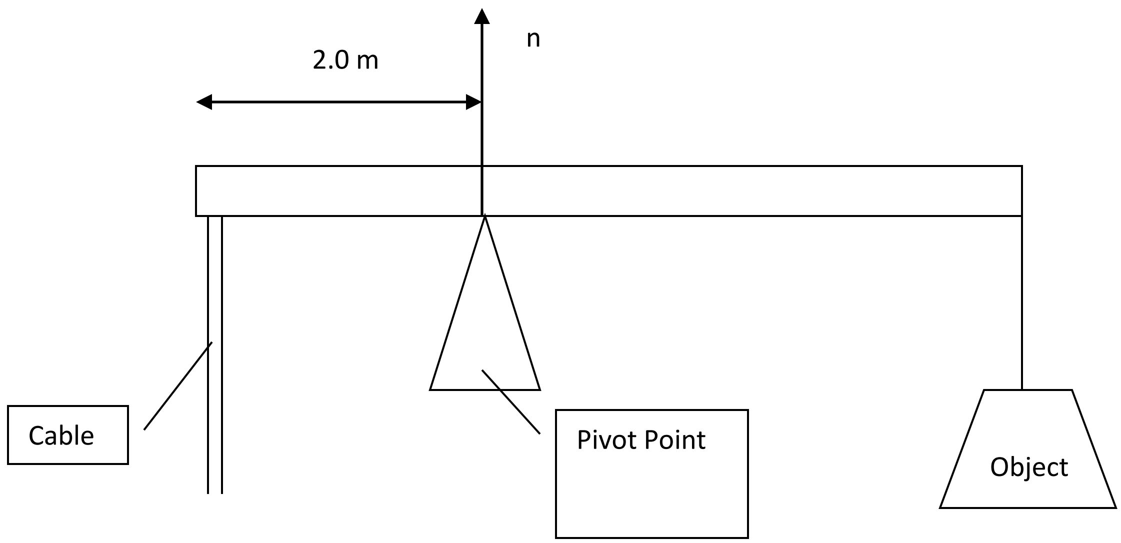
1. Original length of steel cable: 20 m
   Final length of steel cable : 20.001 m
   Diameter of steel cable: 12.5 mm
   Young's modulus for steel cable: 20 x 10^{10} N/m^2
   Mass of uniform beam: 21 kg
   Length of uniform beam: 5.0 m



Find:

a) The mass of the object which would cause the elongation of the cable
b) The magnitude of the upward force n exerted by the pivot point

First we need to determine the force that would cause the elongation of the cable. We are given the modulus, final and initial length, and diameter which we can use to find the area. Using the formula for young's modulus and rearranging to solve for F we get

$$Y = \frac{F/A}{\Delta L / L_i} \Rightarrow F = \frac{Y \Delta L A}{L_i}$$

$Y = 20\,E\,10\ N/m^2$

$\Delta L = 20.001 - 20 = .001\ m$

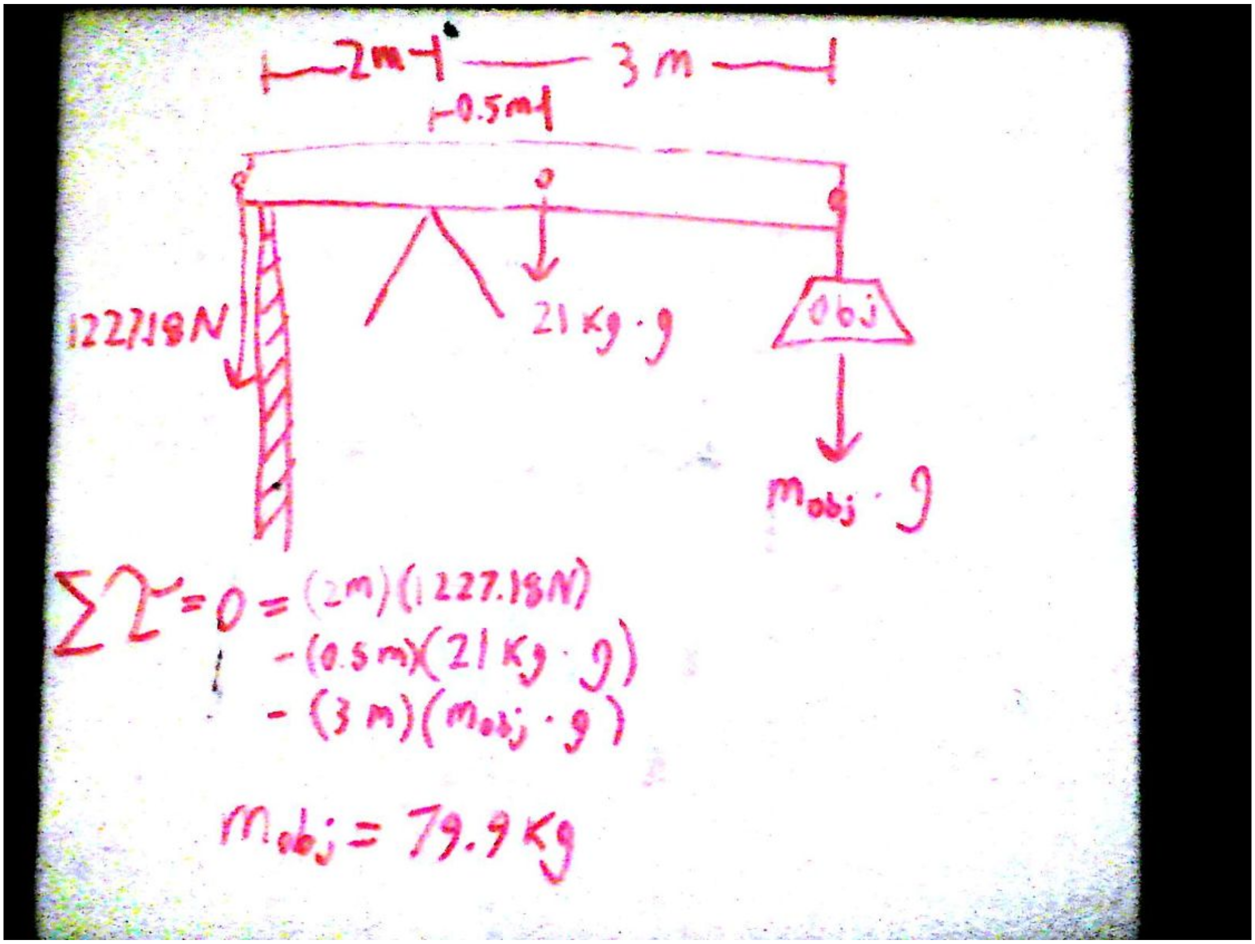
$A = \pi \left(\frac{d}{2}\right)^2 = \pi \left(\frac{.0125}{2}\right)^2$

$L_i = 20\ m$

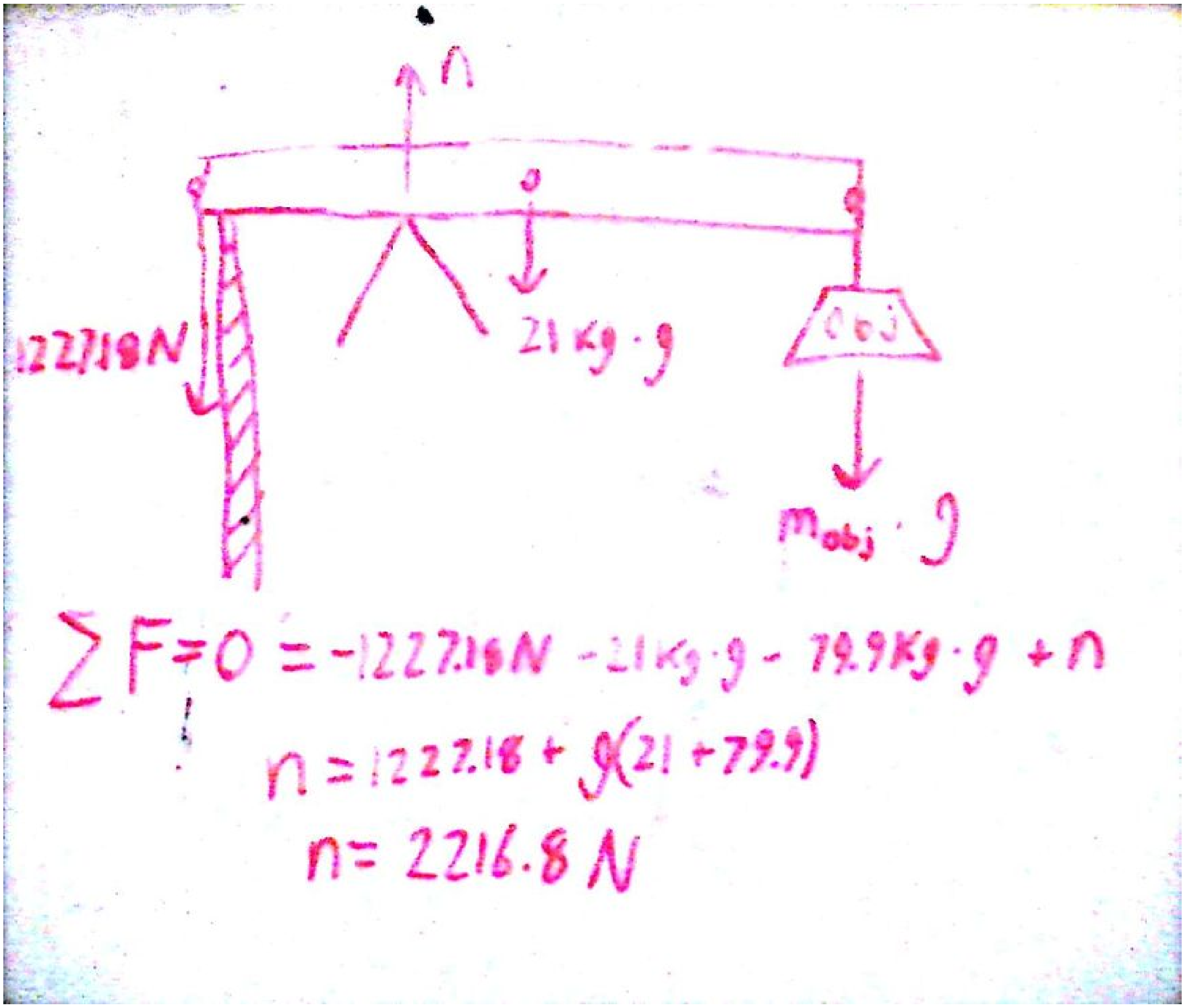
$$F = 1227.18\ N$$

So the cable is subjected to 1227.18 N. However, when we draw the vector it will be down, not up. It is similar to how a cable supporting a weight will have an upward tension to counteract the weight.

For this to be in static equilibrium all torques must balance out to zero so let establish our forces and their distance from the pivot point. We can treat all the forces as vertical. The beam has a mass so we need to find the center of mass and concentrate the entire mass of the beam there. It is a uniform beam so the center of mass is halfway across the beam. Now we can solve for mass of object.



So the object's mass is 79.9 kg. Find n is relatively easy. All the forces must also balance out to be zero. Solving for n leads to



So n = 2216.8 N or close to that depending on your value for g.

2. A meteor is traveling 900 m/s towards the moon when it is 1250 km from its surface. At what velocity will the meteor strike the surface of the moon? Mean radius of moon: $1.74\text{x}10^6$ m
Mass of moon: $7.35\text{x}10^{22}$ kg G: $6.674\text{x}10^{-11}$ N▪m^2/kg^2

In order to solve this you need to use energy conservation. At any point between the beginning of the problem and impact to the moon the meteor has the same amount of energy. The change in energy is zero so the change in kinetic energy plus the change in potential energy also equals zero. Kinetic energy stay ½ mv^2 but gravitational potential energy needs to change to –GMm/r. This is necessary for 2 reasons. First, we are not on earth so we cannot use mgh. Secondly, mgh only works situations near the surface. If we set our final radius to the moon's radius and the final radius to the moon's radius plus the initial height we can rearrange terms to find final velocity.

$$E = KE + PE$$

$$\Delta E = \Delta KE + \Delta PE = 0$$

$$KE_f - KE_i + PE_f - PE_i = 0$$

$$\frac{1}{2} m V_f^2 - \frac{1}{2} m V_i^2 - \frac{G M_m m}{R_m} - \left(-\frac{G M_m m}{R_m + h}\right) = 0$$

$$\frac{1}{2} V_f^2 = \frac{1}{2} V_i^2 + G M_m \left(\frac{1}{R_m} - \frac{1}{R_m + h}\right)$$

$$V_f = \sqrt{V_i^2 + 2 G M_m \left(\frac{1}{R_m} - \frac{1}{R_m + h}\right)}$$

$V_i = 900 \frac{m}{s}$ $\quad G = 6.674E{-11} \frac{N \cdot m^2}{Kg^2}$

$R_m = 1.74E6\ m$ $\quad M_m = 7.35E22\ Kg$

$h = 1.25E6\ m$

$$V_f = 1780 \frac{m}{s}$$

So final velocity is 1780 m/s

3.

4. A car has damaged it suspension so that one of its four springs has half the spring constant of the other three. When the 1200 kg car has no passengers it oscillates at 1.21 Hz. The bad spring is replaced so that it is the same as the other 3. An 85 kg driver enters the car. What is the new frequency? If the driver exits the car and pushes down on the hood so the car drops 32 mm then starts oscillating, what is the car's max vertical speed during this oscillation?

   We are given a mass and a frequency for our initial condition. We can use equations and rearrange to find the spring constant given by 3 good springs and 1 bad spring.

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} \qquad 2\pi f = \sqrt{\frac{K}{m}}$$

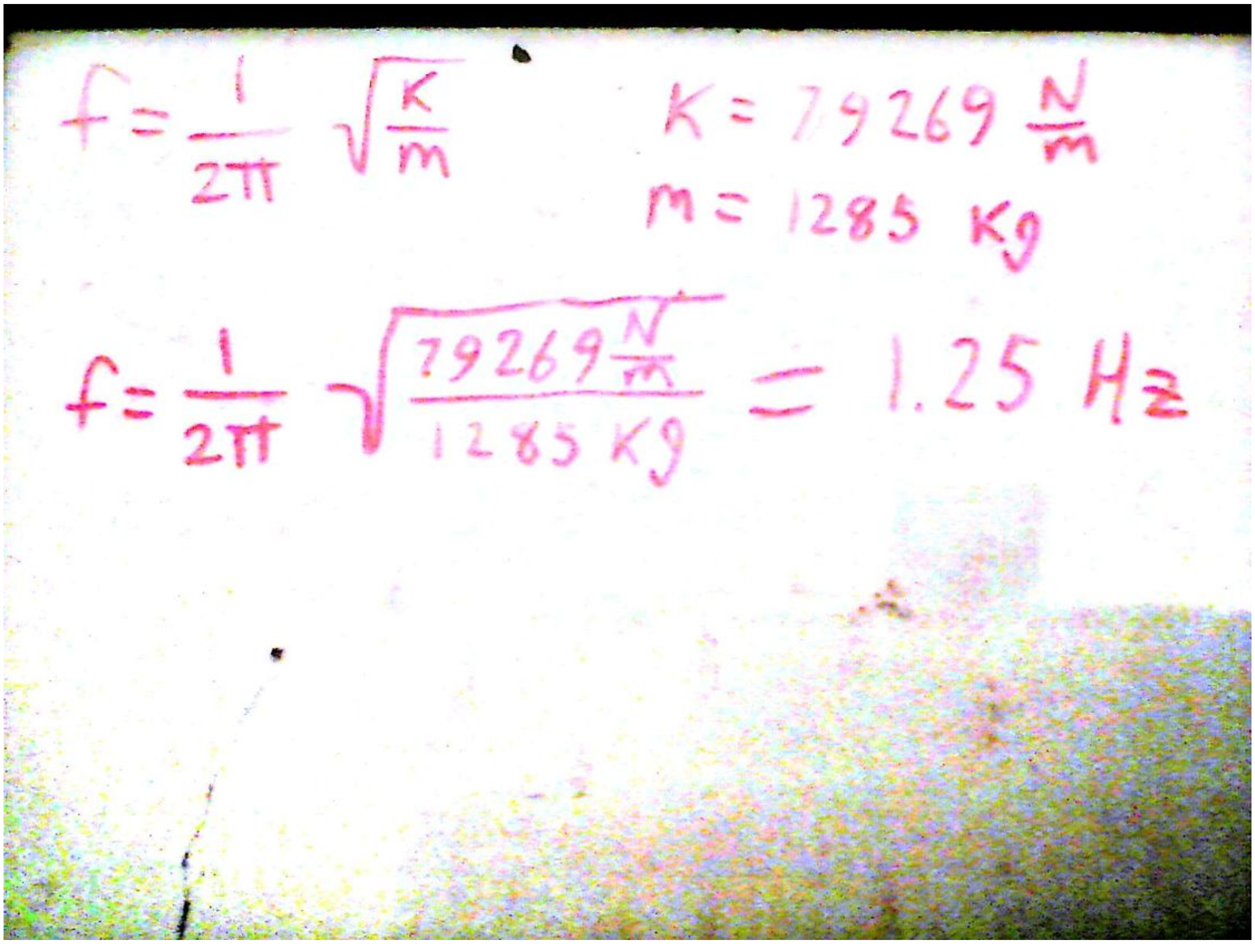
$$4\pi^2 f^2 = \frac{K}{m} \qquad K = 4m\pi^2 f^2$$

$$m = 1200 \text{ Kg}$$

$$f = 1.21 \text{ HZ}$$

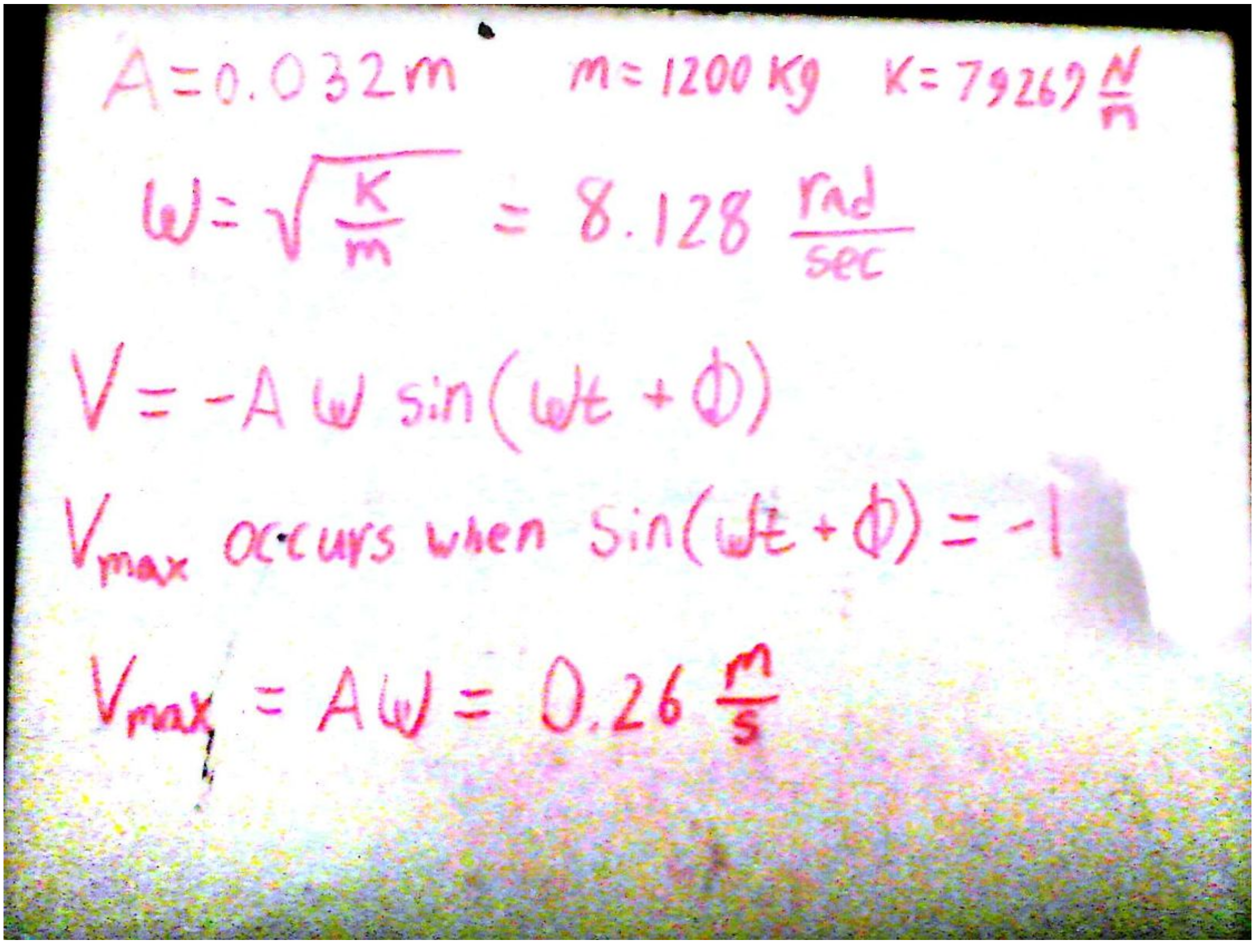
$$K = 69360 \frac{N}{m}$$

So the spring constant for 3 and a half good springs is 69360 N/m. After we replace the bad spring with a good spring we now have 4 good springs instead of 3 and a half. This new spring constant is (4/3.5)*(69360 N/m) or 79269 N/m. We also increase the mass by 85 kilograms. We plug in this new k and m to find frequency.



So our new frequency is 1.25 Hz.

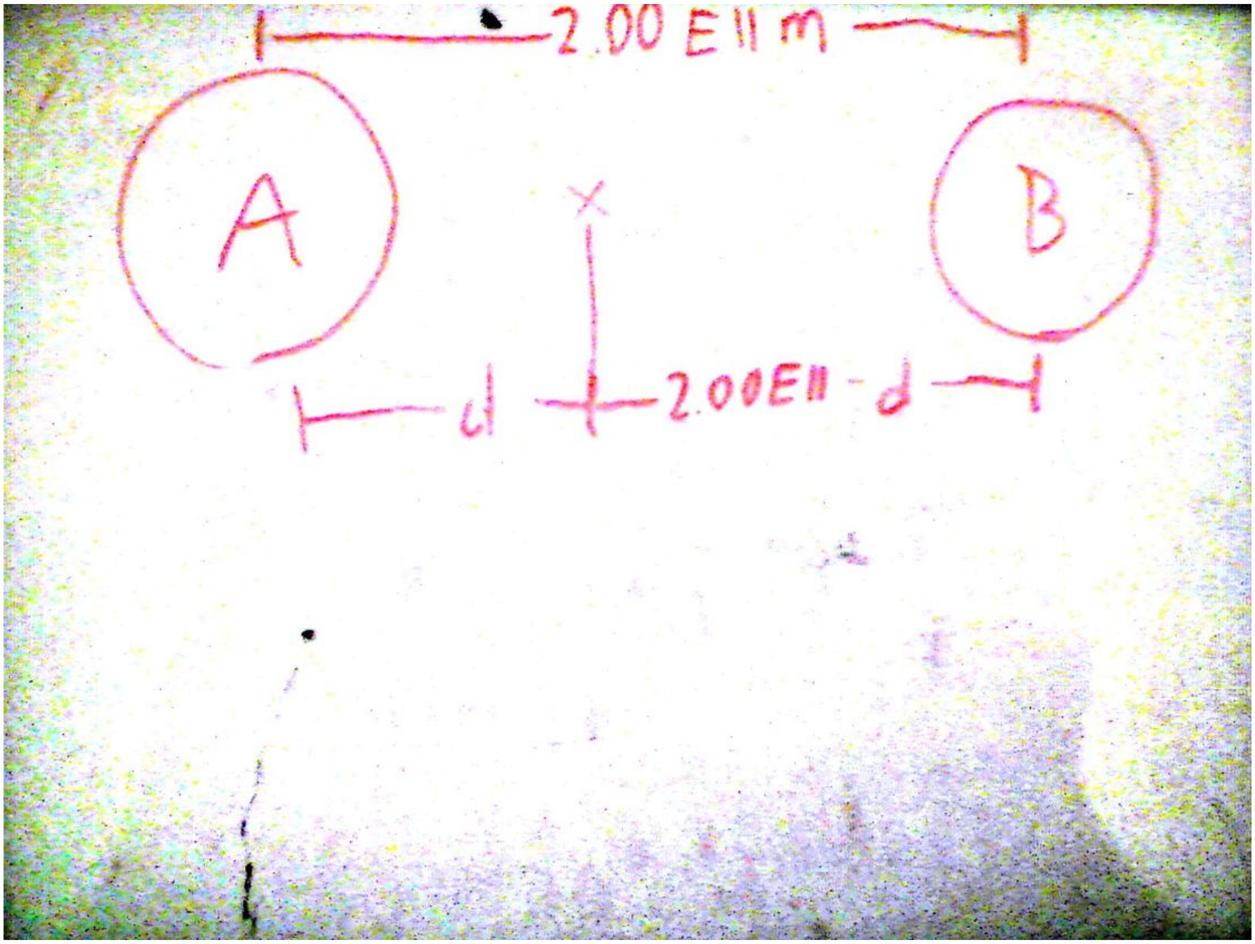
Now we change the mass back to 1200 kg but keep the same spring constant. Our amplitude is how far the mass is displaced from the equilibrium point. In this case it is 32 mm or .032 m. We find our new angular velocity and use that to find v max.



So v max = .26 m/s

5. The distance between the centers of two equally sized planets is 2.00×10^{11} m, planet A has a density of 6.70×10^{3} kg/m^3 and planet B has a density of 4.35×10^{3} kg/m^3. If the radius of both planets is 3.80×10^{6} m, find the distance from the center of planet A towards planet B where there is zero gravitational force acting on an object.

   This can be hard to visualize but an easier example would be if you were place halfway between two identical planets. You would feel an equal amount of gravitational force exerted by each planet in opposite direction so your net force would be zero. Using this we can define a distance from the center of planet A (we'll call it d) and by subtracting d from 2.00×10^{11} m we will have the distance from the center of planet B.



These 2 distances become the radii for our gravitational force equations. For the sake of simplicity we will say planet A pulls our object to the left and Planet B pulls our object to the right. If our net force is zero then the forces must be equal in magnitude. We rearrange our equation to get the following.

$$\sum F = -\frac{G M_A m}{(d)^2} + \frac{G M_B m}{(2.00E11\,m - d)^2} = 0$$

$$\frac{G M_A m}{(d)^2} = \frac{G M_B m}{(2.00E11\,m - d)^2}$$

$$\frac{M_A}{(d)^2} = \frac{M_B}{(2.00E11\,m - d)^2}$$

Now we need the mass of planets A and B. Or do we? The fact that they are equally sized can make this problem much simpler. But let's go with the normal line of thinking for now. Mass is equal to density times volume and the volume of a sphere is (4/3)pi r^3. But both planets are equally sized so the ratio of the masses is proportional to their densities. This lets us simplify our equation and ignore volume.

$$M_A = \rho_A \frac{4}{3}\pi r_A^3 \qquad M_B = \rho_A \frac{4}{3}\pi r_B^3$$

$$\frac{M_A}{d^2} = \frac{M_B}{(2.00E11\,m - d)^2}$$

$$\frac{\rho_A \frac{4}{3}\pi r_A^3}{d^2} = \frac{\rho_B \frac{4}{3}\pi r_B^3}{(2.00E11\,m - d)^2}$$

$$r_A = r_B$$

$$\frac{\rho_A}{d^2} = \frac{\rho_B}{(2.00E11\,m - d)^2}$$

Now we plug in our densities. We can rearrange terms to create a quadratic equation and solve.

$$\frac{6.70E3\,\frac{kg}{m^3}}{d^2} = \frac{4.35E3\,\frac{kg}{m^3}}{(2.00E11m - d)^2}$$

$$6.70E3\,\frac{kg}{m^3}(2.00E11m - d)^2 = 4.35E3\,\frac{kg}{m^3}\,d^2$$

$$6.70E3\,d^2 - 2.68E15\,d + 2.68E26 = 4.35E3\,d^2$$

$$2.35E3\,d^2 - 2.68E15\,d + 2.68E26 = 0$$

$$d = \frac{2.68E15 \pm \sqrt{(2.68E15)^2 - 4(2.35E3)(2.68E26)}}{2(2.35E3)}$$

$$d = 1.10757E11\,m \text{ and } 1.02967E12\,m$$

The distanced has to be in between the 2 planets. If it wasn't then the 2 planets would be pulling the object in the same direction. So our d is 1.10757×10^{11}m.