Practice Problem Set 2

Question 1 (1 point)

What is the difference between velocity and acceleration?

Acceleration is the derivative of velocity with respect to time; velocity is the derivative of position with respect to time.

$$\vec{a} = \frac{d\vec{v}}{dt} \ , \ \vec{v} = \frac{d\vec{x}}{dt}$$

Questions 2-4 involve Hank Huckleberry, who is the quarterback for the Kennesaw Kilowatts in the Metro Metric Football League. In this league all distances on the field are measured in meters.

Question 2 (3 points)

Hank throws the ball with an initial velocity of 12 m/s at an angle which is 35 degrees up from north. He releases the ball from a height of 1.6 m above the field. How far north does the ball travel before it lands on the field?

Horizontal:

$$a_x = 0$$

$$V_{ix} = 12\frac{m}{s} \times \cos(35°) = 9.8\frac{m}{s}$$

$$V_{fx} = V_{ix} = 9.8\frac{m}{s}$$

$$x_f = \cancel{x_i} + V_i \Delta t$$

(choose Δt = 1.6s from vertical solutions)

$$x_f = 0 + 9.8\frac{m}{s} \times 1.6\ s = \boxed{15.7\ m}$$

Vertical:

$$a_y = -9.8\frac{m}{s^2}$$

$$v_{fx} = v_{iy} + ay\Delta t$$

$$v_{iy} = 12\frac{m}{s} \times \sin(35°) = 6.9\frac{m}{s}$$

$$y_f = y_i + v_{iy}\Delta t + \frac{1}{2} a\Delta t^2$$

$$0 = 1.6m + 6.9\frac{m}{s}\ \Delta t + \frac{1}{2}\left(-9.8\frac{m}{s^2}\right)\Delta t$$

$$\Delta t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Delta t = \frac{-\left(6.9\frac{m}{s}\right) \pm \sqrt{\left(6.9\frac{m}{s}\right)^2 - 4\left(\frac{1}{2} \times -9.8\frac{m}{s^2} \times 1.6m\right)}}{-2 \times \left(\frac{1}{2} \times \frac{9.8m}{s^2}\right)}$$

$$\Delta t = \frac{6.9\frac{\mathrm{m}}{\mathrm{s}} \pm \sqrt{47.6\frac{\mathrm{m}^2}{s^2} + 31.36\frac{m^2}{s^2}}}{9.8\frac{\mathrm{m}}{s^2}}$$

$$\Delta t = \frac{6.9\frac{m}{s} \pm 8.9\ \frac{m}{s}}{9.8\ \frac{m}{s^2}} = 1.6\ s\ or - 0.2\ s$$

Question 3 (3 points)

Hank throws the ball with an initial velocity of 16 m/s at an angle which is 48 degrees up from north. He releases the ball from a height of 1.8 m. How long does it take from the time of the throw until it lands?

$a_x = 0$

$V_{ix} = 16\frac{m}{s} \times \cos(48°) = 8.03\frac{m}{s}$

$V_{fx} = V_{ix} = 8.03\frac{m}{s}$

$x_f = \cancel{x_i} + V_i \Delta t$

(don't need to solve for x_f here)

Vertical:

$a_y = -9.8\frac{m}{s^2}$

$v_{fx} = v_{iy} + ay\Delta t$

$v_{iy} = 16\frac{m}{s} \times \sin(48°) = 11.89\frac{m}{s}$

$y_f = y_i + v_{iy}\Delta t + \frac{1}{2}a\Delta t^2$

$0 = 1.8m + 11.89\frac{m}{s}\Delta t + \frac{1}{2}\left(-9.8\frac{m}{s^2}\right)\Delta t$

$$\Delta t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Delta t = \frac{-\left(11.89\frac{m}{s}\right) \pm \sqrt{\left(11.89\frac{m}{s}\right)^2 - 4\left(\frac{1}{2} \times -9.8\frac{m}{s^2} \times 1.8m\right)}}{-2 \times \left(\frac{1}{2} \times \frac{9.8m}{s^2}\right)}$$

$$\Delta t = \frac{11.89\frac{\mathrm{m}}{\mathrm{s}} \pm \sqrt{141.4\frac{\mathrm{m}^2}{s^2} + 35.3\frac{m^2}{s^2}}}{9.8\frac{\mathrm{m}}{s^2}}$$

$$\Delta t = \frac{11.89\frac{m}{s} \pm 10.72\frac{m}{s}}{9.8\frac{m}{s^2}}$$

$\Delta t = \boxed{2.3\ s}\ or - 0.12\ s$ (choose positive solution)

Question 4 (3 points)

Hank throws the ball in exactly the same situation as question 2, except that now there is a wind moving to the west at 4.6 m/s. What is the final speed of the ball when it lands?

For a velocity in three dimensions:

$$|v_f| = \sqrt{v_{fx}^2 + v_{fy}^2 + v_{fz}^2}$$

$v_{fx} = 8.03\frac{m}{s}$ (from previous question)

$$v_{fy} = v_{iy} + a_y \Delta t = 11.89\frac{m}{s} + \left(-9.8\ \frac{m}{s^2} \times 2.3\ s\right) = -10.65\frac{m}{s}$$

$$v_{fz} = 4.6\frac{m}{s}$$

$$|v_f| = \sqrt{\left(8.03\frac{m}{s}\right)^2 + \left(-10.65\frac{m}{s}\right)^2 + \left(4.6\frac{m}{s}\right)^2} = \boxed{14.1\frac{m}{s}}$$