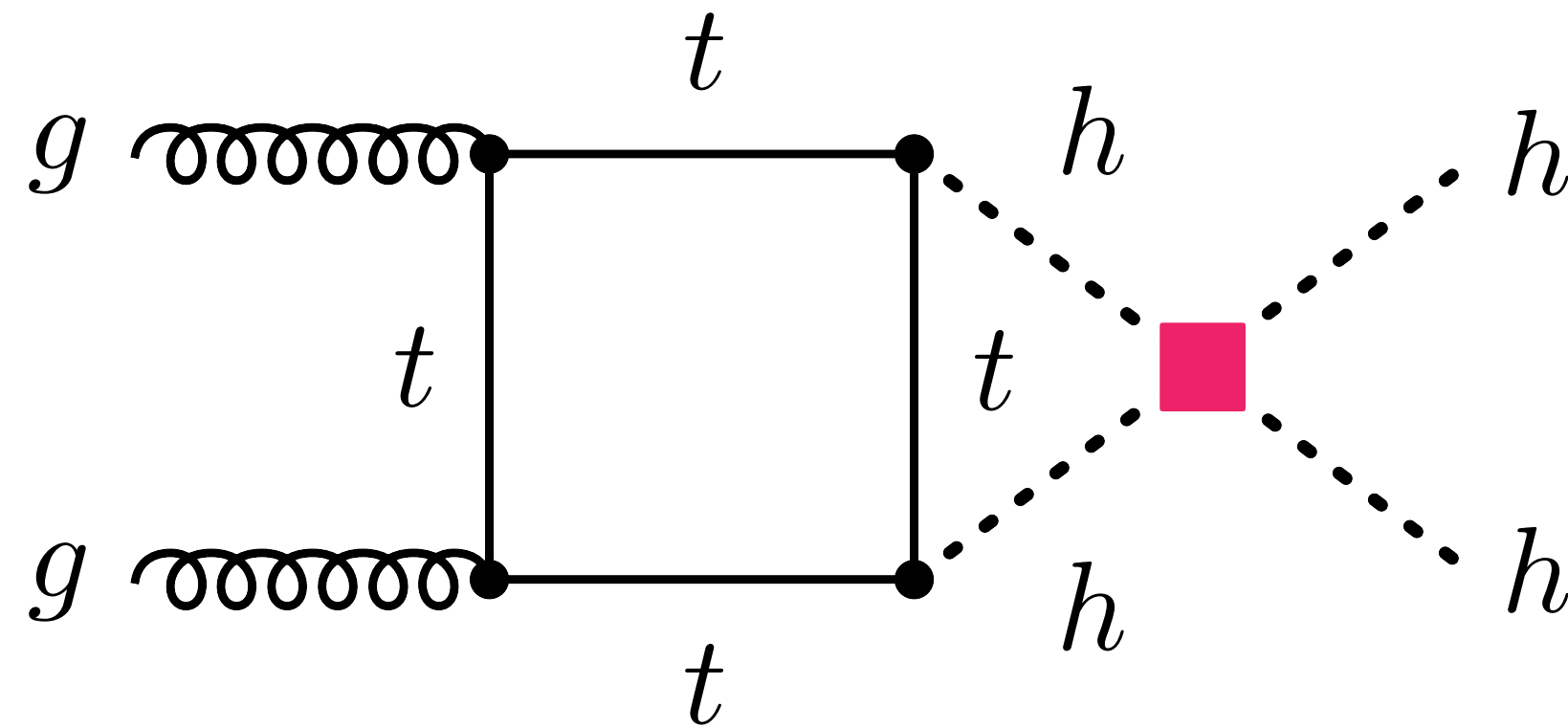
pp collider energy variation
(85 $\rightarrow$ 50 $\rightarrow$ 28 TeV)

hhh 100 TeV / 20 ab^{-1} + combination with
future trilinear constraint,
assuming: $\delta c_3 = 5 \%$

Recall: $\lambda_4/\lambda_4^{\text{SM}} = 1 + d_4 = \kappa_4$

Indirect Constraints on the Quartic

🎩 Obtain indirect constraints on Higgs boson's quartic (λ_4) via hh production, e.g. through electroweak corrections:



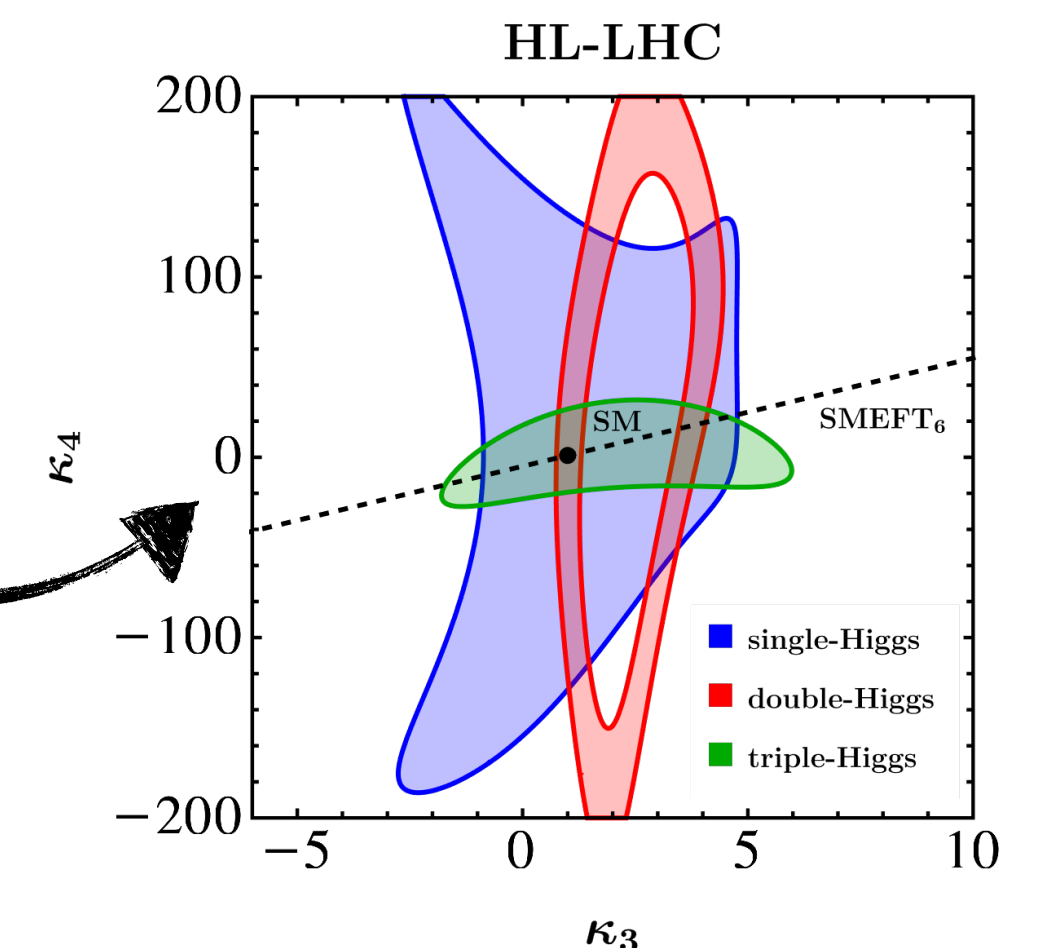
[Bizoń, Haisch, Rottoli, 1810.04665, Li Si, Wang, Zhang, Zhao, 2407.14716
Heinrich, Jones, Kerner, Stone, Vesten, 2407.04653]

$$\lambda_4/\lambda_4^{\text{SM}} \sim [-5, 14]$$

[For $\lambda_3 = \lambda_3^{\text{SM}}$]

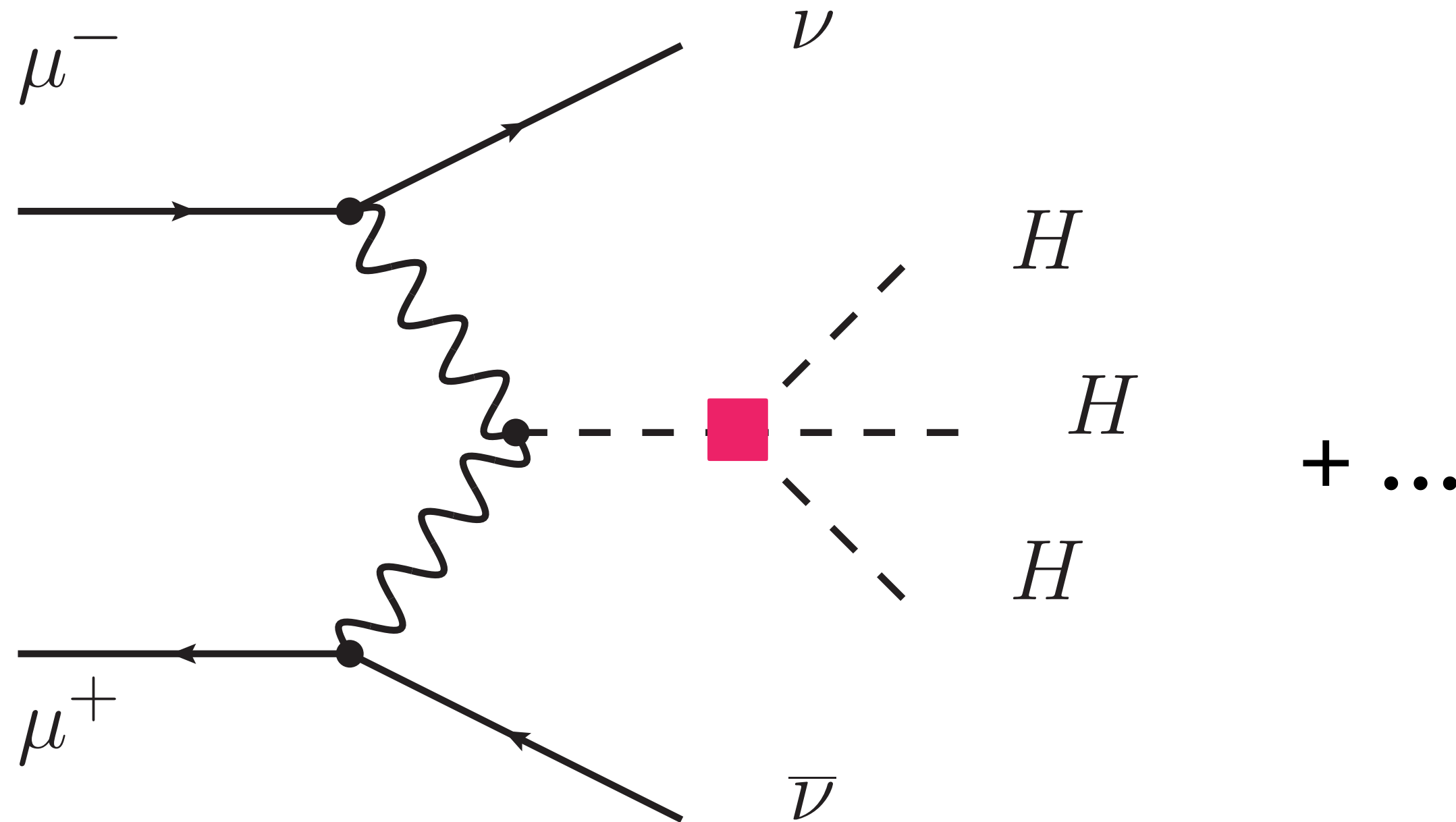
[But note: Full EW corrections to hh are only available for the SM!]

🎩 **See also:** [Haisch, Sankar, Zanderighi, 2505.20463] for single Higgs constraints.



Lepton Colliders

- 🎩 FCC-ee: insufficient energy for Zhh production (allowing **indirect** constraints).
- 🎩 Muon Collider, through VBF:



e.g. for $\sqrt{s} = 14 \text{ TeV}$, 33 ab^{-1} : $\lambda_4/\lambda_4^{\text{SM}} \sim [0.65, 1.55]$ [$\lambda_3 = \lambda_3^{\text{SM}}$]

 [Chiesa, Maltoni, Mantani, Mele, Piccinini, Zhao, 2003.13628]

(Rough) Summary of Self-Coupling Prospects



	HL-LHC	FCC-ee	FCC-hh	Muon Coll.
Trilinear: $\lambda_3/\lambda_3^{\text{SM}} = 1 + c_3 = \kappa_3$	~50%	~20-30%	~5%	~5-30%
Quartic: $\lambda_4/\lambda_4^{\text{SM}} = 1 + d_4 = \kappa_4$	$\mathcal{O}(\textcolor{red}{10}-100)$ [ATL-PHYS-PUB-2025-018 CMS-HIG-25-002]	???	$\mathcal{O}(\textcolor{red}{1}-10)$ [Fuks, AP, Tetlalmatzi-Xolocotzi, 2509.16364]	$\sim 50-100\%$ [Chiesa, Maltoni, Mantani, Mele, Piccinini, Zhao, 2003.13628]

PART IV: EXTENDED SCALAR SECTORS

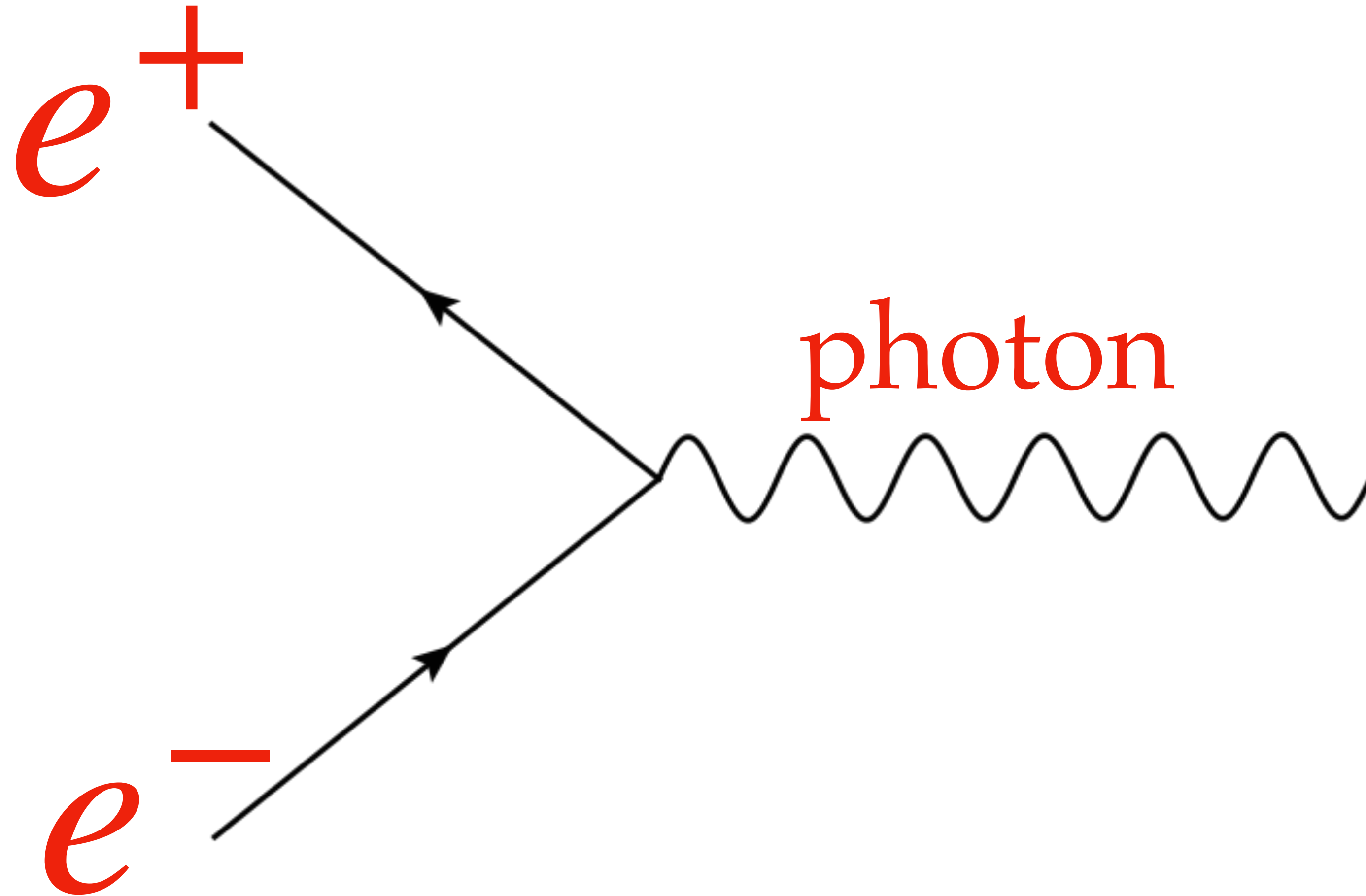


Important Open Question: Why does matter exist?

[or: Why is there so much more matter than anti-matter?]

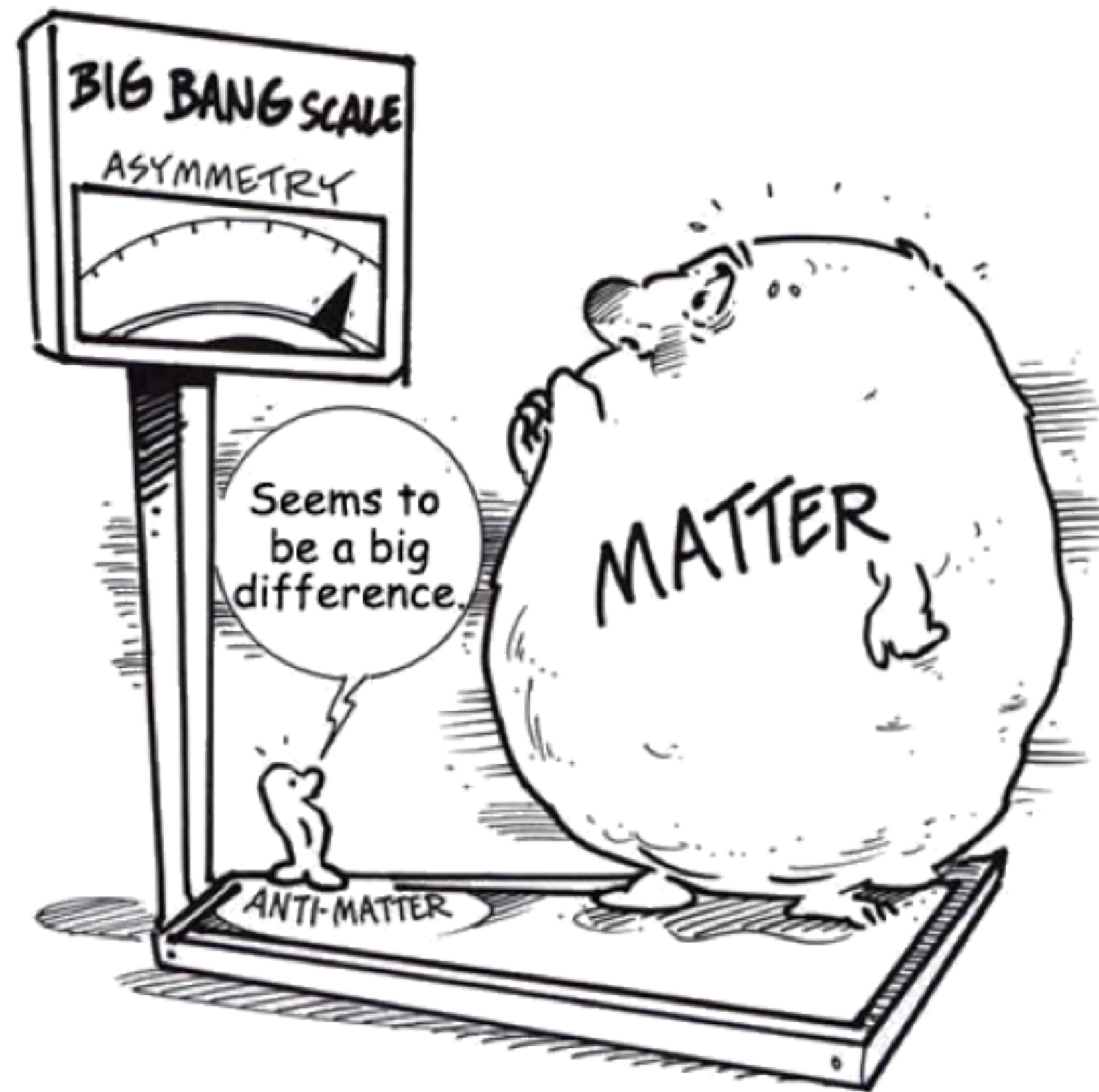
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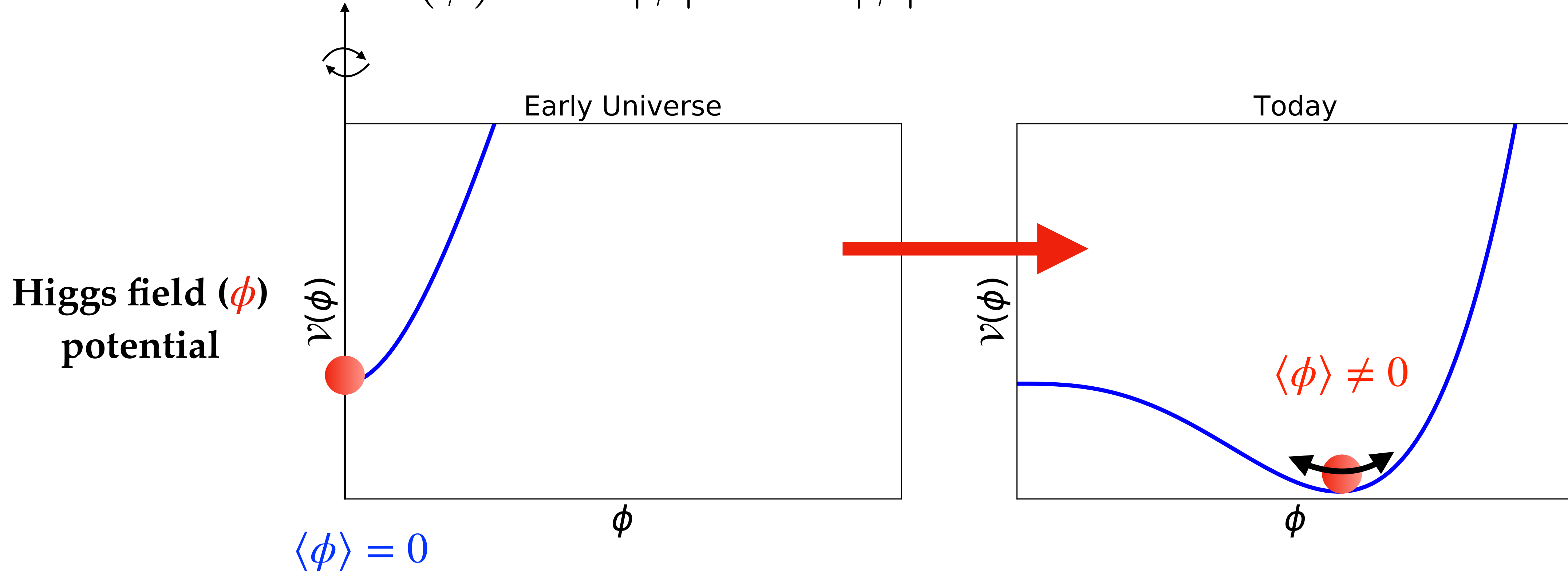
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→ the “**Matter-Anti-Matter Asymmetry**”.

Consider Again: Symmetry Breaking in the SM

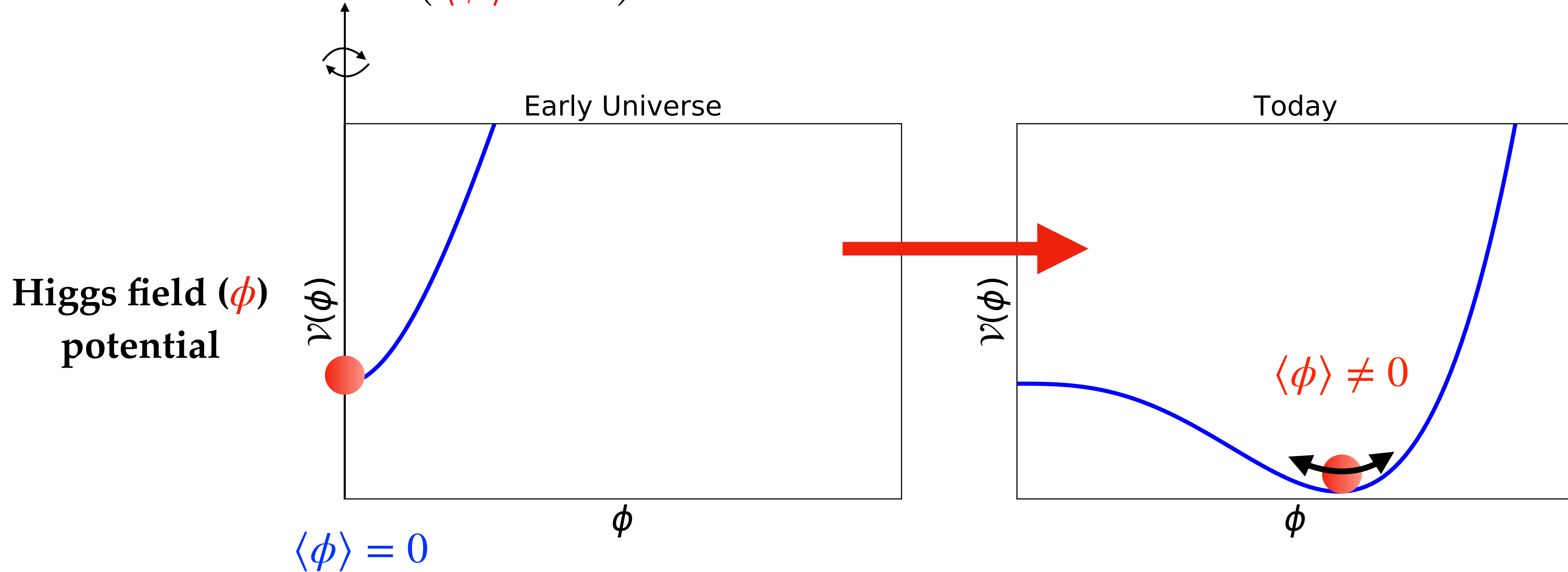
$$\mathcal{V}(\phi) = \text{●} |\phi|^2 + \text{■} |\phi|^4$$



More Symmetry $\rightarrow$ Less (obvious) Symmetry

Consider Again: Symmetry Breaking in the SM

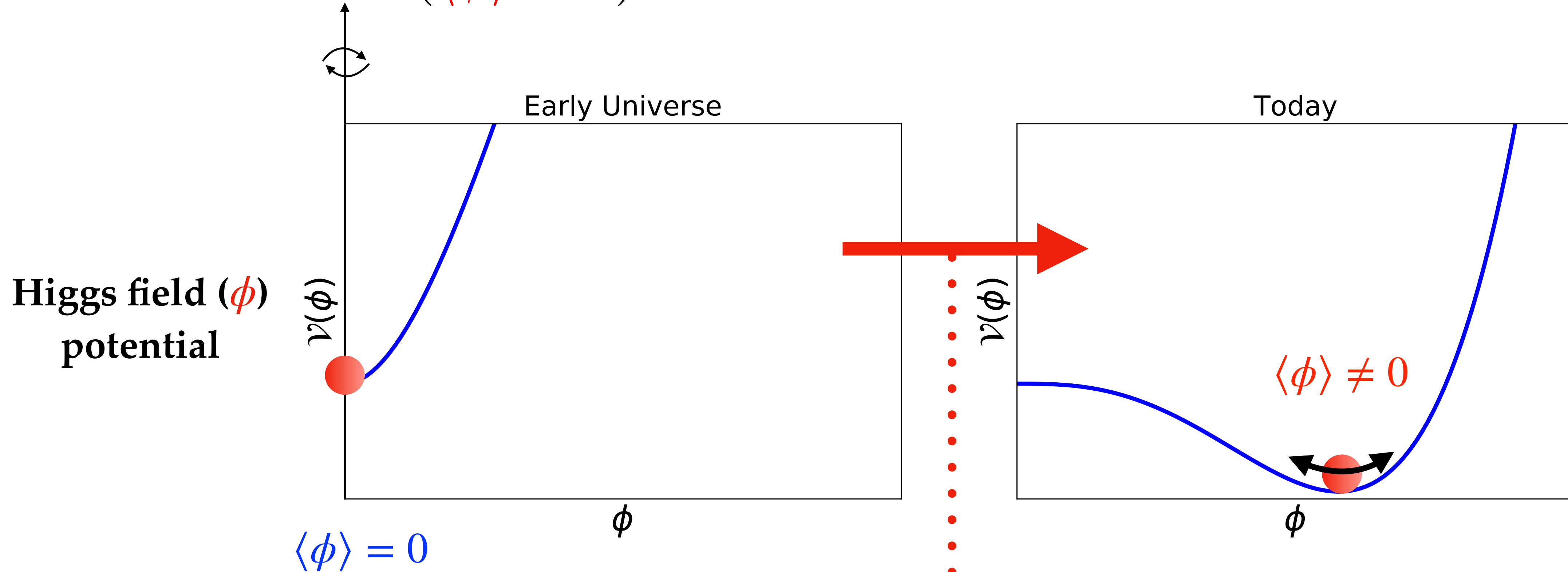
$$\mathcal{V}(\langle\phi\rangle + h) = \text{●} h^2 + \text{▲} h^3 + \text{■} h^4 \rightarrow h \text{ is the Higgs boson}$$



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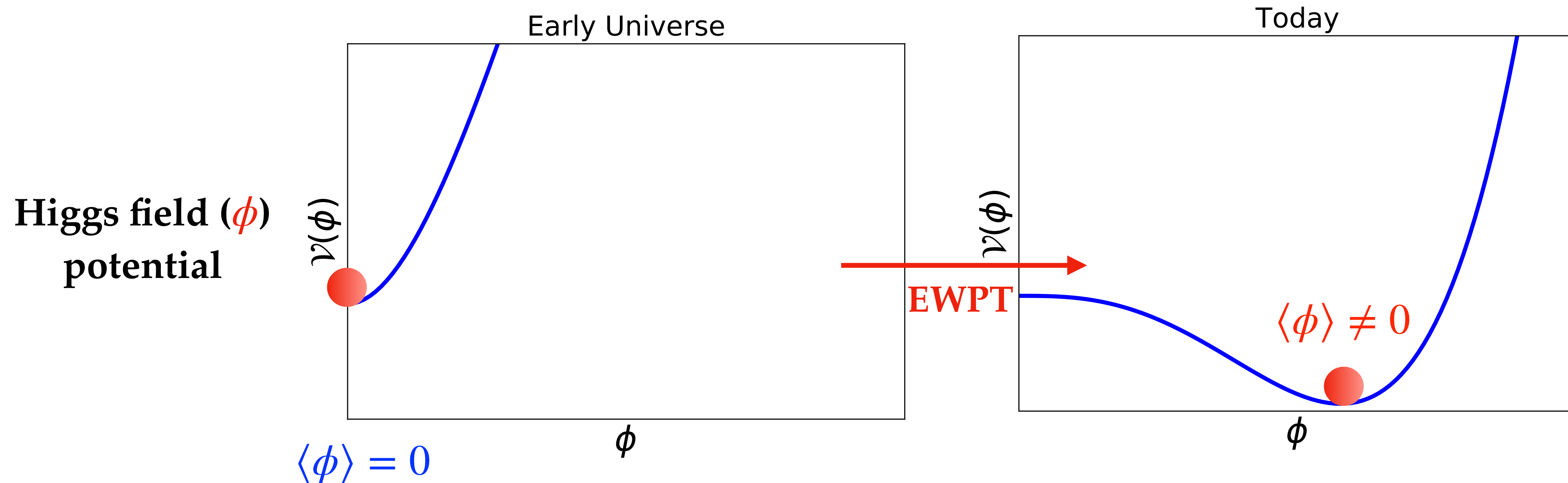
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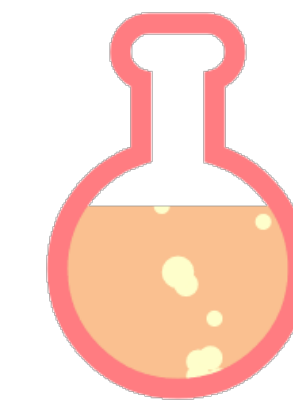
$\equiv$ Electro-Weak Phase Transition (**EWPT**)

Consider Again: Symmetry Breaking in the SM



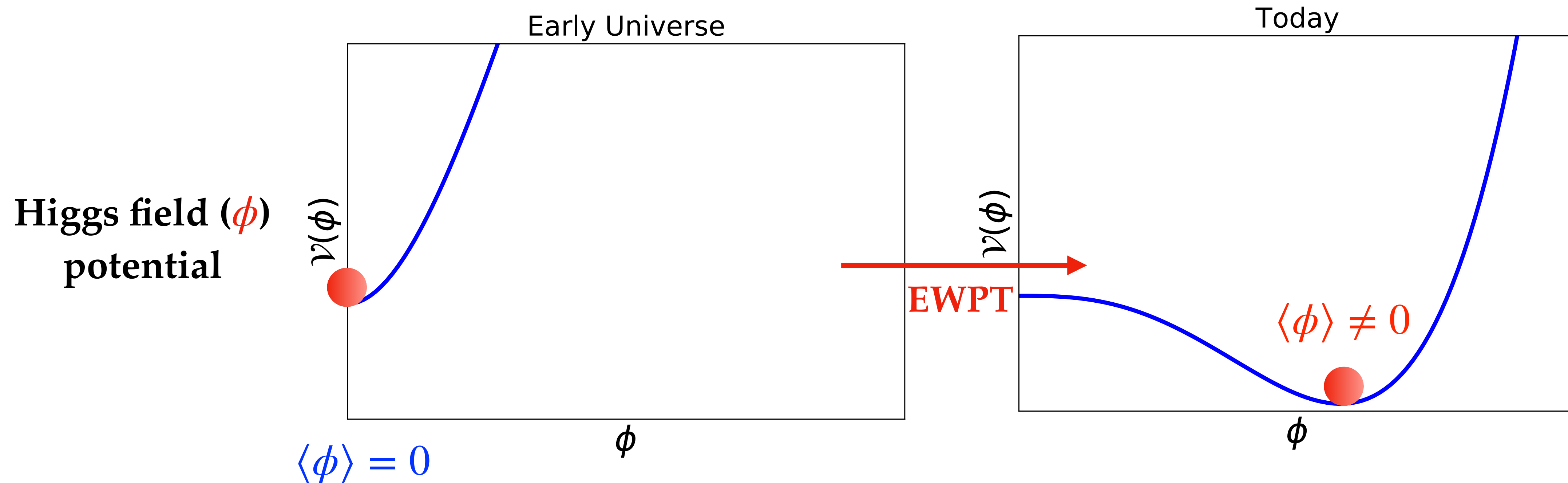
🎩 **Nature** of EWPT → Important open question, e.g. its order:

🎩 A **First-Order transition** (e.g. the boiling of water)?



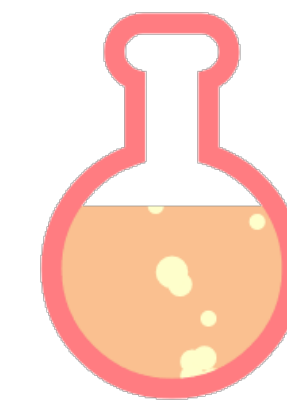
🎩 or a **Second-Order transition** (e.g. the superfluid transition) or **cross-over**?

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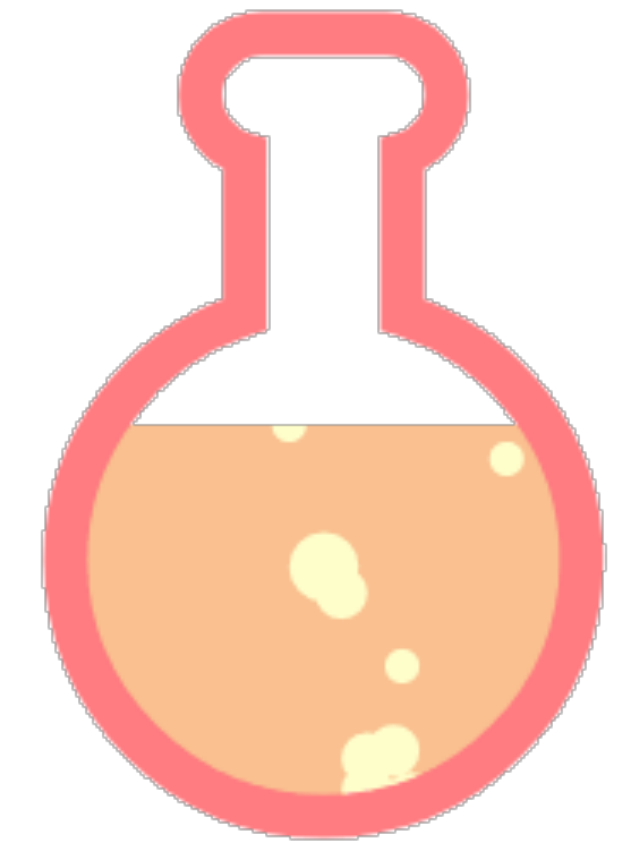
Nature of the Electro-Weak Phase Transition:

→ Clues to the origin of **matter-anti-matter asymmetry**.

👒 Was the asymmetry created **during** the EWPT?

→ “Electro-Weak Baryogenesis” (**EWBG**).

👒 **Pre-requisite: a First-Order transition.** →



👒 *Note:* This **does not** occur in the SM!

[Kajantie, Laine, Rummukainen, Shaposhnikov hep-ph/9605288]

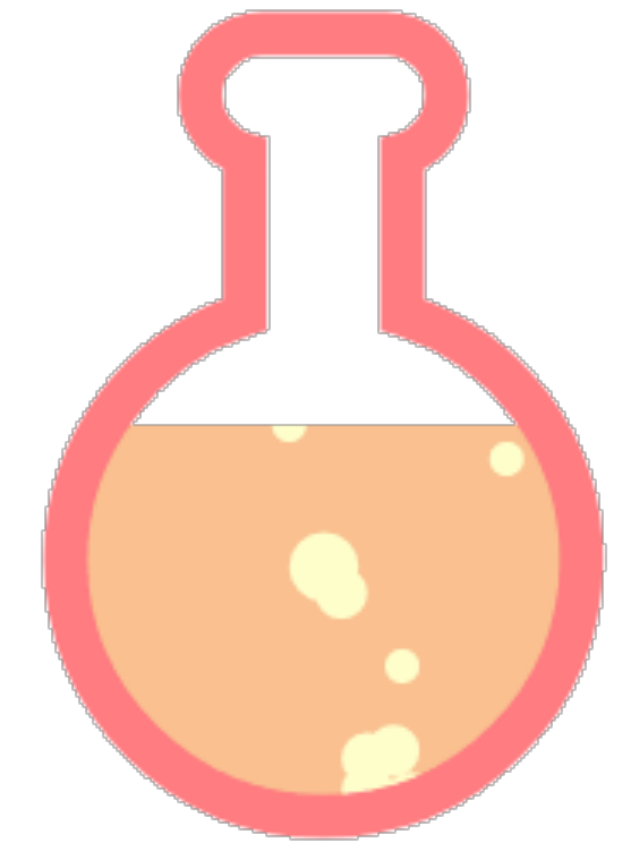
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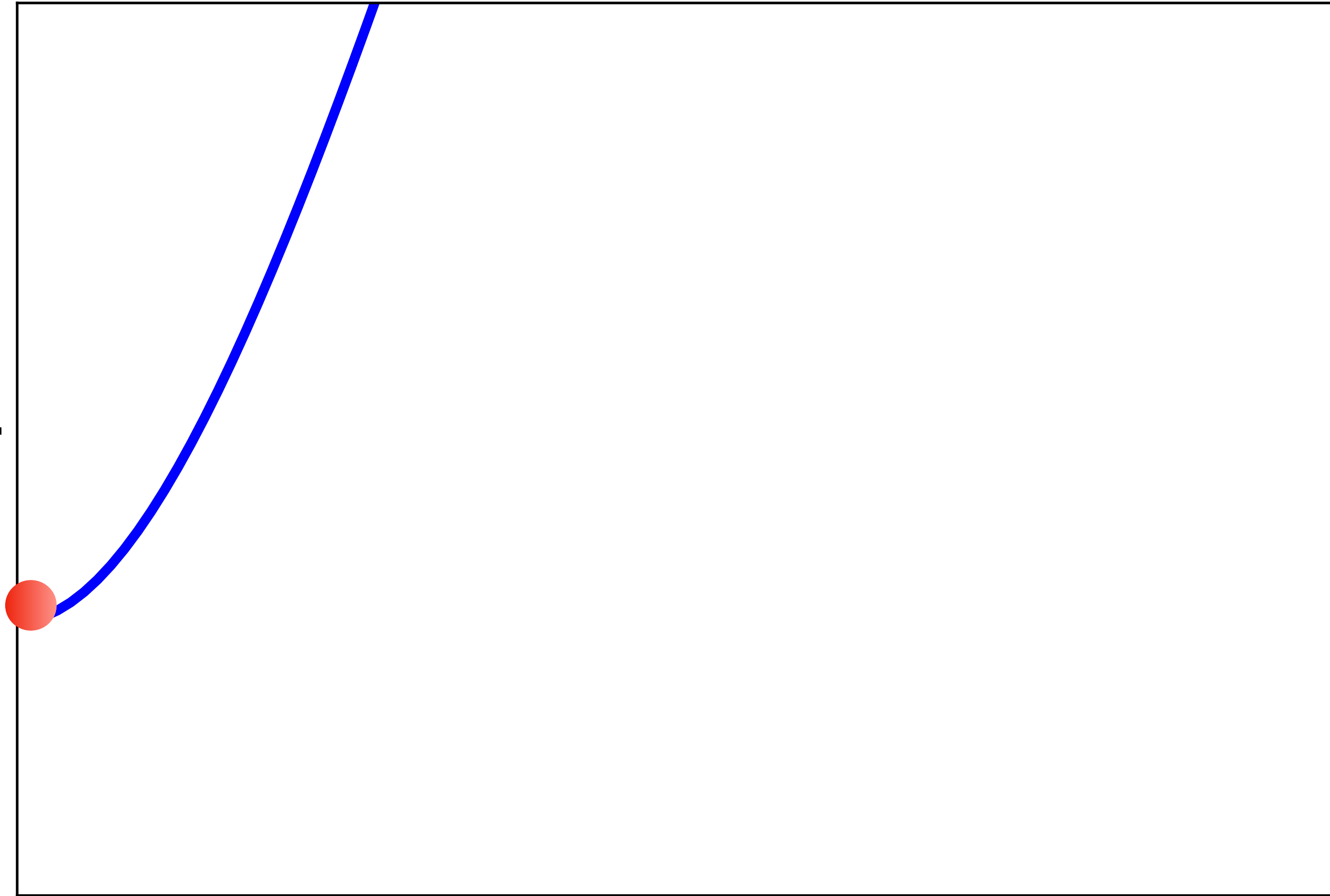
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First-Order Electro-Weak Phase Transition: As the Universe Cools Down!

Early Universe

Higgs field (ϕ)
potential

$V(\phi)$

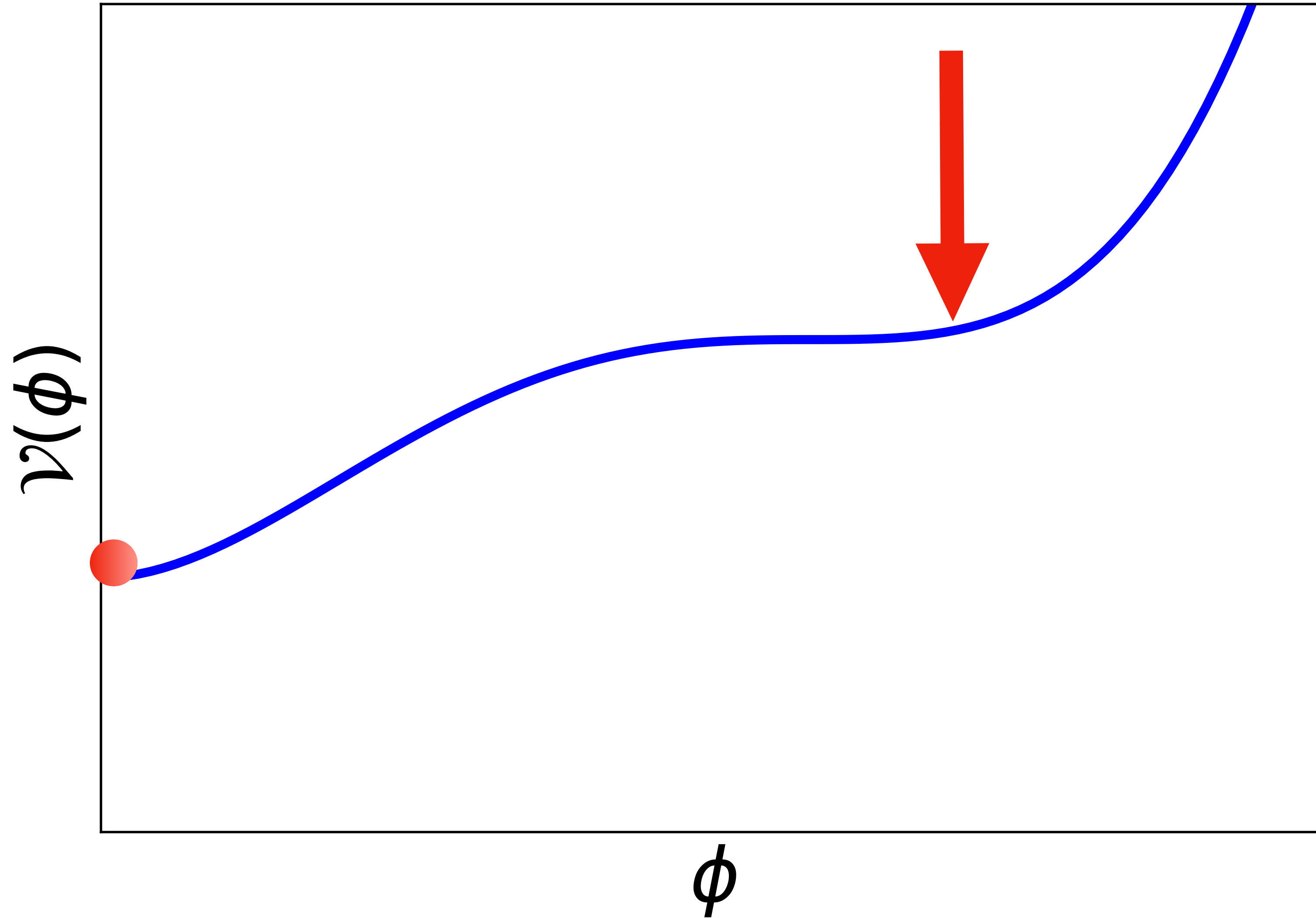


ϕ

First-Order Electro-Weak Phase Transition: As the Universe Cools Down!

Second minimum appears

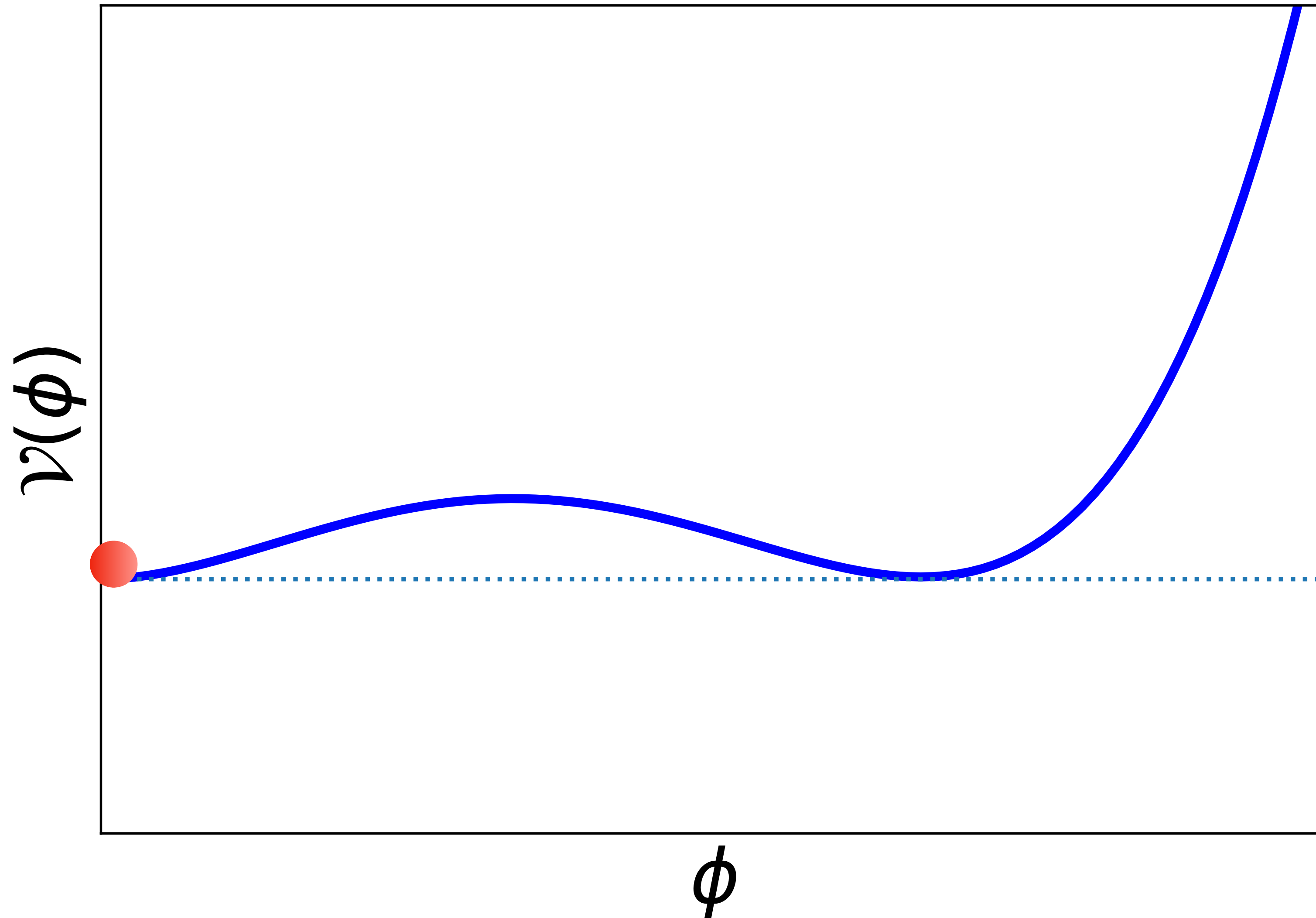
Higgs field (ϕ)
potential



First-Order Electro-Weak Phase Transition: As the Universe Cools Down!

Critical temperature reached

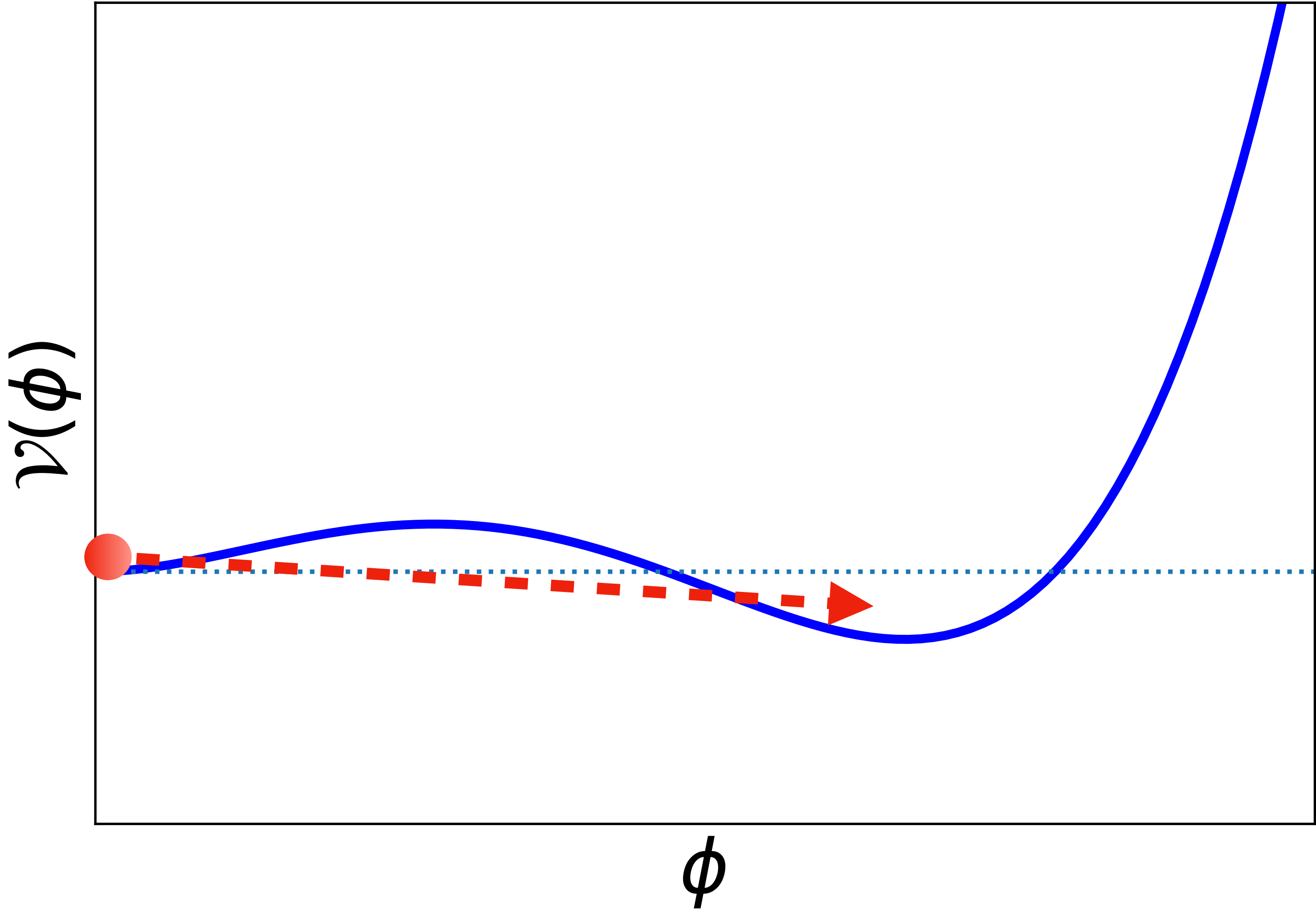
Higgs field (ϕ)
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First-Order Electro-Weak Phase Transition: As the Universe Cools Down!

Tunneling!

Higgs field (ϕ)
potential

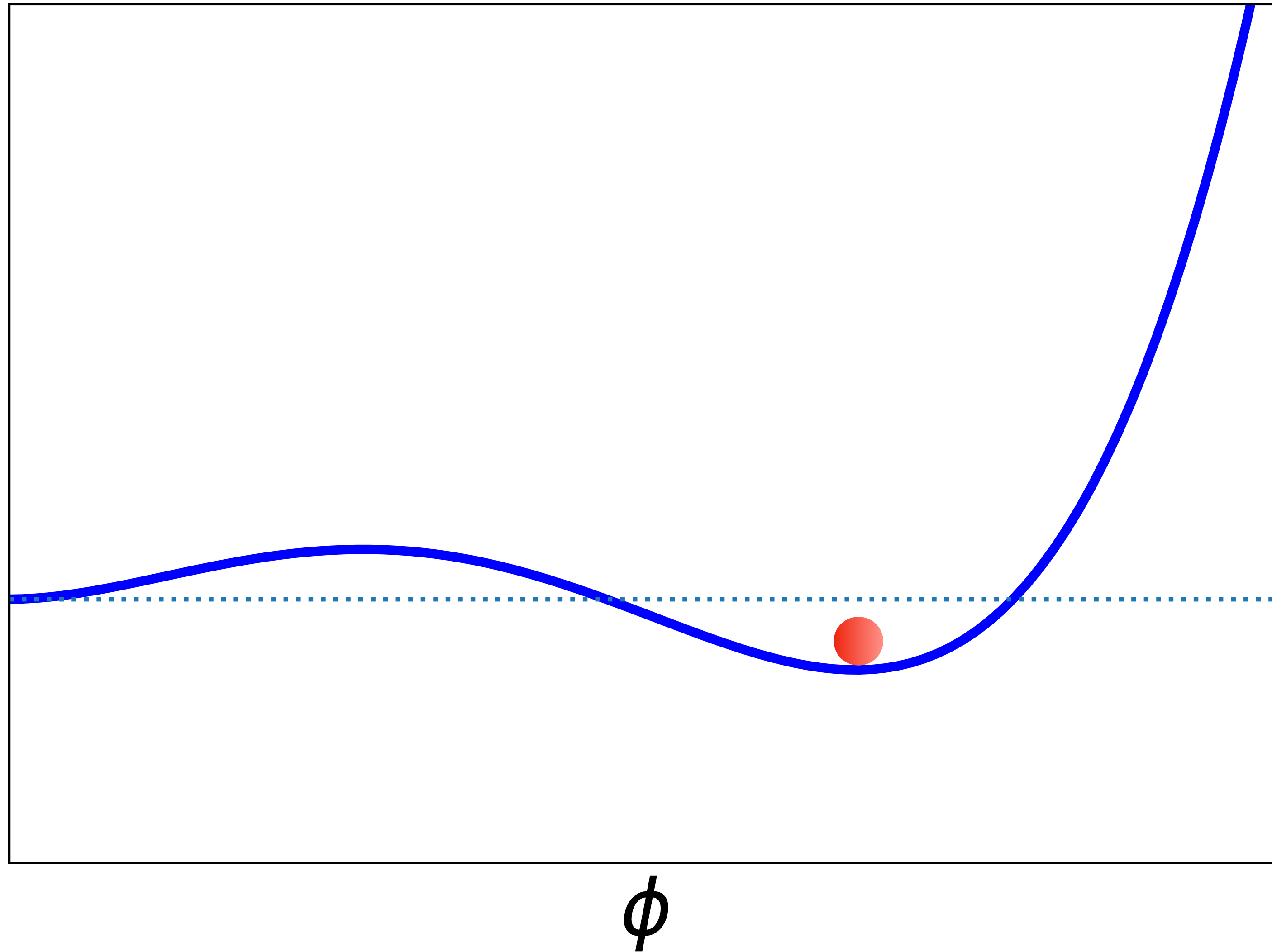


First-Order Electro-Weak Phase Transition: As the Universe Cools Down!

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$V(\phi)$

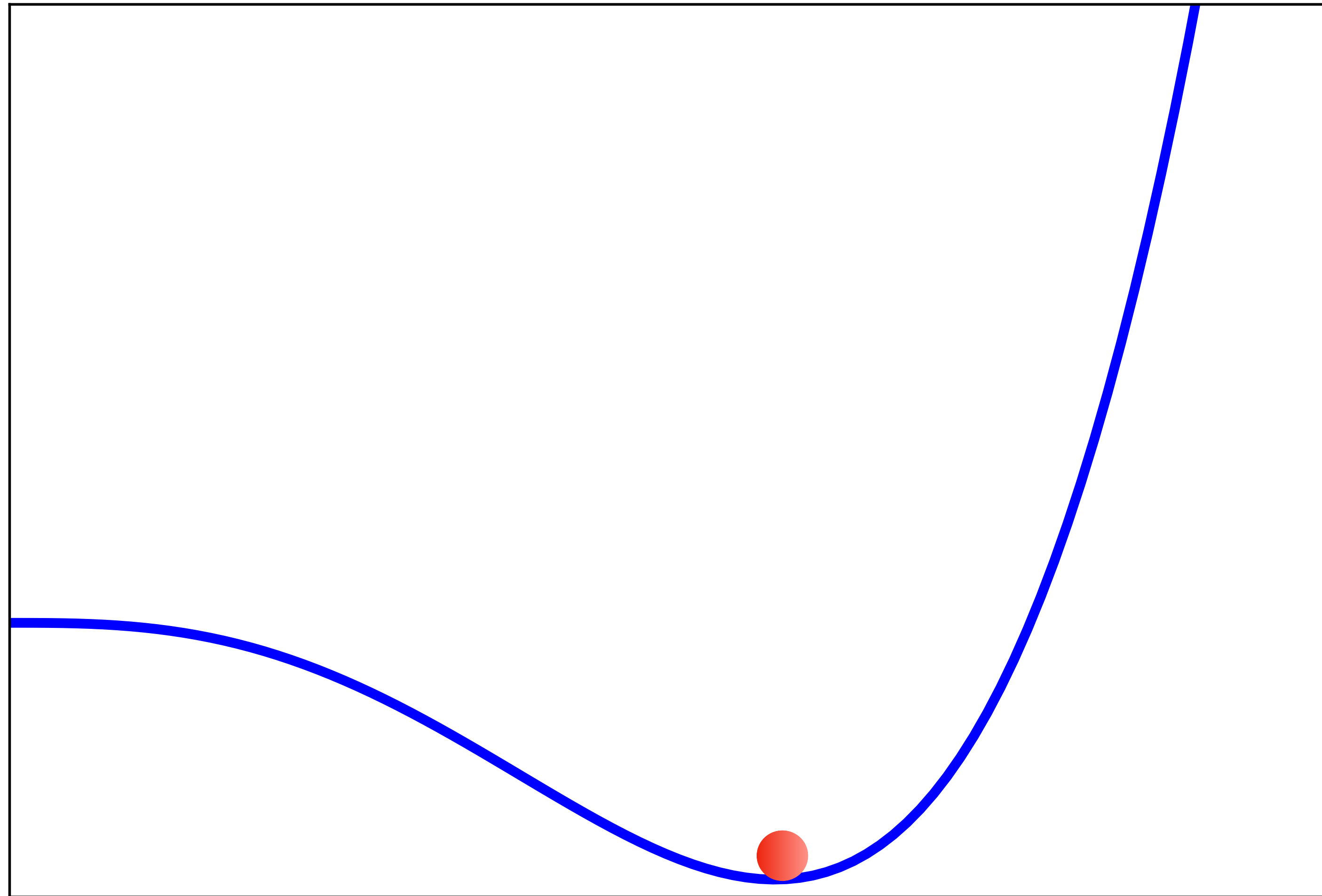


First-Order Electro-Weak Phase Transition: As the Universe Cools Down!

Today

Higgs field (ϕ)
potential

$V(\phi)$



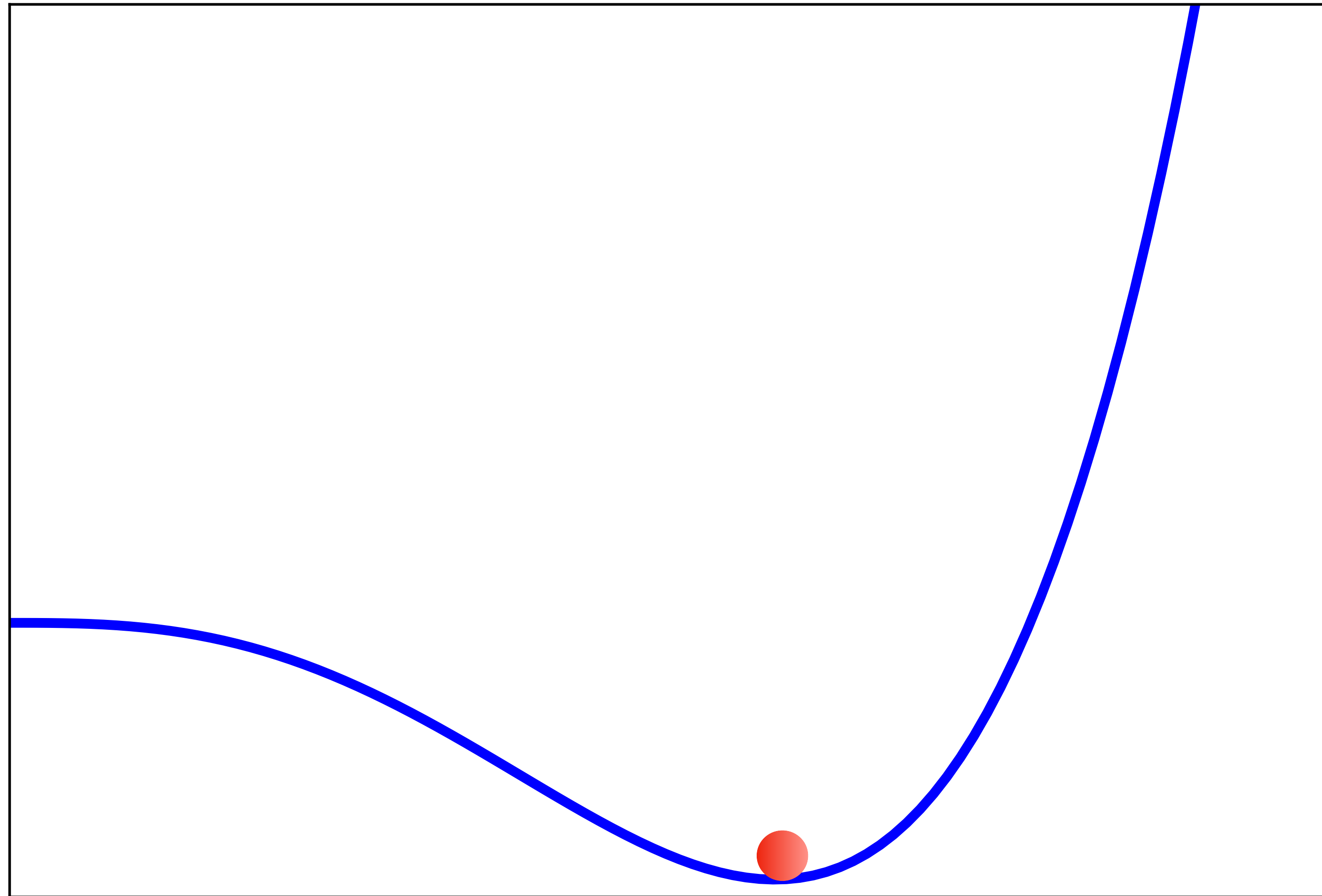
ϕ

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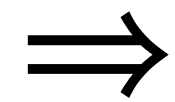
Higgs field (ϕ)
potential

$V(\phi)$



ϕ

$$\langle \phi \rangle \neq 0$$



Symmetry
Breaking.

First-Order Electro-Weak Phase Transition: **As the Universe Cools Down!**

Some time after critical temperature is reached:

→ **Bubbles** of the broken phase nucleate and expand.*



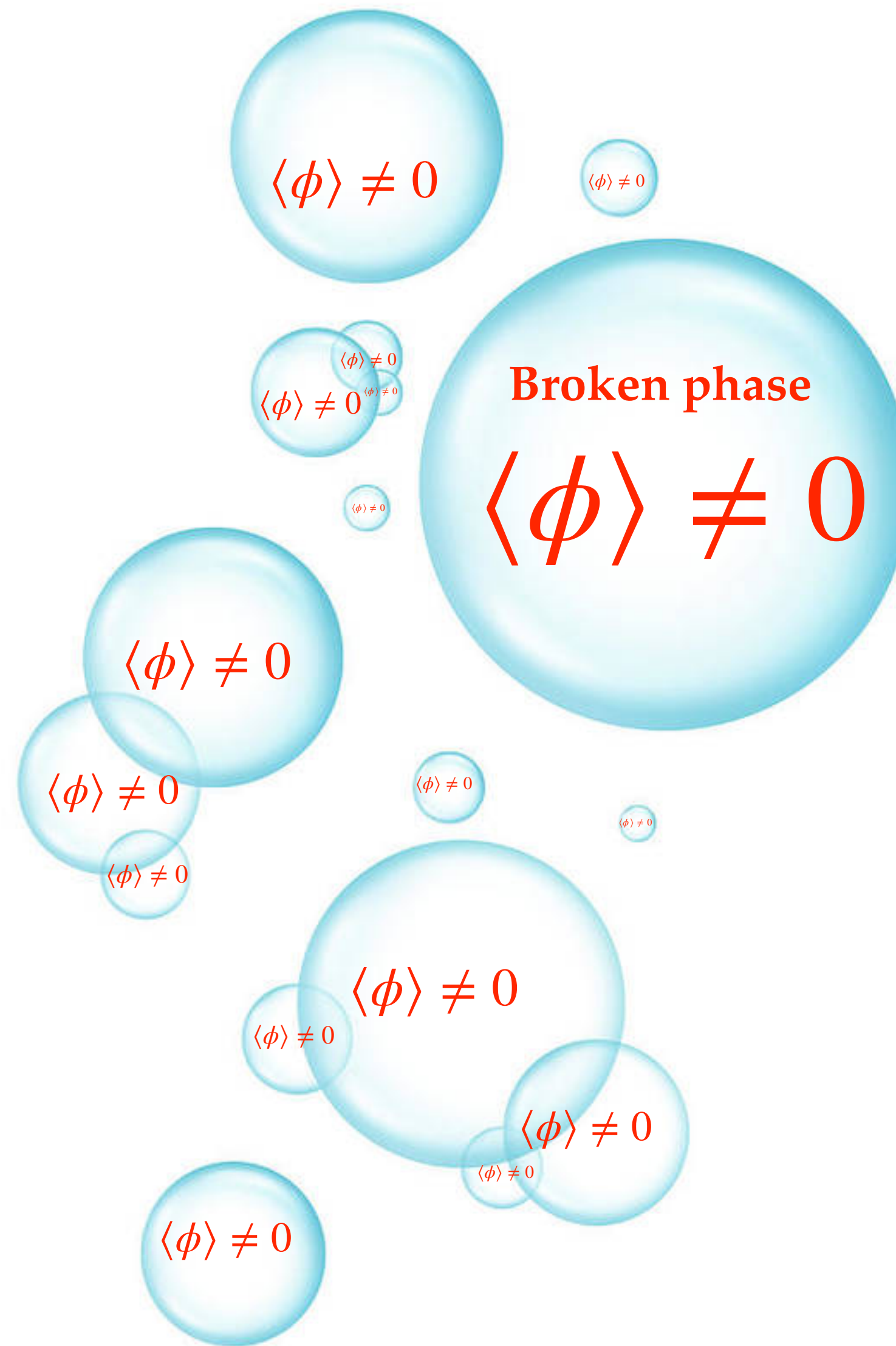
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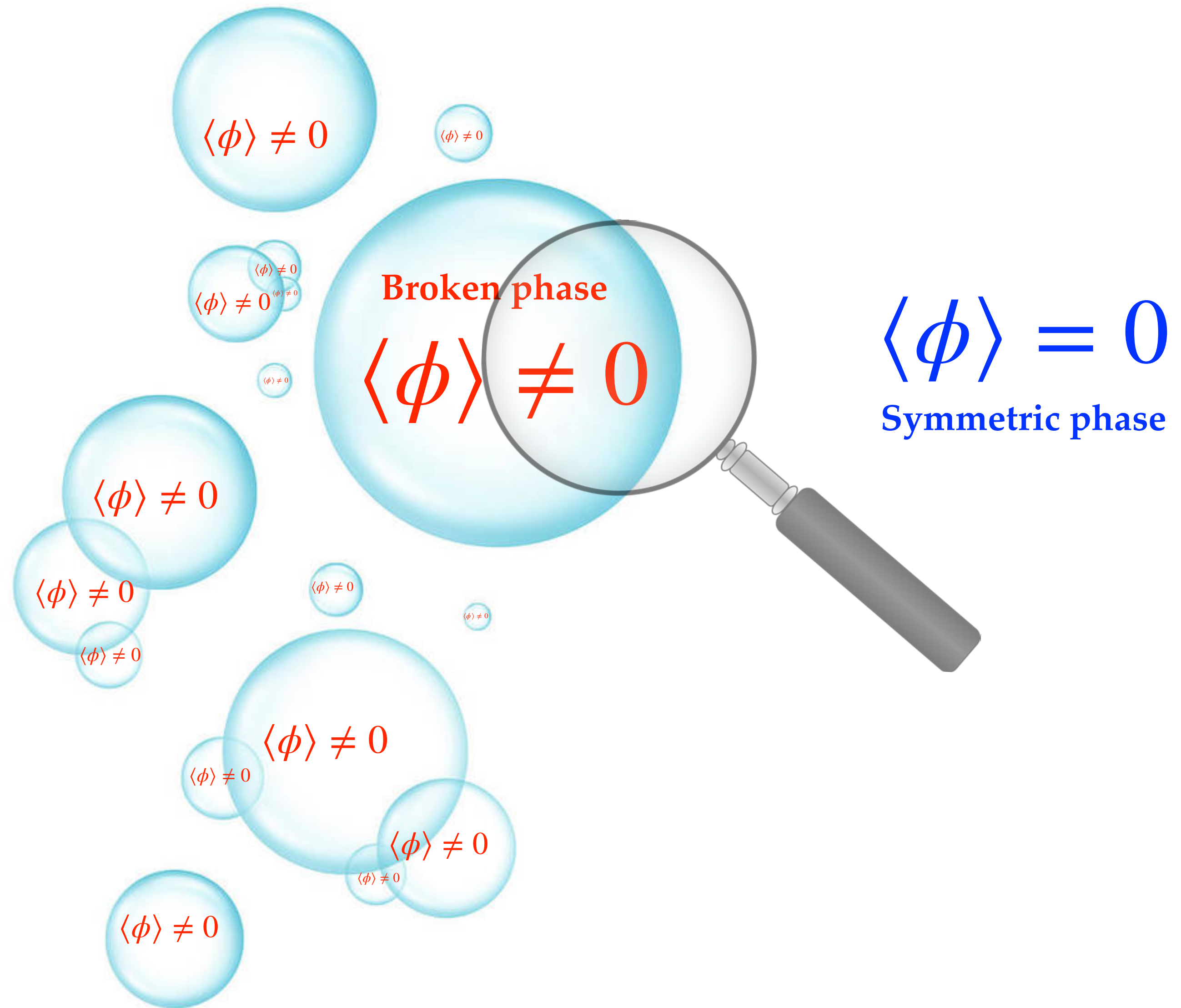
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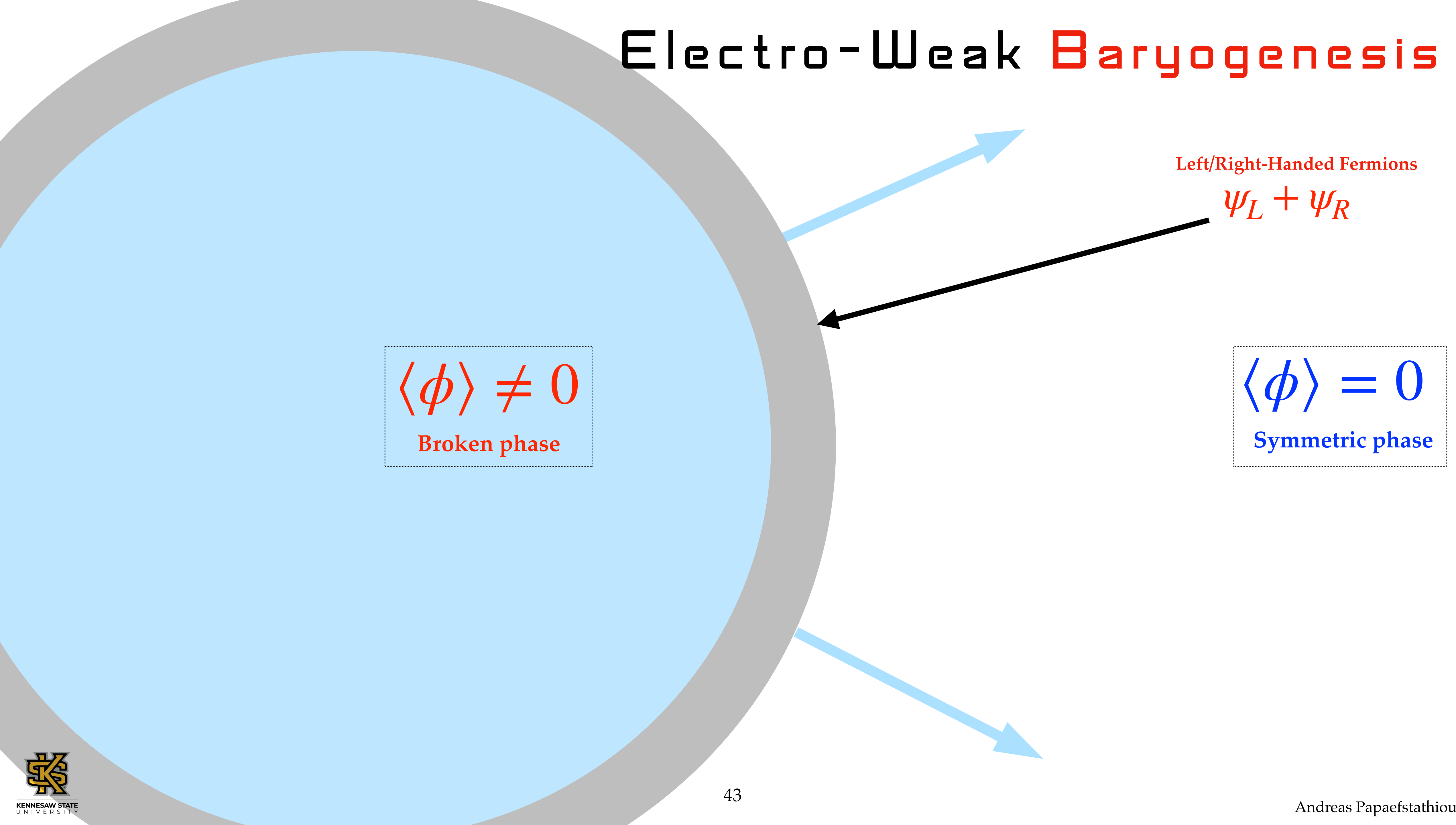


Electro-Weak **Baryogenesis**

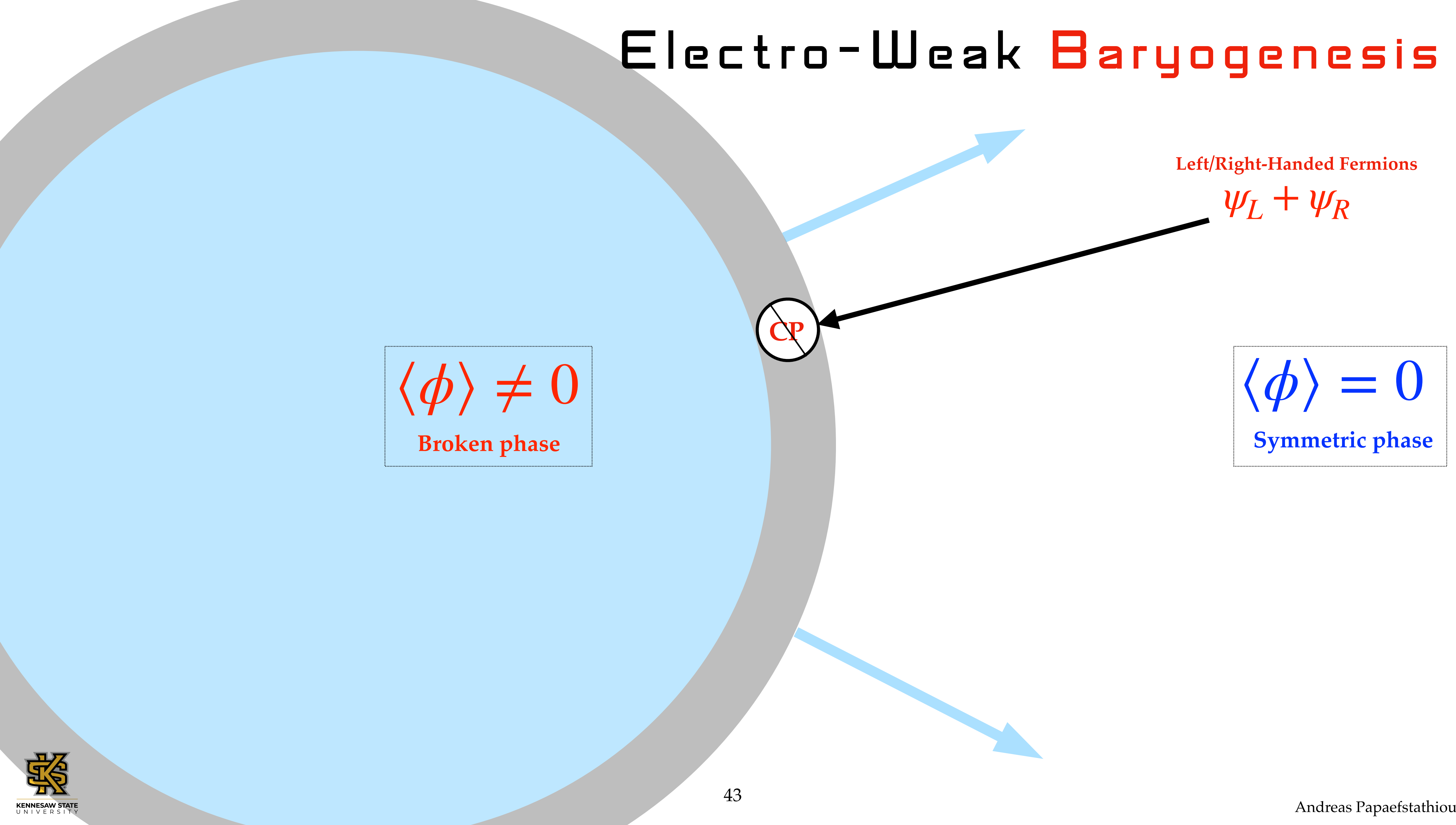
$\langle \phi \rangle \neq 0$
Broken phase

$\langle \phi \rangle = 0$
Symmetric phase

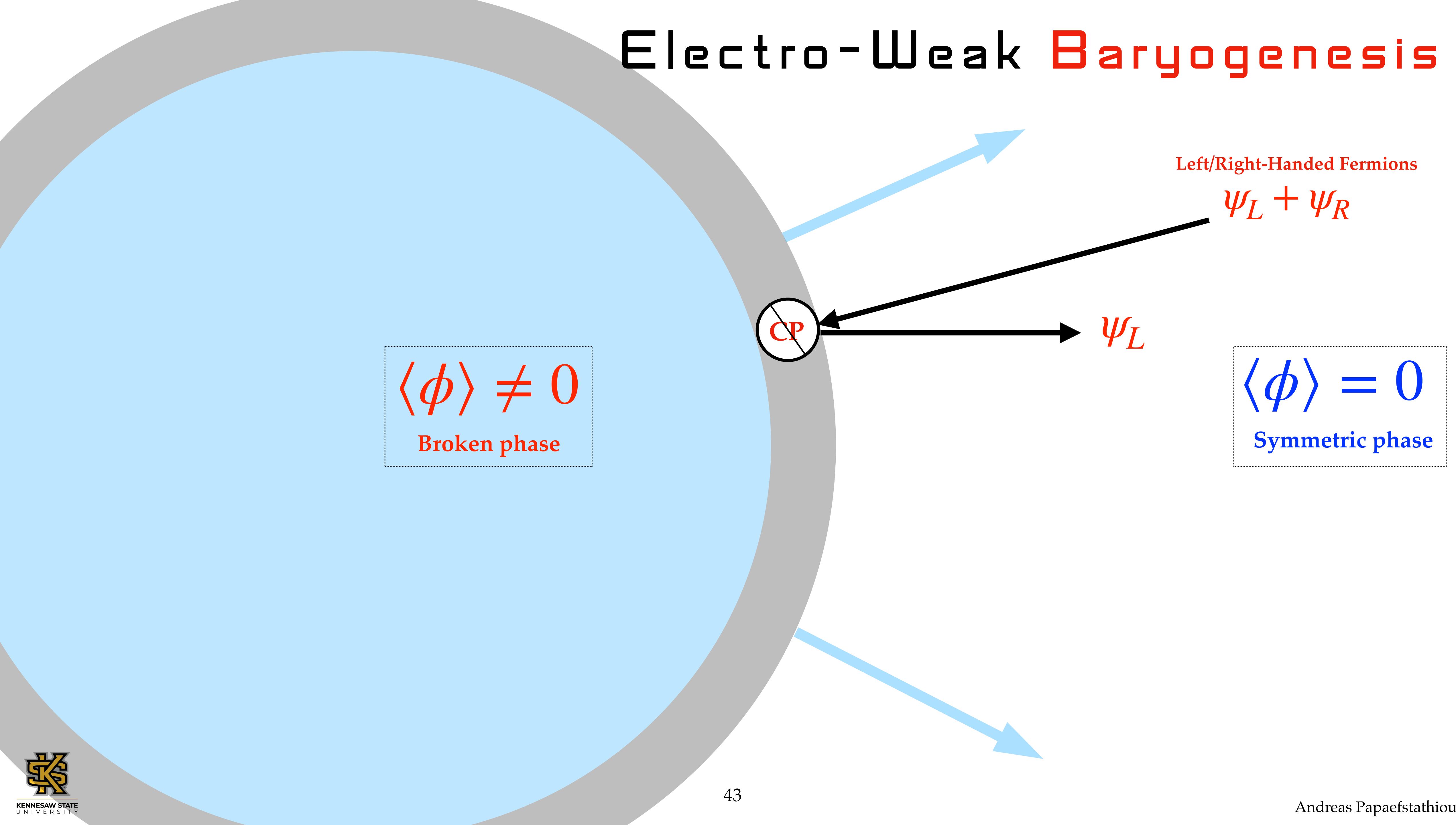
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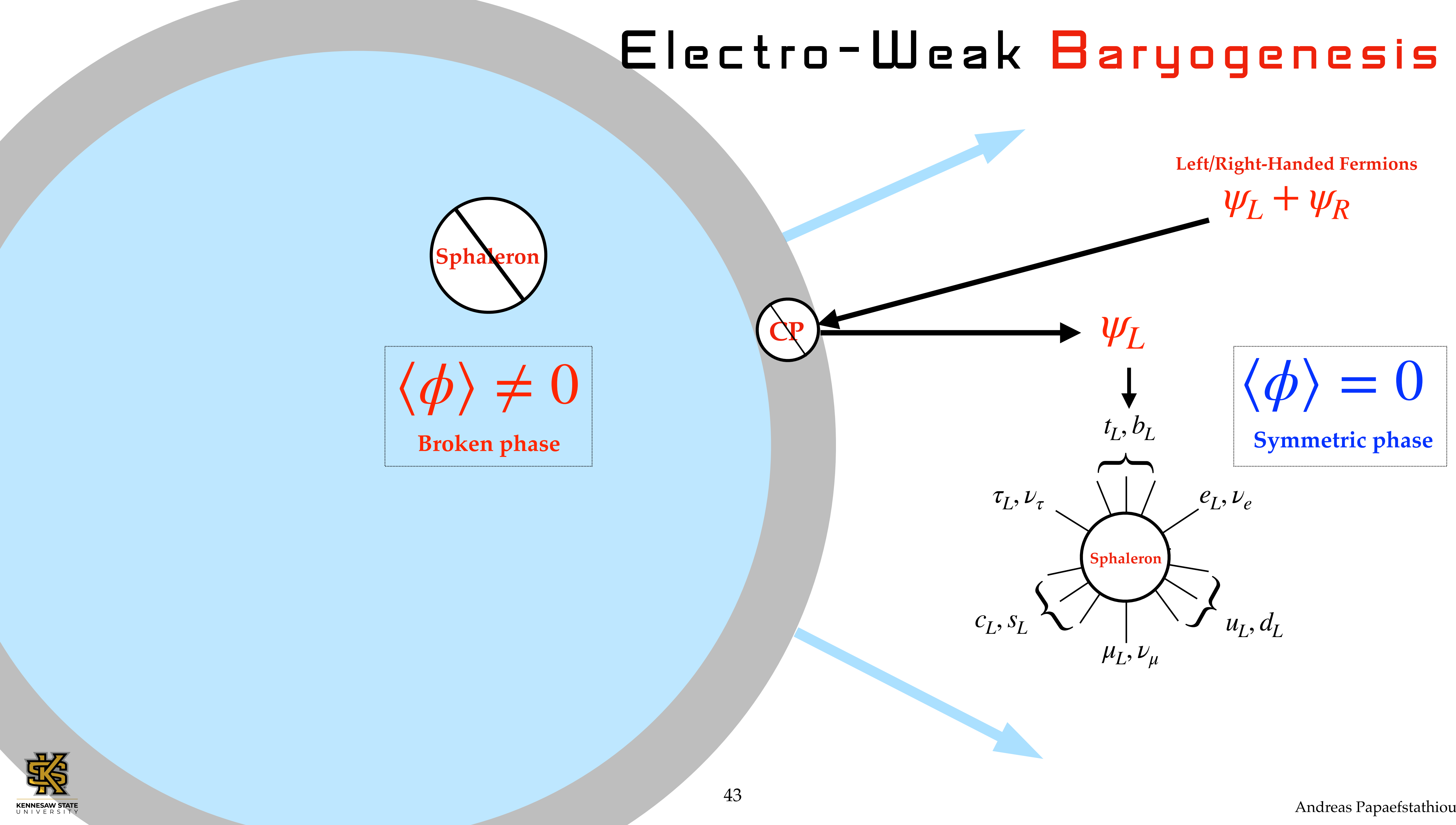
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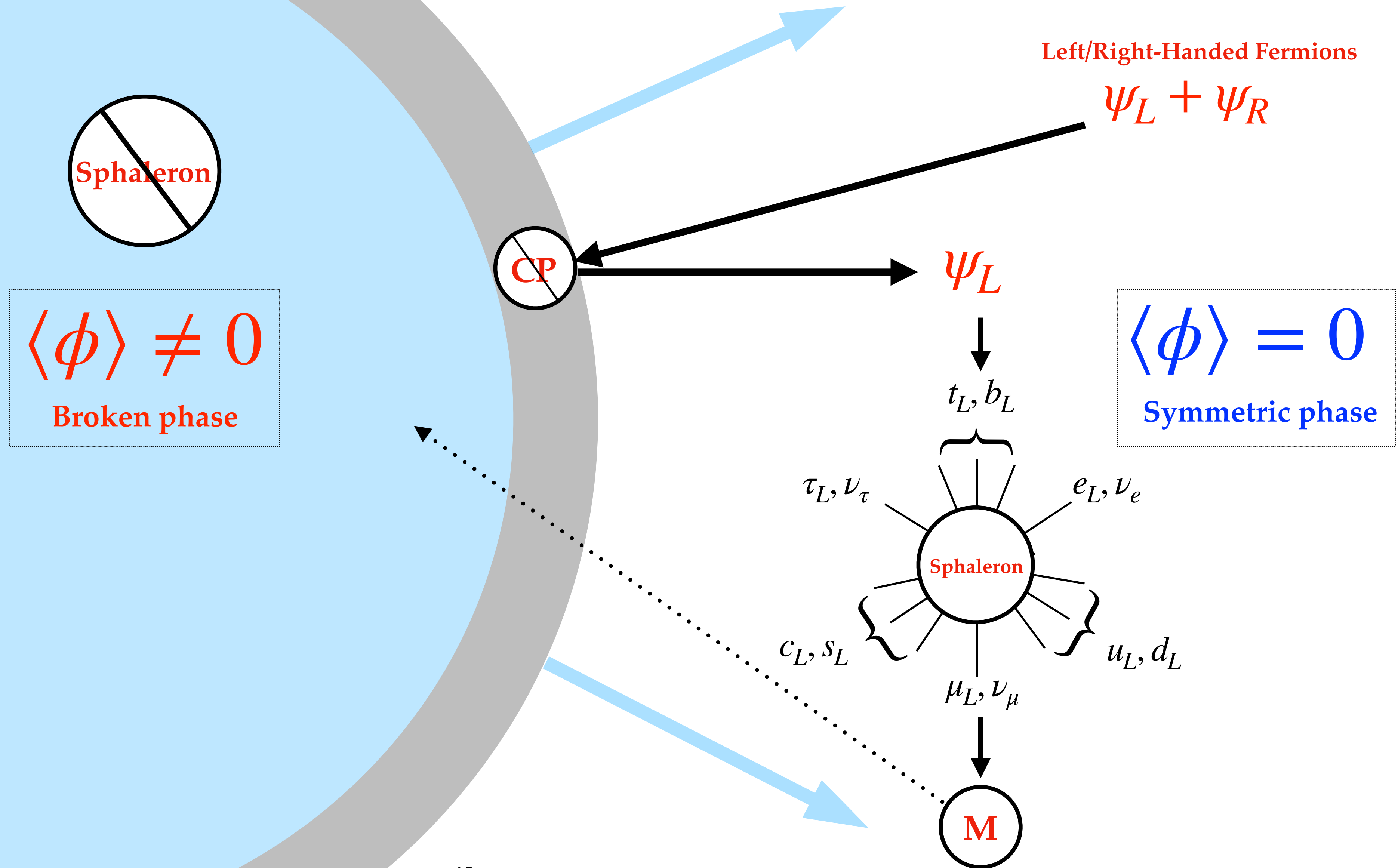
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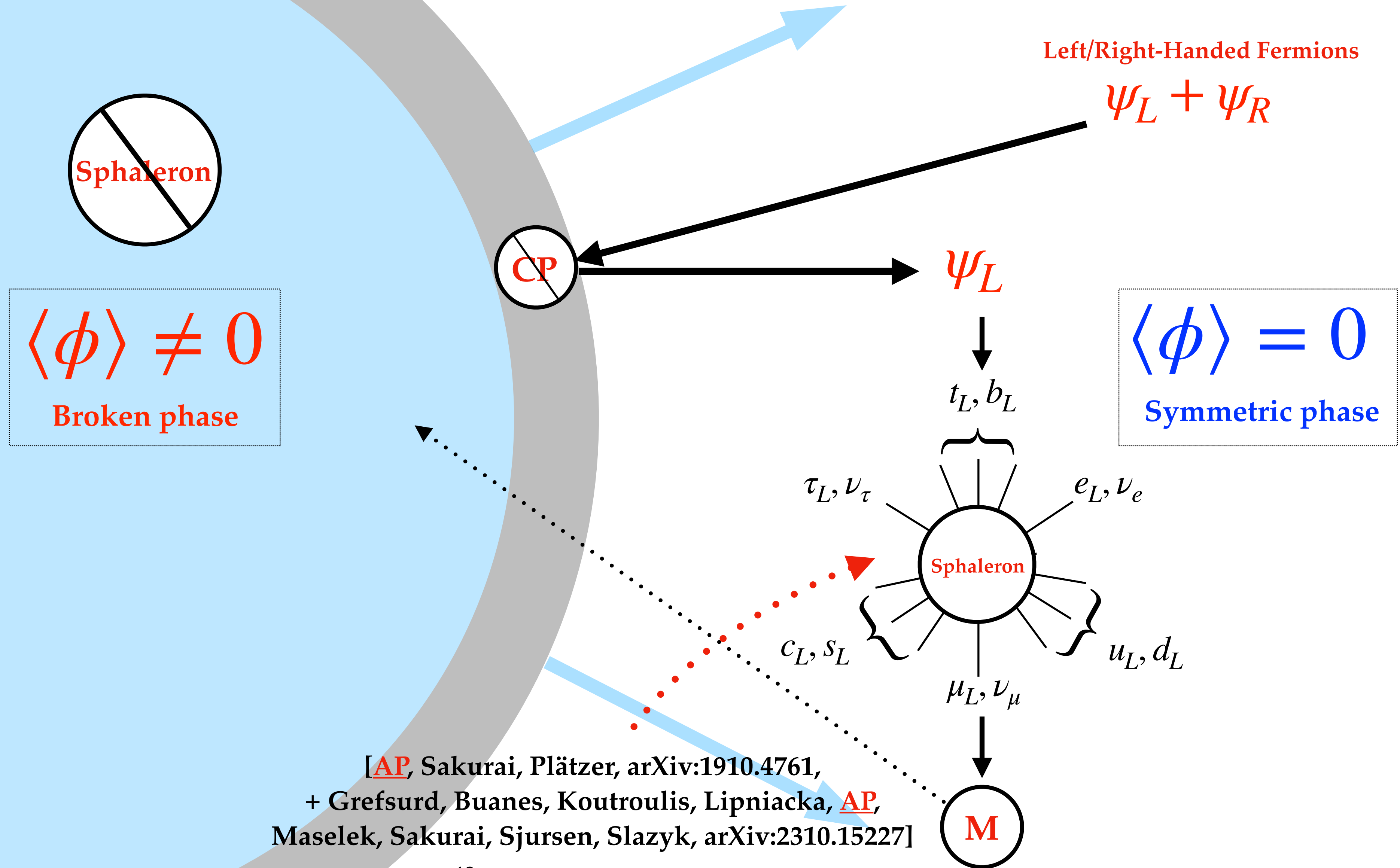
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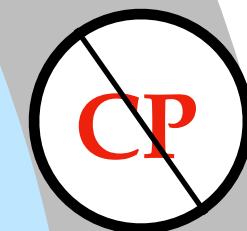


Electro-Weak Baryogenesis



Electro-Weak **Baryogenesis**

**A First-Order
Transition requires
New Phenomena
beyond the SM!**



Left/Right-Handed Fermions

$$\psi_L + \psi_R$$

$$\psi_L$$

$$\langle \phi \rangle = 0$$

Symmetric phase

$$t_L, b_L$$

$$\tau_L, \nu_\tau$$

$$e_L, \nu_e$$

Sphaleron

$$c_L, s_L$$

$$u_L, d_L$$

$$\mu_L, \nu_\mu$$

M

[[AP](#), Sakurai, Plätzer, arXiv:1910.4761,
+ Grefsurd, Buane, Koutroulis, Lipniacka, [AP](#),
Maselek, Sakurai, Sjursen, Slazyk, arXiv:2310.15227]

Electro-Weak Archaeology



We live here!

$$\langle \phi \rangle \neq 0$$

Broken phase



Electro-Weak Archaeology



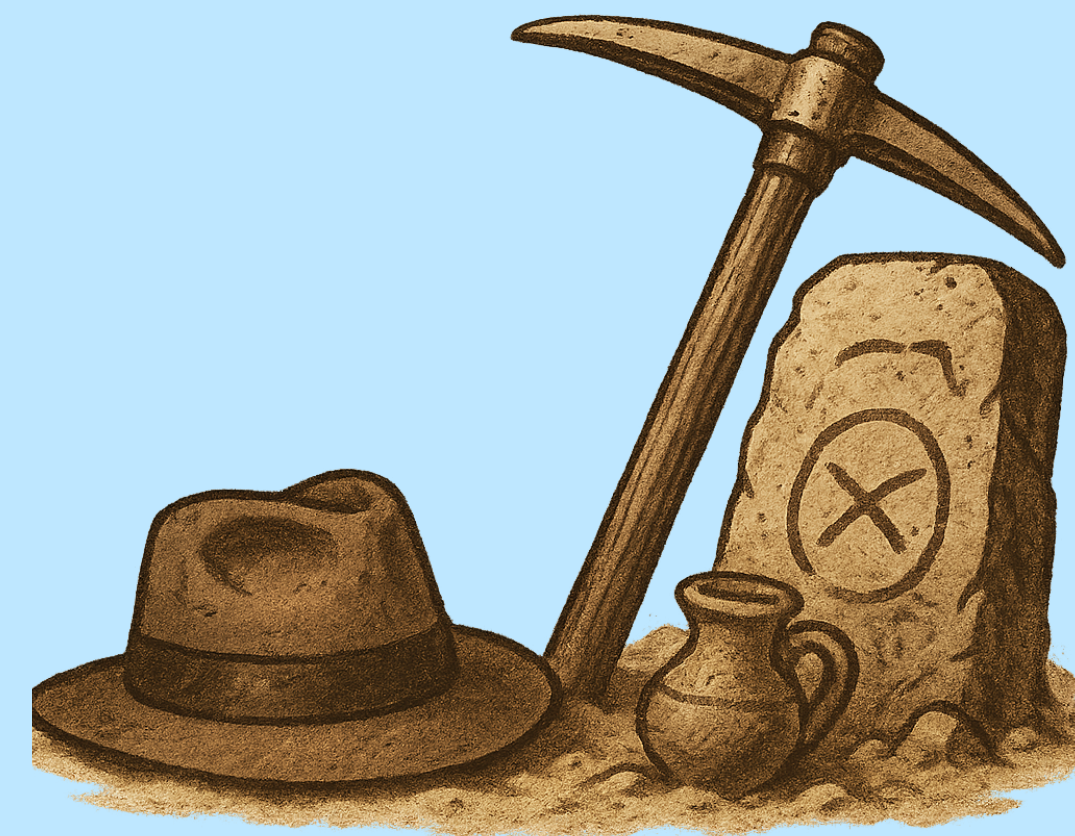
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→ What are the imprints of
Electro-Weak Baryogenesis
at Colliders?



Extending the Scalar Sector

👒 New scalars with significant couplings to the Higgs field $\Rightarrow$ influence early universe evolution of Higgs potential. [e.g. Morrissey, Ramsey-Musolf, 1206.2942]

👒 Consider the **simplest possible extension to the SM:**

$$\mathcal{V}(\phi, S) = \bullet |\phi|^2 + \blacksquare |\phi|^4$$

Add: S , a **new scalar field**,
No SM “charges” $\equiv$ Singlet.

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Extending the Scalar Sector

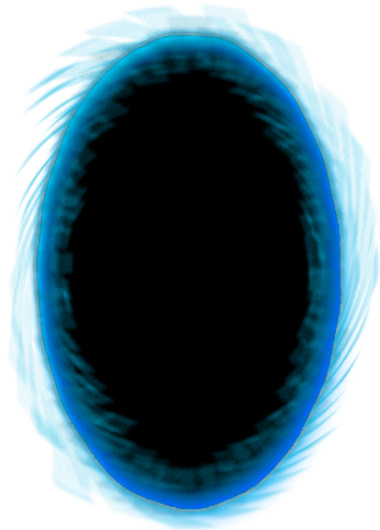
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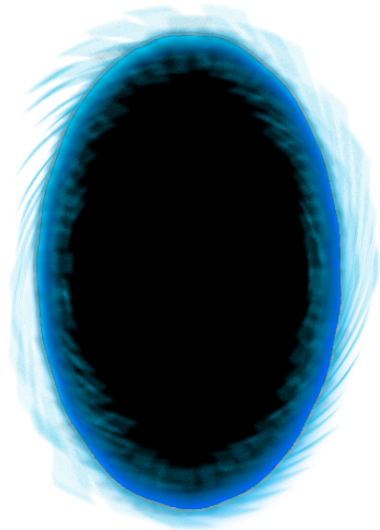
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$$+ \textcolor{red}{\blacktriangle} |\phi|^2 S + \textcolor{purple}{\blacksquare} |\phi|^2 S^2 \leftarrow \text{“Portal” interactions.}$$

$$+ S \times (\text{Hidden Sector}) + \dots \leftarrow \text{Dark Matter?}$$

Extending the Scalar Sector

[e.g. AP, White, 2010.00597]

$$\mathcal{V}(\phi, S) = \textcolor{red}{\bullet} |\phi|^2 + \textcolor{blue}{\blacksquare} |\phi|^4 + \textcolor{green}{\bullet} S^2 + \textcolor{orange}{\blacktriangle} S^3 + \textcolor{brown}{\blacksquare} S^4 + \textcolor{red}{\blacktriangle} |\phi|^2 S + \textcolor{purple}{\blacksquare} |\phi|^2 S^2$$

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EWSB $\leftrightarrow$ VEVs:

$$\phi \rightarrow \langle \phi \rangle + h$$

$$S \rightarrow \langle S \rangle + \chi$$

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$$\mathcal{V}(\phi, S) = \text{●} |\phi|^2 + \text{■} |\phi|^4 + \text{●} S^2 + \text{▲} S^3 + \text{■} S^4 + \text{▲} |\phi|^2 S + \text{■} |\phi|^2 S^2$$

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$$S \rightarrow \langle S \rangle + \chi \quad \Rightarrow \text{Mass (squared) matrix:}$$

$$M^2 = \begin{pmatrix} \frac{\partial^2 \mathcal{V}}{\partial h^2} & \frac{\partial^2 \mathcal{V}}{\partial h \partial \chi} \\ \frac{\partial^2 \mathcal{V}}{\partial h \partial \chi} & \frac{\partial^2 \mathcal{V}}{\partial \chi^2} \end{pmatrix}$$

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$$\phi \rightarrow \langle \phi \rangle + h \quad \Rightarrow \quad \mathcal{V} \supset \text{●} h^2 + \text{●} h\chi + \text{●} \chi^2 \quad \Rightarrow \quad \text{Diagonalize!}$$

$$S \rightarrow \langle S \rangle + \chi \quad \Rightarrow \quad \text{Mass (squared) matrix:} \quad \text{Mass Eigenstates}$$

$$M^2 = \begin{pmatrix} \frac{\partial^2 \mathcal{V}}{\partial h^2} & \frac{\partial^2 \mathcal{V}}{\partial h \partial \chi} \\ \frac{\partial^2 \mathcal{V}}{\partial h \partial \chi} & \frac{\partial^2 \mathcal{V}}{\partial \chi^2} \end{pmatrix}$$

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} h \\ \chi \end{pmatrix}$$

θ : mixing angle

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Mass Eigenstates

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$\textcolor{red}{\theta}$: mixing angle

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$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} h \\ \chi \end{pmatrix} \Rightarrow \begin{aligned} h_1 &\rightarrow \text{“SM-like” Higgs boson.} \\ h_2 &\rightarrow \text{new scalar resonance.} \end{aligned}$$

θ : mixing angle

i.e. choose: $|\theta| \gtrsim 0$, and:

$$h_1 = h \cos \theta + \chi \sin \theta$$

$$h_2 = -h \sin \theta + \chi \cos \theta$$

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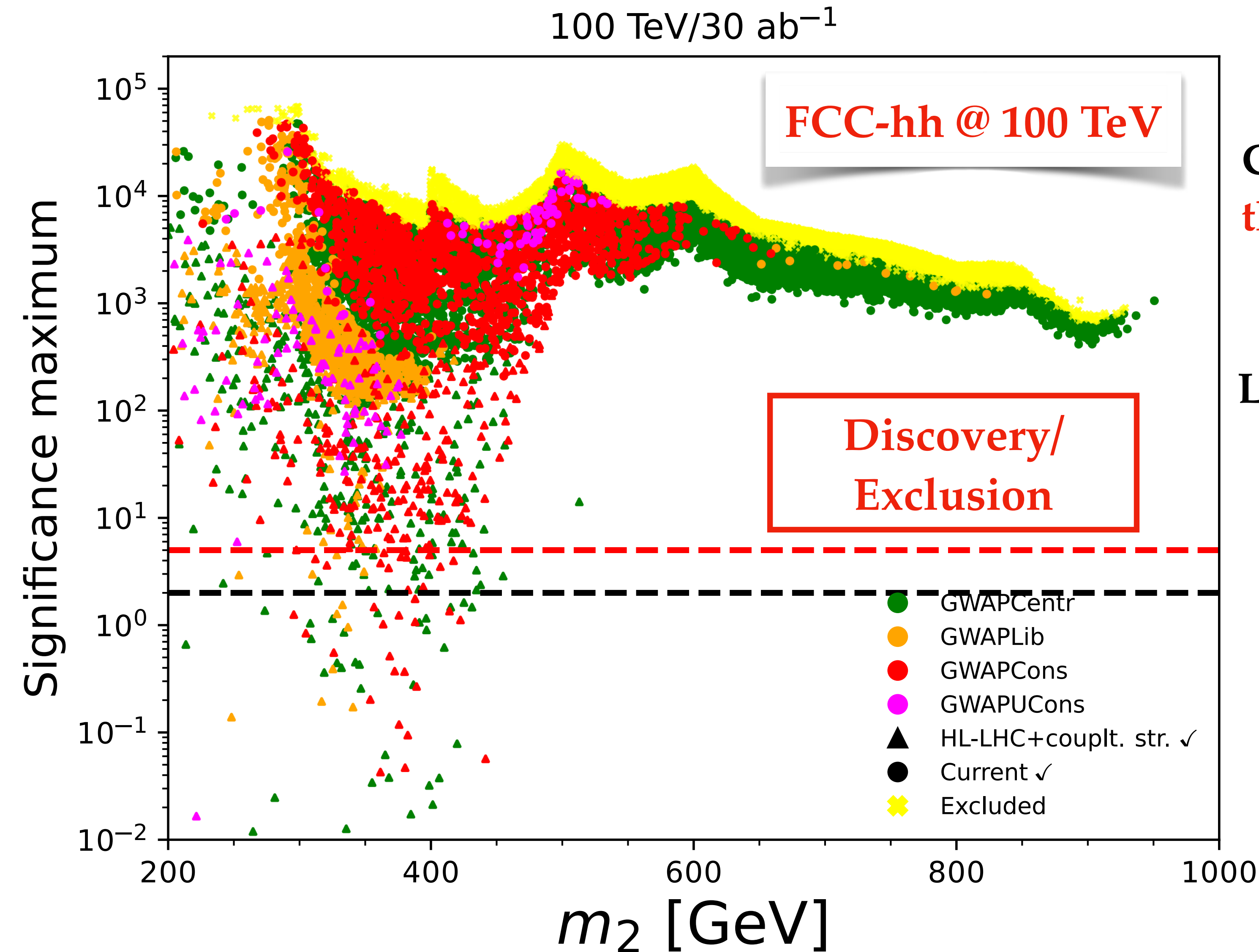
$$h_2 = -h \sin \theta + \chi \cos \theta$$

**Primary targets for
collider studies!**

$h_1 \Rightarrow$ **Reductions** in
Higgs boson rates.

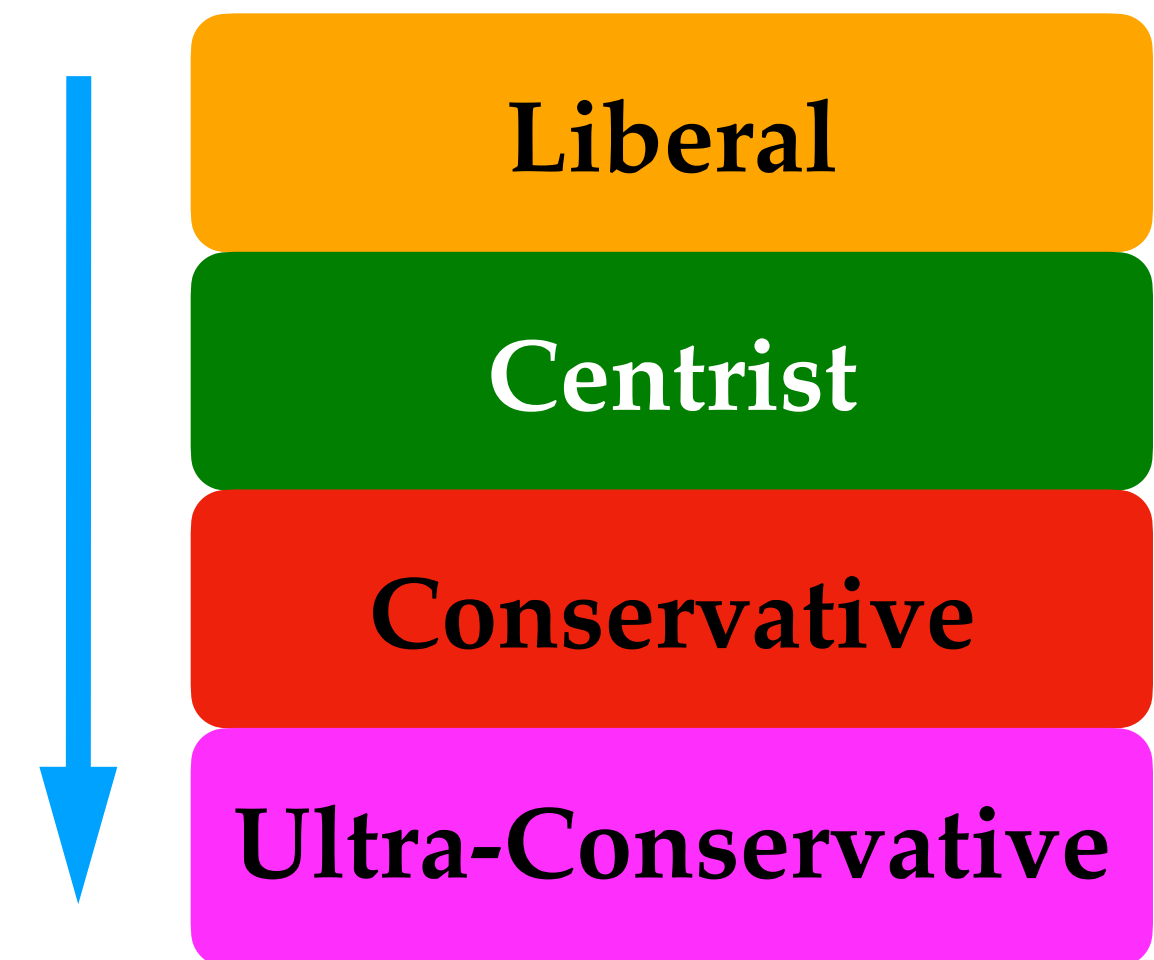
$h_2 \Rightarrow$ **New resonance
searches.**

Constraints from $pp \rightarrow h_2 \rightarrow W^+W^-, ZZ, h_1h_1$ @ Future Colliders



Color-coded parameter-space points denote **theoretical uncertainty for First-Order EWPT:**

Lower uncertainty
 $\Rightarrow$
FO-EWPT
 more certain to
 have occurred.



✕ = Point is excluded today (~2020).

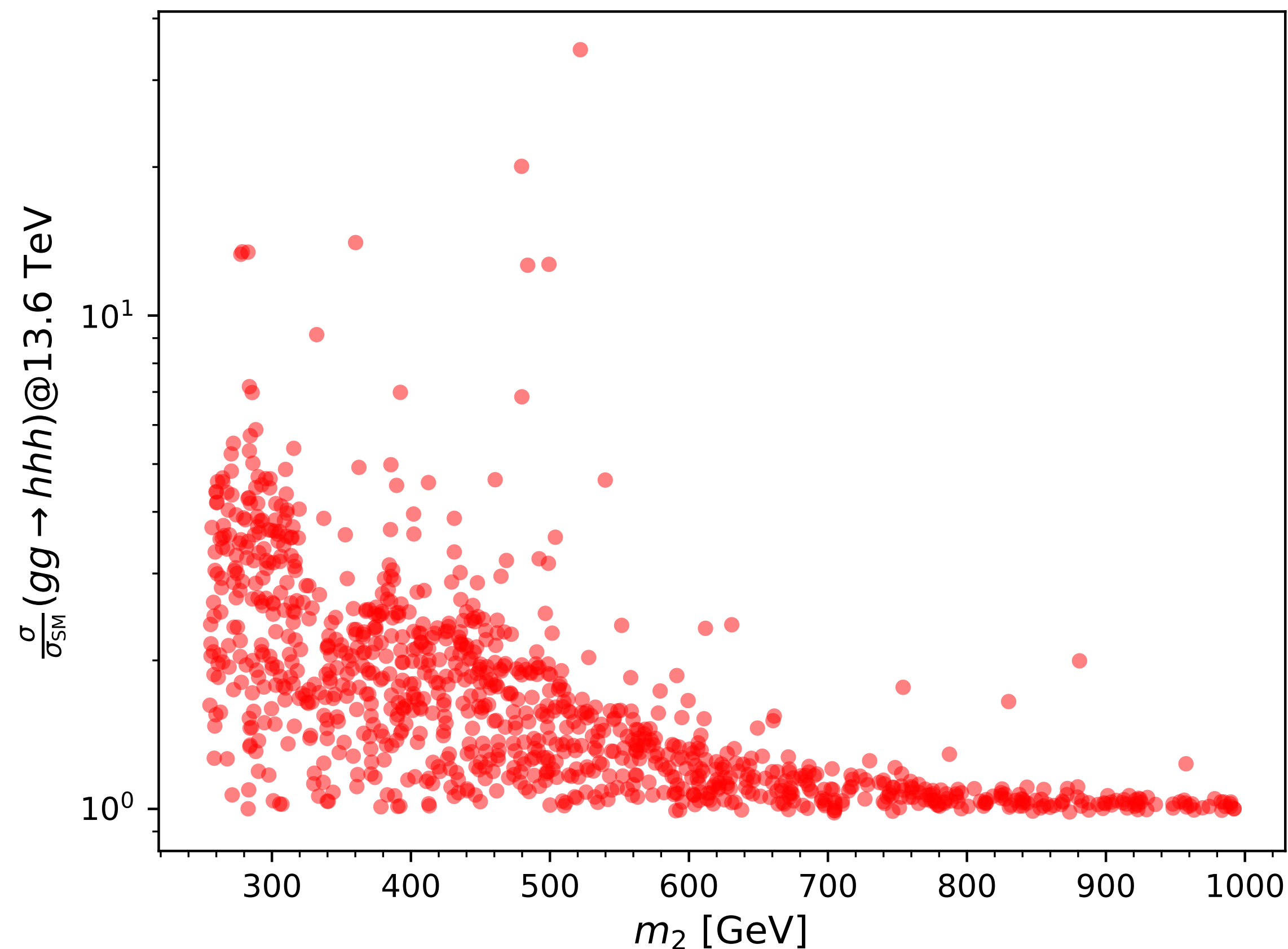
Triple Higgs Production in SM+One Real Singlet [=xSM]

🎩 First-Order EWPT in xSM $\Rightarrow$ Large scalar couplings $\rightarrow$ Enhanced hhh in the xSM???

🎩 (Rough) Example with $\mathbb{Z}_2$ symmetry: $\mathbb{Z}_2 : S \rightarrow -S$ & NO First-Order EWPT!

🎩 Including: boundedness of potential / perturbativity / HiggsTools (i.e. experimental) constraints.

$\mathbb{Z}_2$ xSM - Viable points



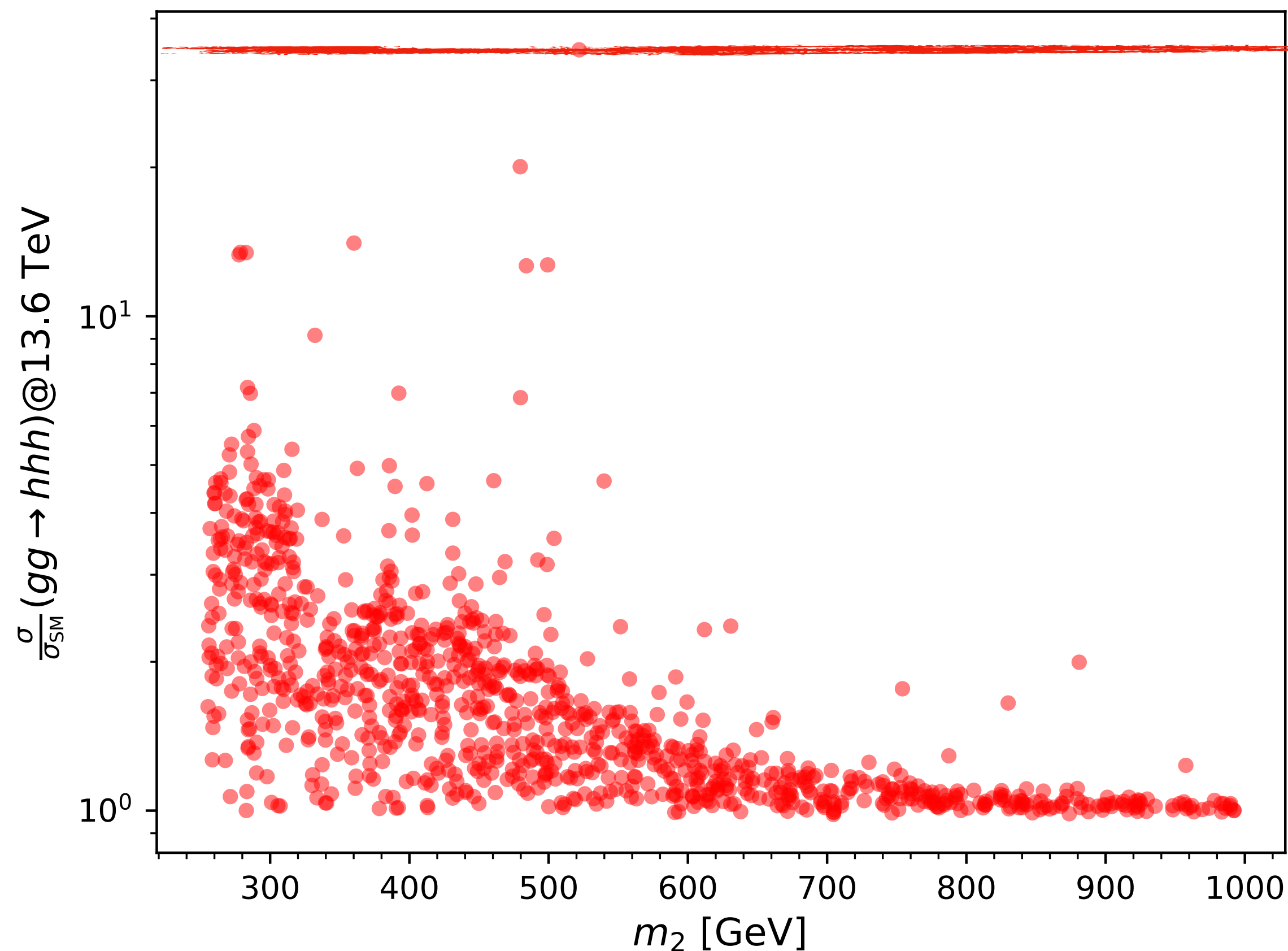
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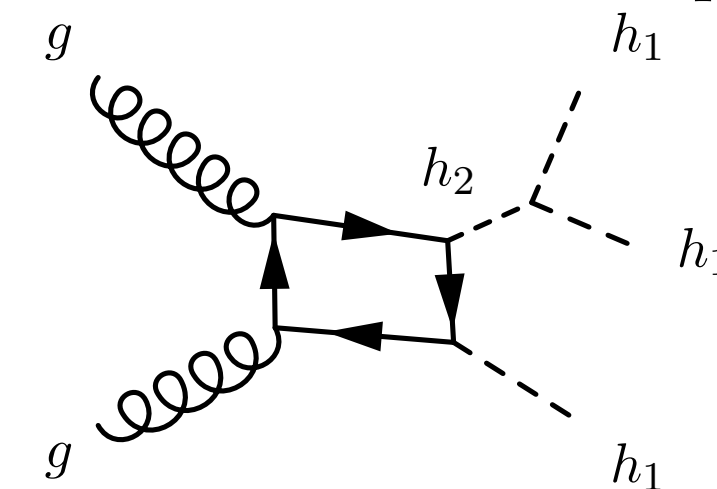
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Maximum enhancement $\sim$
 $\mathcal{O}(30) \times \sigma(hhh)_{\text{SM}}!$

[**But:** I suspect these will be excluded quickly by resonant hh ! Due to:



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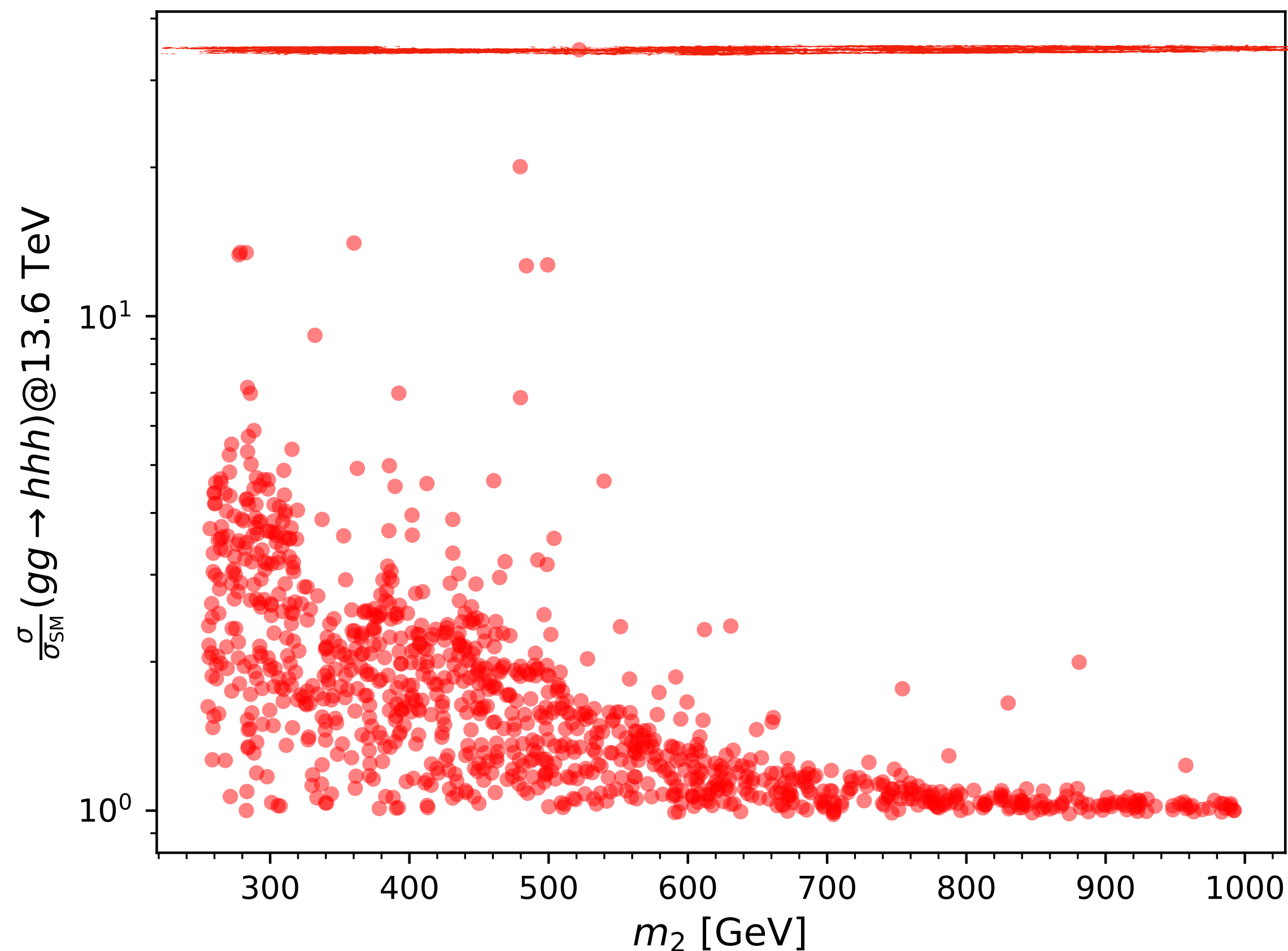
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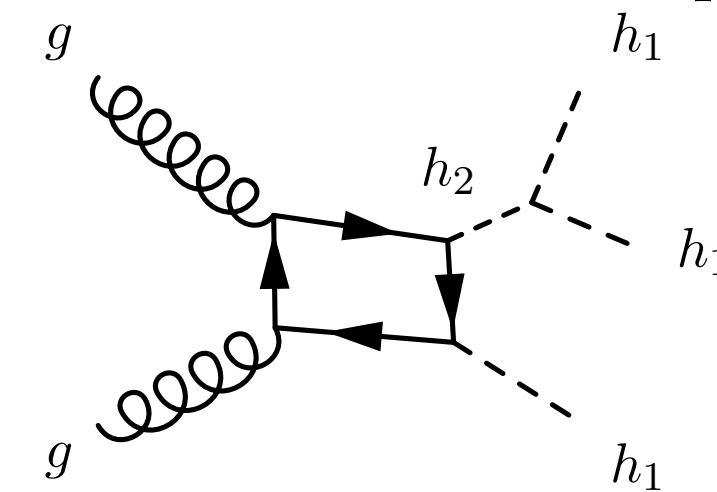
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[But: I suspect these will be excluded quickly by resonant hh ! Due to:



$\Rightarrow$ Look at further extended scalar sectors for enhanced hhh & First-Order EWPT!

SM + Two Real Singlet Scalars [= TRSM]

👒 Consider adding two real singlet scalar fields $S, X \rightarrow$ the **TRSM**.

👒 & impose discrete $\mathbb{Z}_2$ symmetries:

$$\mathbb{Z}_2^S : S \rightarrow -S, \quad X \rightarrow X$$
$$\mathbb{Z}_2^X : X \rightarrow -X, \quad S \rightarrow S$$

$\Rightarrow$ **TRSM** Scalar Potential:

$$\begin{aligned} \mathcal{V}(\phi, S, X) = & \bullet |\phi|^2 + \blacksquare |\phi|^4 + \bullet S^2 + \blacksquare S^4 + \bullet X^2 + \blacksquare X^4 \\ & + \blacksquare S^2 X^2 \\ & + \blacksquare |\phi|^2 S^2 + \blacksquare |\phi|^2 X^2 \end{aligned}$$

SM + Two Real Singlet Scalars [= TRSM]

👒 Electroweak Symmetry Breaking in the TRSM:

⇒ Three scalar bosons: $h_1, h_2, h_3 \rightarrow h_1 \approx \text{SM-like "Higgs boson"}$.

⇒ hhh that may even be detectable at the LHC!

through: $pp \rightarrow h_3 \rightarrow h_2 h_1 \rightarrow h_1 h_1 h_1$

[Robens, Stefaniak, Wittbrodt, 1908.08554,
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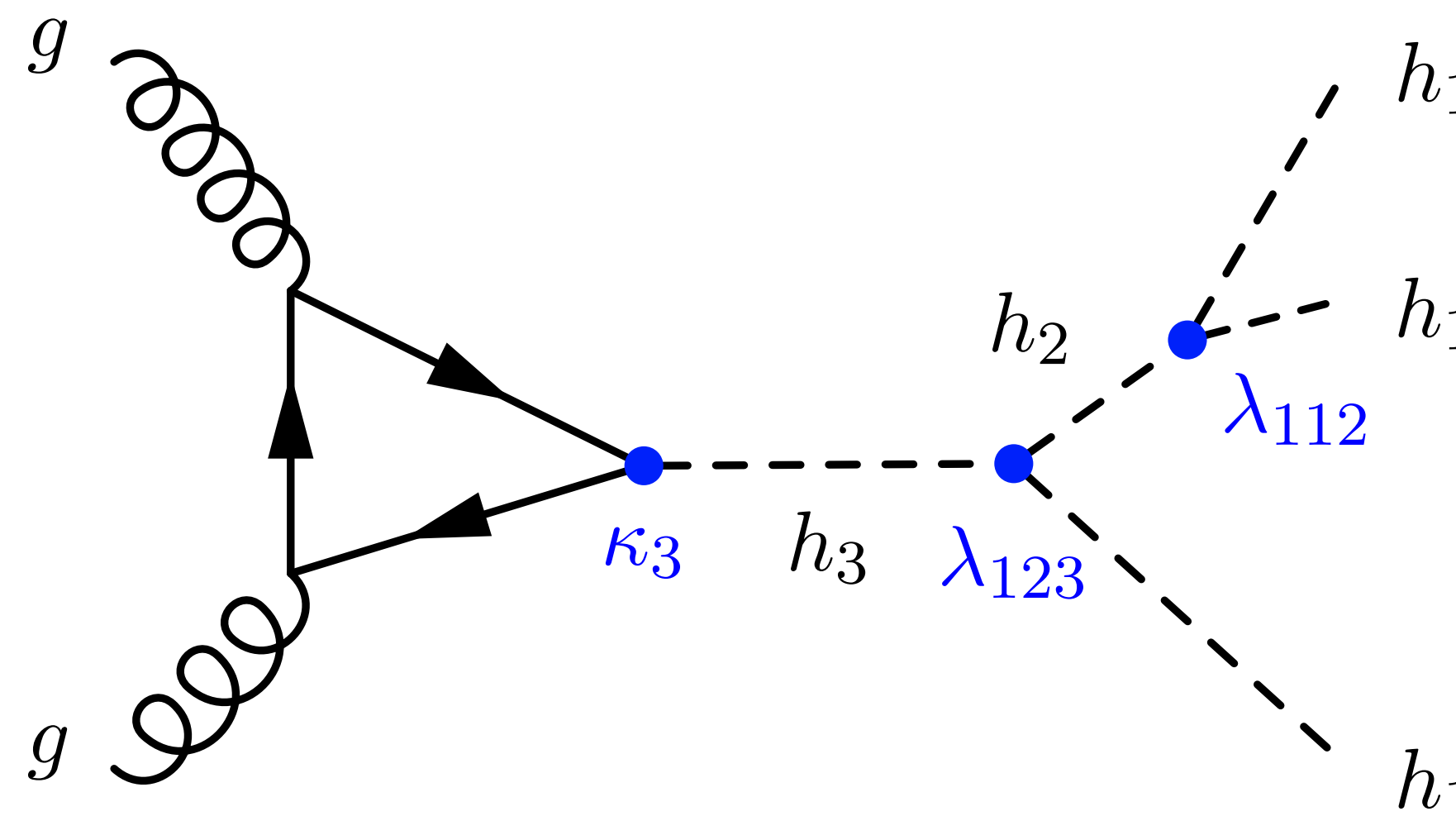
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→ Double-Resonant enhancement!



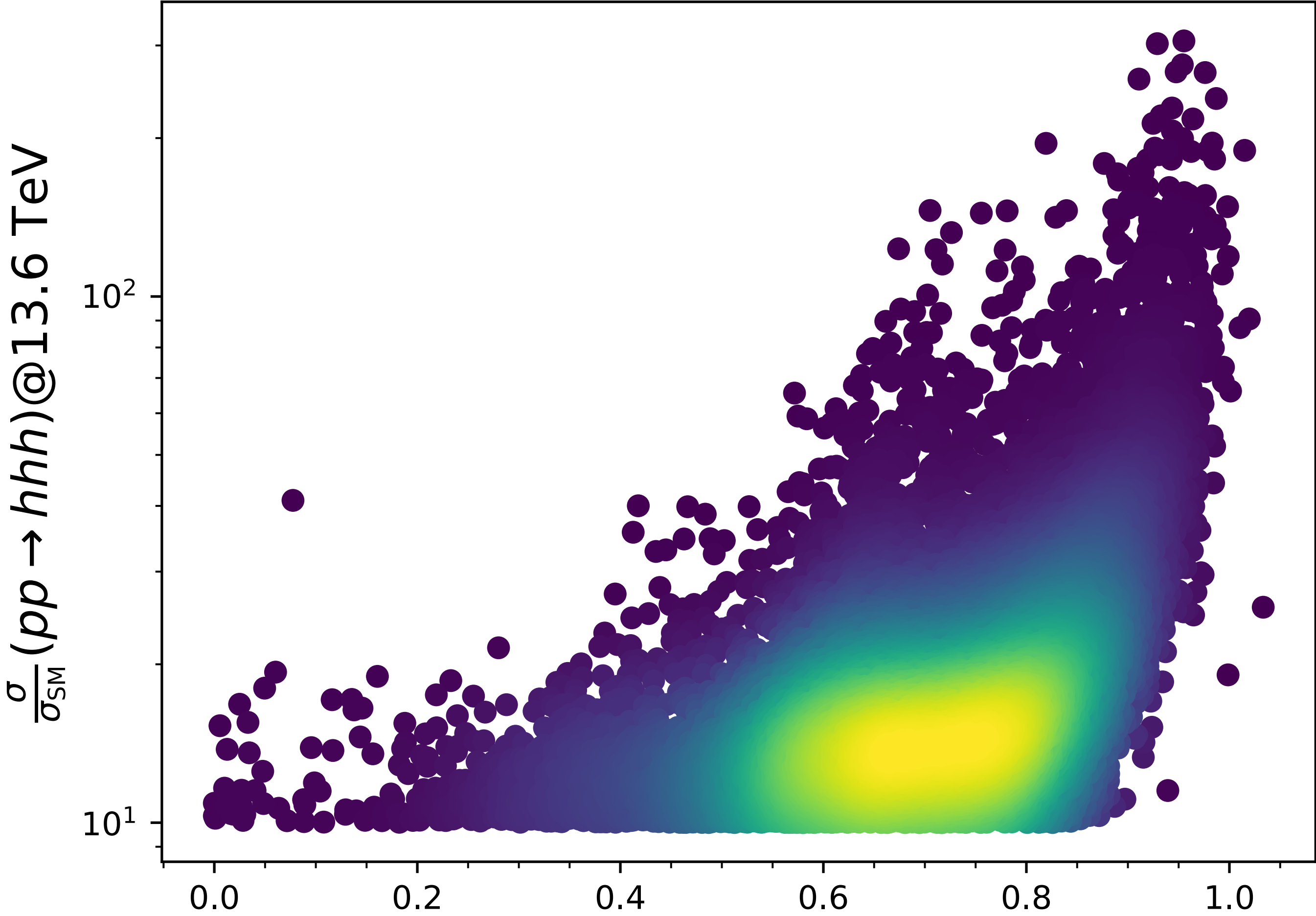
Requires:
 $m_3 > m_2 + m_1, m_2 > 2m_1$

Double-Resonant hhh in the TRSM

🎩 Enhancement of hhh $\Leftrightarrow$ Large “fraction” of double-resonant process!

Viable points with $\sigma > 10 \times \sigma_{\text{SM}}(pp \rightarrow hhh)@13.6 \text{ TeV}$

Enhancement
over SM

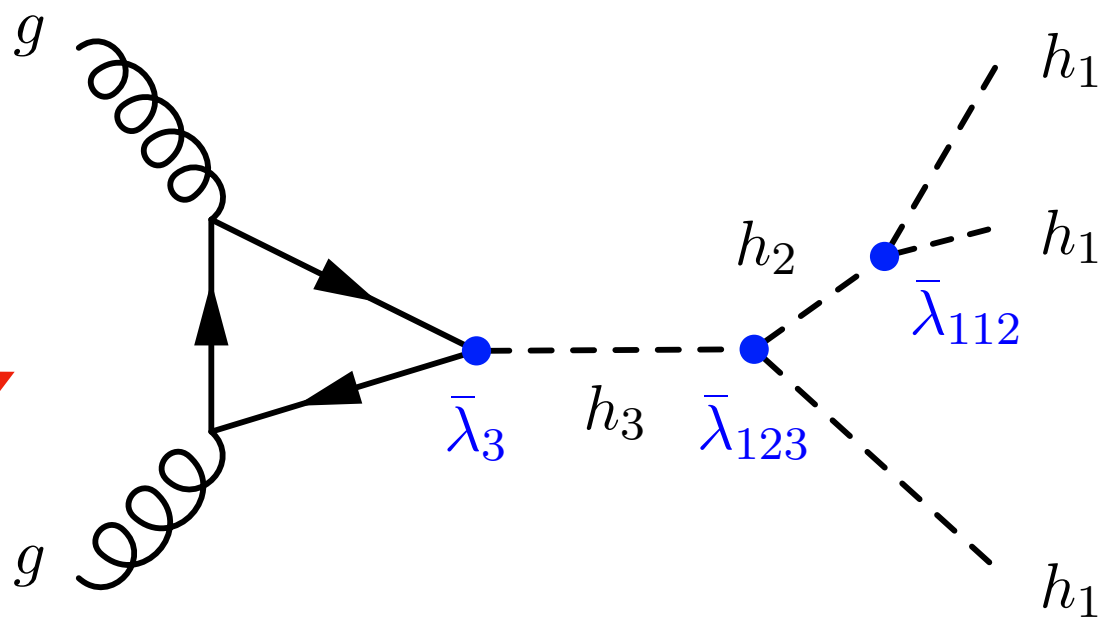


[Karkout, AP, Postma, Tetlalmatzi-Xolocotzi, van de Vis, du Pree, 2404.12425]

Including boundedness of potential + perturbativity + **HiggsTools** constraints.

BUT: NO First-Order EWPT! (see shortly!)

Resonant Fraction (R.F.) =
How much of the total cross section comes from... ?

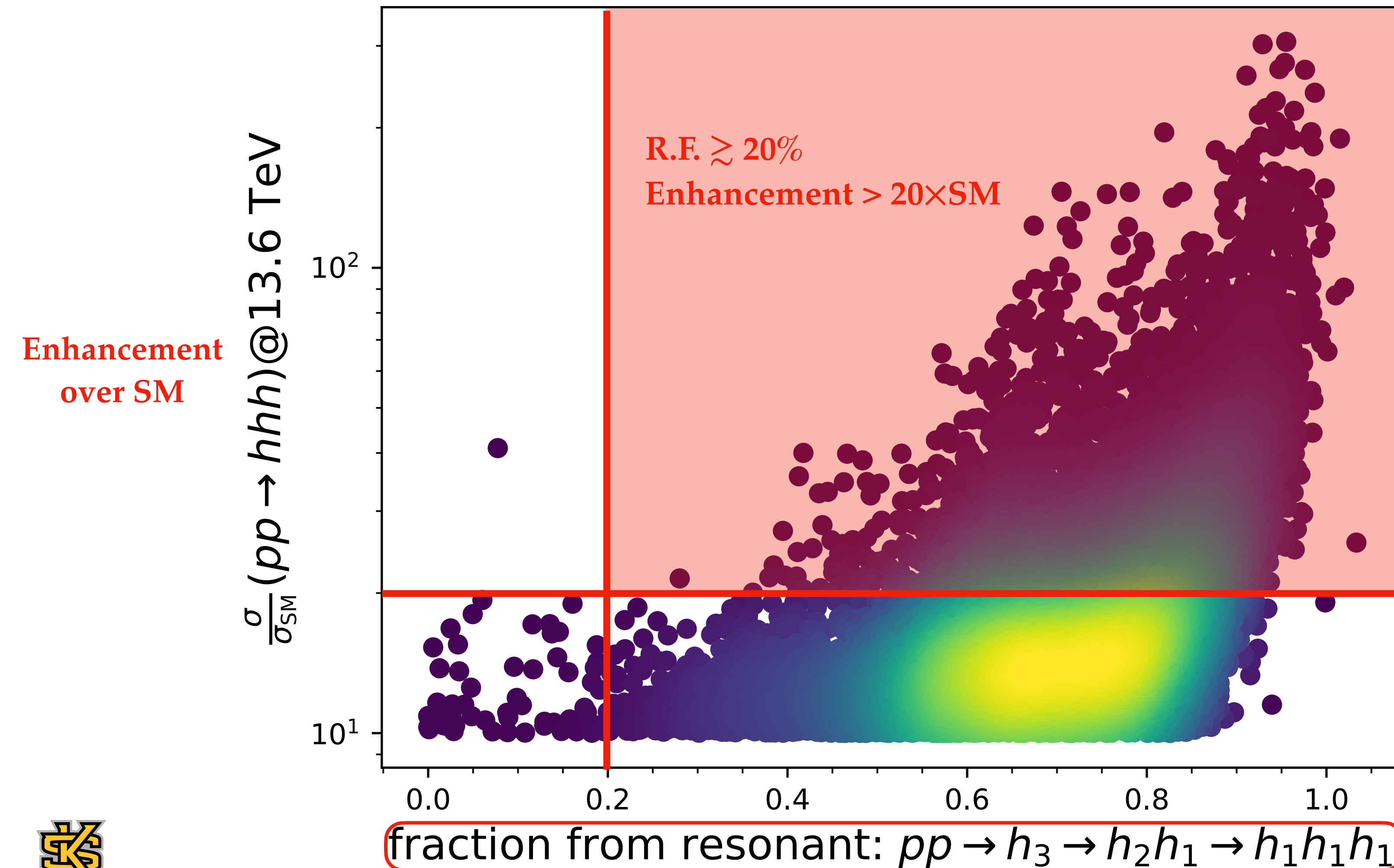


fraction from resonant: $pp \rightarrow h_3 \rightarrow h_2 h_1 \rightarrow h_1 h_1 h_1$

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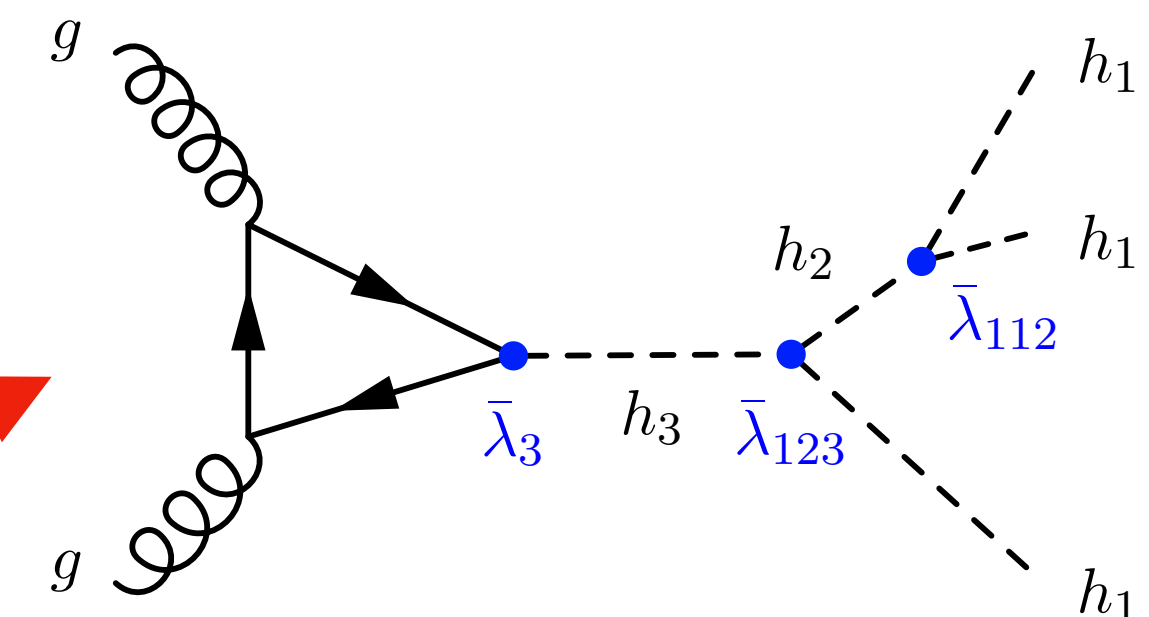


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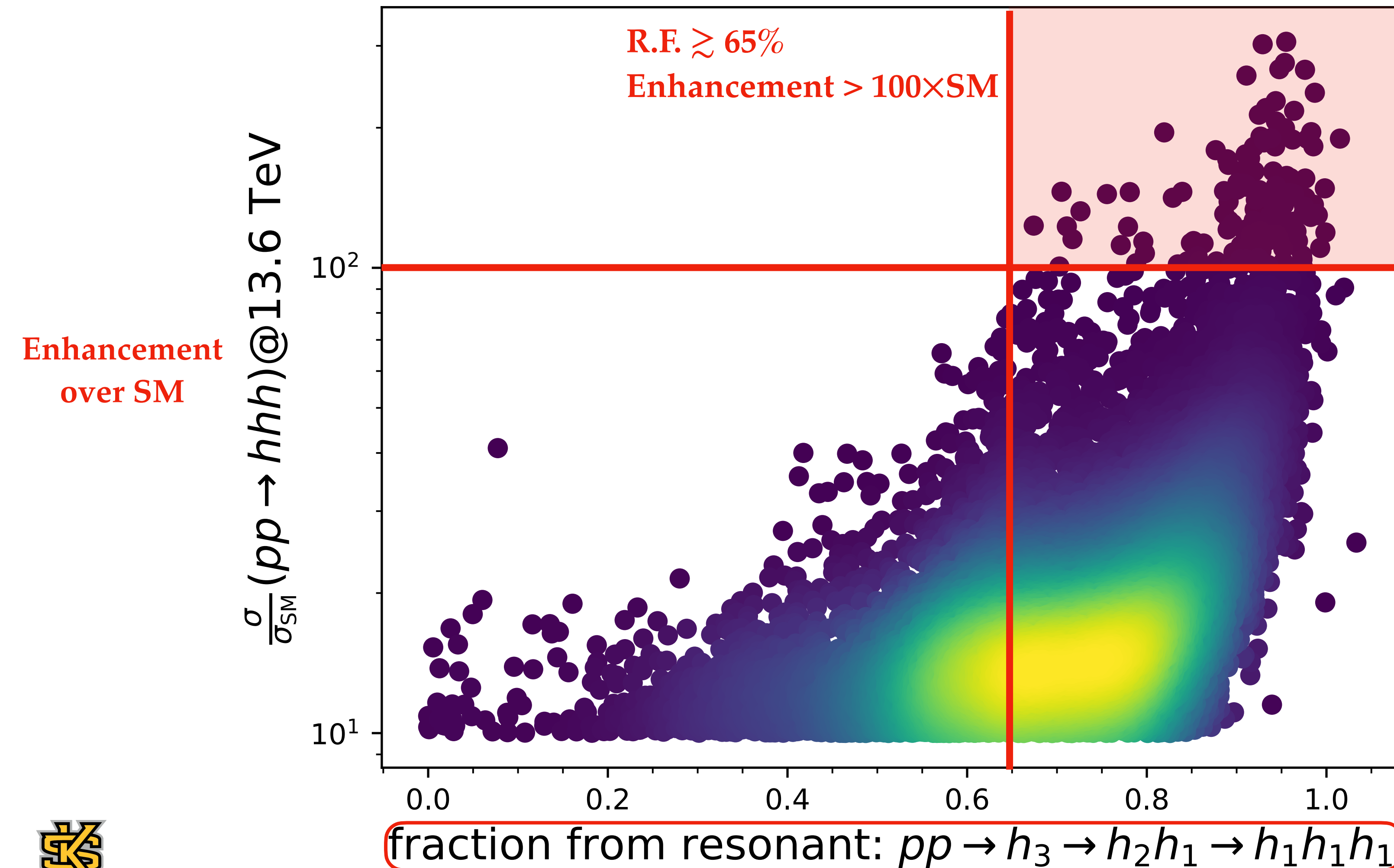
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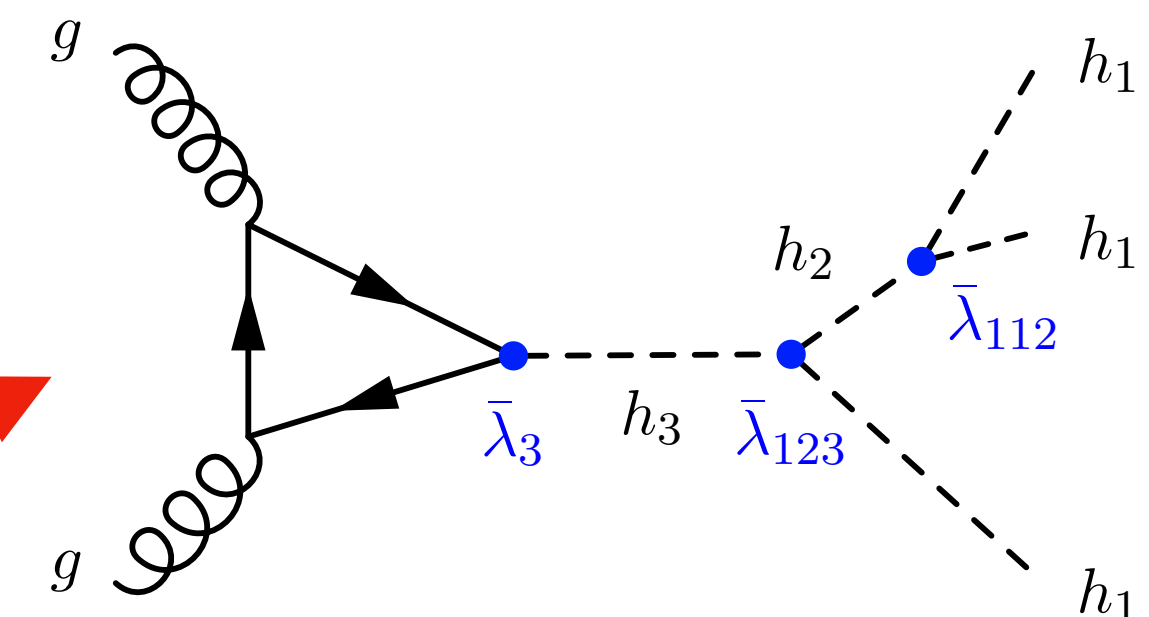


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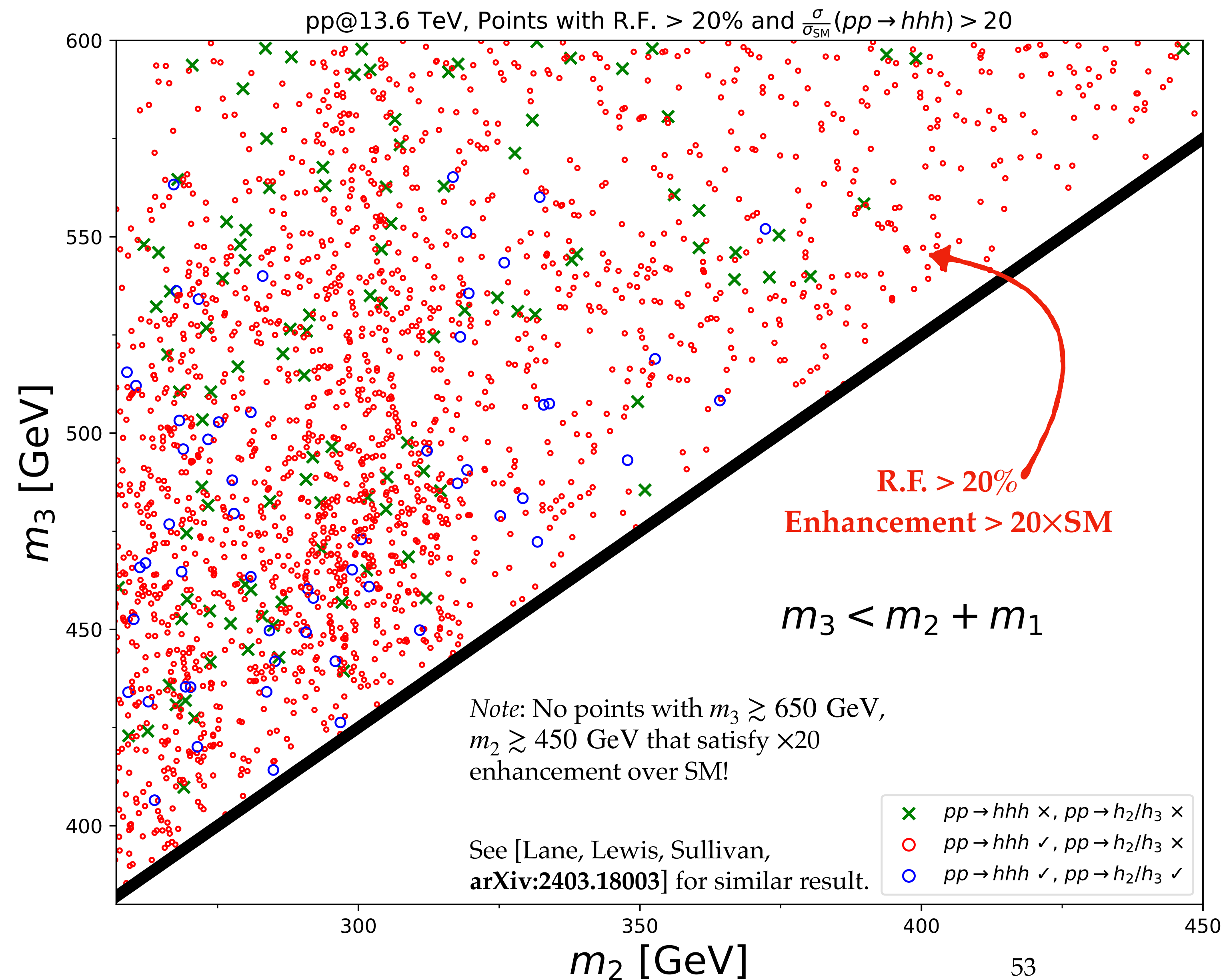
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Pheno of TRSM Benchmark Points @ HL-LHC



HL-LHC results:

$\circ$: Excluded ONLY by single h_2 and h_3 production, NOT by hhh ,

$\times$: Excluded ALSO by hhh ,

$\circ$: NOT excluded @ HL-LHC

($\Rightarrow$ Future Colliders?)

Notice: NO hhh exclusion without single h_2 and h_3 exclusion!

$\Rightarrow$ in TRSM, hhh is *unlikely* to be a “discovery” channel [$\sim$ *but: inverse problem?*]

EW Baryogenesis and hhh in the TRSM

👒 Q : Can there be a First-Order Electroweak Phase Transition in the TRSM?

👒 and if so, will this lead to enhanced hhh at the LHC?

[Karkout, AP, Postma, Tetlalmatzi-Xolocotzi,
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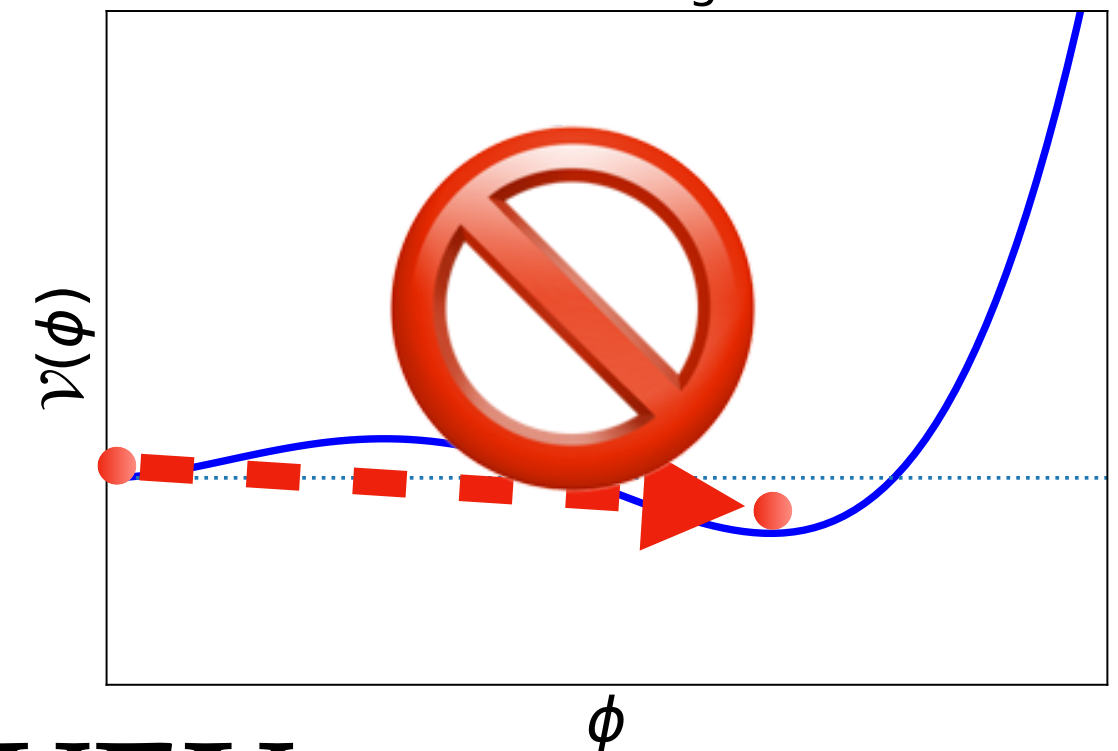
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[Karkout, AP, Postma, Tetlalmatzi-Xolocotzi, Tunneling!]

👒 **NO!**

- FO-EWPT & enhanced hhh are mutually exclusive!
- “barrier” not generated if both new scalars attain a VEV,
- and non-zero VEVs are necessary for sufficient mixing (and double-res hhh !)



👒 Removing the $\mathcal{F}_2$ restrictions might help!



CONCLUSIONS

- Higgs boson's properties & couplings are already very well determined @ LHC!
- But: Higgs potential** remains a flagship target for LHC & future colliders:
 - hh production will allow measurement of the trilinear ($\sim 50\%$ @LHC/ $\sim 5\%$ @FCC-hh).
 - hhh production of the quartic: but VERY challenging at ALL colliders:
 - *even* with large anom. couplings (c_3, d_4) & digging out the quartic is *hard*!
- hhh recommendation: Experiments should consider:
 - **Truncation** of cross section within the context of EFTs (?).
 - **More anomalous couplings.**



CONCLUSIONS

- Extended Scalar Sectors are phenomenologically rich:
 - can generate EW baryogenesis (e.g. $xSM + \text{many more}$).
 - & enhance hhh in $TRSM$ $\leadsto$ observable @ LHC!
 - e.g. $TRSM \rightarrow$ double-resonant enhancement:
 - $\rightarrow hhh$ (probably) *not* a $TRSM$ discovery channel!
- But: Info on inverse problem in $TRSM$!
- First-Order EWPT & enhanced hhh in $TRSM$ not simultaneous,
 - but could be achieved by relaxing $\mathcal{L}_2$ restrictions!



CONCLUSIONS

- Extended Scalar Sectors are phenomenologically rich:
 - can generate EW baryogenesis (e.g. xSM + *many more*).
 - & enhance hhh in $TRSM$ $\leadsto$ observable @ LHC!
- e.g. $TRSM \rightarrow$ double-resonant enhancement:
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- First-Order EWPT & enhanced hhh in $TRSM$ not simultaneous,
- but could be achieved by relaxing $\mathcal{L}_2$ restrictions!

THANKS! QUESTIONS?



APPENDICES

Parameter-space Categories

1. Define two conditions:

- i. VEV at 1-loop: $\langle \phi(T = 0) \rangle = 246 \pm 30 \text{ GeV}$ & deepest minimum.
- ii. $\langle \phi(T_c) \rangle / T_c > 1$ & no other transition with higher T_c .

2. Define four mutually-exclusive categories*:

SFO-EWPT more certain

*An alternative classification appears in our article: see Appendix.

Liberal

Centrist

Conservative

Ultra-Conservative

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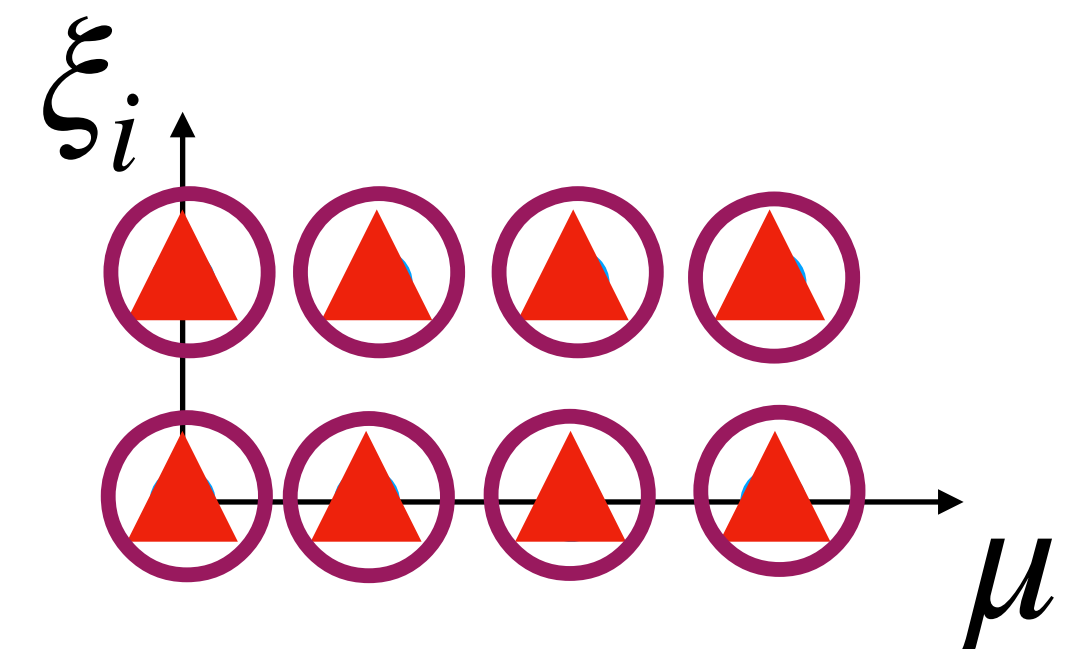
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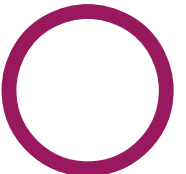
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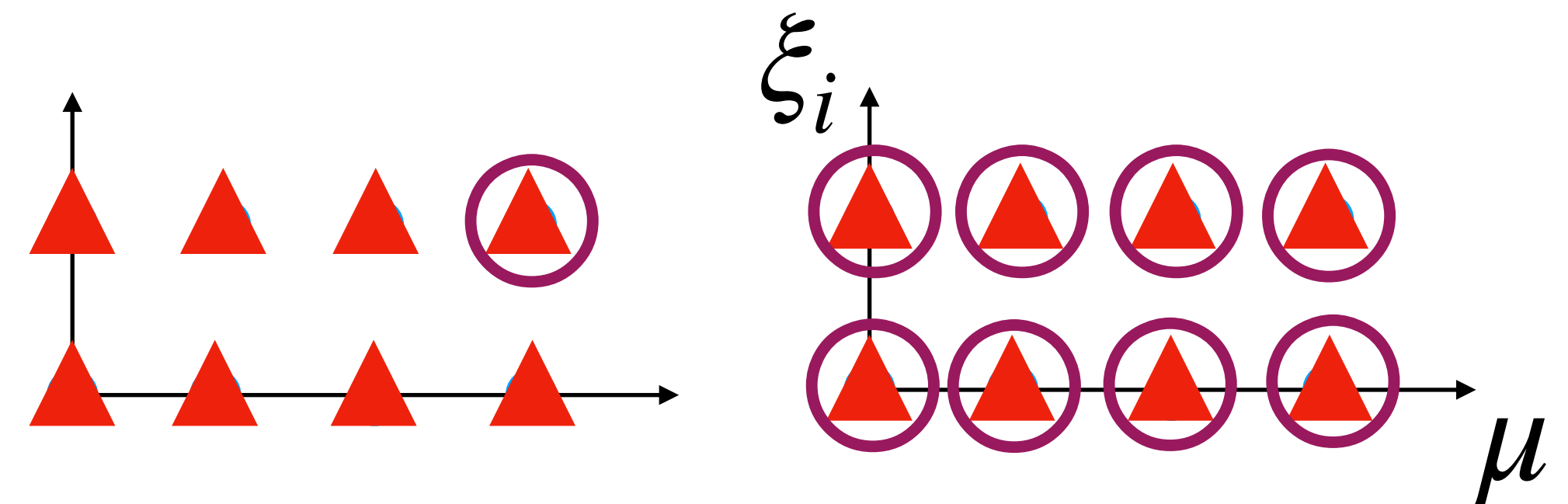
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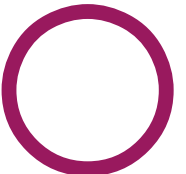
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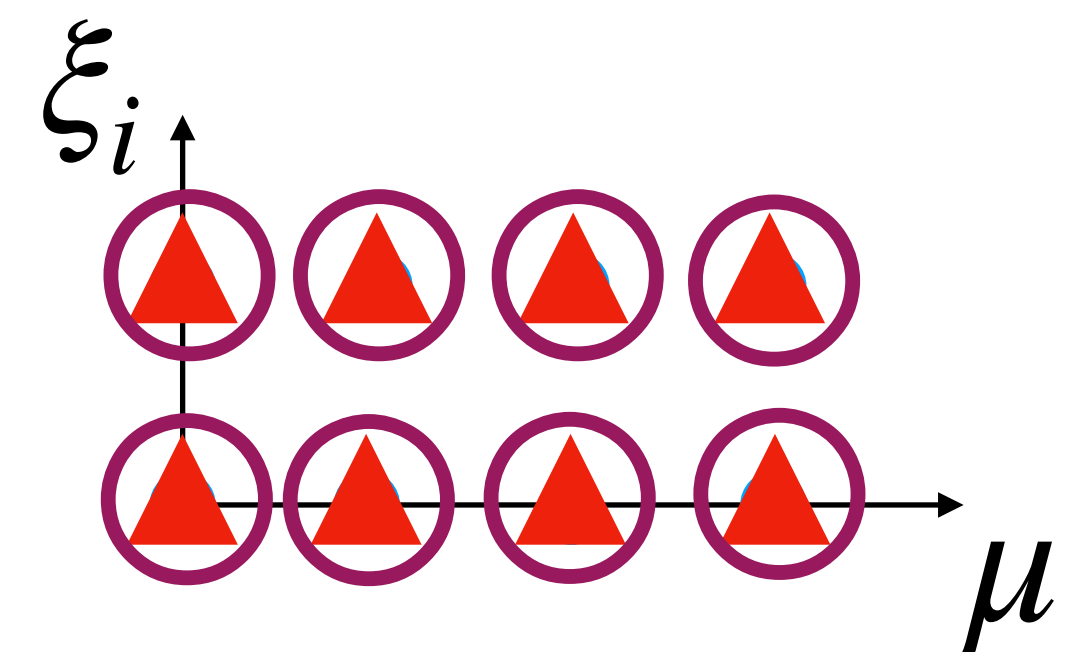
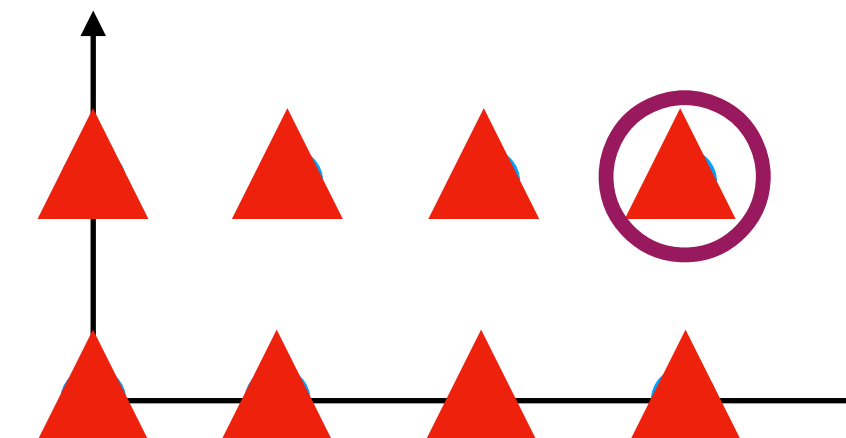
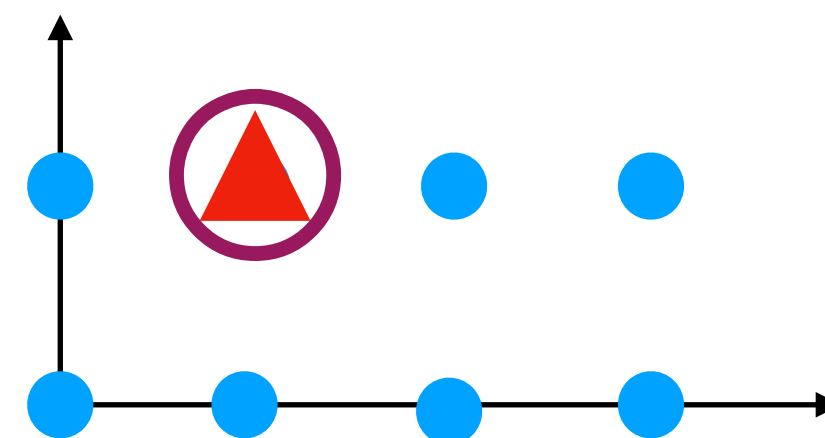
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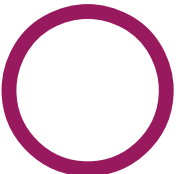
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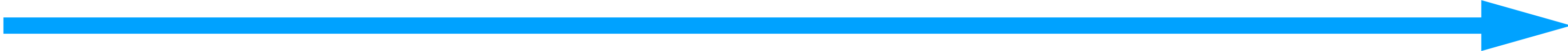
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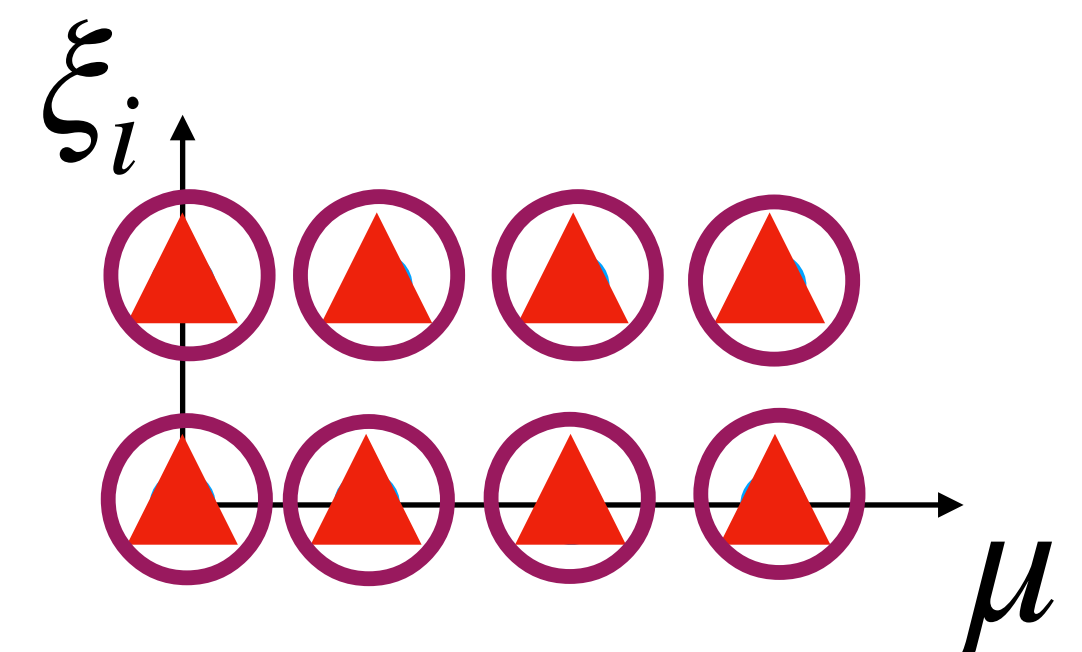
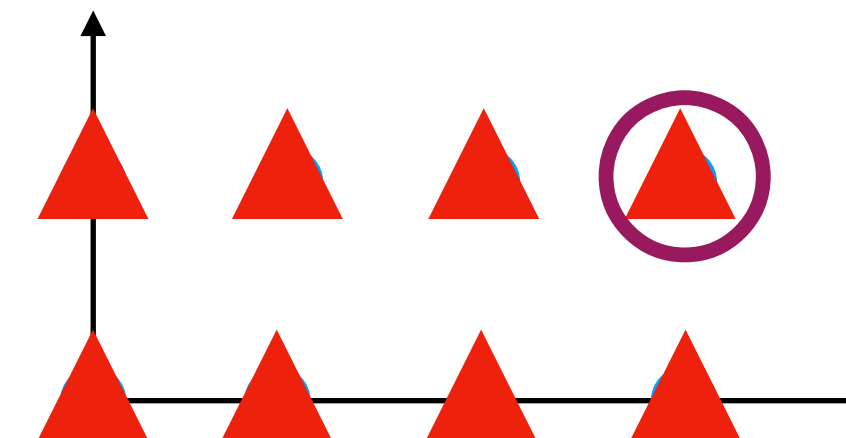
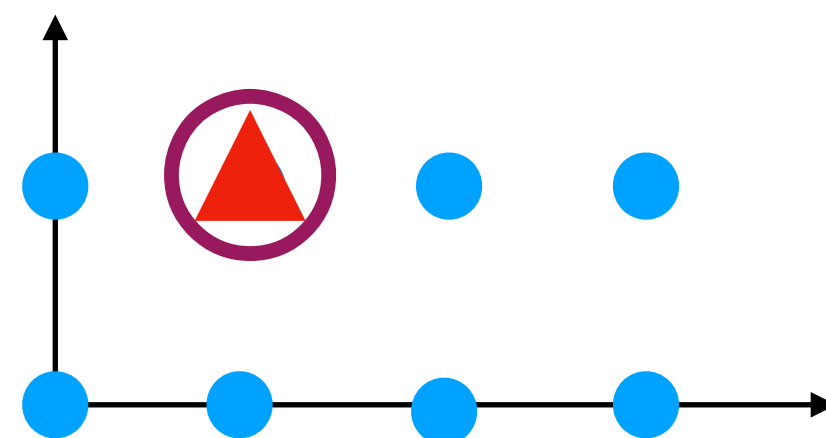
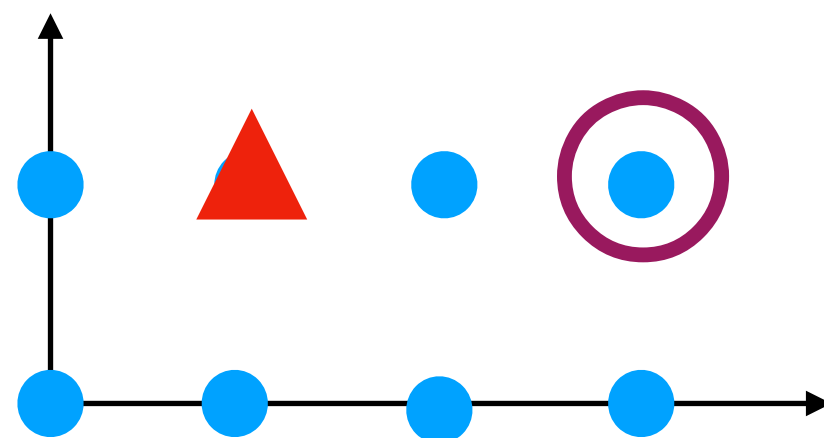
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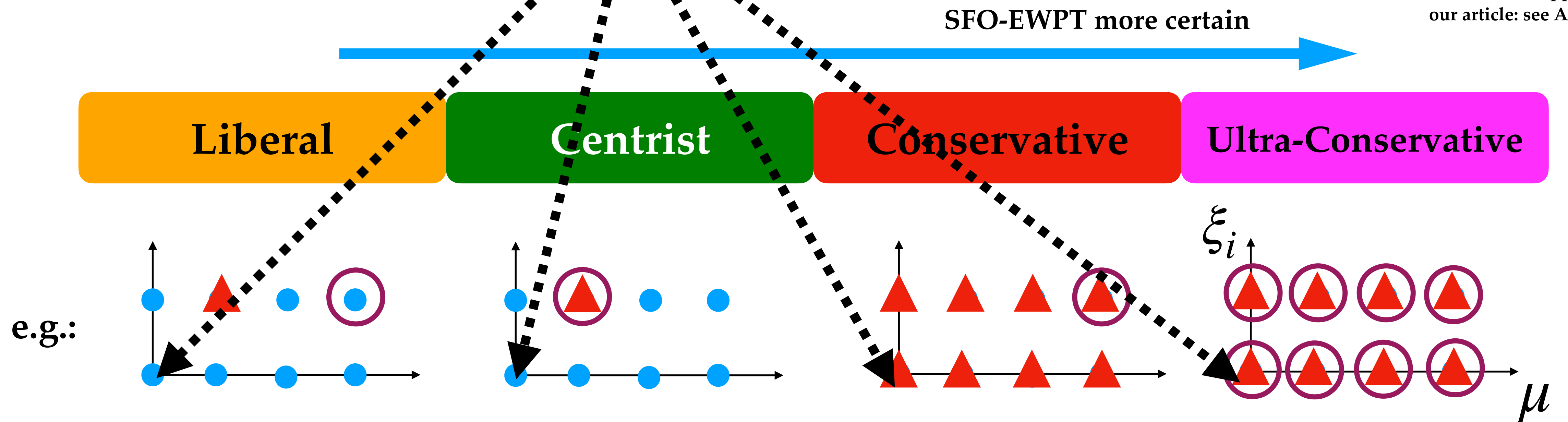
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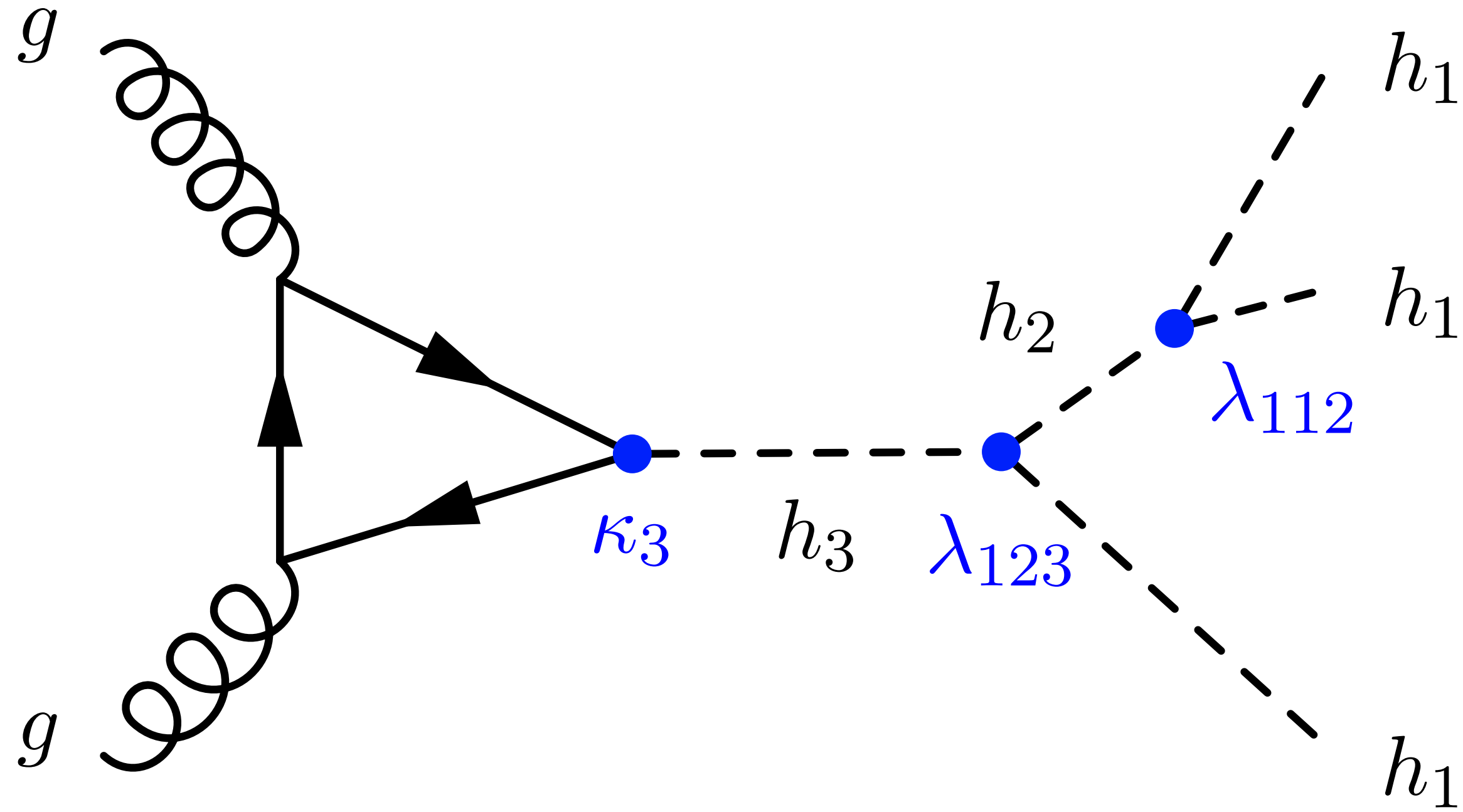
Note: For phenomenological analyses, take “central” μ and ξ_i .

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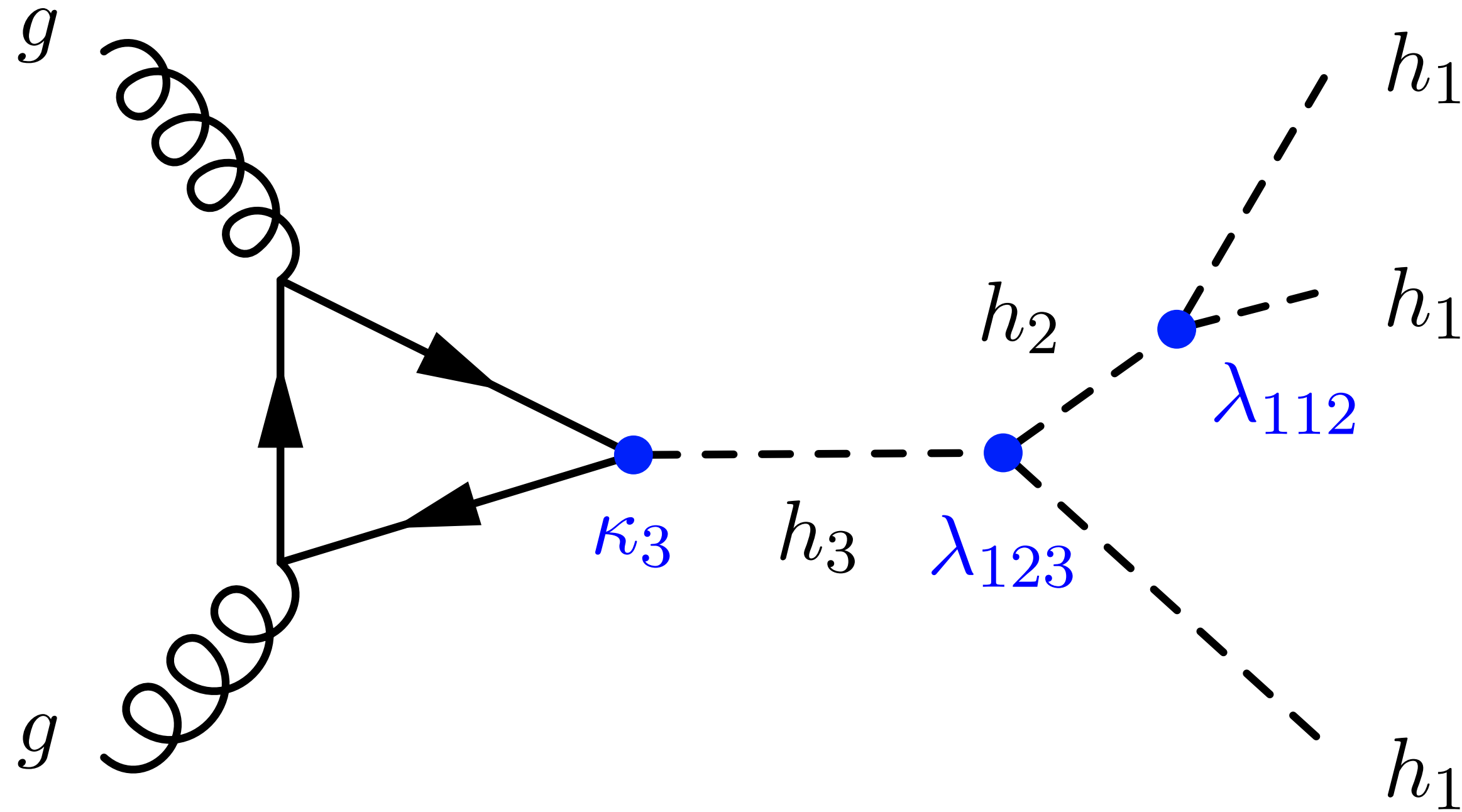
*An alternative classification appears in our article: see Appendix.



Anatomy of Double-Resonant *hhh*



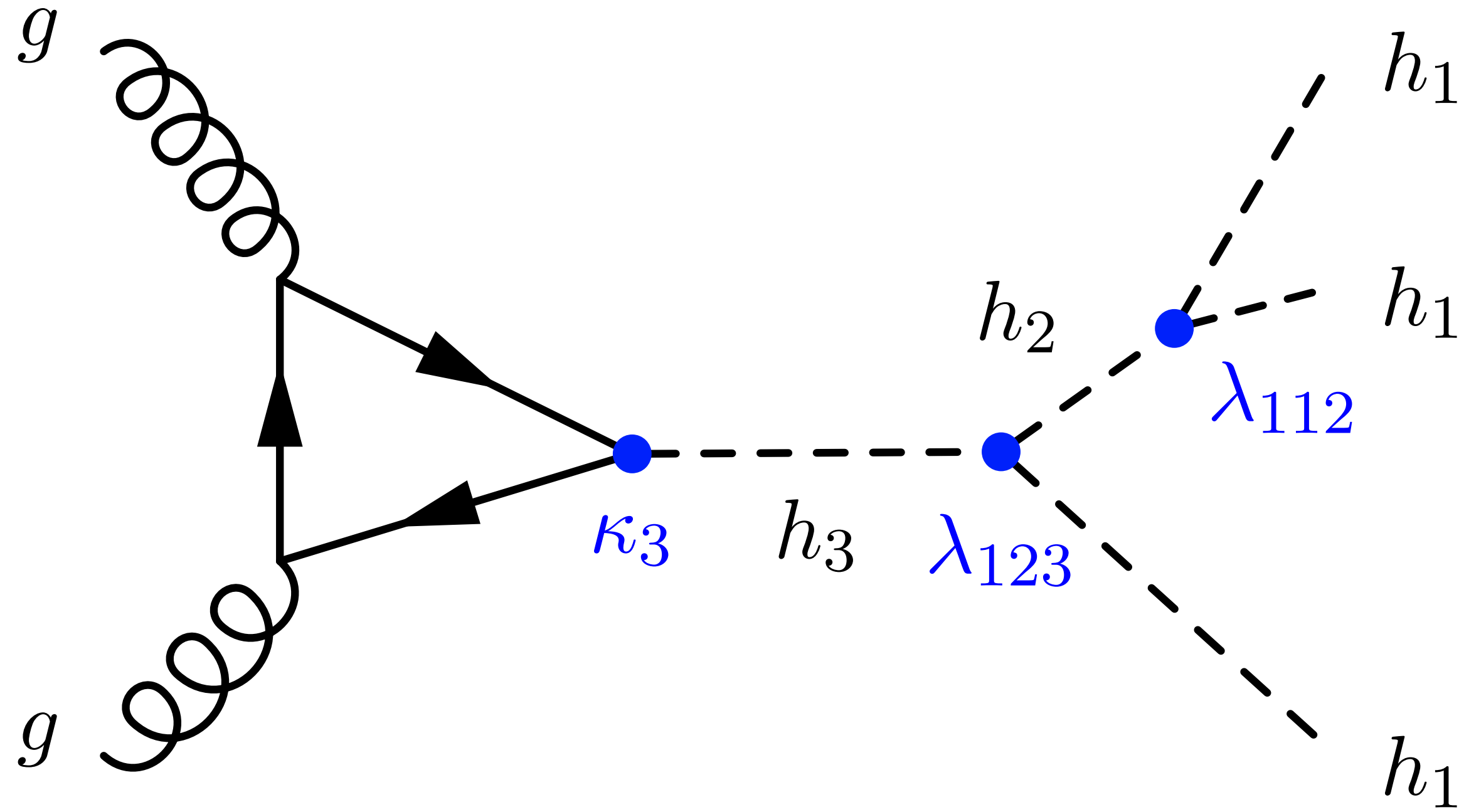
Anatomy of Double-Resonant *hhh*



$$\sigma(m_2, m_3) = \sigma_u(m_2, m_3) \times \underbrace{\kappa_3^2 \lambda_{123}^2 \lambda_{112}^2}_{\text{Factor out couplings}}$$

① Factor out couplings

Anatomy of Double-Resonant hhh



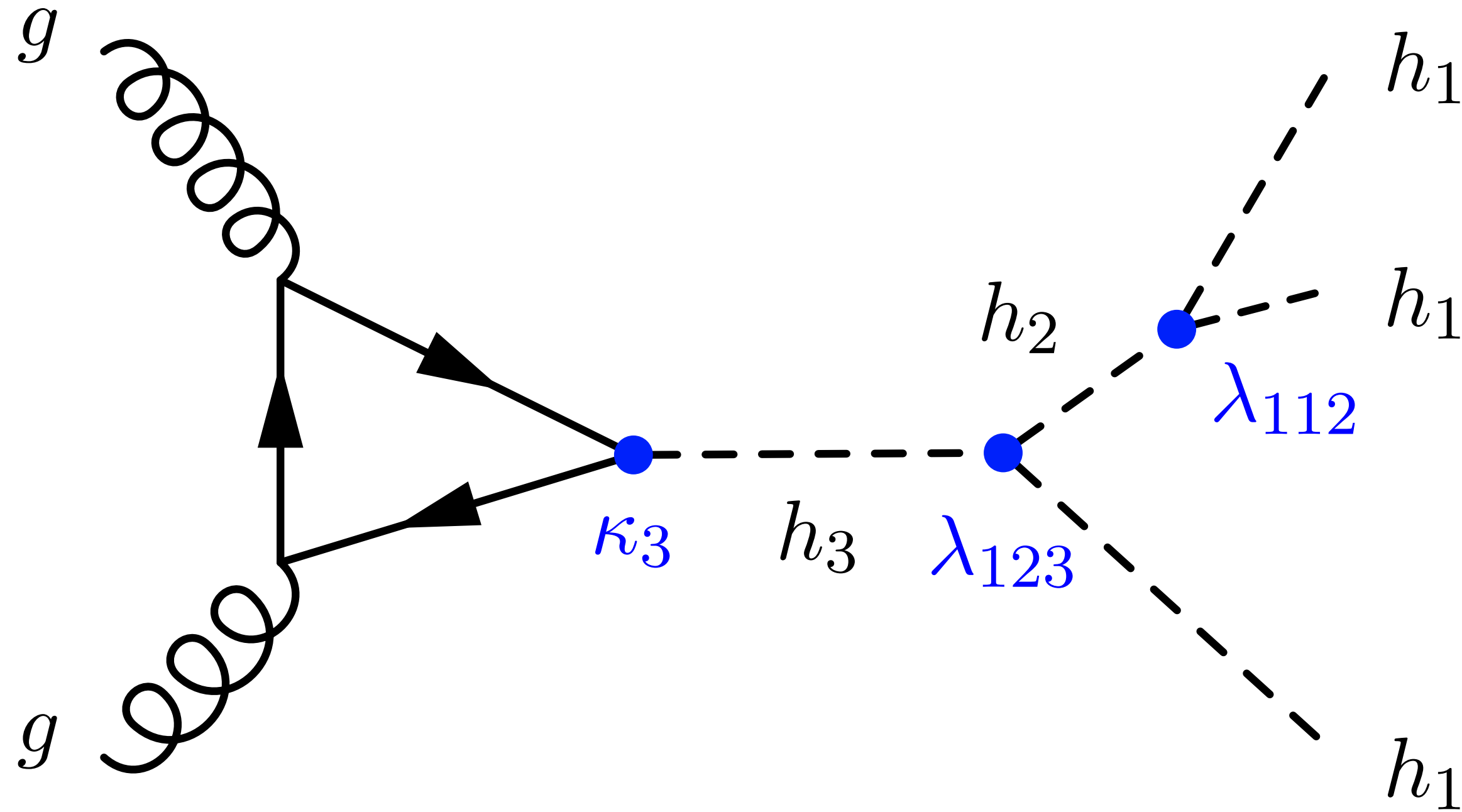
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① Factor out couplings

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$$\frac{dq_i^2}{(q_i^2 - m_i^2)^2 + m_i^2 \Gamma_i^2} \rightarrow \frac{\pi}{m_i \Gamma_i} \delta(q_i^2 - m_i^2) dq_i^2$$

Anatomy of Double-Resonant $h h h$



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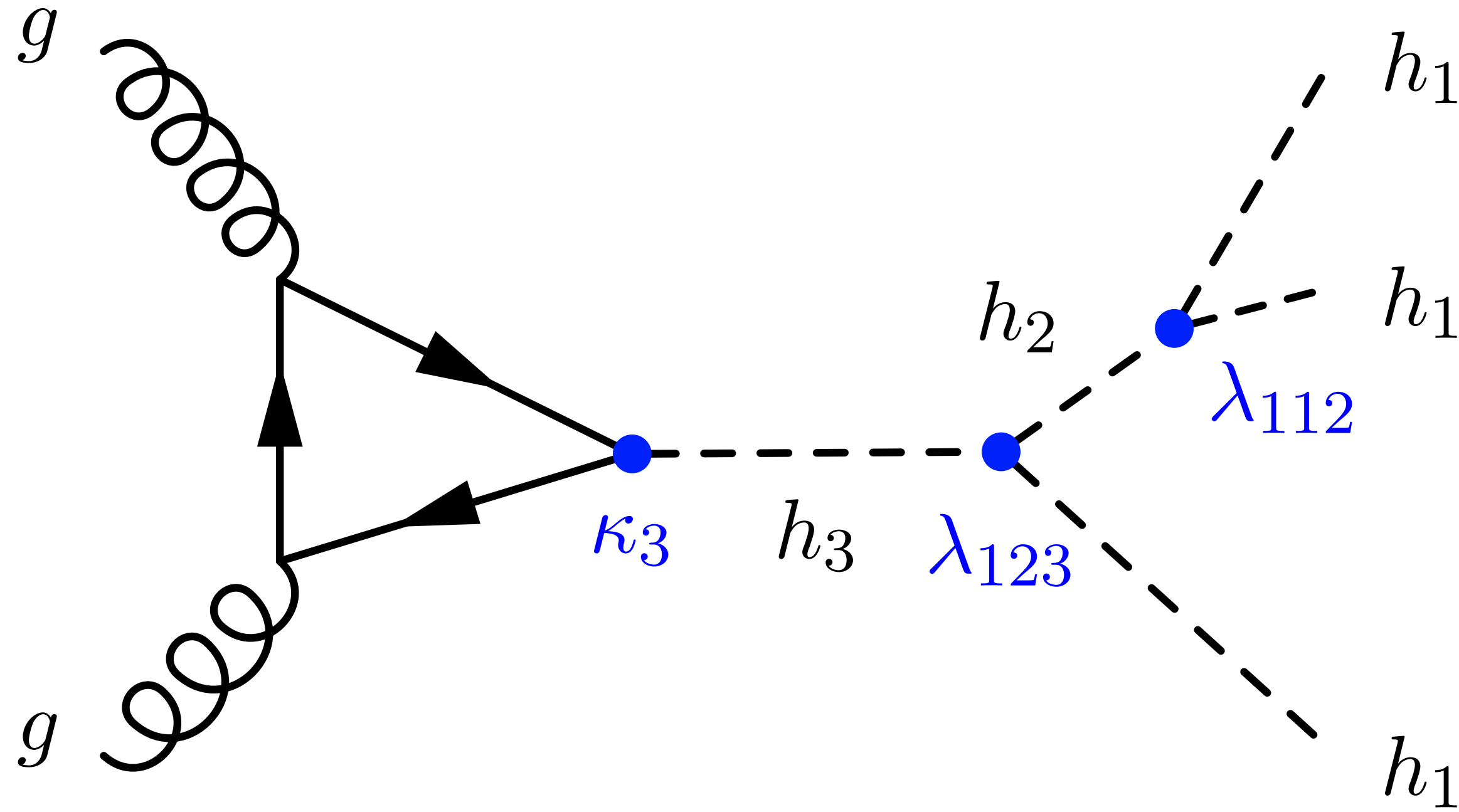
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depends only on masses

③ Obtain: $\sigma(m_2, m_3) = \hat{\sigma}_u(m_2, m_3) \times \frac{\kappa_3^2 \lambda_{123}^2 \lambda_{112}^2}{\Gamma_2 \Gamma_3}$

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depends only on masses

③ Obtain: $\sigma(m_2, m_3) = \hat{\sigma}_u(m_2, m_3) \times \left(\frac{\kappa_3^2 \lambda_{123}^2 \lambda_{112}^2}{\Gamma_2 \Gamma_3} \right)$

④ Define: $\rho^2 \equiv \kappa_3^2 \lambda_{123}^2 \lambda_{112}^2 / (\Gamma_2 \Gamma_3)$

“rescaling factor”

*Narrow width: See appendix!

Andreas Papaefstathiou

What if...?

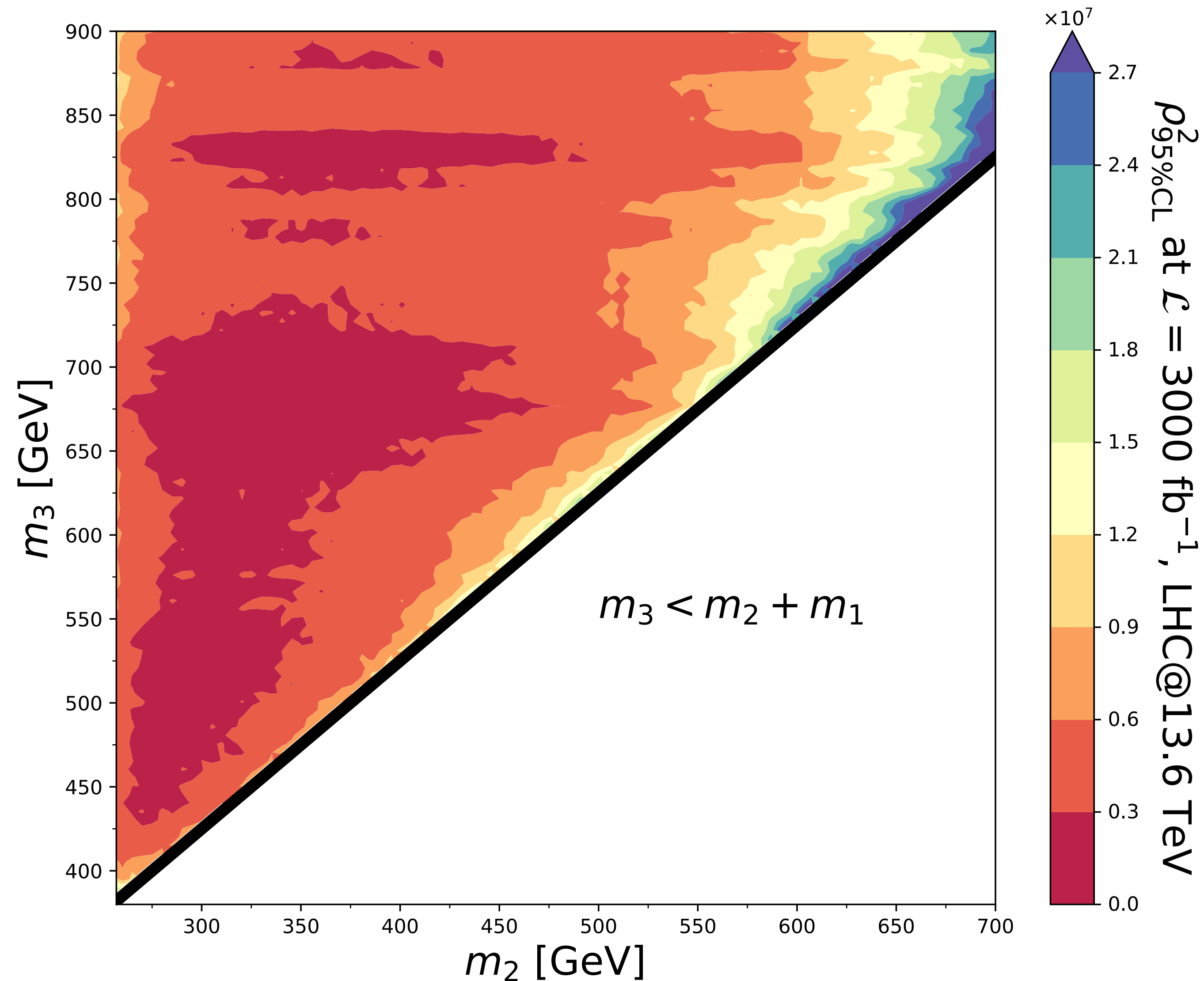
- Let's suppose two new scalars h_2 and h_3 are discovered (🍾🍾):
 - ➡ m_2, m_3 [and possibly] the widths Γ_2, Γ_3 would be known.
- hhh can provide relevant **information** on the theoretical parameter space.
 - ➡ An important contribution to solving the inverse problem!
 - ➡ through rescaling factor $\rho^2 = \kappa_3^2 \lambda_{123}^2 \lambda_{112}^2 / (\Gamma_2 \Gamma_3)$ (if narrow width!*)
- We derived **constraints** on ρ^2 via: $pp \rightarrow (b\bar{b})(b\bar{b})(b\bar{b}) \rightarrow 6 \text{ b-jets}$.
 - ➡ [$\sim 20\%$ of the hhh final state.]



[AP, Tetlalmatzi-Xolocotzi, Zaro,
arXiv:1909.09166,
AP, Robens, Tetlalmatzi-Xolocotzi,
arXiv:2101.00037,
AP, Tetlalmatzi-Xolocotzi,
arXiv:2501.14866]

*Narrow width: See appendix!

Constraints on $\rho^2 = \kappa_3^2 \lambda_{123}^2 \lambda_{112}^2 / (\Gamma_2 \Gamma_3)$ on the (m_2, m_3) -plane



95% C.L. Constraint on ρ^2
at HL-LHC.

[AP, Tetlalmatzi-Xolocotzi, arXiv:2501.14866]

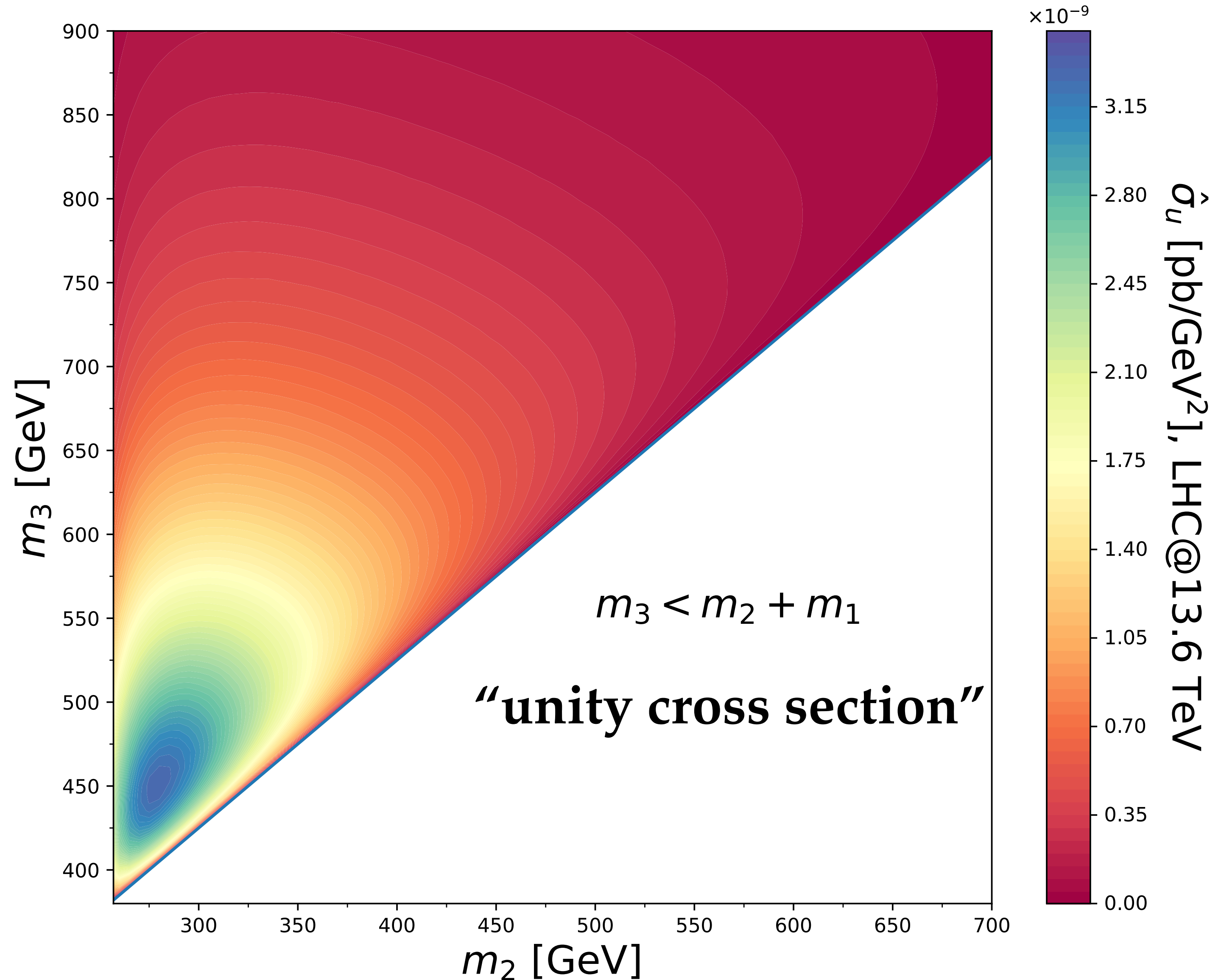
Anatomy of Double-Resonant *hhh*

$$\sigma(m_2, m_3) = \underbrace{\hat{\sigma}_u(m_2, m_3)}_{\text{"unity cross section"}} \times \rho^2$$

“unity cross section”:
depends only on *m*₂, *m*₃.
Derived once and for all!
(at fixed collider energy)

with: $\rho^2 \equiv \kappa_3^2 \lambda_{123}^2 \lambda_{112}^2 / (\Gamma_2 \Gamma_3)$

“rescaling factor”:
couplings and widths



TRSM Benchmarks from: [Karkout, AP, Postma, Tetlalmatzi-Xolocotzi, van de Vis, du Pree, **arXiv:2404.12425**]

Benchmark quantities relevant for double-resonant triple Higgs boson production									
Name	m_2	m_3	Γ_2	Γ_3	κ_3	λ_{123}	λ_{112}	$\frac{[\text{GeV}^2]}{\rho^2}$ $[\times 10^6]$	$\frac{[\text{pb/GeV}^2]}{\hat{\sigma}_u}$ $[\times 10^{-9}]$
BM0	259.0	495.0	0.003514	3.927	0.1854	-191.8	8.167	6.11	2.018
BM1	270.6	444.7	0.5078	2.586	0.1571	-204.3	67.52	3.574	3.408
BM2	268.6	452.7	0.3805	3.142	0.1741	-203.6	57.78	3.509	3.165
BM3	272.6	480.7	0.2009	4.758	0.2024	-224.6	41.39	3.703	2.908
BM4	269.0	409.8	0.2836	1.995	0.1713	-180.3	48.89	4.031	2.663
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BM7	283.7	575.0	0.001056	5.587	0.1884	193.5	-3.578	2.885	1.711
BM8	264.3	469.3	0.3916	2.941	0.1746	-144.3	55.88	1.721	2.789
BM9	266.5	461.9	0.3092	2.042	0.1635	142.8	39.98	1.381	3.29
BM10	259.2	399.7	0.2188	0.9312	0.1463	121.2	35.41	1.936	2.159

- Including:
- **EXP constraints** through HiggsTools.
 - **TH constraints:** perturbativity & boundedness from below.

[AP, Tetlalmatzi-Xolocotzi, **arXiv:2501.14866**]

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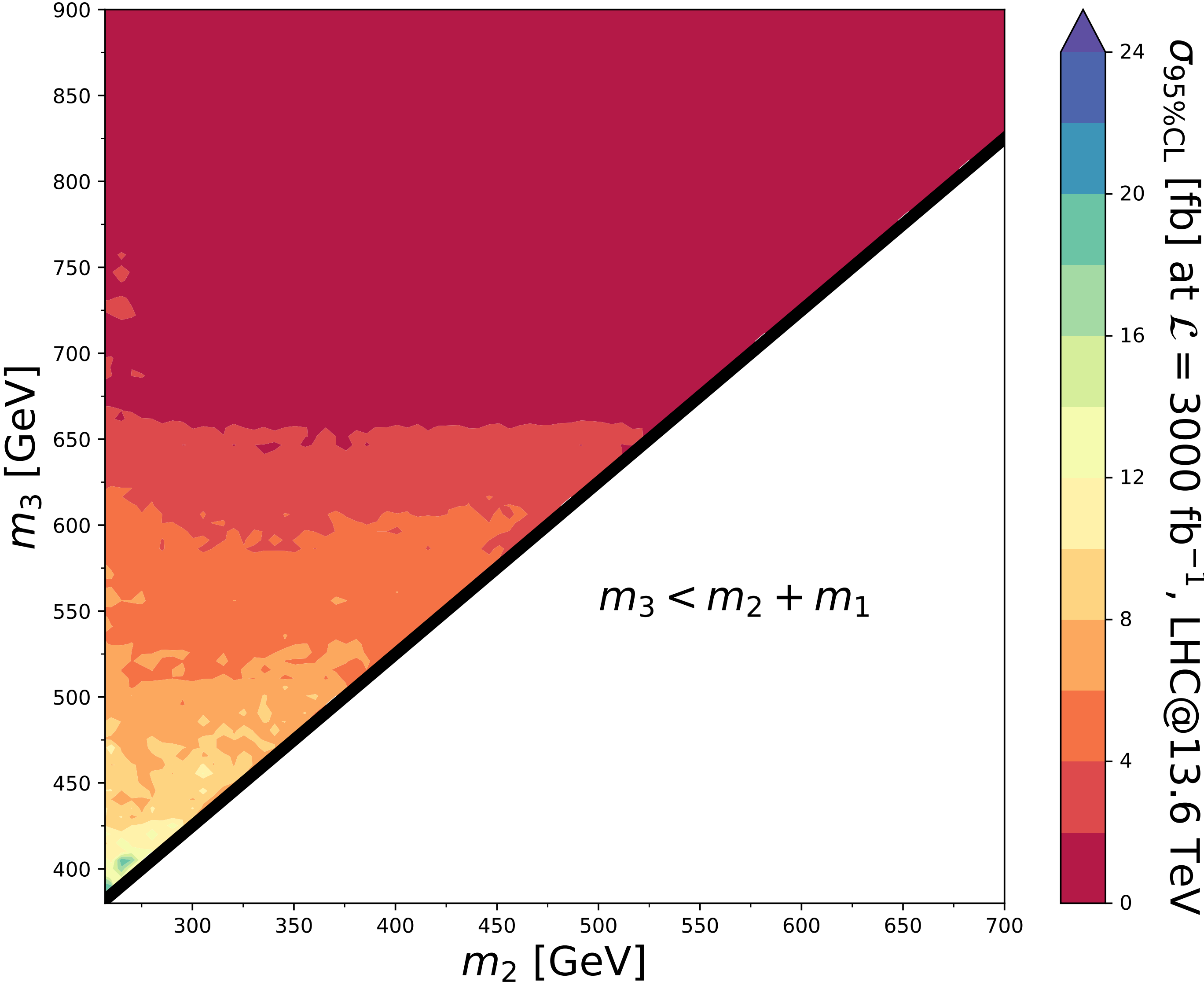
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➡ $\sigma(13.6 \text{ TeV}) \gtrsim \mathcal{O}(10) \text{ fb}$
[AP, Tetlalmatzi-Xolocotzi, arXiv:2501.14866]

➡ **Narrow width OK!**

Constraints on the Cross Section



95% C.L. Constraint on
the cross section
at HL-LHC.

[AP, Tetlalmatzi-Xolocotzi, arXiv:2501.14866]

Comment: Validity of the Narrow-Width Approximation

- In the TRSM: h_2 and h_3 are **constrained** to possess small mixing angles.

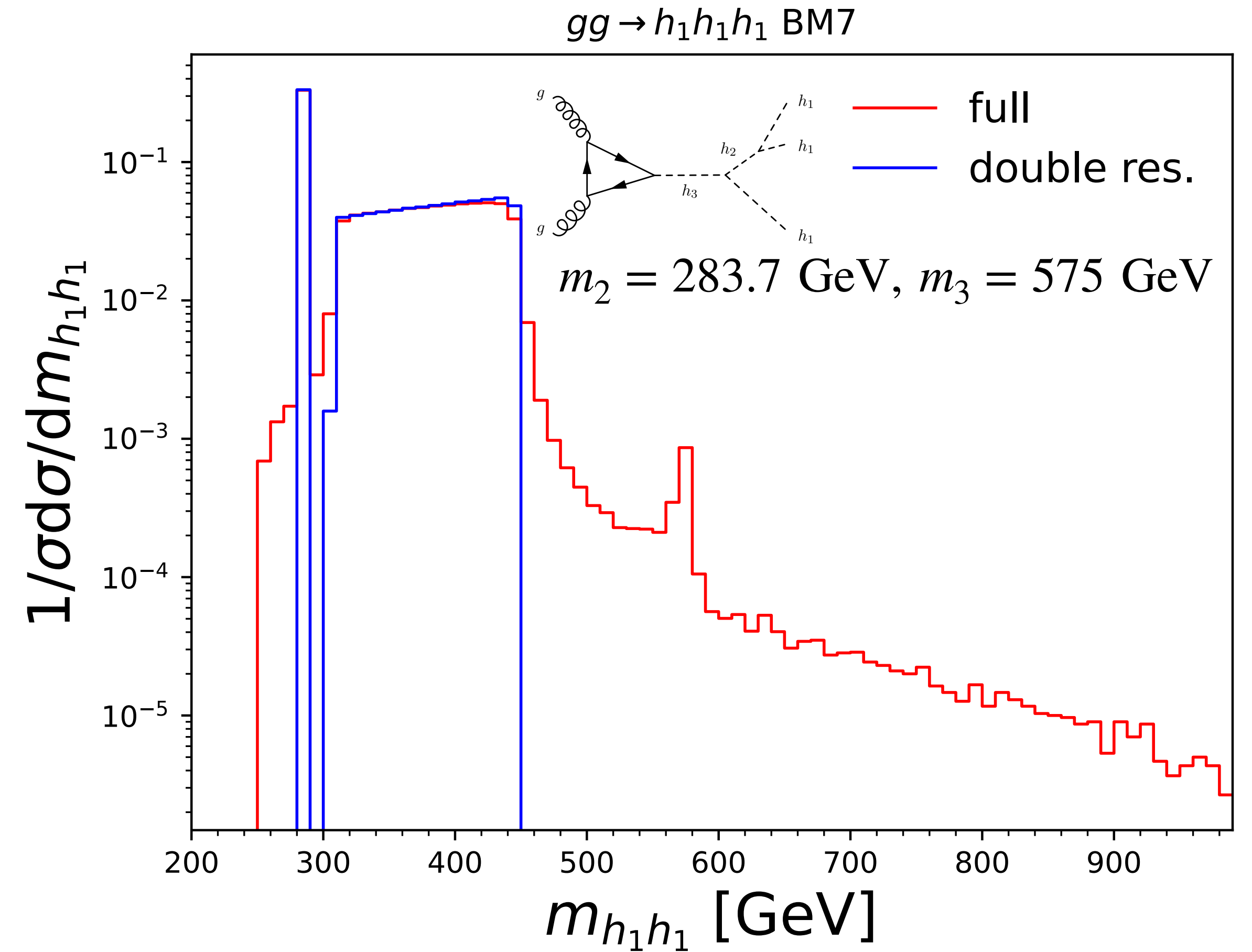
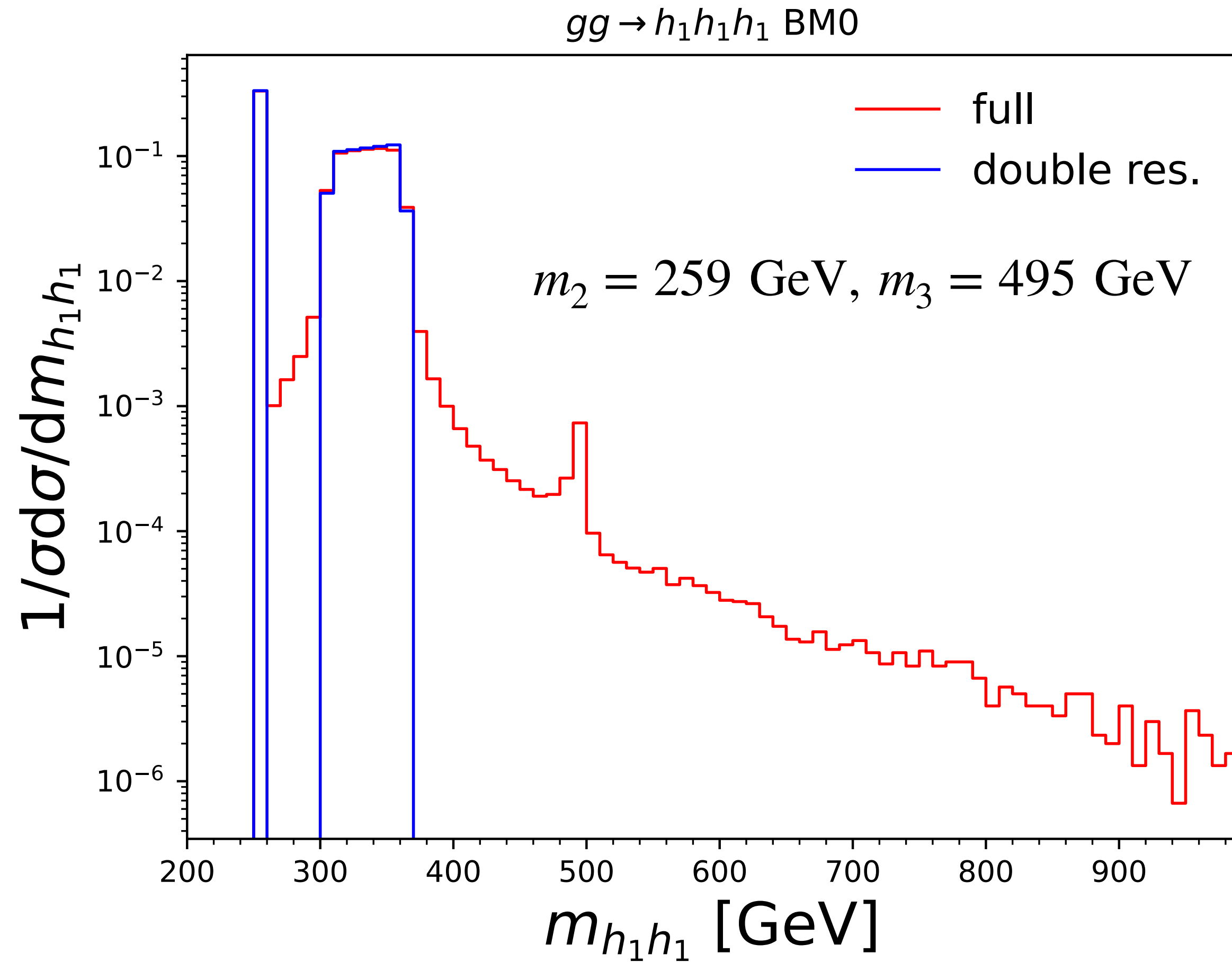
(e.g. we know this from h_1 signal strength)

⇒ Contributions to width from $h_{2,3} \rightarrow f\bar{f}, VV$ are small!

- In general: Can increase width of h_2 and h_3 through scalar-to-scalar decays ⇒ no guarantee in generic models for the narrow width.
- If h_2 and h_3 are already discovered, Γ_2, Γ_3 would be known (or limited) ⇒ the narrow-width approximation validity should be checked!
- **TL;DR**: The narrow width approximation is OK due to mixing constraints in the TRSM, but this statement is somewhat model dependent! [***see appendix for a case study!]

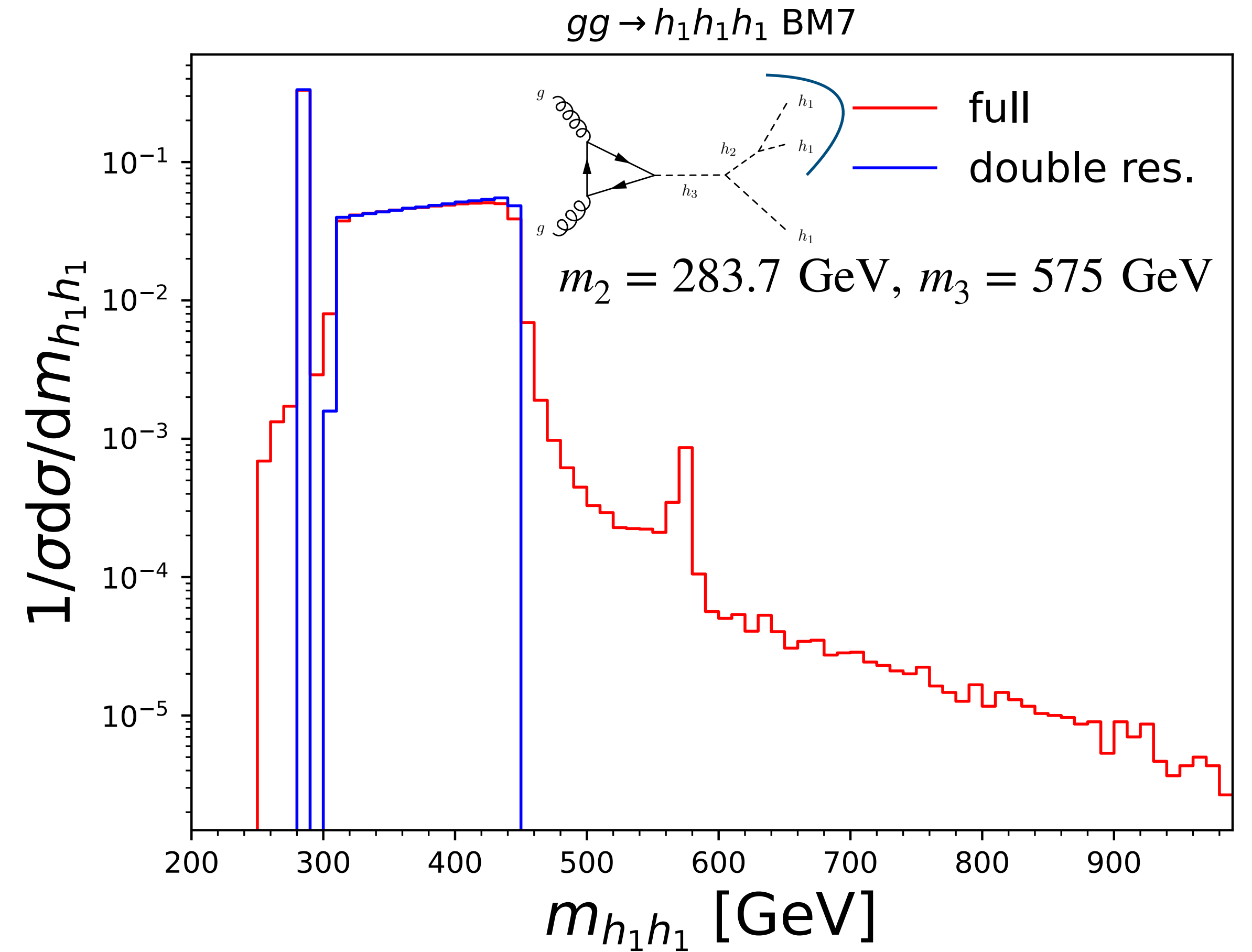
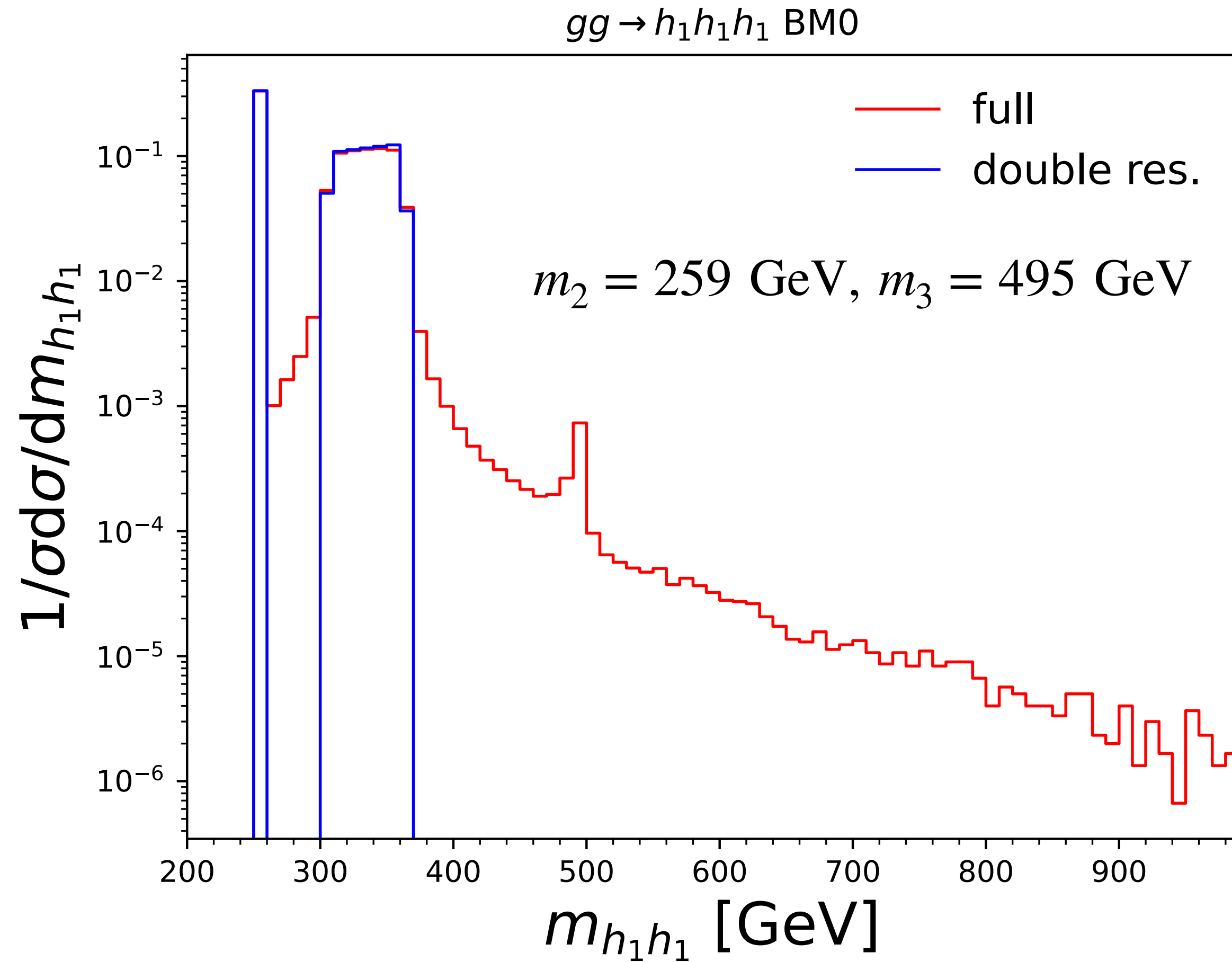
Non-Resonant Effects in hhh in the TRSM

(any) two Higgs invariant mass distributions in hhh (“parton level”, all combinations)



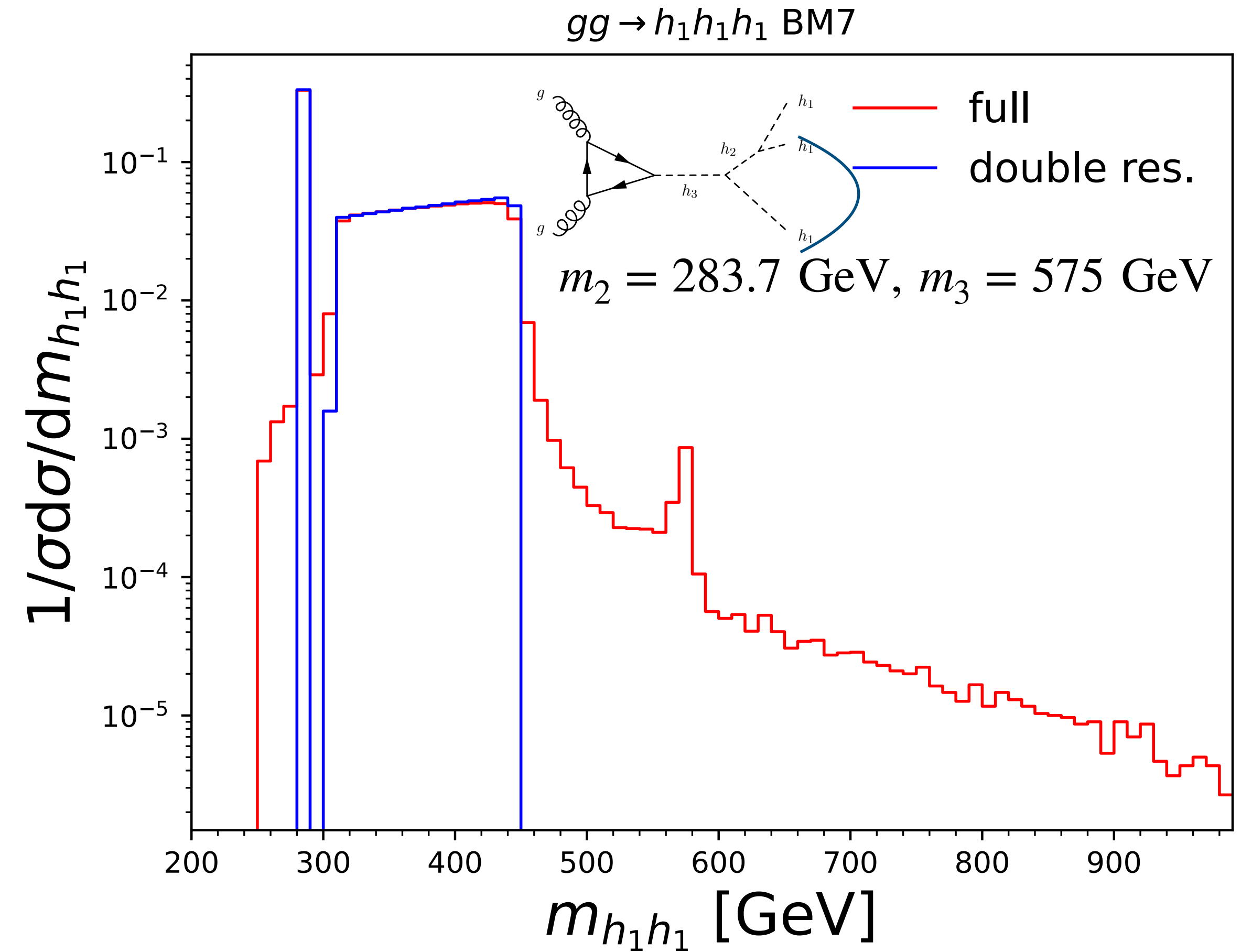
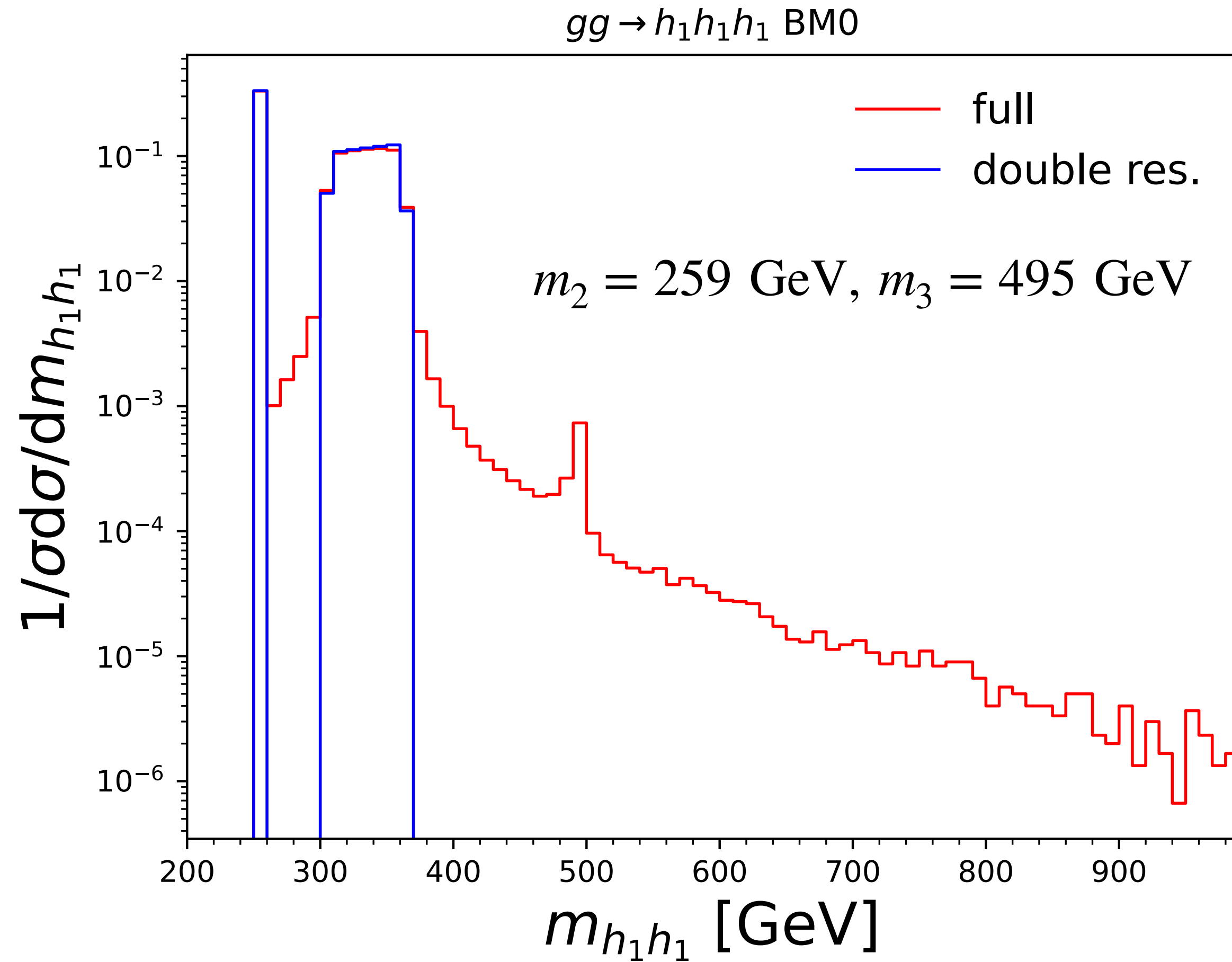
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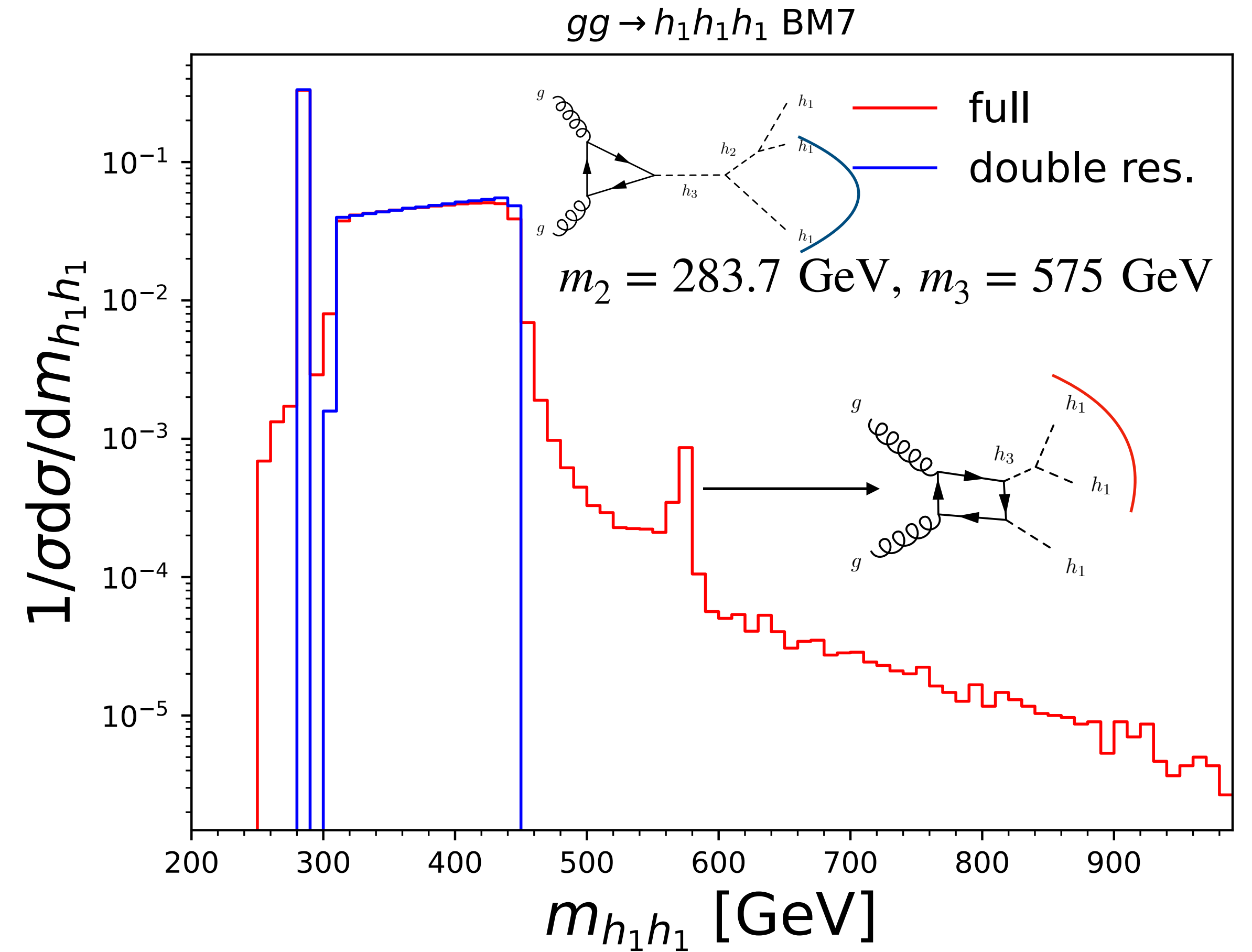
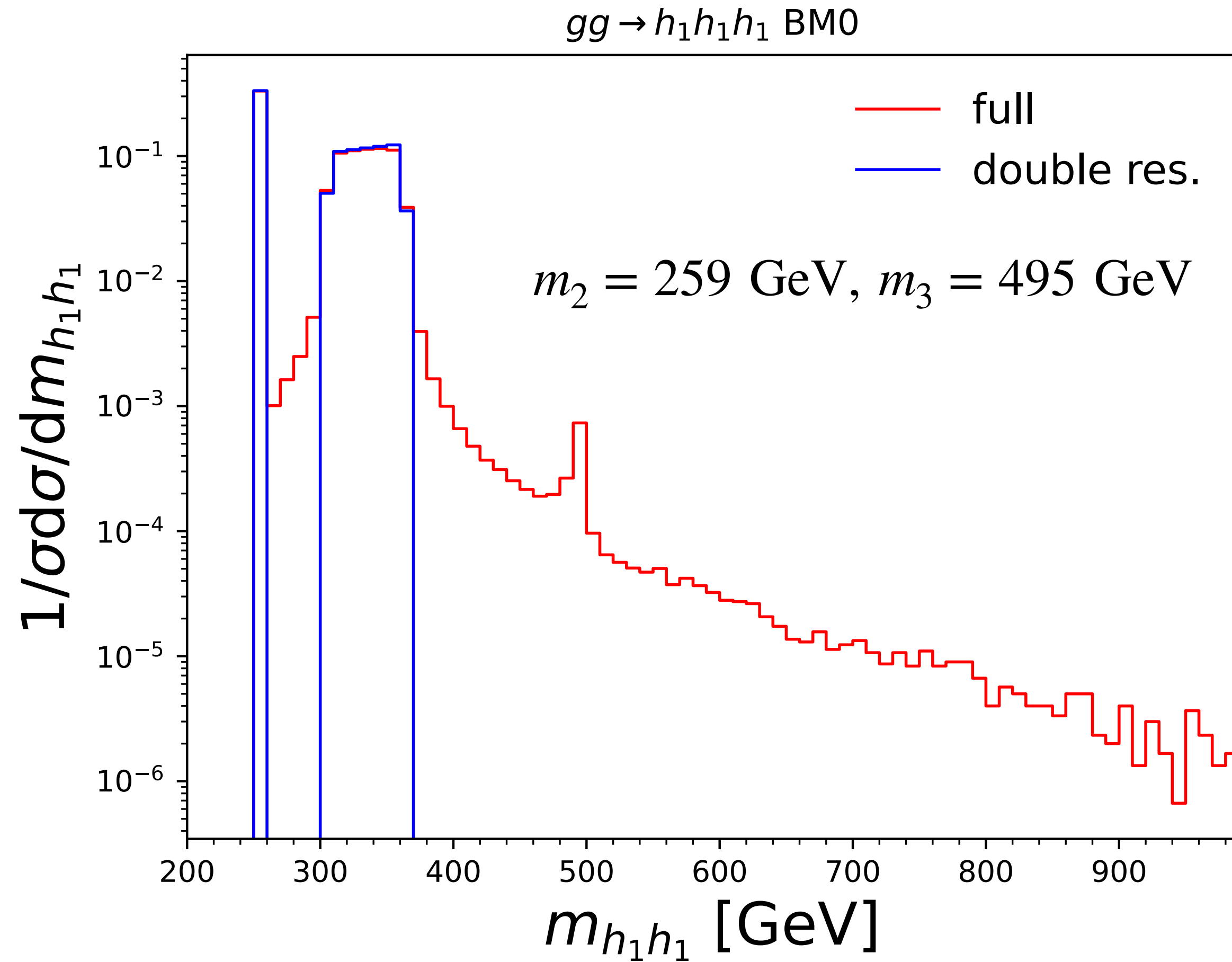
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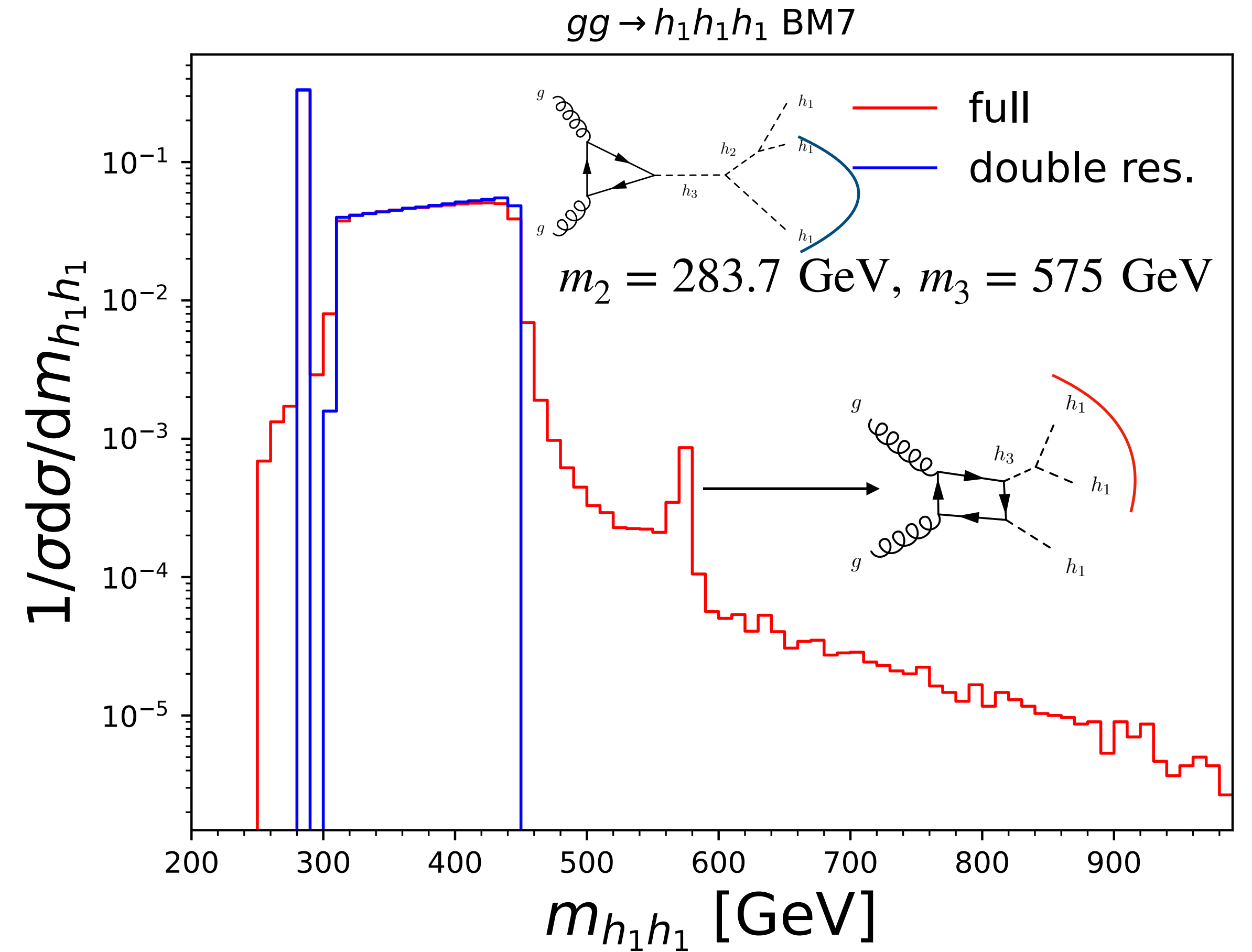
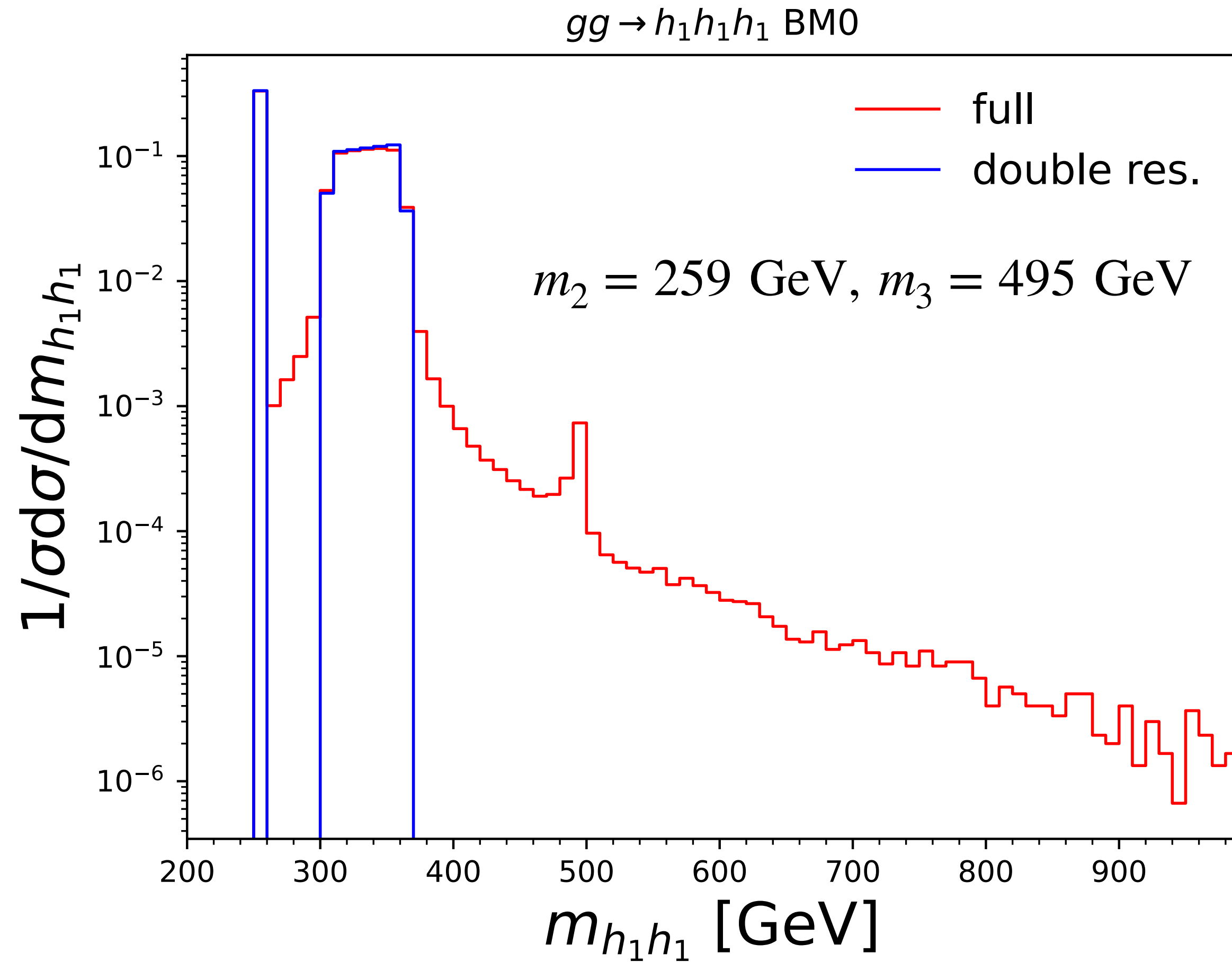
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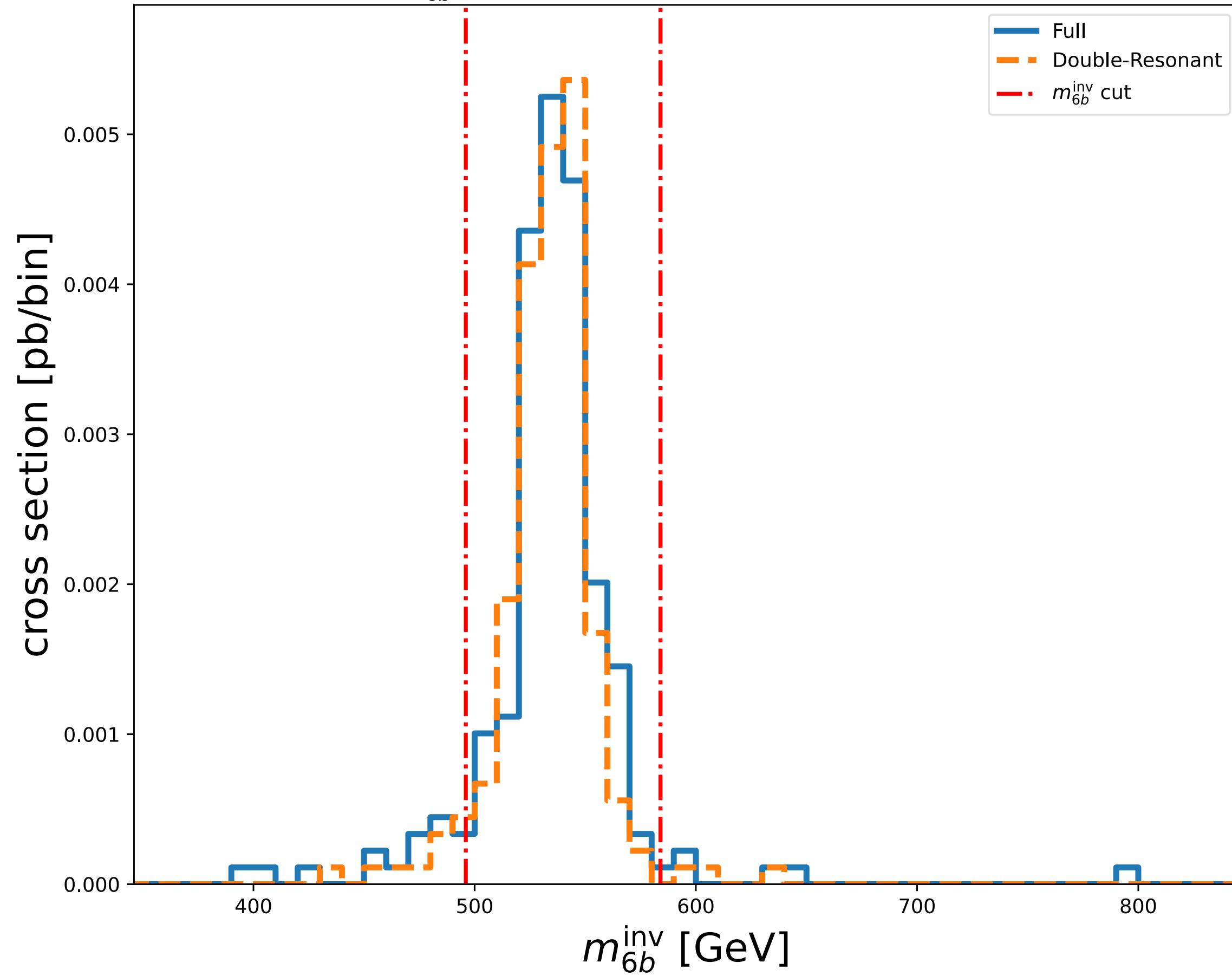


➡ Bulk of cross section from double-resonant process.

Non-Resonant Effects in *hhh* in the TRSM

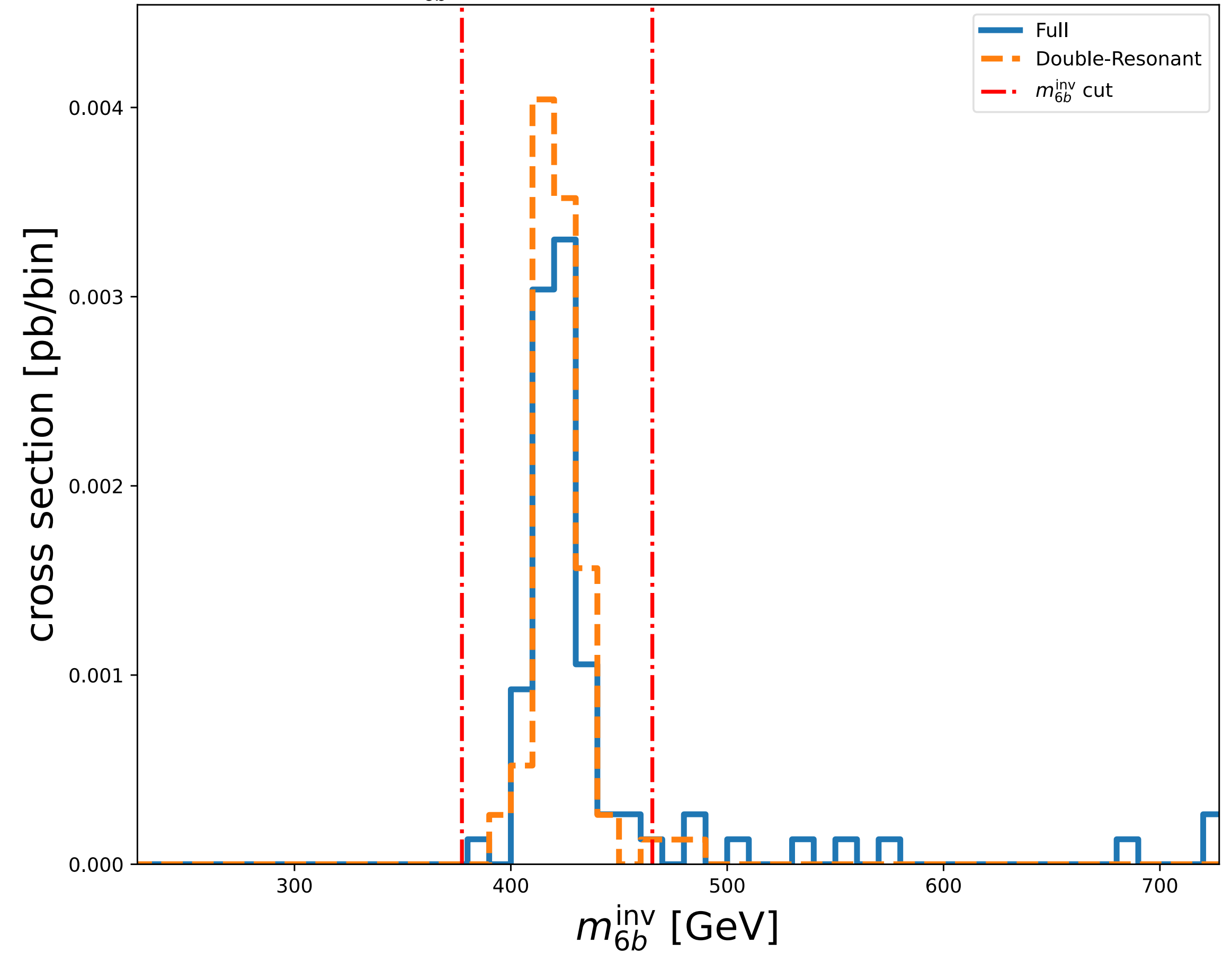
Sample 6 *b*-jet invariant mass distributions in our analysis (all other cuts applied):

Comparison of m_{6b}^{inv} after all other cuts, $(m_2, m_3) = (264.5, 546.0)$ GeV



m_{6b} [GeV]

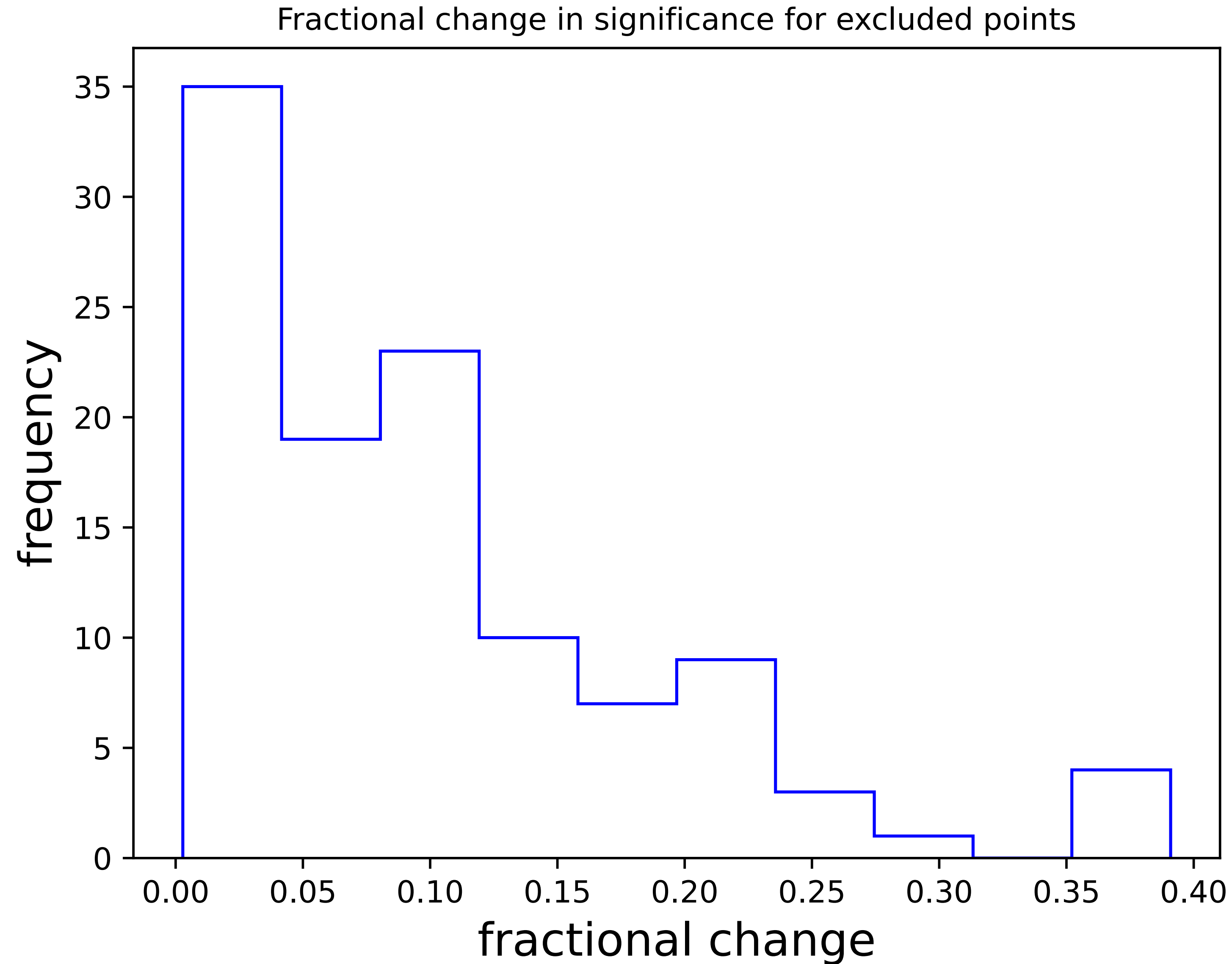
Comparison of m_{6b}^{inv} after all other cuts, $(m_2, m_3) = (270.9, 427.4)$ GeV



m_{6b} [GeV]

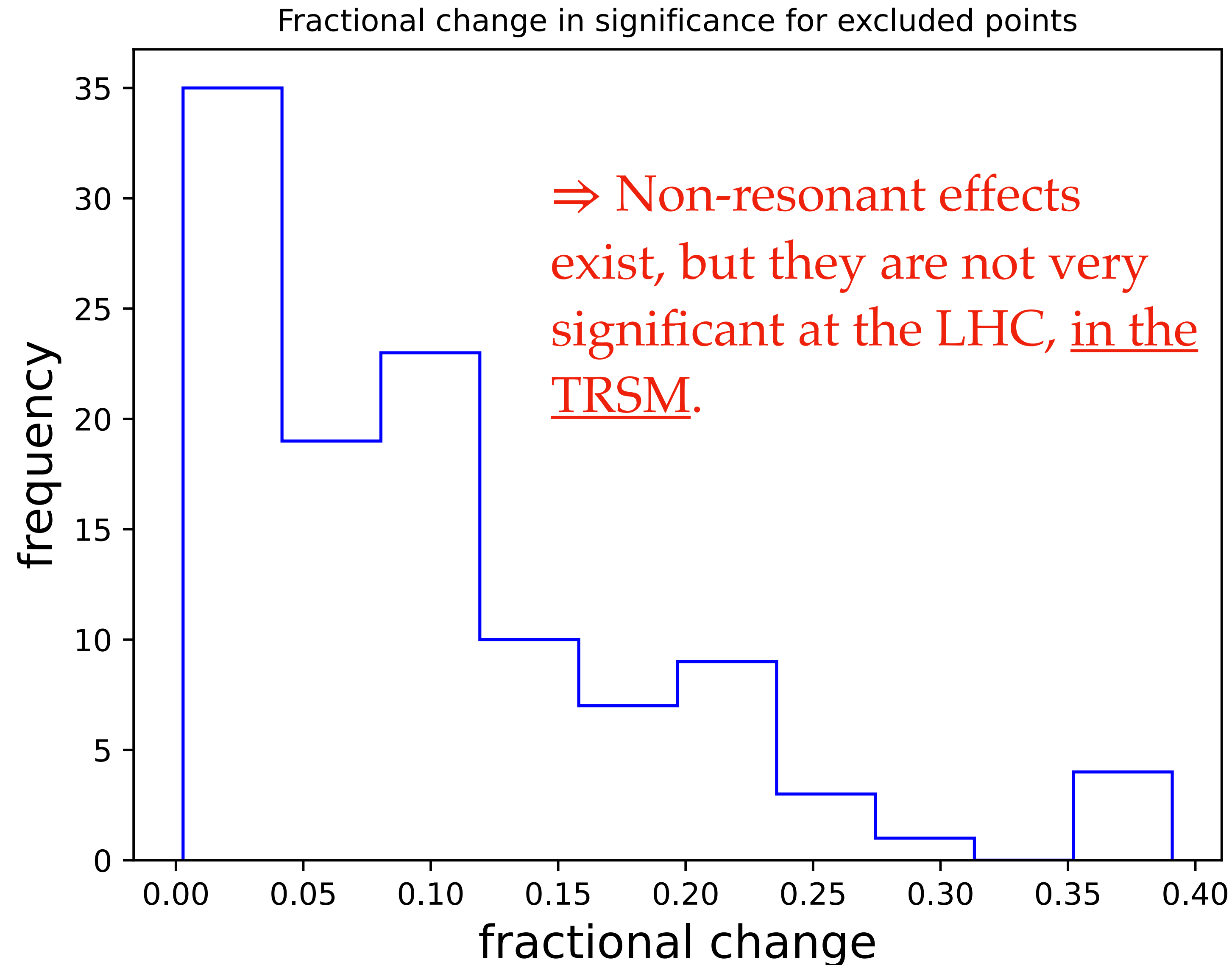
Non-Resonant Effects in *hhh* in the TRSM

= Number of Parameter
Space Points in our
Scan
that can be excluded at
95% C.L.



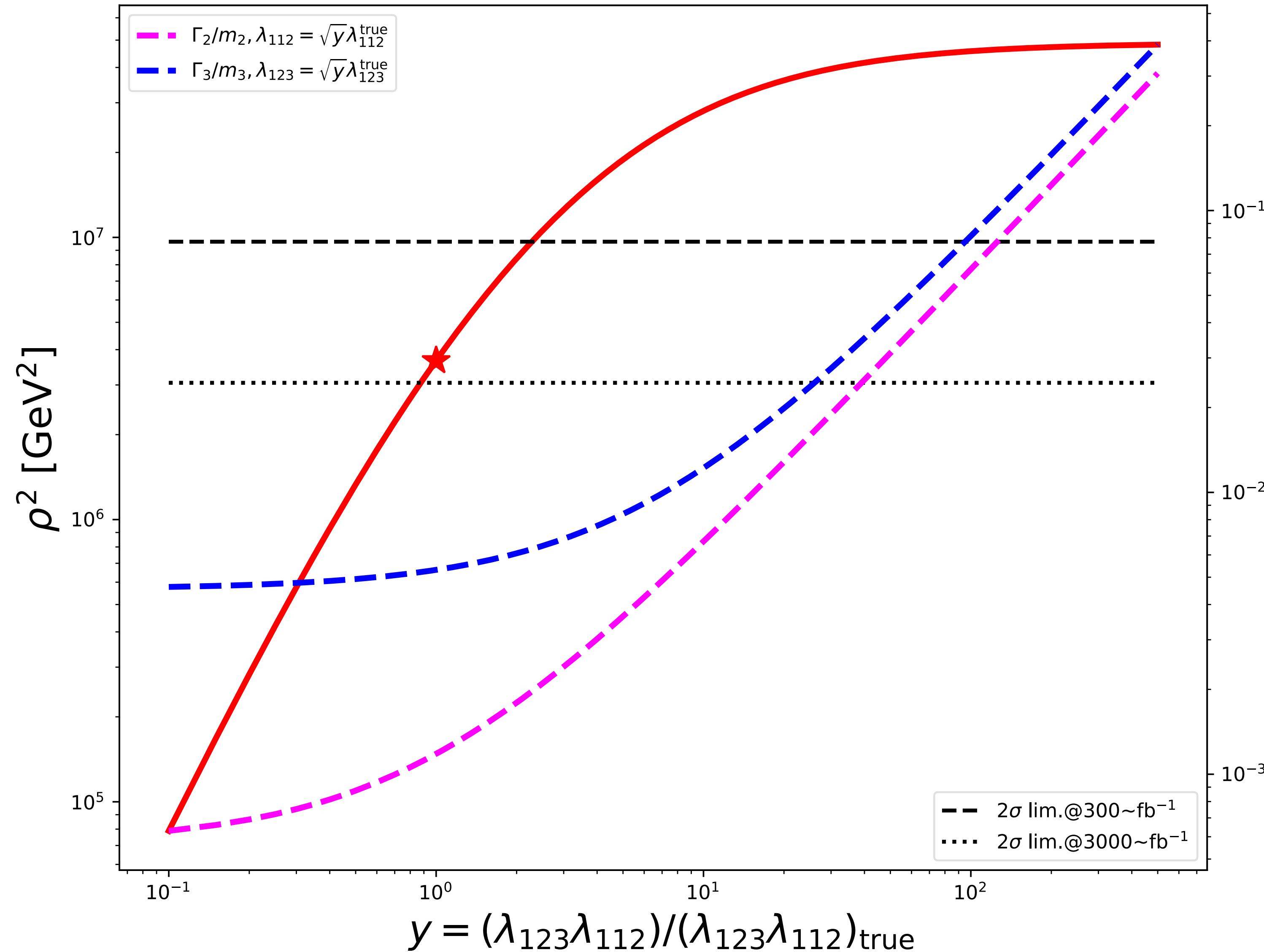
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The Narrow Width Approximation: An Example

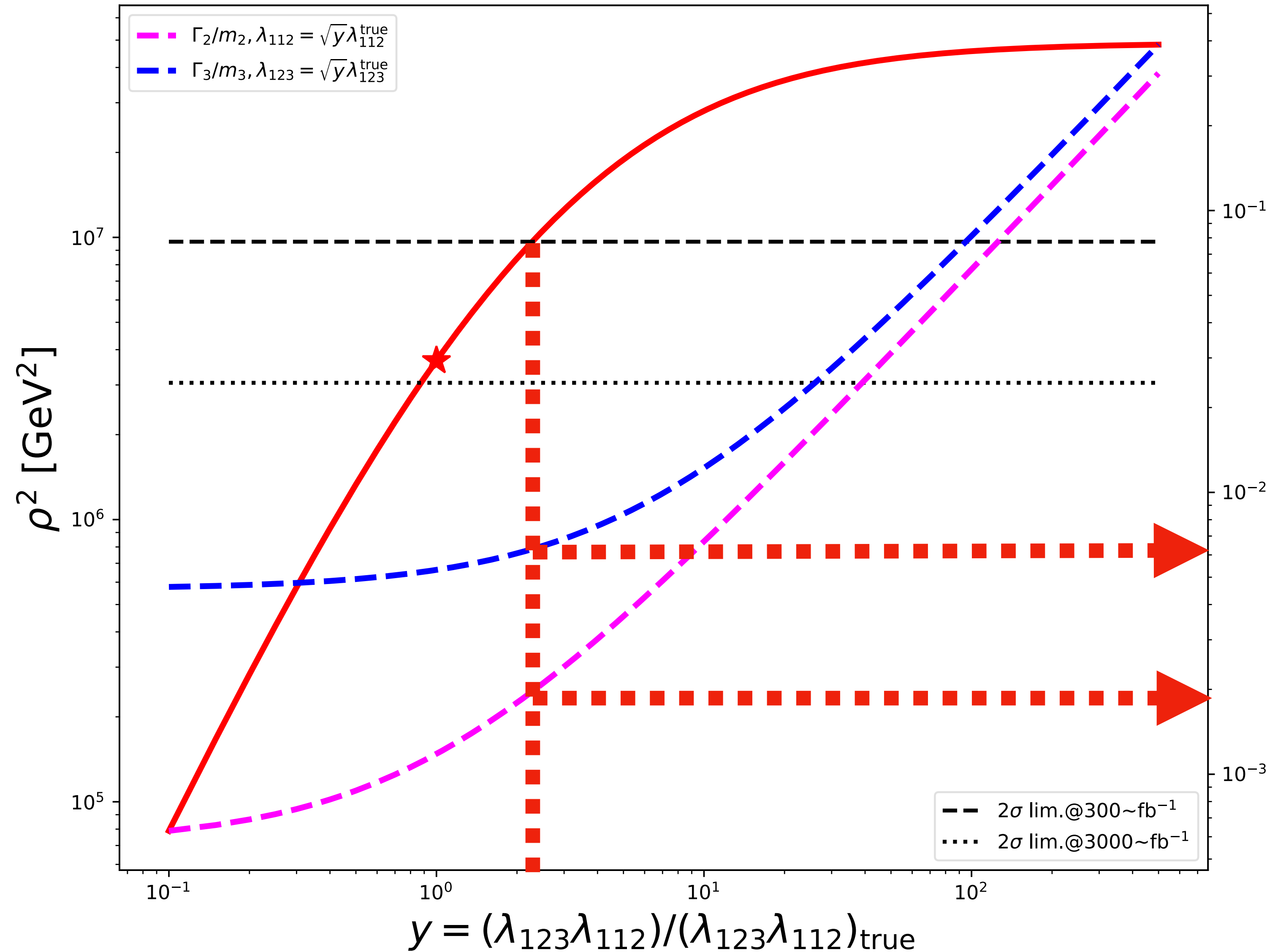
$$m_2 = 308.9 \text{ GeV}, m_3 = 468.5 \text{ GeV}, k_2 = 0.138, k_3 = 0.165$$



- Pick a parameter space point, **fix:** m_2, m_3 and scalar coupling λ_{113} .
- **Rescale:** $\lambda_{112} = \sqrt{y}\lambda_{112}^{\text{true}}$ and $\lambda_{123} = \sqrt{y}\lambda_{123}^{\text{true}}$.
- **Plot:** ρ^2 , Γ_2/m_2 and Γ_3/m_3 versus y .
- **Red star** = **true** parameter point value.
- Plot pheno analysis limits (dashed: 300 fb⁻¹, dotted: 3000 fb⁻¹).
- **For both limits, NWA approximation is valid!**

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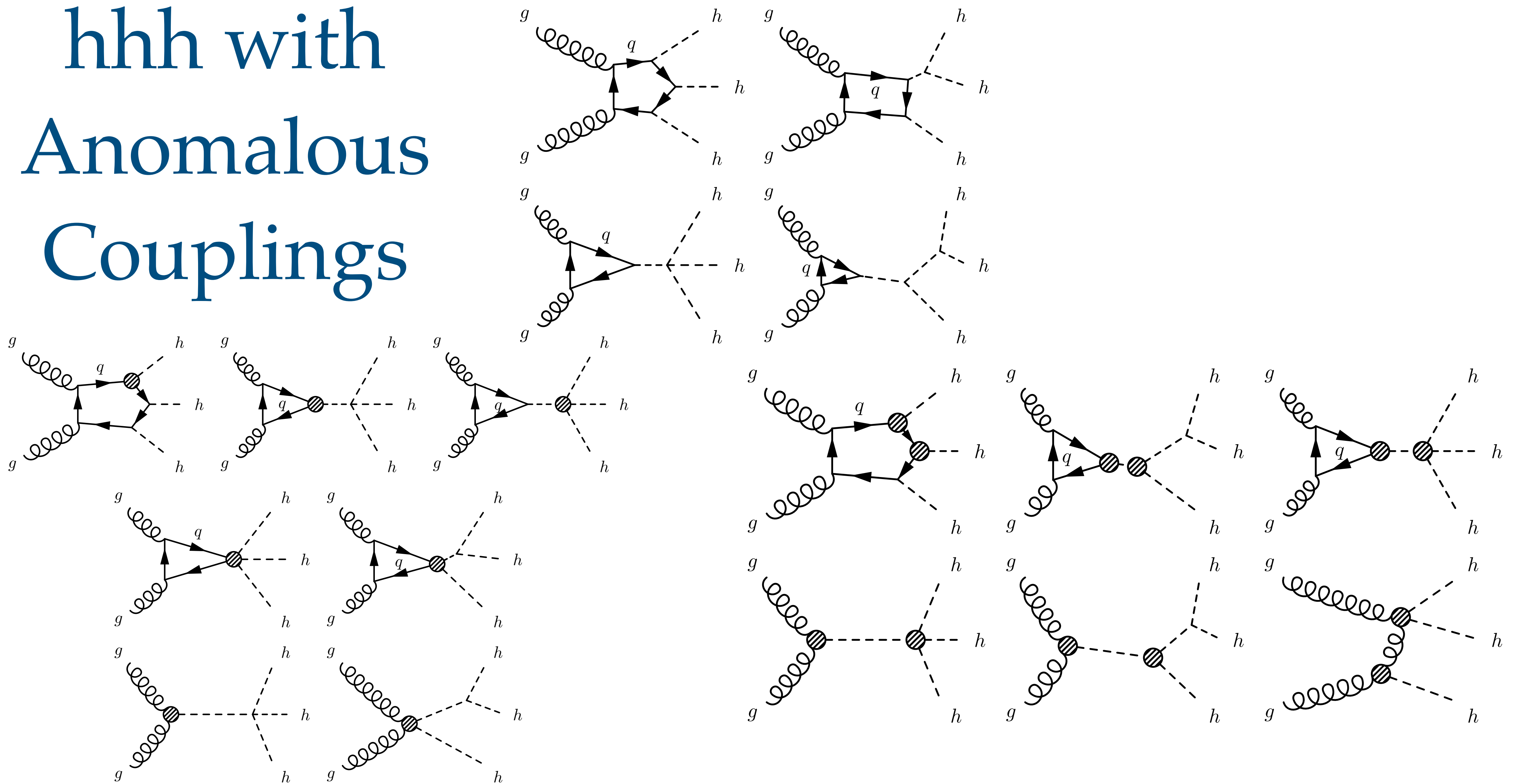
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TRSM Monte Carlo Event Generation

- We have implemented a MadGraph5_aMC@NLO (MG5_aMC) “loop” model for the TRSM:
 - **MG5_aMC input parameters:** the three mixing angles, two masses / widths and **all** the scalar couplings (only 7 are independent in TRSM).
 - Comes with a **Python script** that:
 - allows conversion of M_2, M_3 + three mixing angles + two VEVs to the MG5_aMC model input,
 - calculates several single-production cross sections, branching ratios, widths,
 - and writes associated MG5_aMC parameter card (**param_card.dat**) automatically.
- **Get it at:** <https://gitlab.com/apapaefs/twosinglet>.

[[AP](#), Tania Robens, Gilberto Tetlalmatzi-Xolocotzi, arXiv:2101.00037]

hhh with Anomalous Couplings



hhh: Final states

Assume: K-factor = 2.

[Maltoni, Vryonidou, Zaro, 1408.6542]

$hhh \rightarrow$ final state	BR (%)	$N_{20\text{ab}}^{-1}$	
$(b\bar{b})(b\bar{b})(b\bar{b})$	19.21	22207	
$(b\bar{b})(b\bar{b})(WW_{1\ell})$	7.20	8328	
$(b\bar{b})(b\bar{b})(\tau\bar{\tau})$	6.31	7297	→ Fuks, Kim, Lee, 1510.07697, Fuks, Kim, Lee, 1704.04298.
$(b\bar{b})(\tau\bar{\tau})(WW_{1\ell})$	1.58	1824	
$(b\bar{b})(b\bar{b})(WW_{2\ell})$	0.98	1128	
$(b\bar{b})(WW_{1\ell})(WW_{1\ell})$	0.90	1041	→ Kilian, Sun, Yan, Zhao, Zhao, 1702.03554.
$(b\bar{b})(\tau\bar{\tau})(\tau\bar{\tau})$	0.69	799	
$(b\bar{b})(b\bar{b})(\gamma\gamma)$	0.23	263	→ <u>AP</u> , Sakurai, 1508.06524, Chen, Yan, Zhao, Zhao, Zhong, 1510.04013, Fuks, Kim, Lee, 1510.07697.

[AP, Sakurai, 1508.06524]

The 6b final state, analysis [[AP](#), Gilberto Tetlalmatzi-Xolocotzi, Marco Zaro, arXiv:1909.09166]

- What can we learn about the anomalous couplings via **hhh** at 13.6 TeV?
- Begin by using the 6 **b-jet final state!**

1. Require 6 tagged b-jets.

1

2

3

4

5

6

The 6b final state, analysis [[AP](#), Gilberto Tetlalmatzi-Xolocotzi, Marco Zaro, arXiv:1909.09166]

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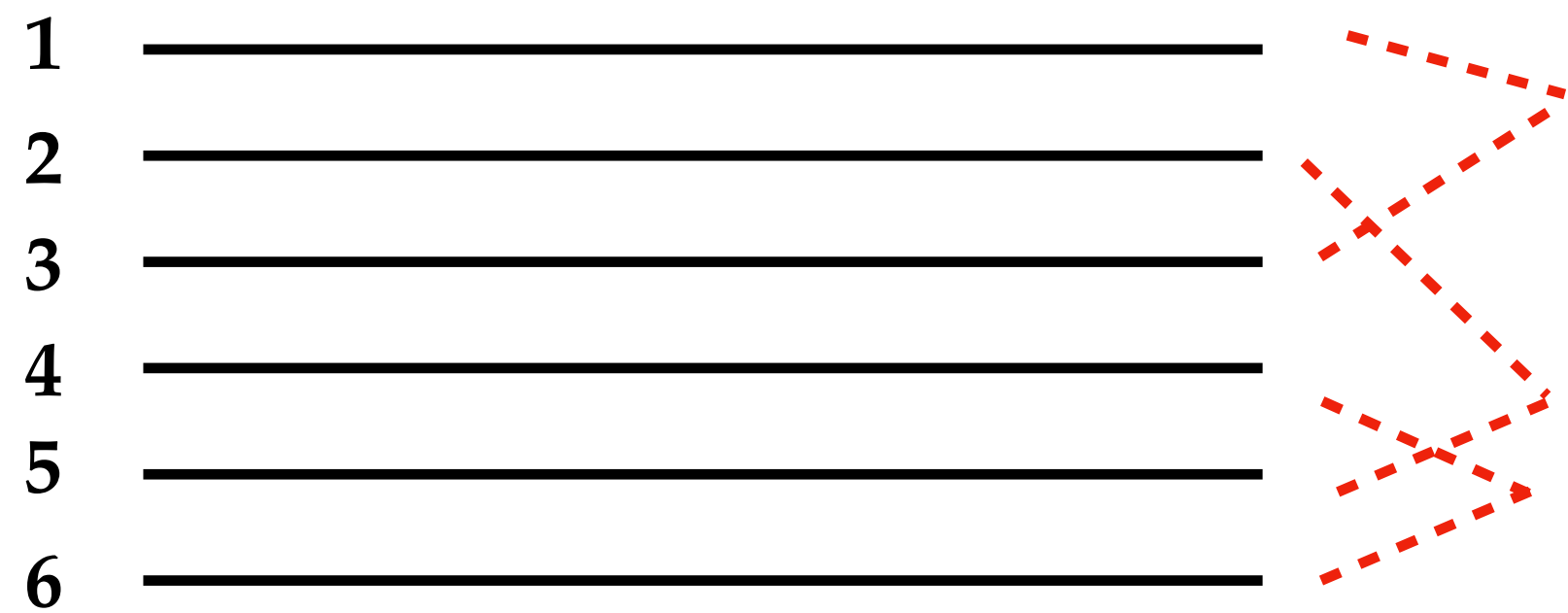


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$$\chi^2 = \sum_{qr \in \text{pairings}} (M_{qr} - m_h^2)^2$$

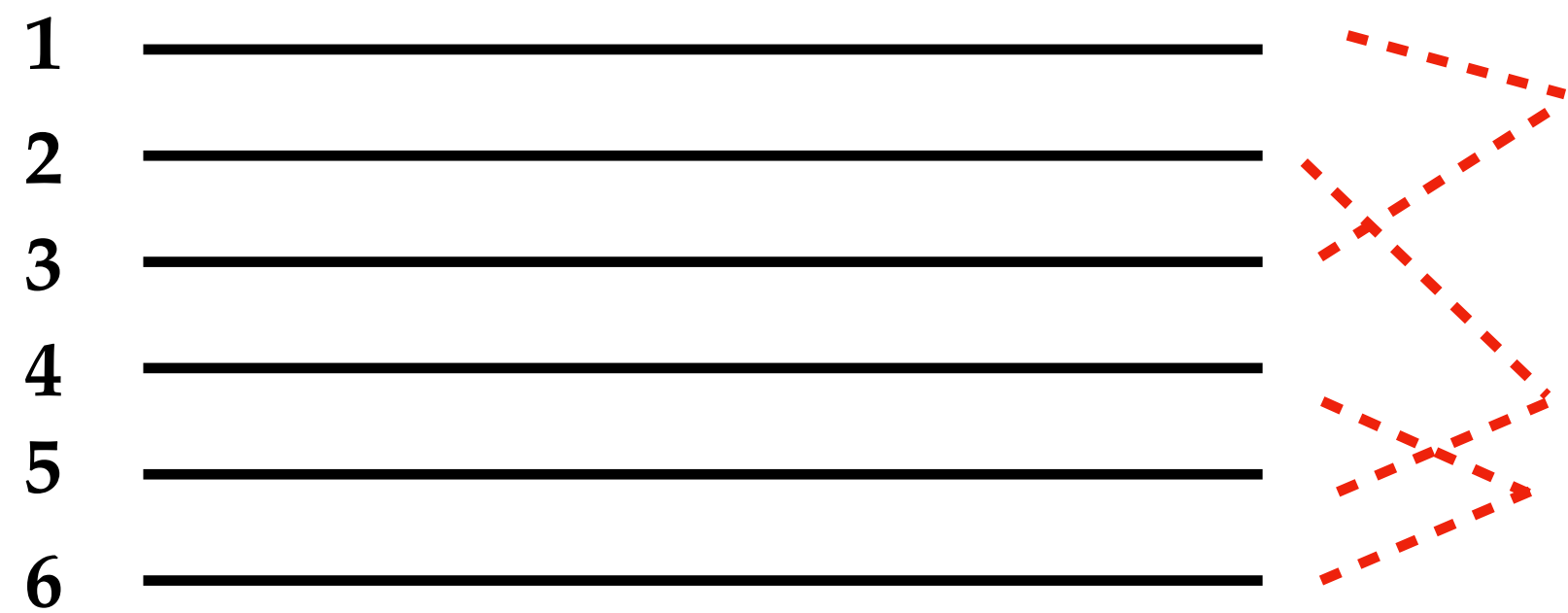
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⇒ 4. Pairing that gives minimum χ^2 determines “reconstructed Higgs boson”.

$$\chi_{\min}^2$$

The 6b final state, analysis

$$h_r^i \rightarrow \text{Higgs boson candidates}$$

observable

cut

$$p_{T,b}$$

$$> 45 \text{ GeV}$$

$$|\eta_b|$$

$$< 3.2$$

$$\Delta R_{b,b}$$

$$> 0.3$$

$$p_T(h_r^i)$$

$$> [170, 120, 0] \text{ GeV}, i = 1, 2, 3$$

$$\chi_{\min}^2$$

$$< 17 \text{ GeV}$$

$$\Delta m_{\min, \text{mid}, \text{max}}$$

$$< 8, 8, 11 \text{ GeV} \quad \leftarrow \text{the three terms in } \chi_{\min}^2.$$

$$\Delta R(h_r^i, h_r^j)$$

$$< [3.5, 3.5, 3.5], (i, j) = [(1, 2), (1, 3), (2, 3)]$$

$$\Delta R_{bb}(h_r^i)$$

$$< [3.5, 3.5, 3.5], i = 1, 2, 3$$



signal/backgrounds after analysis

Process	σ_{GEN} (pb)	$\sigma_{\text{NLO}} \times \text{BR}$ (pb)	$\epsilon_{\text{analysis}}$	$N_{20 \text{ ab}^{-1}}^{\text{cuts}}$
hhh (SM)	2.88×10^{-3}	1.06×10^{-3}	0.0131	278
QCD $(b\bar{b})(b\bar{b})(b\bar{b})$	26.15	52.30	2.6×10^{-5}	27116
$q\bar{q} \rightarrow hZZ \rightarrow h(b\bar{b})(b\bar{b})$	8.77×10^{-4}	4.99×10^{-4}	1.8×10^{-4}	~ 2
$q\bar{q} \rightarrow ZZZ \rightarrow (b\bar{b})(b\bar{b})$	7.95×10^{-4}	7.95×10^{-4}	1.2×10^{-5}	< 1
ggF $hZZ \rightarrow h(b\bar{b})(b\bar{b})$	1.08×10^{-4}	1.23×10^{-4}	$\mathcal{O}(10^{-3})$	~ 2
ggF $ZZZ \rightarrow (b\bar{b})(b\bar{b})$	1.36×10^{-5}	2.73×10^{-5}	2×10^{-5}	$\ll 1$
$h(b\bar{b})(b\bar{b})$	1.46×10^{-2}	1.66×10^{-2}	5.4×10^{-4}	179
$hh(b\bar{b})$	1.40×10^{-4}	9.11×10^{-5}	2.8×10^{-4}	~ 1
$hhZ \rightarrow hh(b\bar{b})$	4.99×10^{-3}	1.61×10^{-3}	7.2×10^{-4}	23
$hZ(b\bar{b}) \rightarrow h(b\bar{b})(b\bar{b})$	9.08×10^{-3}	1.03×10^{-2}	1.4×10^{-4}	29
$ZZ(b\bar{b}) \rightarrow (b\bar{b})(b\bar{b})(b\bar{b})$	2.87×10^{-2}	5.74×10^{-2}	1×10^{-5}	11
$Z(b\bar{b})(b\bar{b}) \rightarrow (b\bar{b})(b\bar{b})(b\bar{b})$	0.93	1.87	3×10^{-5}	1121
Σ backgrounds				2.8×10^4

Reducible backgrounds

process	σ_{GEN} (pb)	$\sigma_{\text{GEN}} \times \mathcal{P}(6\ b\text{-jets})$ (pb)
$(b\bar{b})(b\bar{b})(c\bar{c})$	76.8	0.768
$(b\bar{b})(c\bar{c})(c\bar{c})$	75.6	0.00756
$(c\bar{c})(c\bar{c})(c\bar{c})$	22.5	22.5×10^{-5}
$(b\bar{b})(b\bar{b})(jj)$	1.32×10^4	1.32
$(b\bar{b})(jj)(jj)$	9.79×10^5	0.00979
$(jj)(jj)(jj)$	1.37×10^6	1.37×10^{-6}

c.f. $\sigma_{\text{GEN}}(6b) = 26.15\text{ pb}$

↑
applied:

$$\mathcal{P}_{c \rightarrow b} = 0.1$$

$$\mathcal{P}_{j \rightarrow b} = 0.01$$

⇒ Assuming perfect b-tagging + identical analysis efficiency to QCD 6b:

→ **~10% contribution from reducible backgrounds.**

for $P(\text{b-tagging}) = 0.8$:

→ **~30% contribution.**

TRSM hhh $\rightarrow$ 6b analysis details

Introduce two observables: $\chi^{2,(4)} = \sum_{qr \in I} \left(M_{qr} - M_1 \right)^2$

$$\chi^{2,(6)} = \sum_{qr \in J} \left(M_{qr} - M_1 \right)^2$$

$\rightarrow$ constructed from different pairings of 4 and 6 b-tagged jets, M_{qr} is the invariant mass of the pairing qr .

Monte Carlo Implementation of Anomalous Couplings

- Get the **MG5_aMC model** at: https://gitlab.com/apapaefs/multihiggs_loop_sm.
- [A patch to MG5_aMC to enable **Loop × Tree** is included].
- Can generate events either at:

- **SM²** + interference of [**SM × One-Insertion** diagrams], i.e.:

$$|\mathcal{M}|^2 = |\mathcal{M}_{\text{SM}}|^2 + 2\text{Re}\{\mathcal{M}_{\text{SM}}^* \mathcal{M}_{1-\text{ins.}}\} \propto 1 + c_i$$

or

- **SM²** + interference of [**SM × One or Two** insertion diagrams] + [**One Insertion**]², i.e.:

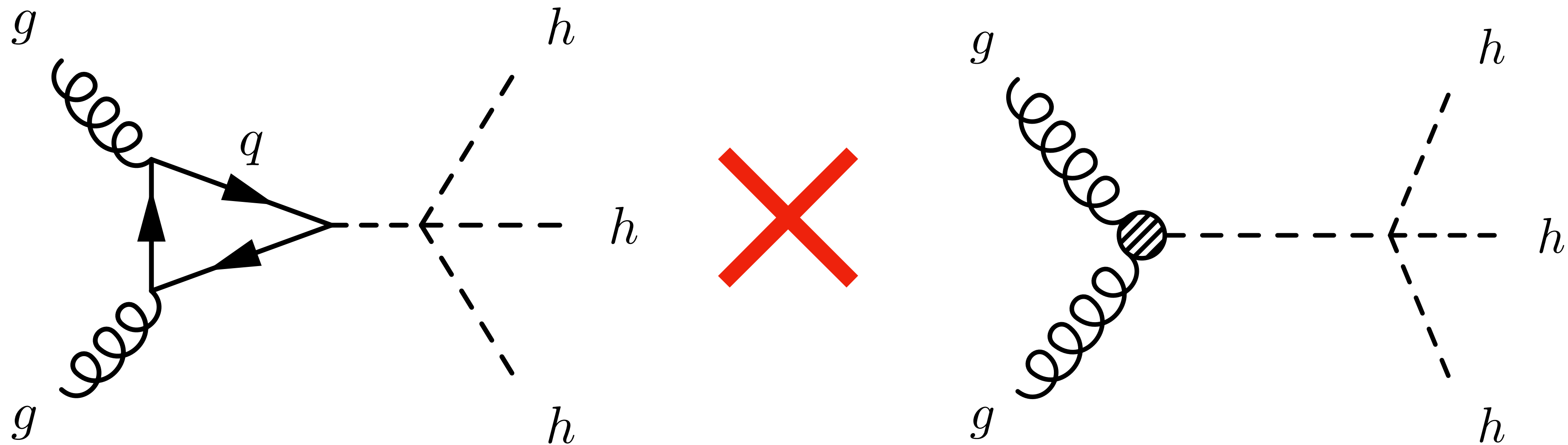
$$|\mathcal{M}|^2 = |\mathcal{M}_{\text{SM}}|^2 + 2\text{Re}\{\mathcal{M}_{\text{SM}}^* \mathcal{M}_{1-\text{ins.}}\} + 2\text{Re}\{\mathcal{M}_{\text{SM}}^* \mathcal{M}_{2-\text{ins.}}\} + |\mathcal{M}_{1-\text{ins.}}|^2 \\ \propto 1 + c_i + c_j c_k + c_\ell^2$$

Monte Carlo Implementation of Anomalous Couplings

- We have implemented a MadGraph5_aMC@NLO “loop” model for $\mathcal{L}_{\text{PhenoExp}}$.
- Includes Loop $\times$ Tree level interference between the various diagrams.

[see: Hirschi, <https://cp3.irmp.ucl.ac.be/projects/madgraph/wiki/LoopInducedTimesTree>].

- e.g.:



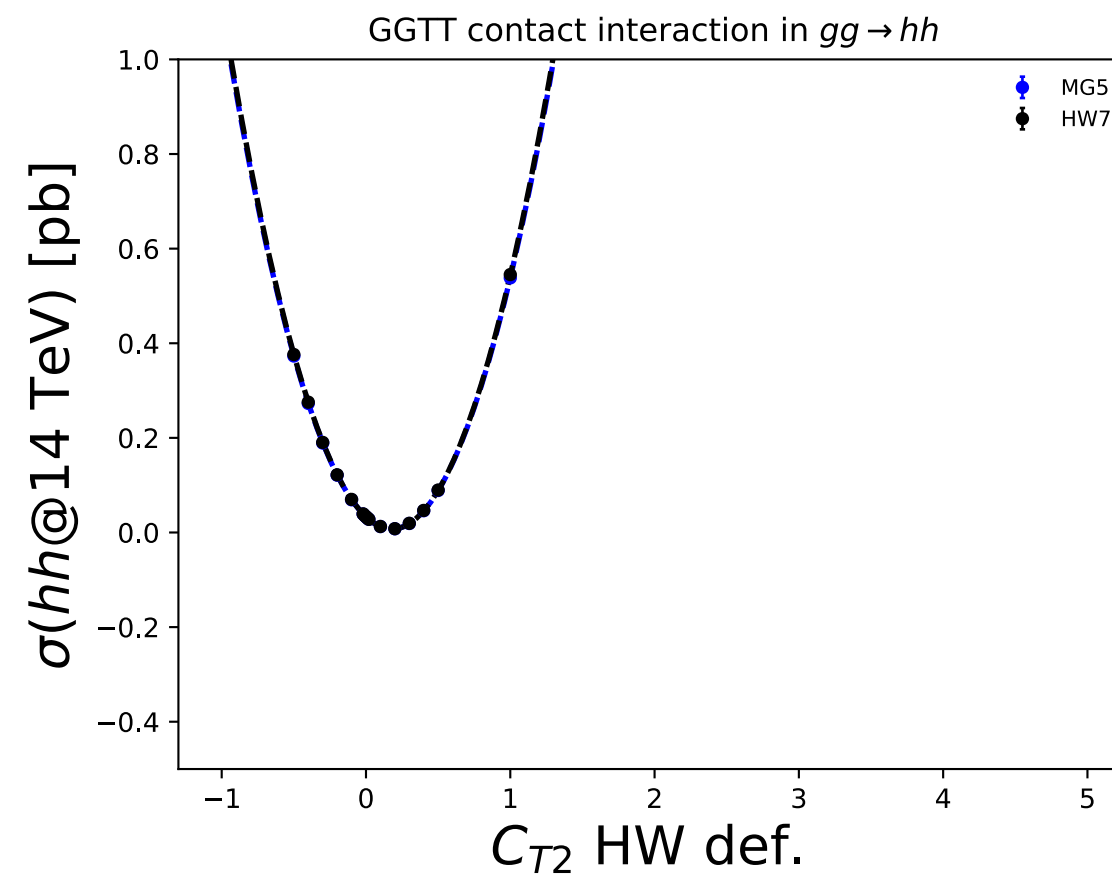
[AP, Tetlalmatzi-Xolocotzi, arXiv:2312.13562]

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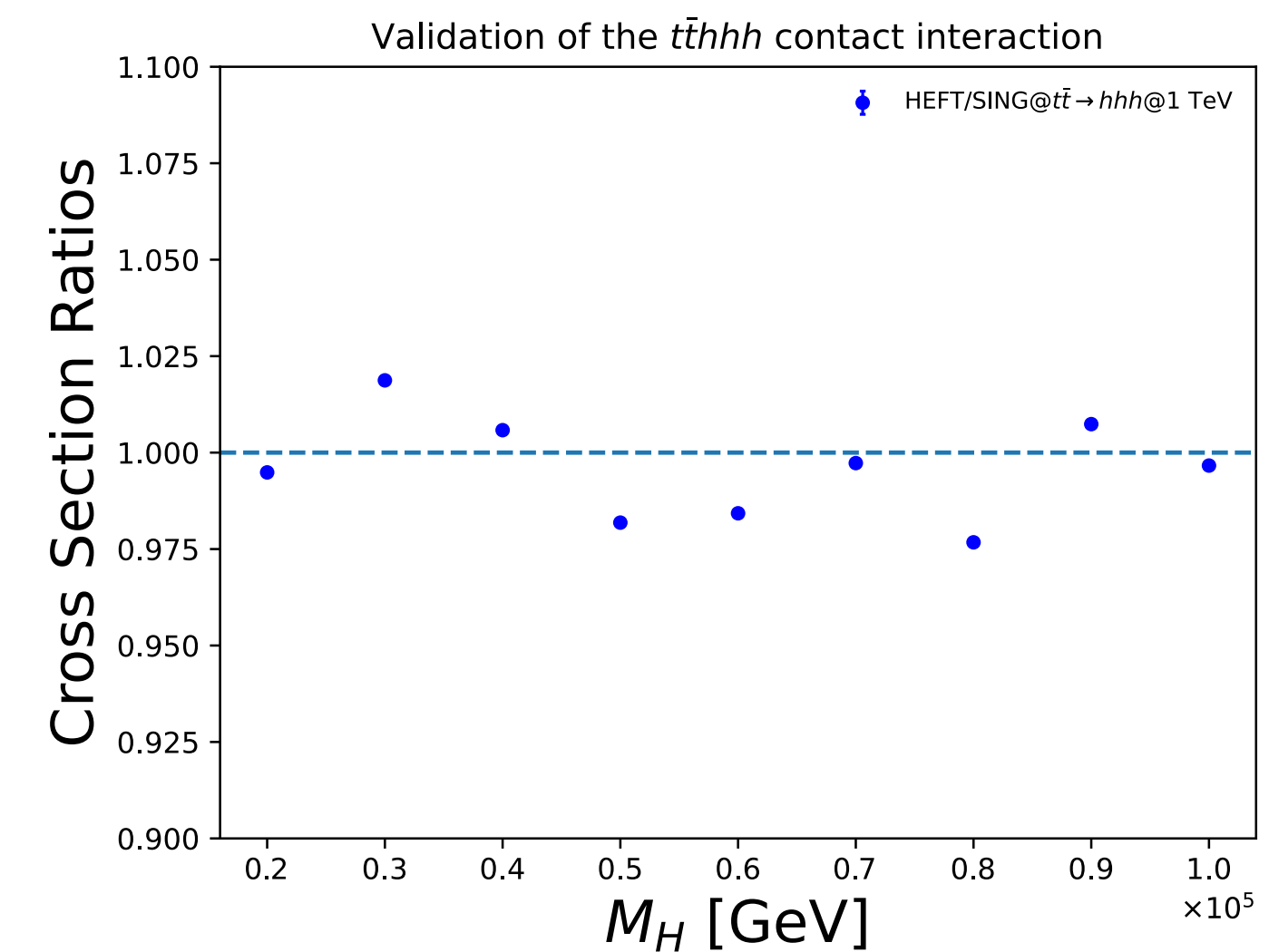
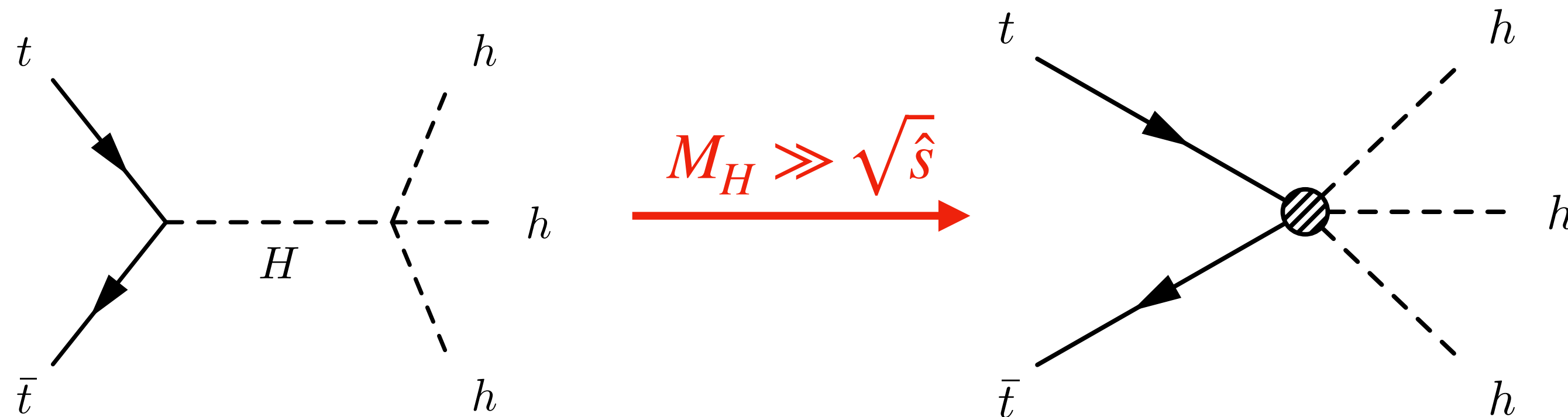
Model Validation

[AP, Tetlalmatzi-Xolocotzi, arXiv:2312.13562]

- Most couplings validated vs. a Herwig 7 $pp \rightarrow hh$ implementation, e.g.:



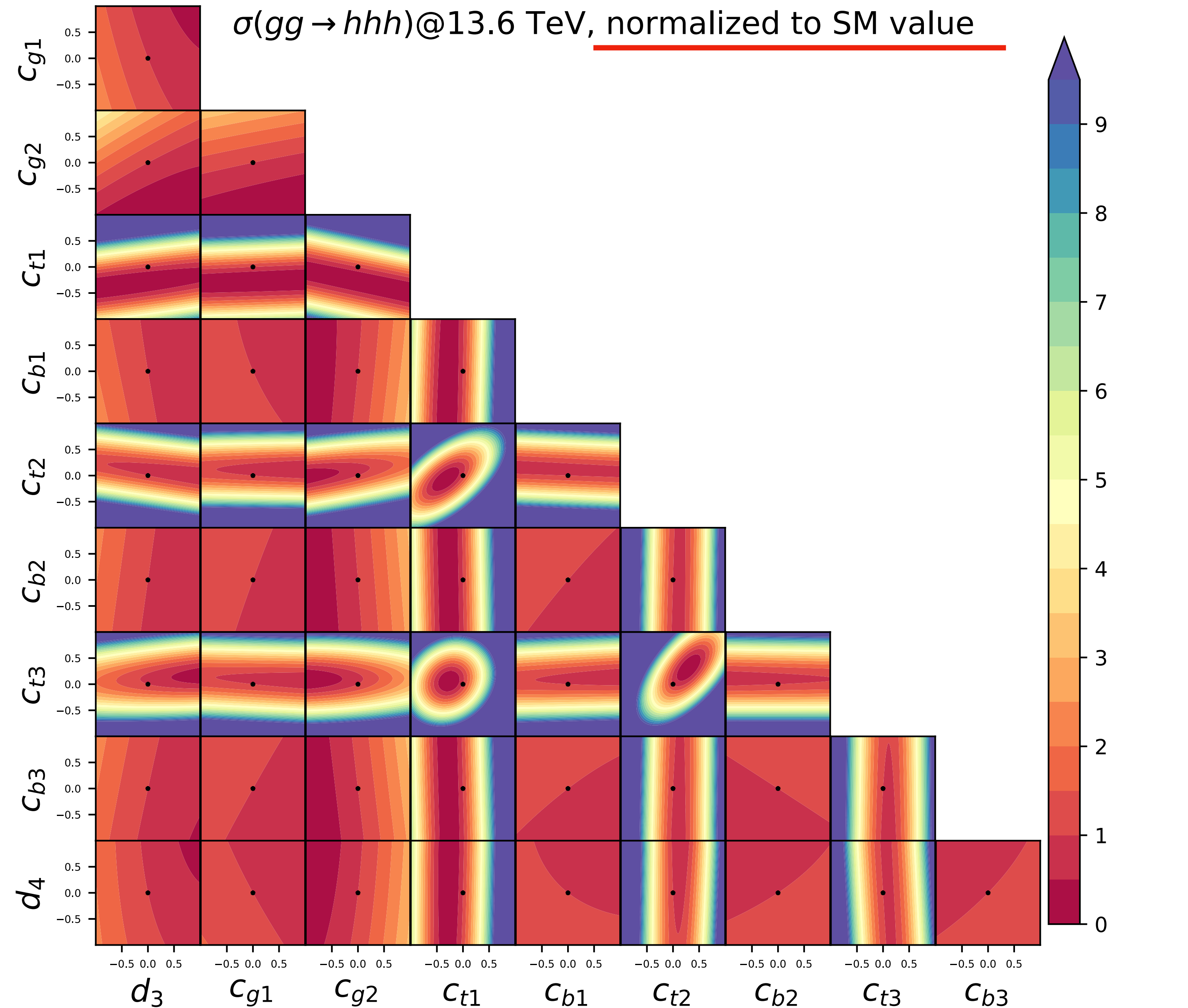
- The one “new” non-trivial coupling that appears, $\propto c_{t3} t\bar{t}h^3$ has been validated via an “EFT” limit, in the $t\bar{t} \rightarrow hhh$ process:



hhh Cross Sections @ 13.6 TeV

[AP, Tetlalmatzi-Xolocotzi, arXiv:2312.13562]

- Cross section as a multiple of the SM
- ($\sigma_{\text{SM}} \sim 0.04 \text{ fb}$ at LO@13.6 TeV).
- In each 2D panel shown: **all other coefficients set to zero!**



Anomalous Couplings Constraints

- Other processes constrain (at LO) all coefficients except $\{c_{t3}, d_4\}$ (**only in hhh**).
- **Projected constraints:**

Percentage uncertainties			
	HL-LHC	FCC-hh	Ref.
$\delta(d_3)$	50	5	[145] (table 12)
$\delta(c_{g1})$	2.3	0.49	[145] (table 3)
$\delta(c_{g2})$	5	1	[140] (Figure 12, right)
$\delta(c_{t1})$	3.3	1.0	[145] (table 3)
$\delta(c_{t2})$	30	10	[140] (Figure 12, right)
$\delta(c_{b1})$	3.6	0.43	[145] (table 3)
$\delta(c_{b2})$	30	10	assumed same as c_{t2}

[See **AP**, Tetlalmatzi-Xolocotzi, arXiv:2312.13562 for the references]

Digression: Non-Resonant hhh @ LHC

- Anomalous couplings can enhance hhh ! e.g. $\kappa_3 \neq 1, \kappa_4 \neq 1$, but also others!

$$\begin{aligned} \mathcal{L}_{\text{PhenoExp}} \supset & -\lambda_{\text{SM}} v (1+c_3) h^3 - \frac{\lambda_{\text{SM}}}{4} (1+d_4) h^4 \\ & + \frac{\alpha_s}{12\pi} \left(c_{g1} \frac{h}{v} - c_{g2} \frac{h^2}{2v^2} \right) G_{\mu\nu}^a G_a^{\mu\nu} \\ & - \left[\frac{m_t}{v} (1+c_{t1}) \bar{t}_L t_R h + \frac{m_b}{v} (1+c_{b1}) \bar{b}_L b_R h + \text{h.c.} \right] \\ & - \left[\frac{m_t}{v^2} c_{t2} \bar{t}_L t_R h^2 + \frac{m_b}{v^2} c_{b2} \bar{b}_L b_R h^2 + \text{h.c.} \right] \\ & - \left[\frac{m_t}{v^3} \left(\frac{c_{t3}}{2} \right) \bar{t}_L t_R h^3 + \frac{m_b}{v^3} \left(\frac{c_{b3}}{2} \right) \bar{b}_L b_R h^3 + \text{h.c.} \right] \end{aligned}$$

[AP, Tatlalmatzi-Xolocotzi, arXiv:2312.13562]

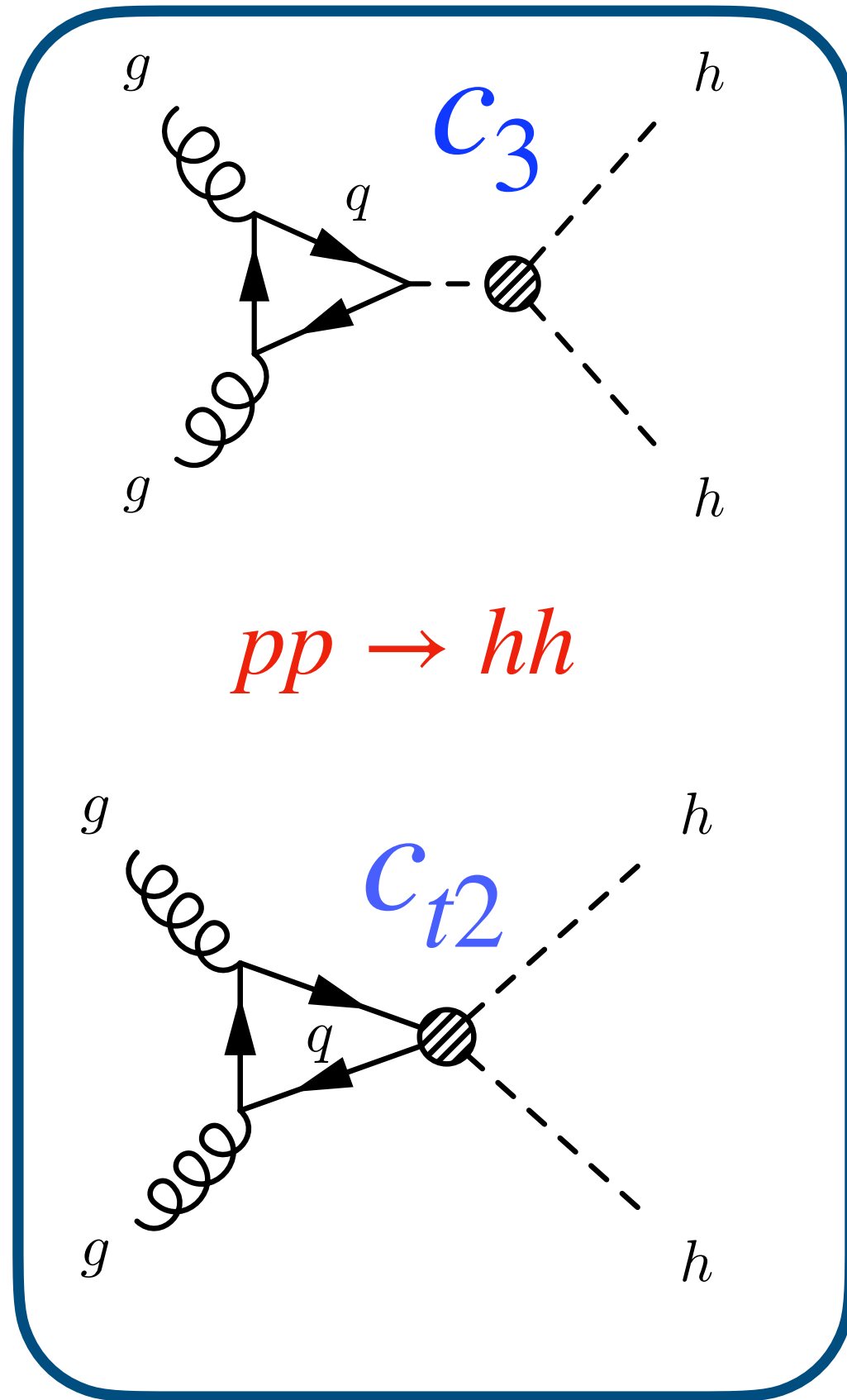
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➡ Here: Examine models with new scalar resonances!

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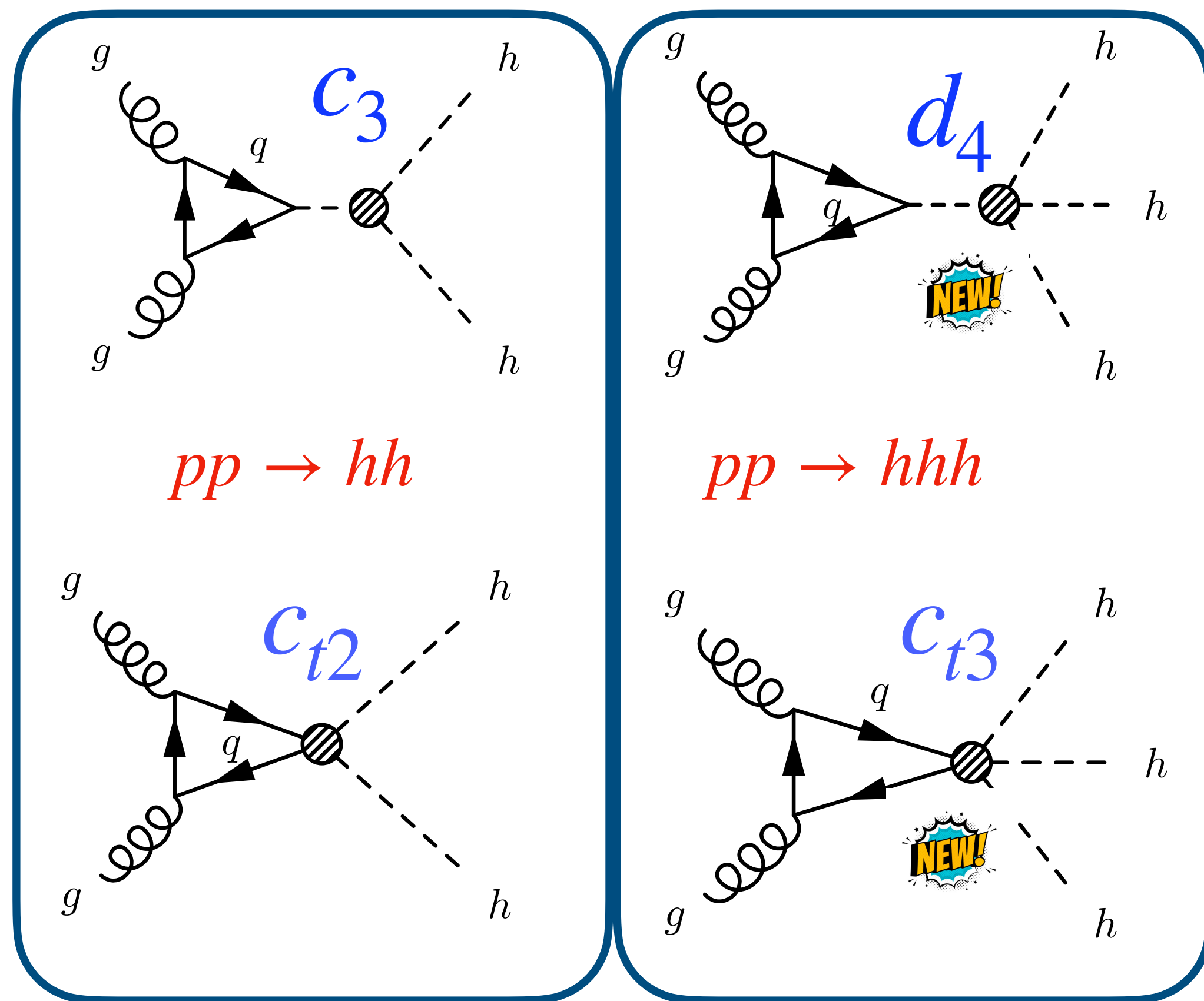
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Anomalous Couplings Constraints

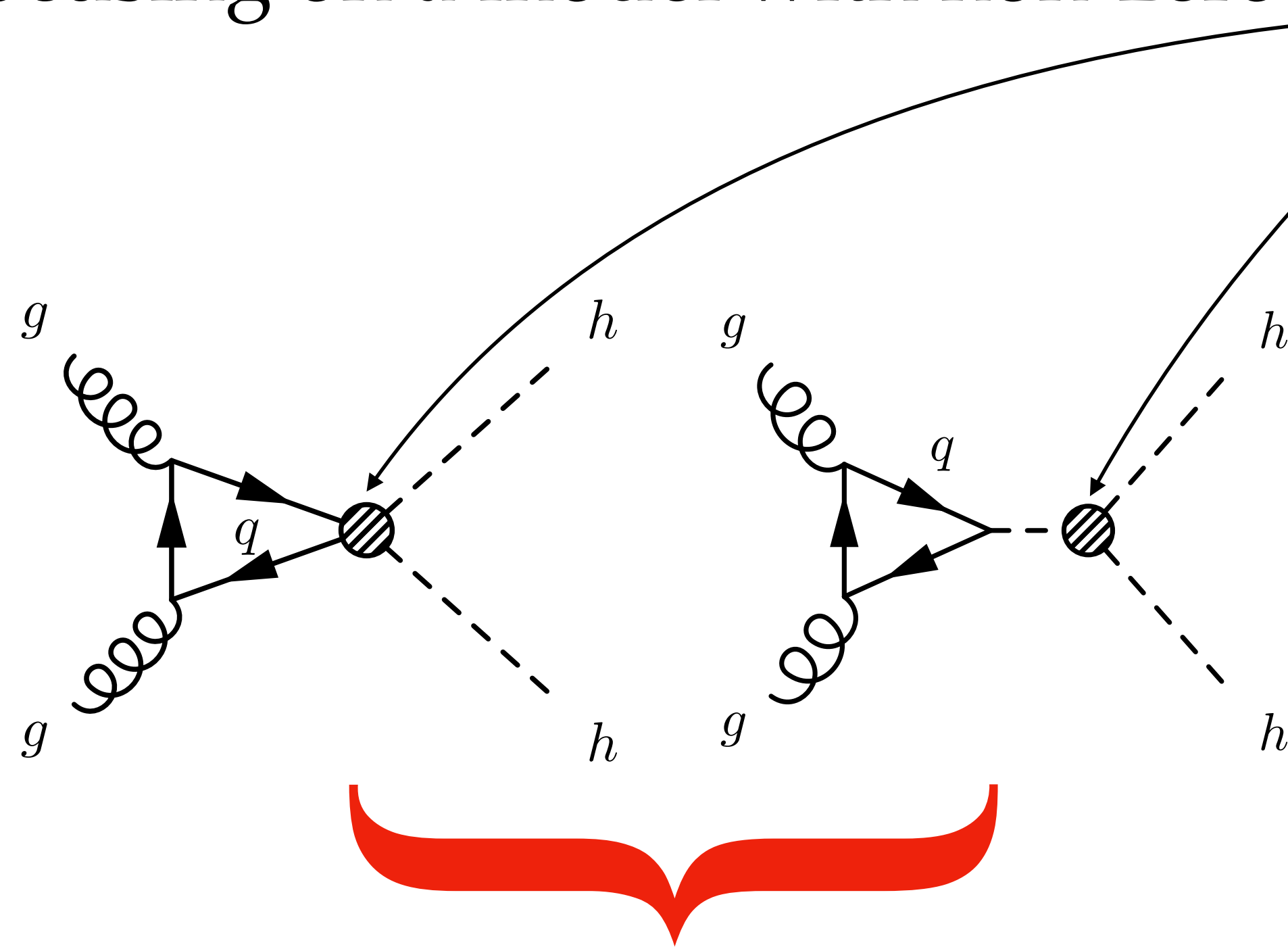
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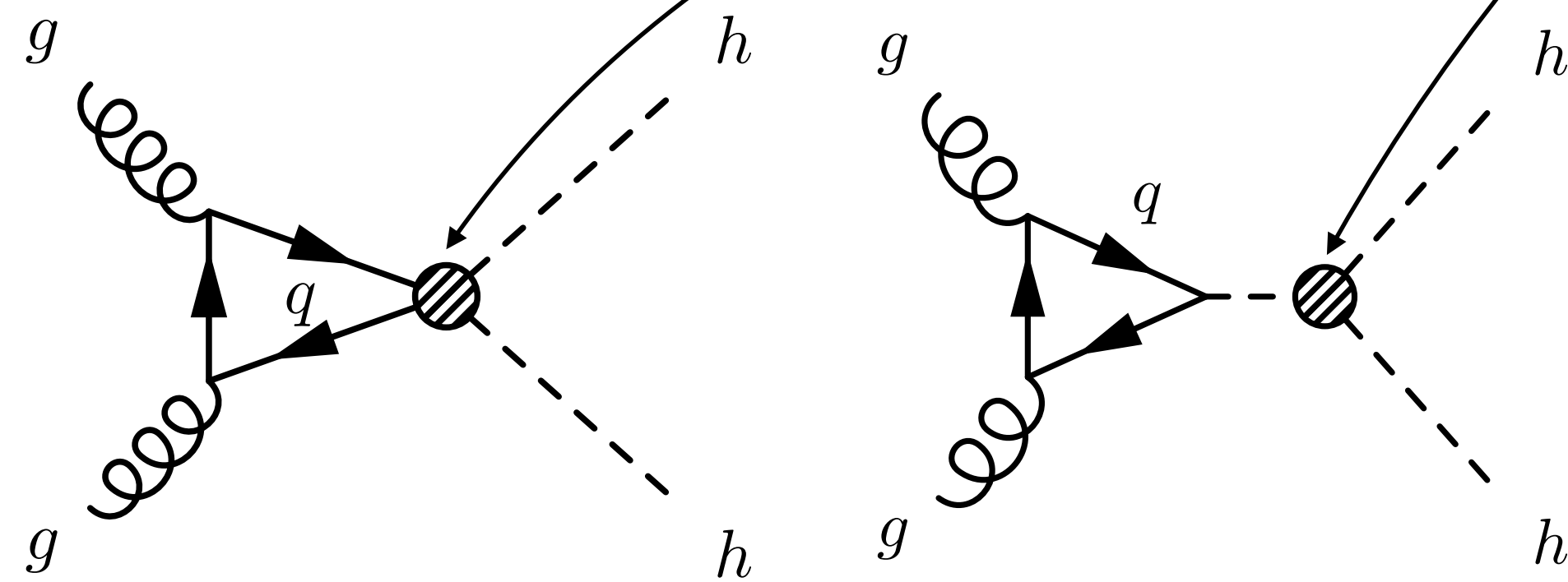


constrained by $pp \rightarrow hh$

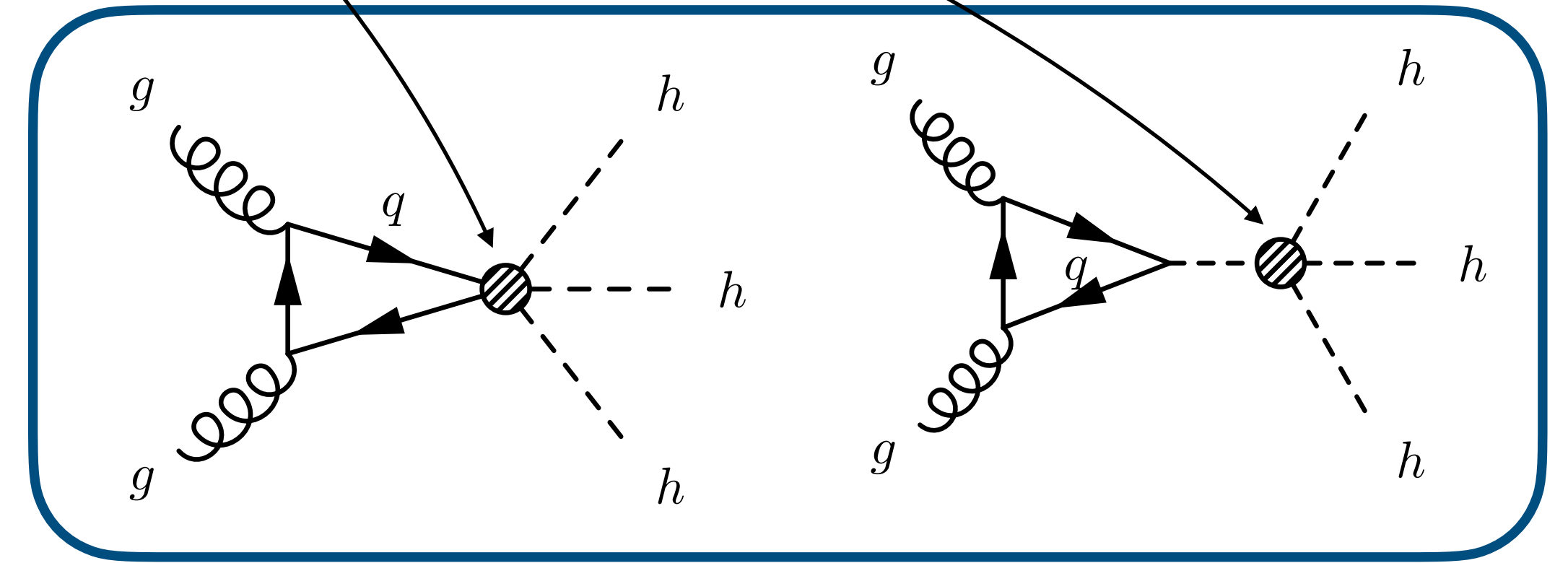
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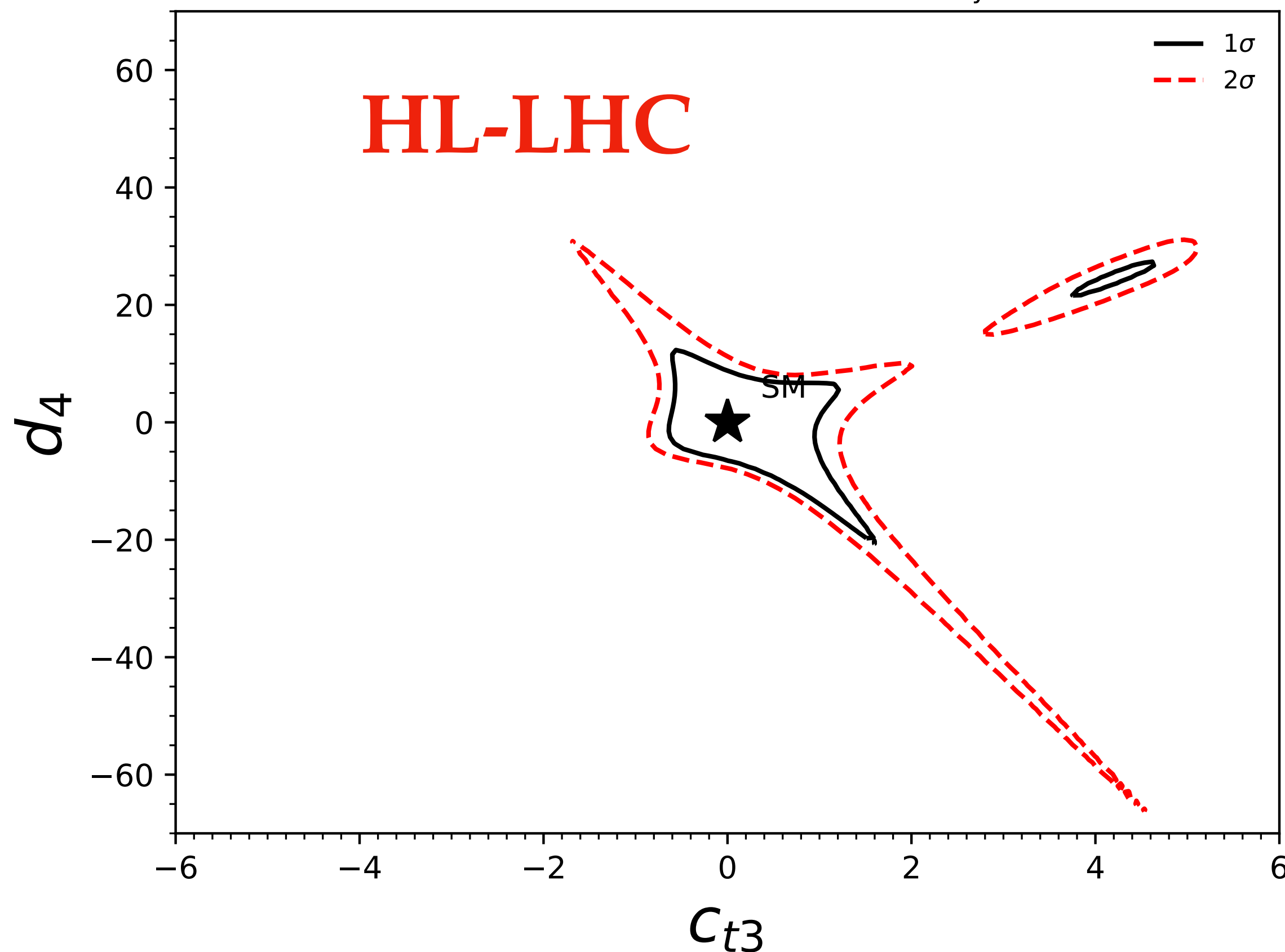
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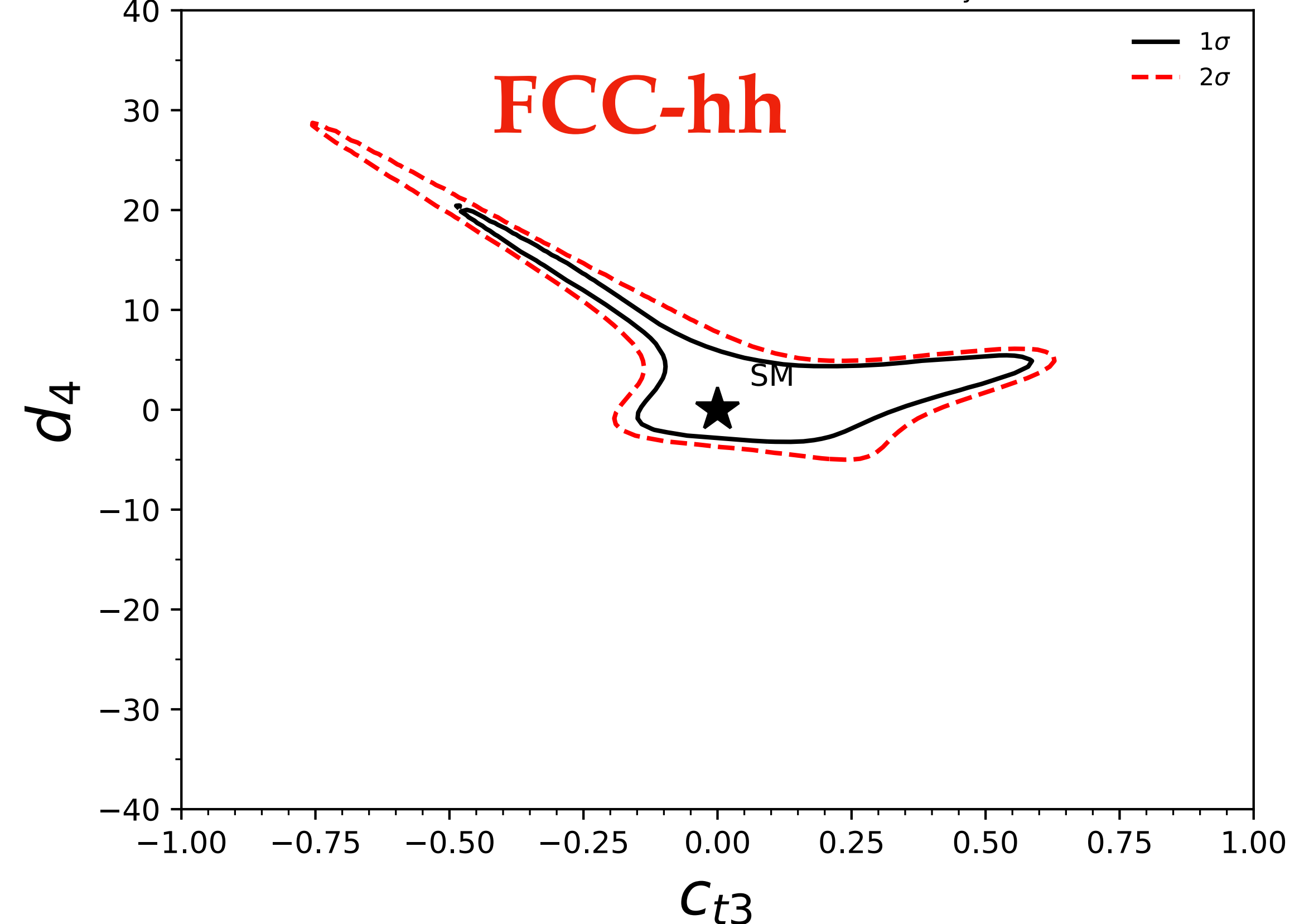
[AP, Tetlalmatzi-Xolocotzi, arXiv:2312.13562]

- Focusing on a model with only $\{c_{t2}, d_3, c_{t3}, d_4\}$,
- Using the **6 b-jet final state**, and marginalizing over $\{c_{t2}, d_3\}$ within projected constraints:

$gg \rightarrow hhh@13.6 \text{ TeV}, L=3000 \text{ fb}^{-1}, \alpha_{\text{syst.}} = 5.0\%$



$gg \rightarrow hhh@100 \text{ TeV}, L=20000 \text{ fb}^{-1}, \alpha_{\text{syst.}} = 5.0\%$

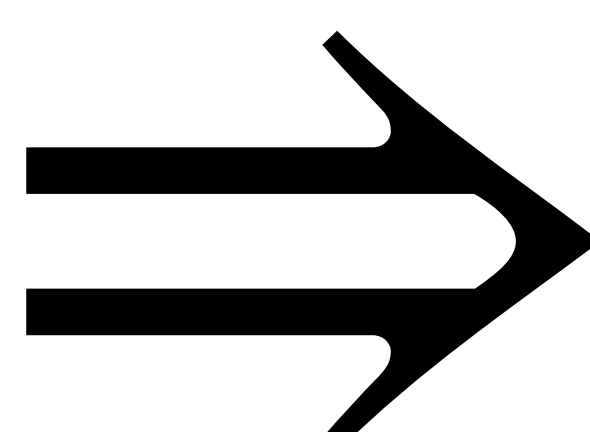


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- Focusing on a model with only $\{c_{t2}, d_3, c_{t3}, d_4\}$,
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	HL-LHC 68%	HL-LHC 95%	FCC-hh 68%	FCC-hh 95%
d_4	$[-6.6, 12.4]$	$[-10.0, 21.3]$	$[-3.9, 10.5]$	$[-10.6, 18.8]$
c_{t3}	$[-0.6, 1.1]$	$[-0.9, 3.6]$	$[-0.1, 0.3]$	$[-0.4, 0.6]$


$$c_{t3} \sim \mathcal{O}(0.1 - 1)$$
$$d_4 \sim \mathcal{O}(10)$$

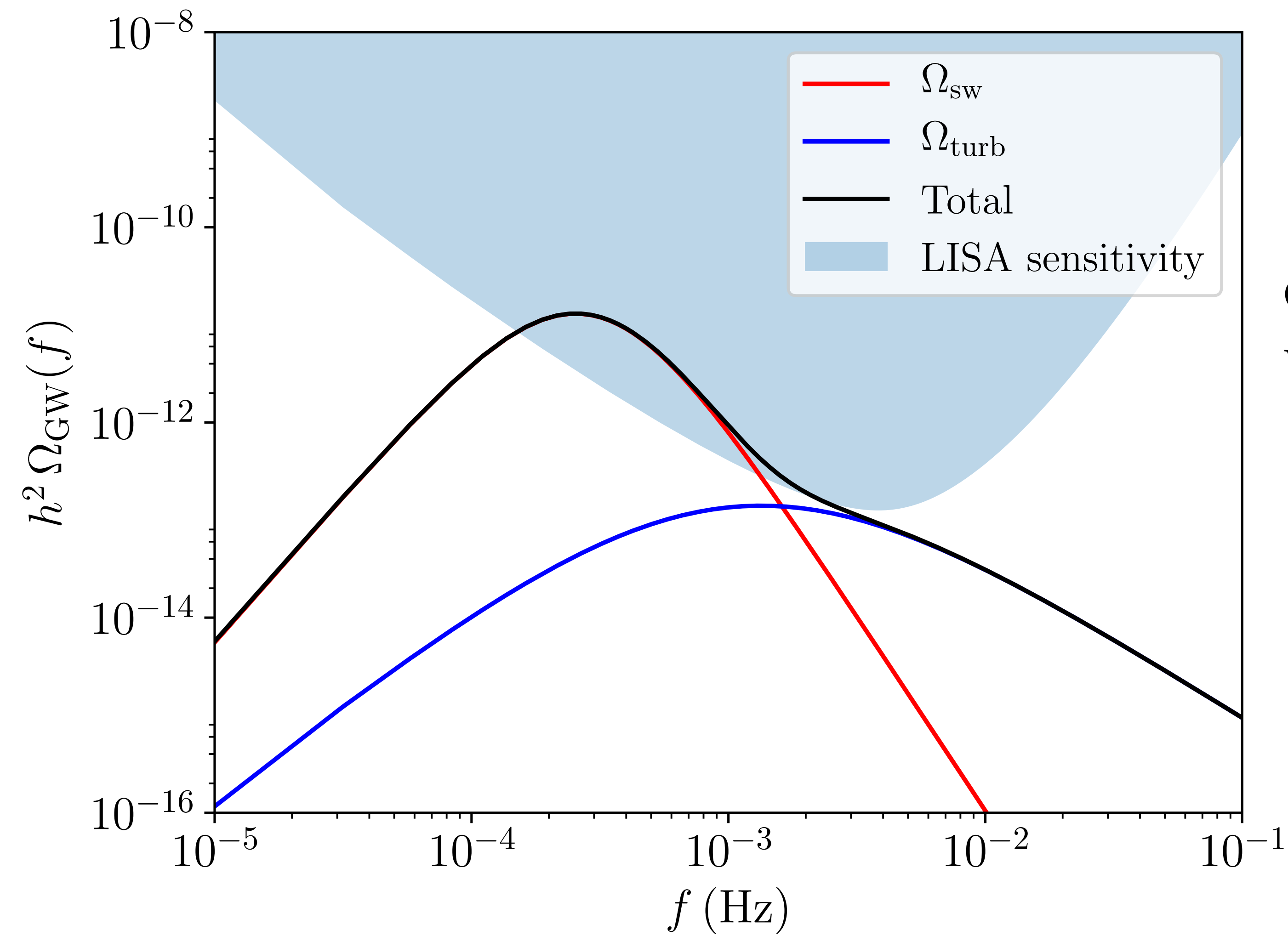
Gravitational Waves from a First-Order EWPT

- First-order EWPT: a violent event in the history of the Universe.
- Nucleation & Collision of bubbles converts symmetric vacuum $\rightarrow$ broken.
- Process far from equilibrium $\Rightarrow$ large transfers of energy!
 $\Rightarrow$ Creation of Gravitational Waves (GWs).
- Main sources of GWs:
 - Collisions of bubble walls,
 - Sound waves generated after transition,
 - Magneto-hydrodynamical turbulence in the plasma.

Gravitational Waves from a First-Order EWPT

- Such GWs could be observed at future space-based detectors:
 - Laser Interferometer Space Antenna (LISA),
 - Big Bang Observer (BBO),
 - DECi-hertz Interferometer Gravitational wave Observatory (DECIGO),
 - Taiji,
 - Tianqin,
- Online in the next decade or so!

Gravitational Waves from a First-Order EWPT

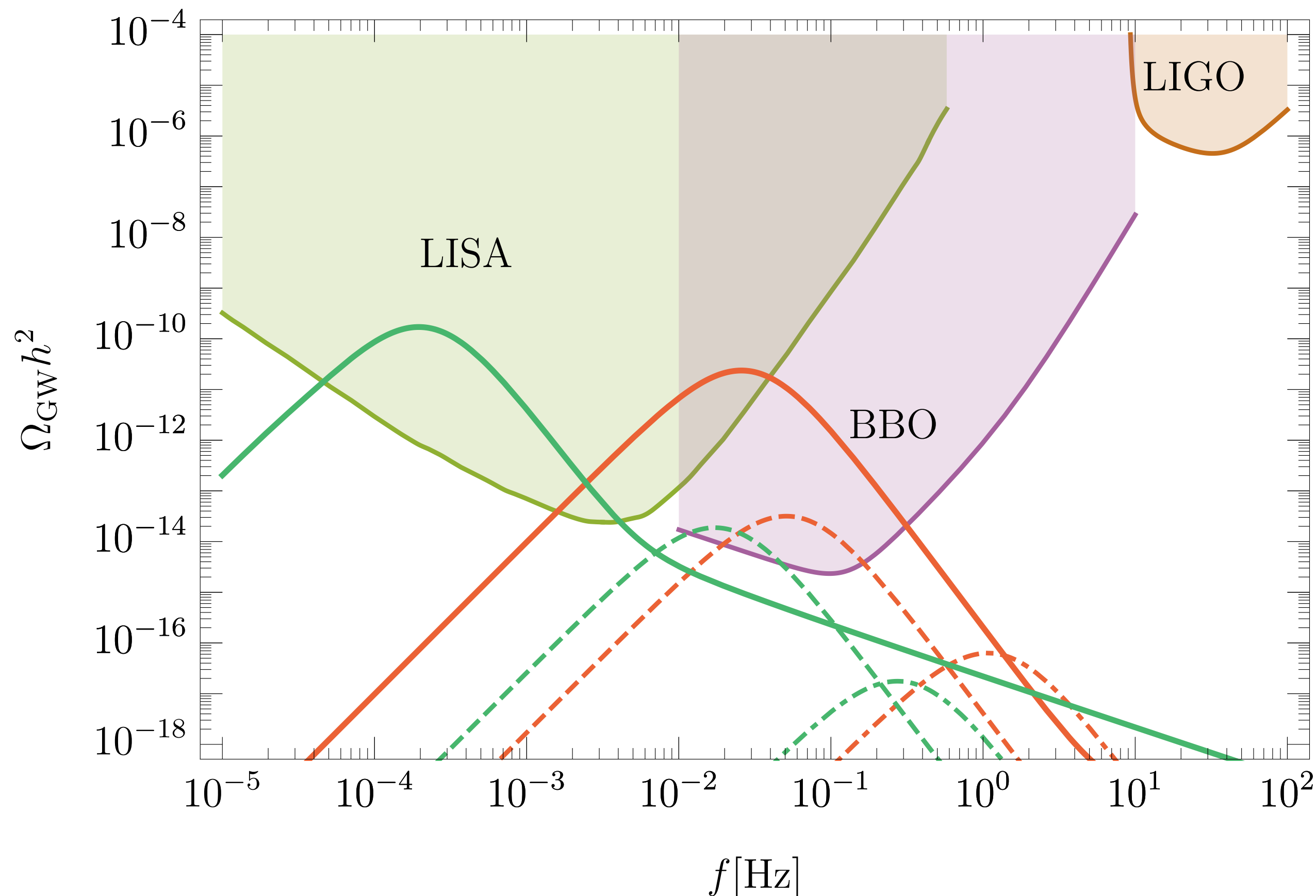


Gravitational wave power spectrum
with a specific set of parameters.

[Weir, arXiv:1705.01783]

Gravitational Waves from a First-Order EWPT

Gravitational wave power spectrum
with a specific set of parameters: Z_2 -
symmetric real singlet.



[Beniwal, Lewicki, Wells, White, Williams, arXiv:1702.06124]

- • $\lambda_{HS} = 0.44, m_S = 80 \text{ GeV}$
- - - + $\lambda_{HS} = 0.38, m_S = 80 \text{ GeV}$
- . - . X $\lambda_{HS} = 0.32, m_S = 80 \text{ GeV}$
- • $\lambda_{HS} = 5.66, m_S = 600 \text{ GeV}$
- - - + $\lambda_{HS} = 5.26, m_S = 600 \text{ GeV}$
- . - . X $\lambda_{HS} = 4.55, m_S = 600 \text{ GeV}$

Such coupling
could be probed
at colliders via:
 $pp \rightarrow h_1 h_2!$

[Carmona, Goertz, [AP](#),
arXiv:1606.02716]

Sphaleron/Instanton Processes

instantons and baryon-# violation

- toy model:

(1+1)-dimensions, Abelian gauge field A^μ , complex scalar Φ , Dirac fermion of unit charge Ψ .

- Euclidean space action:

$$S = \int d^2x \left[\frac{1}{4} F_{\mu\nu}^2 + |(\partial_\mu - ieA_\mu)\Phi|^2 + V(\Phi) + i\bar{\Psi}(\partial_\mu - ieA_\mu)\gamma^\mu\Psi \right]$$

“Higgs potential”: $V(\Phi) = \lambda(\Phi^*\Phi - v^2)^2 \implies \text{“EWSB”} \implies M_A, M_h$

instantons and baryon-# violation

- consider the current:

$$K_\mu = \frac{e}{2\pi} \epsilon_{\mu\nu} A_\nu$$

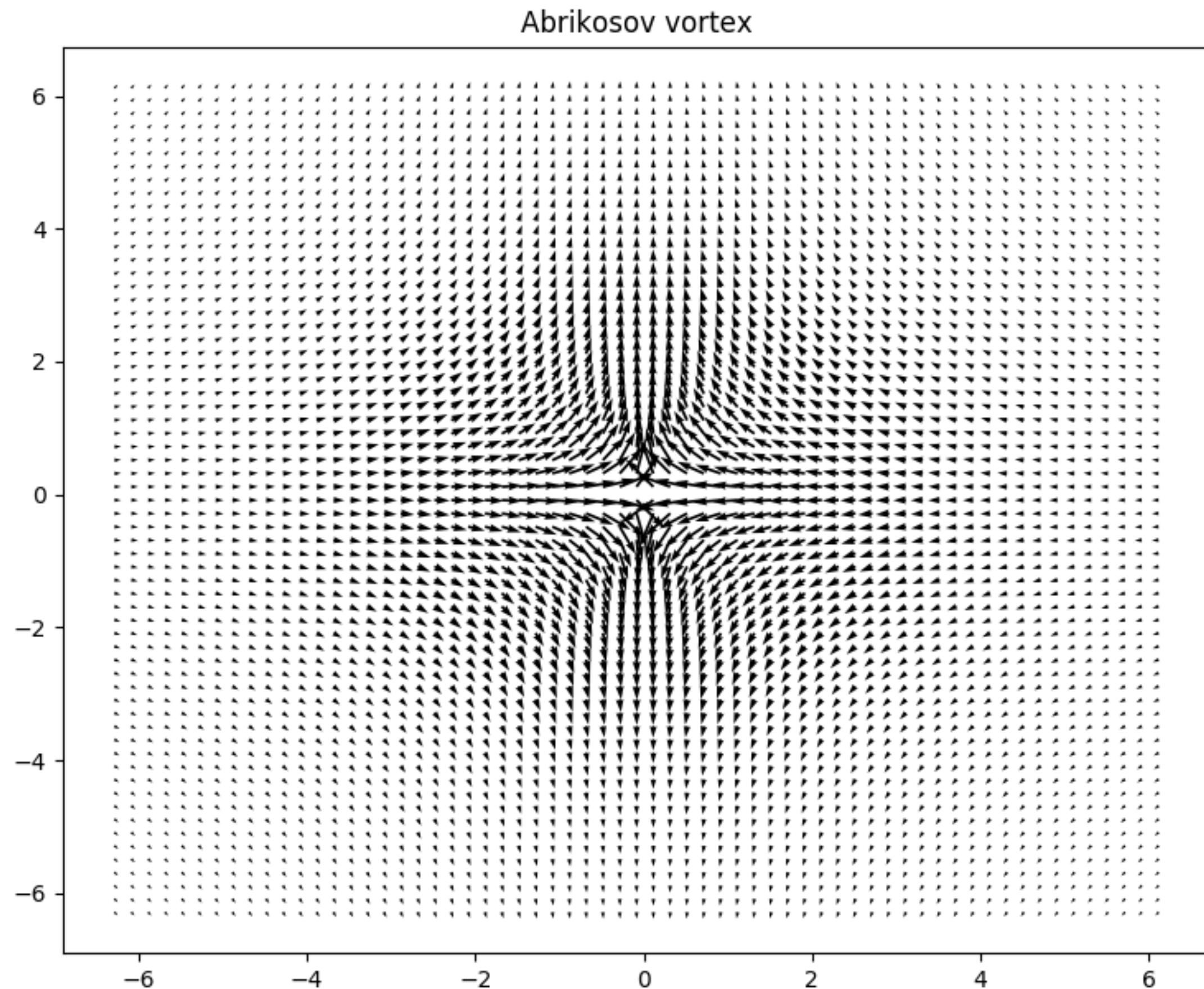
- corresponds to “charge density”:

$$N_{\text{CS}} = \int dx \, K_0 = \frac{e}{2\pi} \int dx \, A_1$$

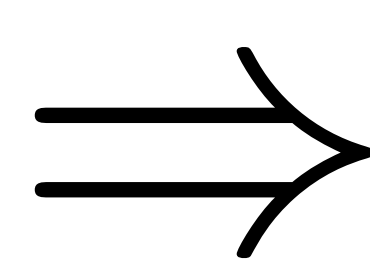
- known as the “winding” or “Chern-Simons” number.

instantons and baryon-# violation

- a classical solution to equations of motion is the “Abrikosov vortex”:



$$A_r = 0, \quad A_\theta = \frac{1}{er} f(r),$$
$$f(0) = 0, \quad 1 - f(r) \sim e^{-M_A r}$$



changes the Chern-Simons
number by one unit:

$$\Delta N_{\text{CS}} = \int d^2x \, \partial_\mu K^\mu = \boxed{\frac{e}{4\pi} \int d^2x \, \epsilon_{\mu\nu} F^{\mu\nu}} = 1$$

instantons and baryon-# violation

- “instanton” transition necessarily accompanied by change of chirality of fermions by two units:

$$j_\mu^5 = \bar{\Psi} \gamma_\mu \gamma_5 \Psi$$

$$\frac{1}{2} \partial_\mu j^{5\mu} = \frac{e}{4\pi} \epsilon_{\mu\nu} F^{\mu\nu} \quad \text{anomalous divergence of the axial-vector current.}$$

$$\Delta N_{\text{CS}} = \int d^2x \partial_\mu K^\mu = \boxed{\frac{e}{4\pi} \int d^2x \epsilon_{\mu\nu} F^{\mu\nu}} = 1$$

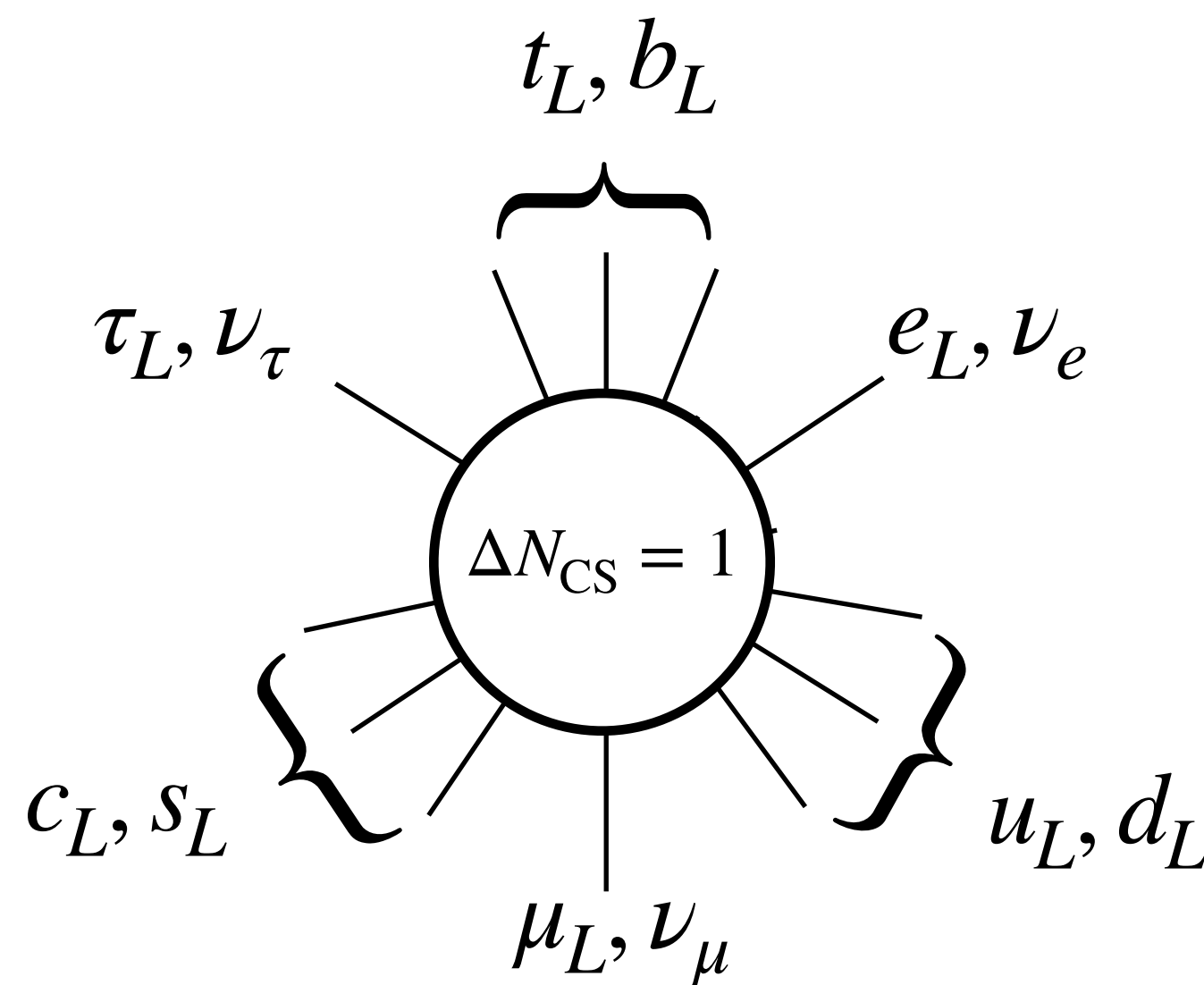
$$\Rightarrow \Delta Q_5 = \Delta \int dx j_0^5 = 2$$

EW Sphalerons at colliders?

- Rate and observability of sphaleron processes at colliders debated.

e.g. [Bezrukov, Levkov, Rebbi, Rybakov, Tinyakov, hep-ph/0304180] VS. [Tye, Wong, 1505.0360, 1710.07223].

- Ponder: **Sphaleron-induced interactions at hadron colliders:**

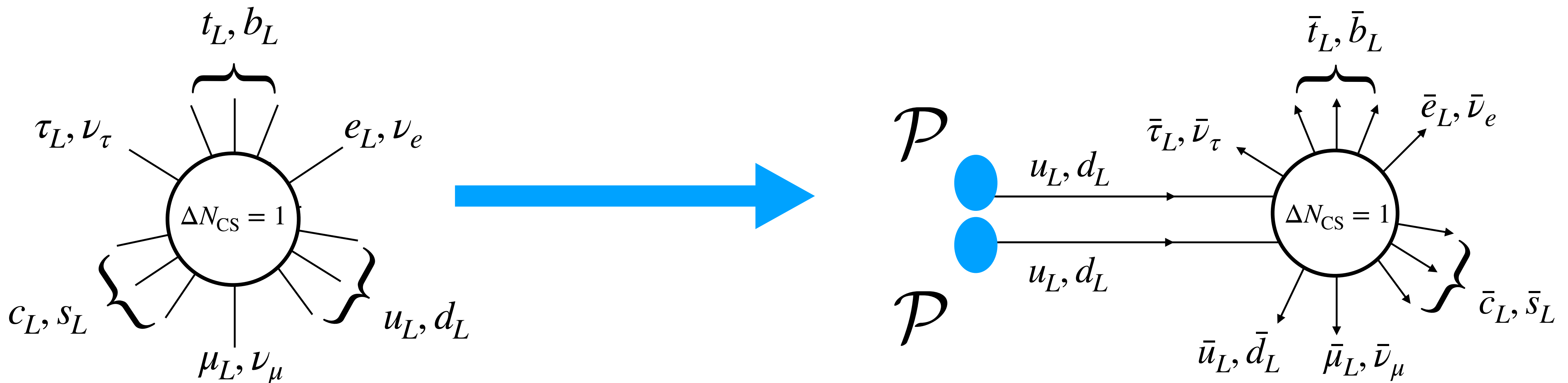


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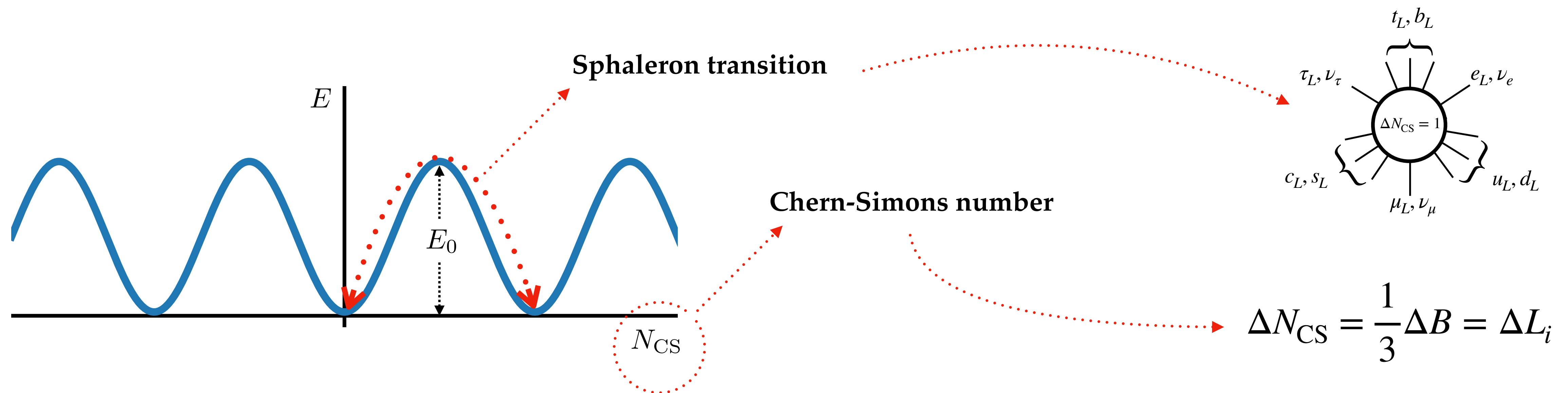
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What is the Sphaleron?



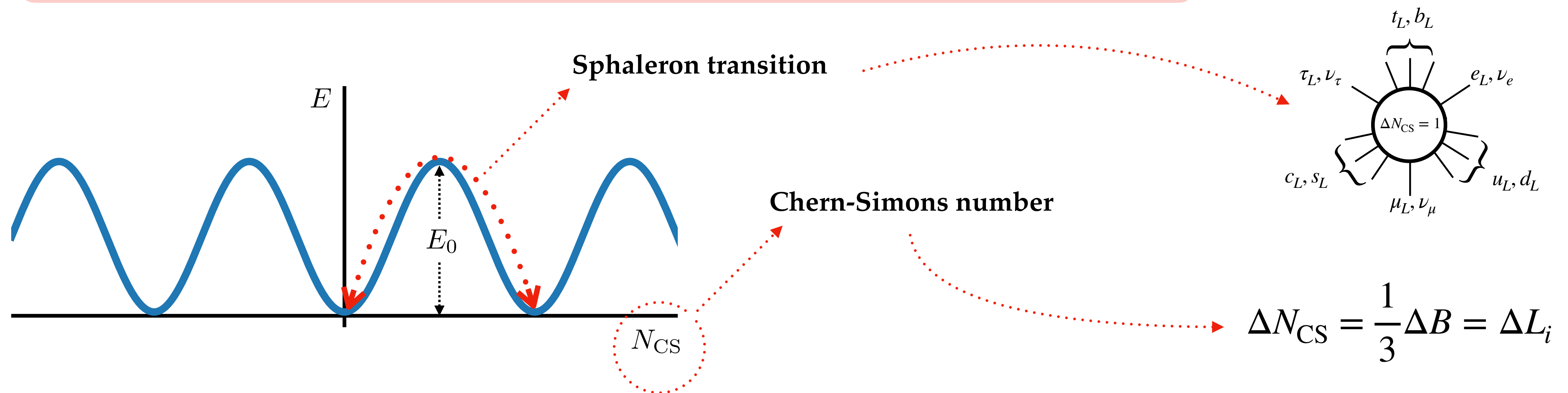
- $E_0 \sim \mathcal{O}(10)$ TeV, separates **degenerate** Electro-Weak vacua.



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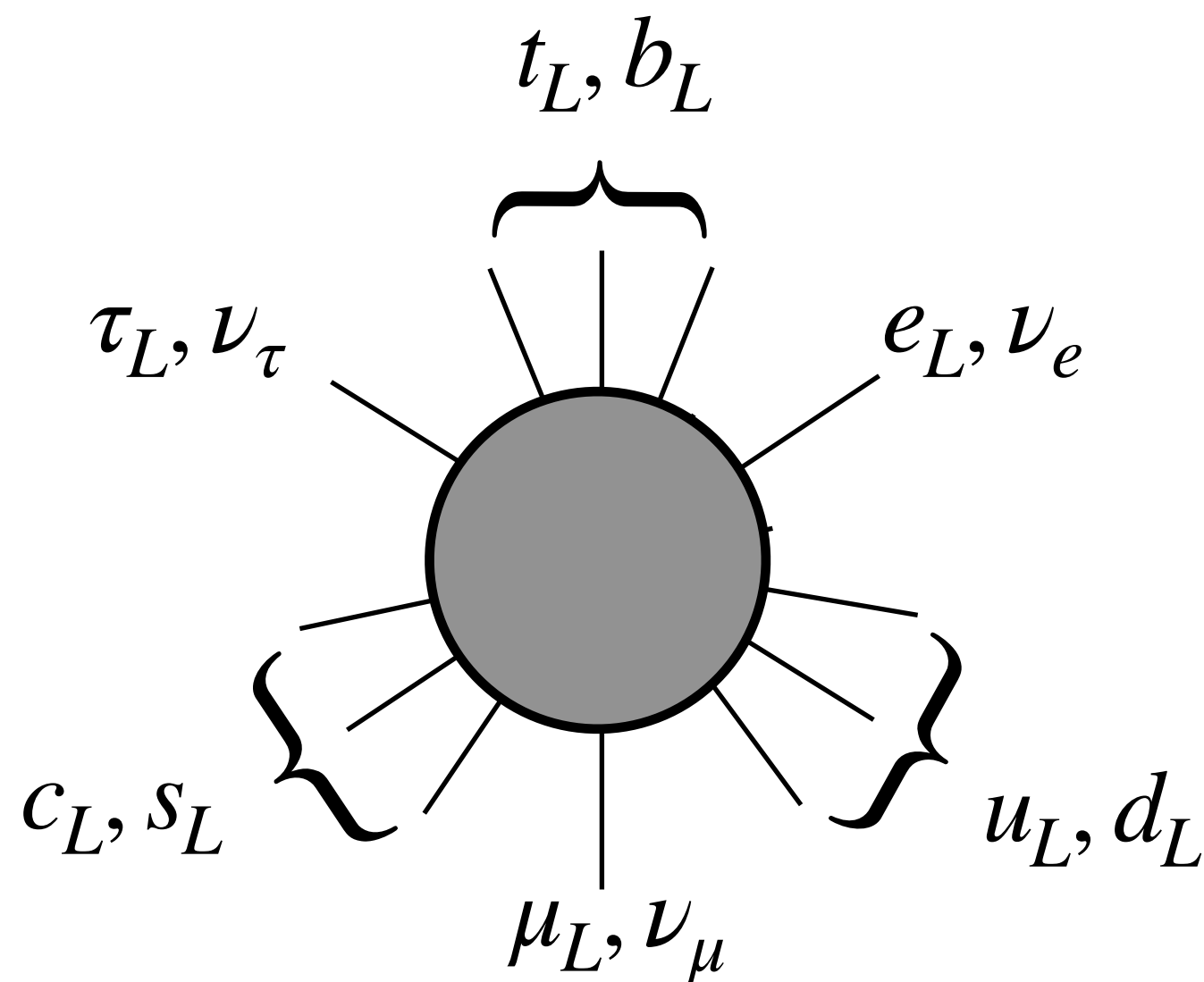
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**The Sphaleron energy depends
crucially on the Higgs sector!**

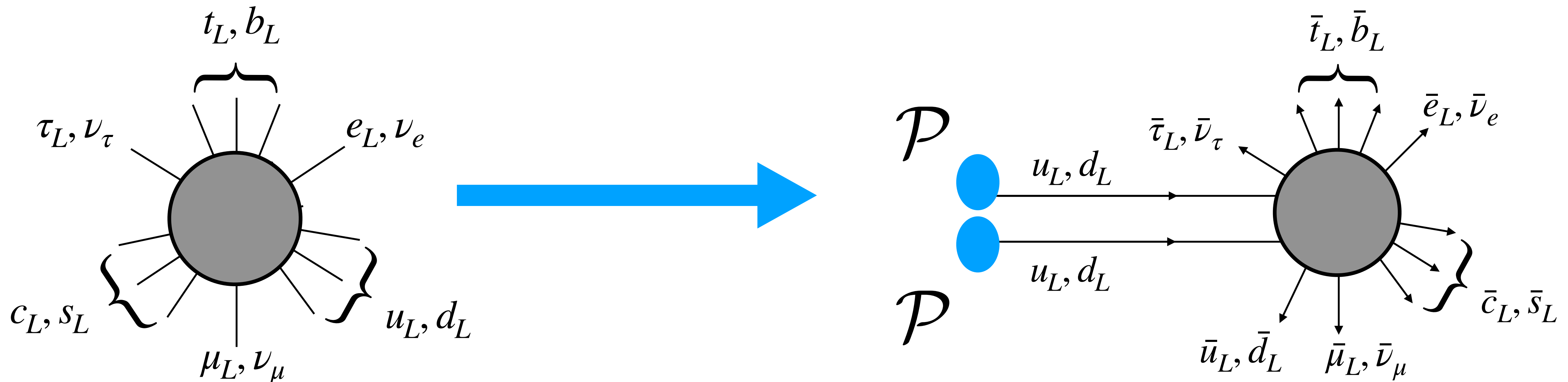
Sphaleron Suppression

- Inside the bubble:  Sphaleron
- Suppression requires “**Strong**” First-Order EWPT (**SFO-EWPT**).
- **Despite suppression:** Sphalerons @ colliders?

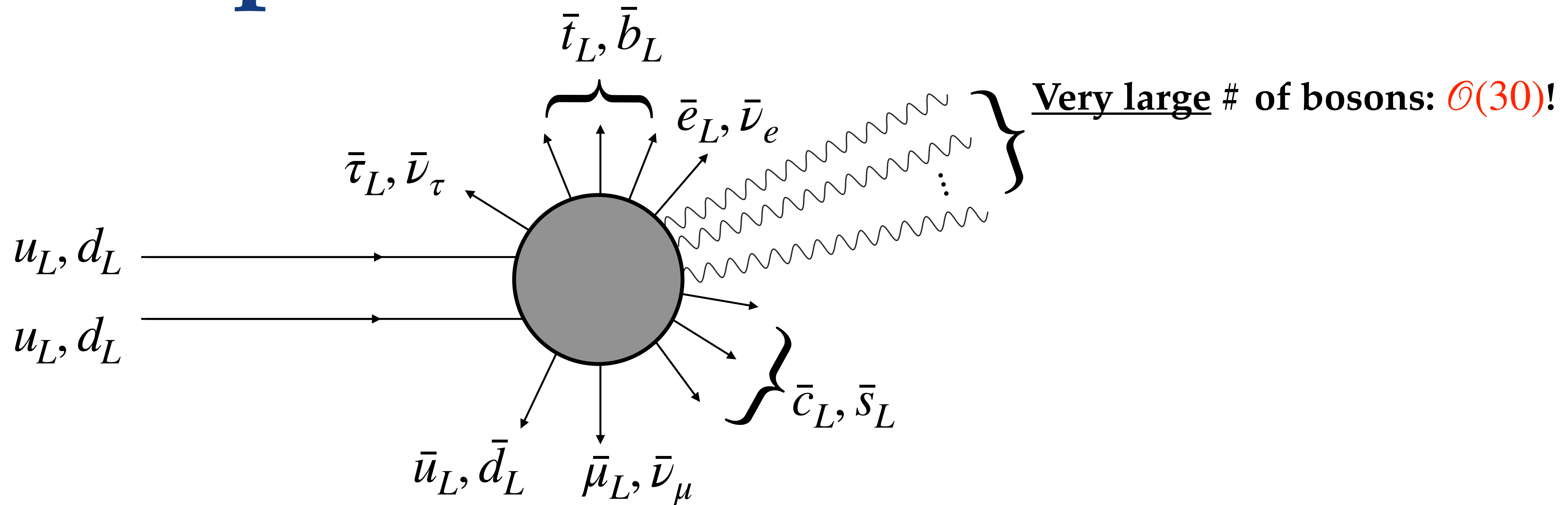


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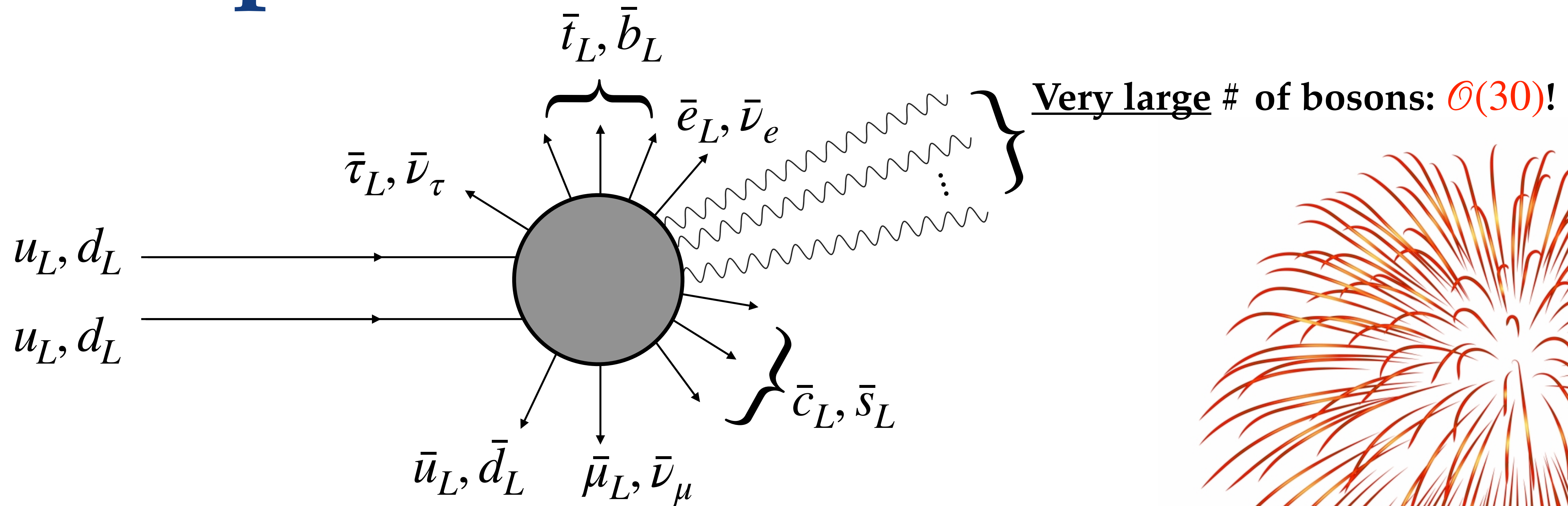
EW Sphalerons at colliders?



- Possible enhancement if large number of bosons,

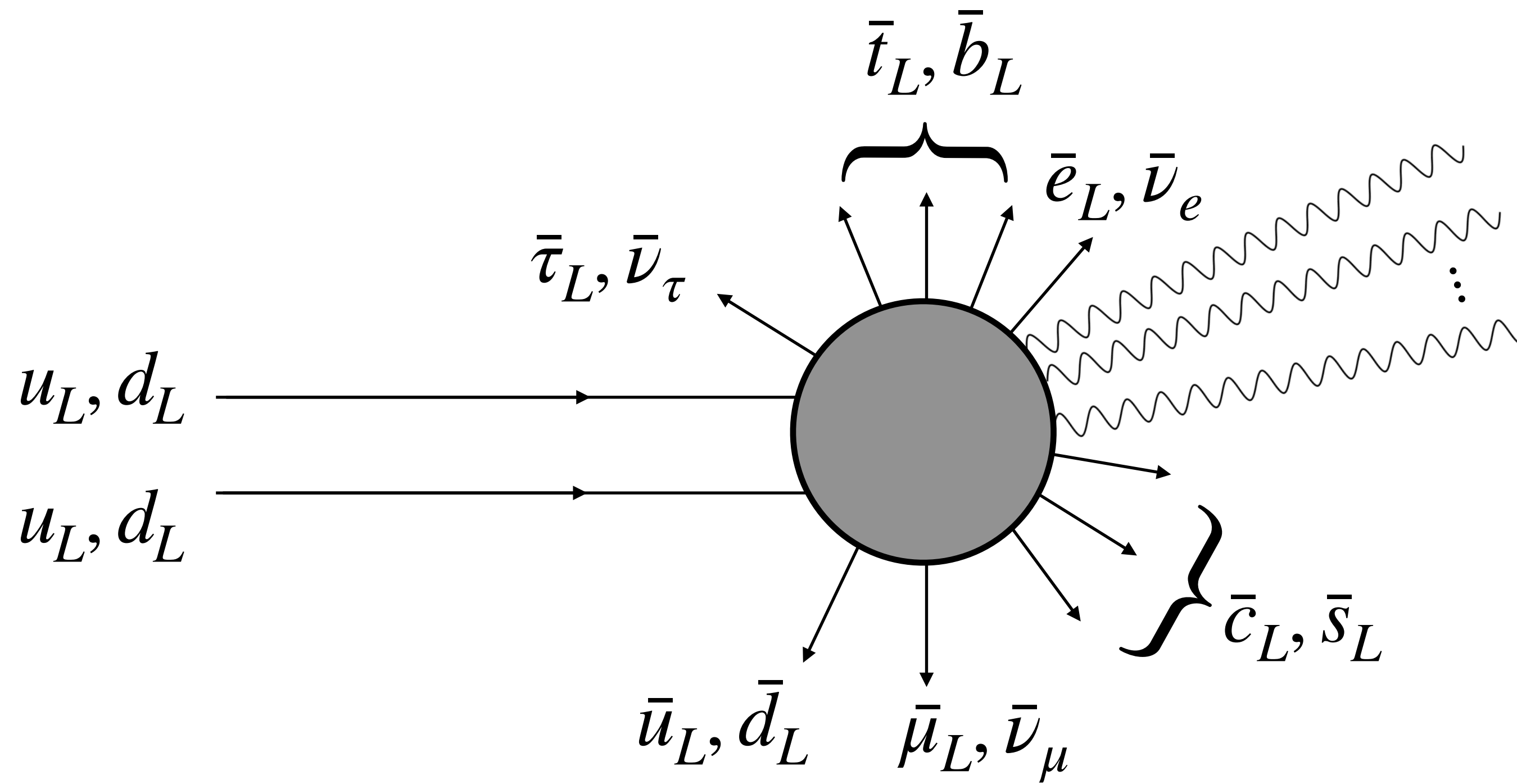
$\Rightarrow$ Events would **spectacularly light up detectors** at experiments!

EW Sphalerons at colliders?



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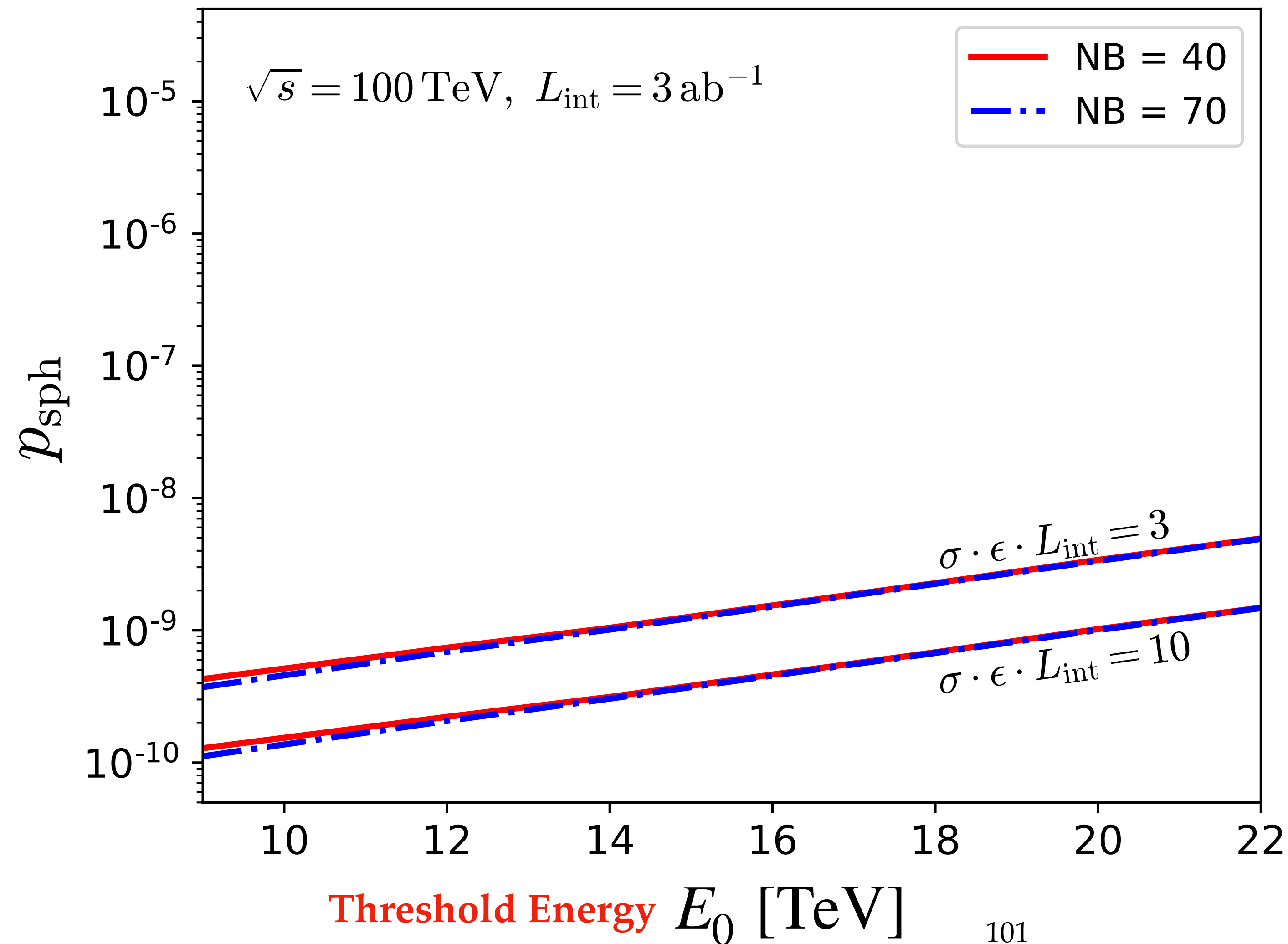
⇒ Events would **spectacularly light up detectors** at experiments!



Sphalerons at the FCC

- Parametrise parton-parton cross section by p_{sph} :

$$\hat{\sigma}(E) = \frac{p_{\text{sph}}}{m_W^2} \Theta(E - E_0)$$



→ Event Generator within HERWIG 7.

[[AP](#), Sakurai, Plätzer, [arXiv:1910.4761](#)]

EW Sphalerons at colliders?

[[AP](#), Sakurai, Plätzer, [arXiv:1910.4761](#)]

⋮



EW Sphalerons at colliders?

[[AP](#), Sakurai, Plätzer, [arXiv:1910.4761](#)]

Homework:

- (i) What can we learn about the Higgs sector and EWBG?
- (ii) New theoretical features in Sphaleron MC.
- (iii) Model discrimination, e.g. VS micro-black holes.
- (iv) Collaboration with experimentalists for measurements.

