1. The first thing you need to find is the mass of piece three. In order to find it you need to realize that the masses of the three pieces must be equal to the initial mass of the starting rocket.

$$\vec{P} = mV = 900\ kg\ \frac{m}{s}$$

$$KE = \frac{1}{2} m V^2 = 20250\ J$$

$$\frac{KE = \frac{1}{2} m V^2 = 20250\ J}{\vec{P} = mV = 900\ kg\ \frac{m}{s}}$$

$$\frac{1}{2} V = \frac{20250\ J}{900\ kg\ \frac{m}{s}}$$

$$V = 45\ m/s$$

$$m = 20\ kg$$

$$m_1 + m_2 + m_3 = m_{start}$$

$$11\ kg + 6\ kg + m_3 = 20\ kg$$

$$m_3 = 3\ kg$$

Now we need to find the velocity of piece three by using conservation of linear momentum . This is a 2 dimensional problem so we'll be splitting momentum into x and y coordinates. Let's start with x because since the initial velocity of the rocket was 0 in the x direction, the sum of the x components of momentum for the three pieces will be 0. Set right as positive and left as negative.

$$\vec{P}_{1x} + \vec{P}_{2x} + \vec{P}_{3x} = 0$$

$$(11)(90)(\cos(50^\circ)) - (6)(100)(\cos(20^\circ)) + \vec{P}_{3x} = 0$$

$$\vec{P}_{3x} = -72.54 \text{ kg}\frac{m}{s}$$

$$\vec{P}_{3x} = m_3 V_{3x}$$

$$-72.54 \text{ kg}\frac{m}{s} = (3 \text{ kg})(V_{3x})$$

$$V_{3x} = -24.18 \frac{m}{s}$$

So our x component of velocity for piece three is -24.18 m/s. The negative means it is going to the left. Let’s repeat with the y components but remember before the rocket exploded it had a y component of momentum. Set up as positive and down as negative.

$$\vec{P}_{1y} + \vec{P}_{2y} + \vec{P}_{3y} = 900\ kg\frac{m}{s}$$

$$(11)(90)(\sin(50^\circ)) + (6)(100)(\sin(20^\circ)) + \vec{P}_{3y} = 90$$

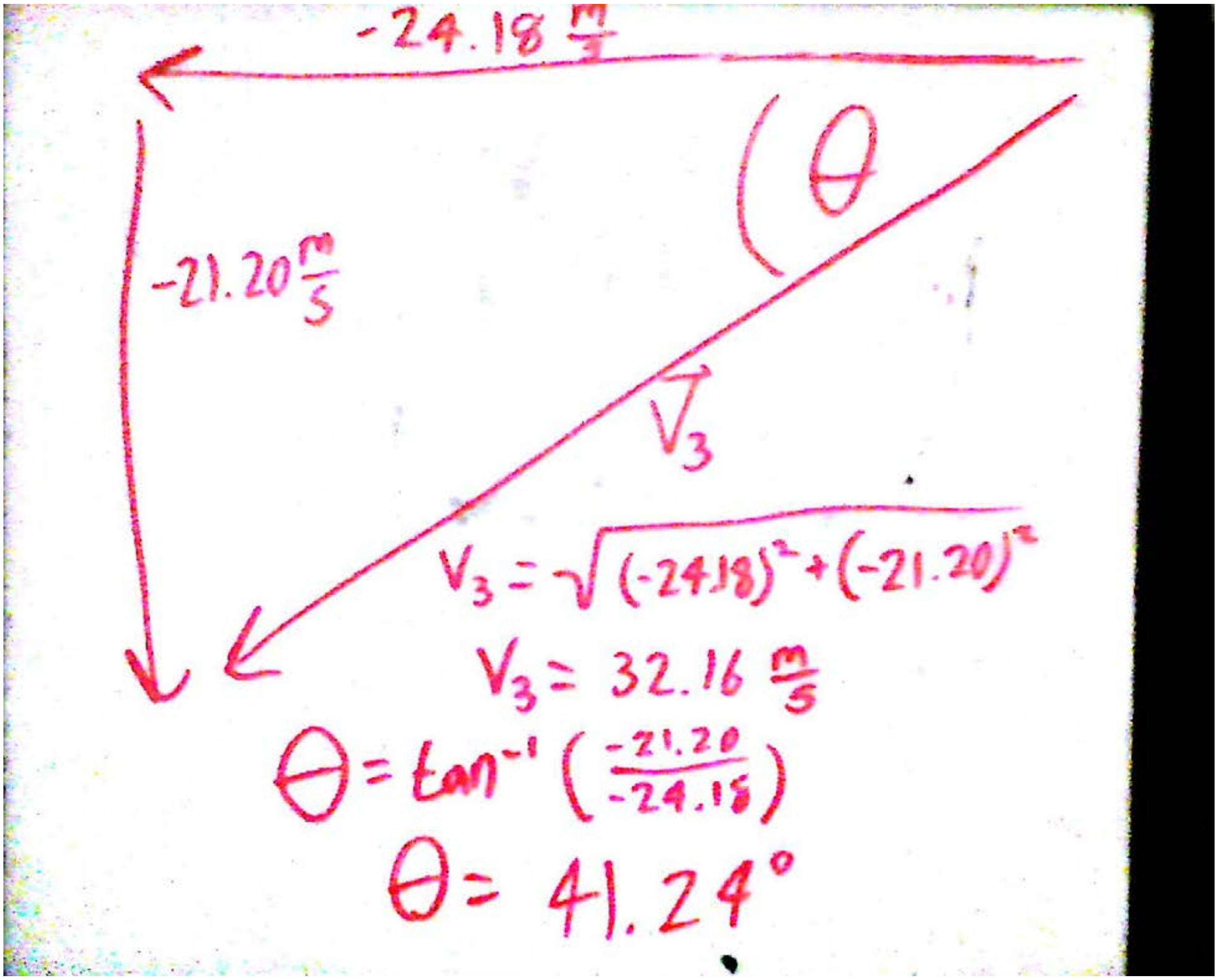
$$\vec{P}_{3y} = -63.60\ kg\frac{m}{s}$$

$$\vec{P}_{3y} = m_3 \vec{V}_{3y}$$

$$-63.60\ kg\frac{m}{s} = (3\ kg)(\vec{V}_{3y})$$

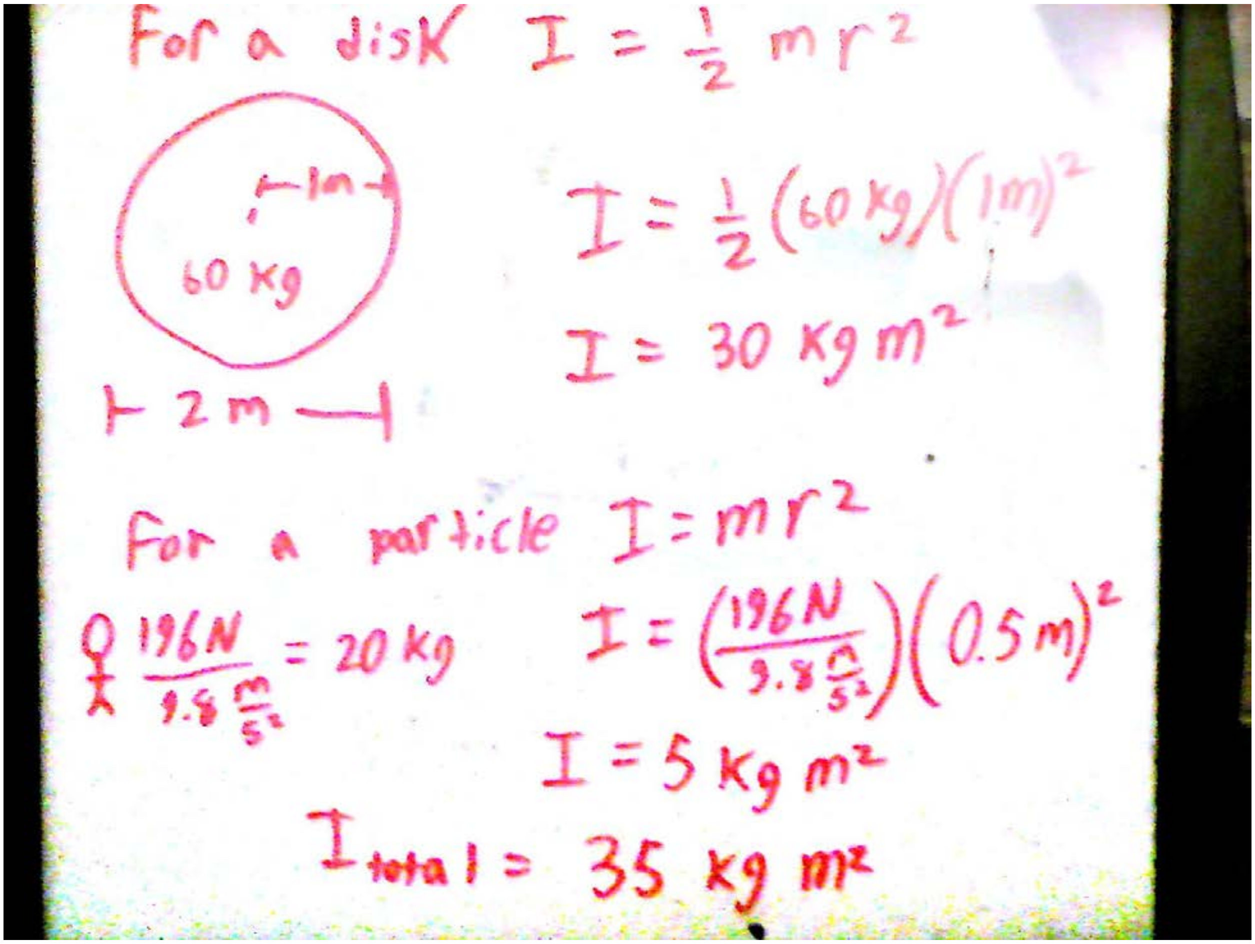
$$\vec{V}_{3y} = -21.20\ \frac{m}{s}$$

So our y component of velocity for piece three is -21.20 m/s. The negative means that it is down. We now have what we need to find total velocity and direction.

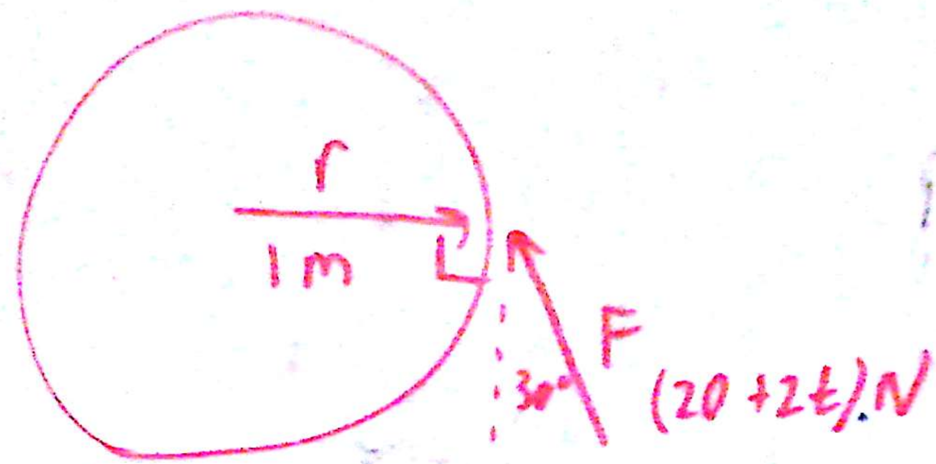


So our final answer is 3kg, 32.16 m/s, and moving left and 41.24 degrees below the x axis.

2. After writing down everything we know we have what we need to find the moment of inertia for the system.



So now that we know the moment of inertia is 35 kg m^2. Now let's focus on the applied force. If we have a force, a radius, and an angle between them we can find a torque. If the force is 30 degrees off the perpendicular it means the angle is either 90 degrees minus 30 degrees or 90 degrees plus 30 degrees. Either value works.



$$\tau = \vec{r} \times \vec{F} = |r||F| \sin\theta$$

$$\tau = (1m)(20+2t)(\sin(120°))$$

$$\tau = \frac{\sqrt{3}}{2}(20+2t)\, N\cdot m$$

We now know our torque is (sqrt(3)/2) times (20 + 2t) N. We now have what we need to find L, α, and ω after 5 seconds. We need to find L before ω but we could also start with α. Let’s start with α.

$$\tau = \vec{r} \times \vec{F} = I\alpha$$

$$\alpha = \frac{\tau}{I} = \frac{\frac{\sqrt{3}}{2}(20+2t)\,N}{35\ kg\ m^2}$$

when $t=5$

$$\alpha = \frac{\frac{\sqrt{3}}{2}(20+2(5))\,N}{35\ kg\ m^2}$$

$$\alpha = .742\ \frac{rad}{s^2}$$

So after 5 seconds α = .742 rad/s^2. Finding L is harder but we know it is the integral of torque. Let's integrate between 0 and 5 seconds.

$$\tau = \frac{\sqrt{3}}{2}(20 + 2t)$$

$$L = \int \tau \, dt$$

$$L = \int_0^5 \frac{\sqrt{3}}{2}(20 + 2t)\, dt$$

$$L = \frac{\sqrt{3}}{2}\left(20t + t^2 \Big|_{t=0}^{t=5}\right)$$

$$L = \frac{\sqrt{3}}{2}(125) = 108.25 \; kg \frac{m^2}{s}$$

Now we know L = 108.25 kg m^2/s. Finding ω is relatively easy because L is also equal to moment of inertia times angular velocity. We know both at 5 seconds so we need to rearrange terms and solve.

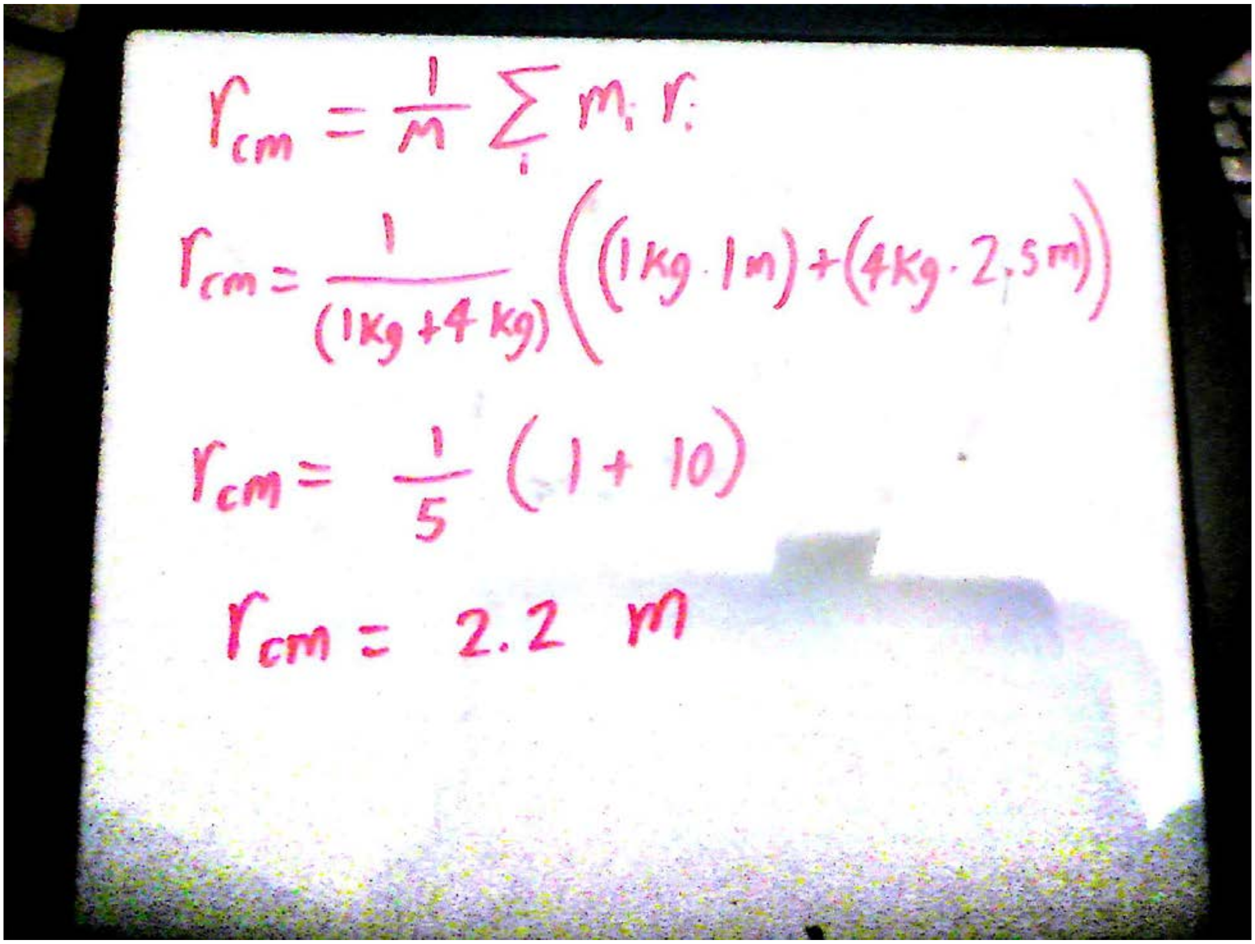
$$L = \int \tau \, dt = I \omega$$

$$\omega = \frac{L}{I} = \frac{108.25 \text{ kg} \frac{m^2}{s}}{35 \text{ kg } m^2}$$

$$\omega = 3.093 \frac{rad}{s}$$

So after 5 seconds L =108.25 kg m^2/s, α = .742 rad/s^2, and ω = 3.093 rad/s

3. This problem needs to be solved in two parts. First we need to find where the third mass is placed. We know that adding a third mass moves the center of mass towards the axis of rotation by 0.1 meters so first we need to find the center of mass when only the original two masses are present. Set our axis of rotation as our reference point.



So if the original center of mass was 2.2 meters away from the axis of rotation and the new mass moves the center of mass 0.1 meters towards the axis of rotation, our new center of mass must be 2.1 meters from the axis of rotation. Let's set up our center of mass equation to find the place of mass 3.

$$r_{cm} = \frac{1}{M} \sum_i m_i r_i$$

$$2.1\,m = \frac{1}{(1kg + 4kg + 5kg)} \left((1kg \cdot 1m) + (4kg \cdot 2.5m) + (5kg \cdot r_3) \right)$$

$$21 = 1 + 10 + 5 r_3$$

$$10 = 5\, r_3$$

$$r_3 = 2\ m$$

So the 5 kg mass is 2 meters from the axis of rotation. If we want to find the angular velocity we need to find angular acceleration. To find angular acceleration we need torque and moment of inertia. Let's find moment of inertia first.

$$I = \sum_i m_i r_i^2$$

$$I = (1\,kg)(1\,m)^2 + (4\,kg)(2.5\,m)^2 + (5\,kg)(2\,m)^2$$

$$I = 1 + 25 + 20$$

$$I = 46\ kg\ m^2$$

So moment of inertia is 46 kg m^2. Now we need to find torque. We have a radius of 1 meter and a constant force of 11.5 N acting perpendicular to the rod so the angle between the radius and the force is 90 degrees. Let's find torque.

$$\tau = \vec{r} \times \vec{F} = |r||F| \sin\theta$$

$$\tau = (1\,m)(11.5\,N)(\sin(90^\circ))$$

$$\tau = 11.5\ N \cdot m$$

So torque is a constant 11.5 N * m. But torque isn't just defined as r X F, it is also equal to moment of inertia times angular acceleration. Also, if our torque is constant then our angular acceleration is also constant.

$$\tau = \vec{r} \times \vec{F} = I\alpha$$

$$\alpha = \frac{\tau}{I} = \frac{11.5\ N\cdot m}{46\ Kg\,m^2}$$

$$\alpha = 0.25\ \frac{rad}{sec^2}$$

So if our angular acceleration is a constant 0.25 rad/s^2, we can use the rotational kinematic equations. The best one to use would be final angular velocity equals initial angular velocity plus angular acceleration times time. We know it starts from rest.

$$\omega_f = \omega_i + \alpha t$$

$$\omega_f = 0 + \left(0.25\frac{rad}{s^2}\right)(6\,s)$$

$$\omega_f = 1.5\,\frac{rad}{s}$$

So our angular velocity after 6 seconds is 1.5 rads/s.

4. Because the two vehicles become intertwined it means we have an inelastic collision and therefore a single final velocity. By using law of conservation for inelastic collisions we can find the velocity of the car.

$$V_f = \frac{m_1 V_1 + m_2 V_2}{m_1 + m_2}$$

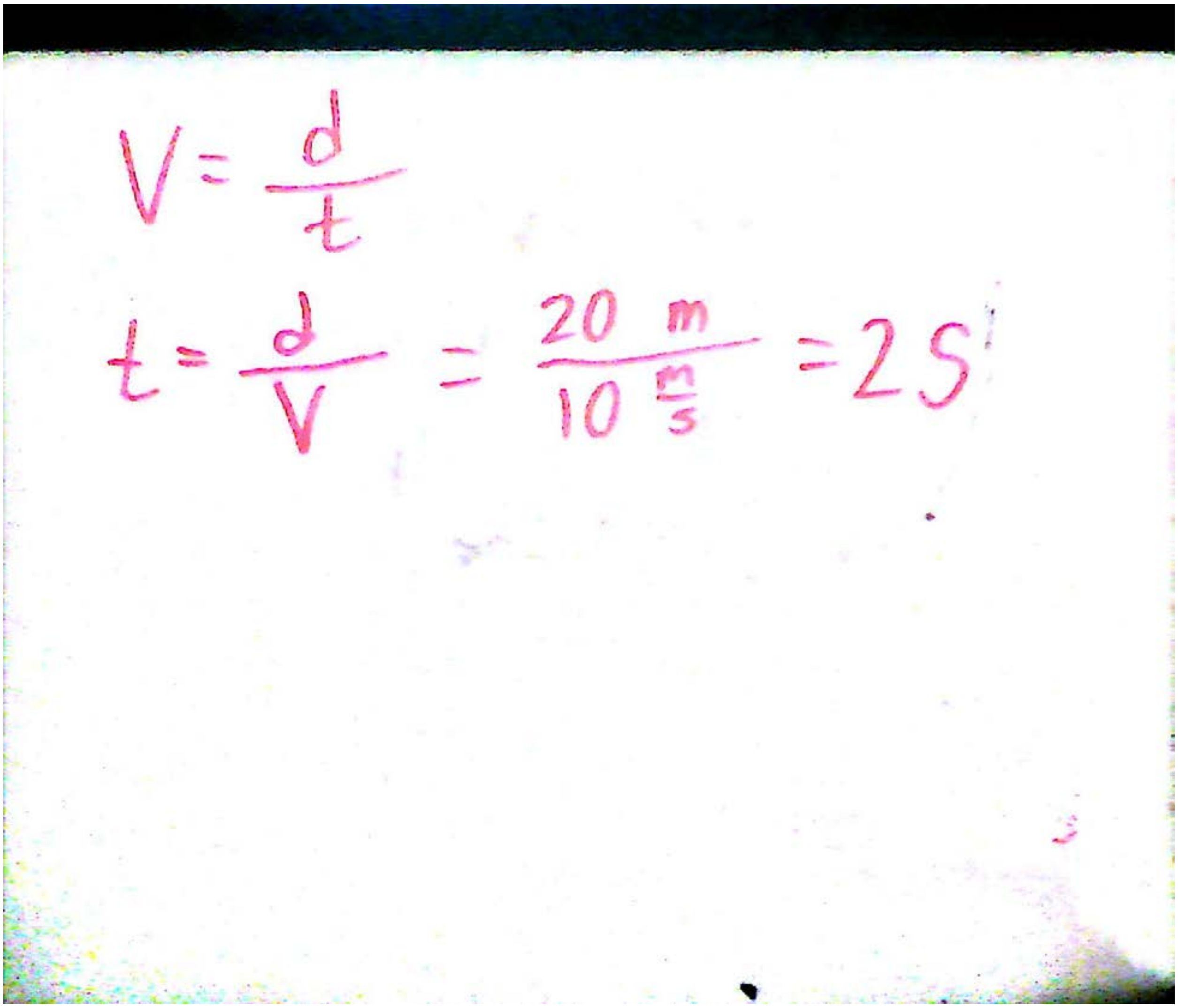
$$V_f(m_1 + m_2) = m_1 V_1 + m_2 V_2$$

$$V_f(m_1 + m_2) - m_1 V_1 = m_2 V_2$$

$$\frac{V_f(m_1 + m_2) - m_1 V_1}{m_2} = V_2$$

$$V_2 = \frac{25(2700 + 1800) - (2700 \cdot 21)}{1800} = 31 \frac{m}{s}$$

So the car is moving at 31 m/s prior to the crash. The important thing here is to realize that the car is moving 10 m/s faster than the truck. Knowing that we can find time by using this relative velocity and the distance.



So our time is 2 seconds. This is the case for all versions of the problems because each of those versions uses a different inertial frame for reference. Even though each of those frames will have different values for the momentums involved, they will each have that momentum conserved. The velocities will be different but the relative velocity of the car to the truck will always be 10 meters per second (at least at velocities not approaching light speed).

5. We are not given V1 initial or V2 initial but we are given V2 initial in terms of V1 initial. If we set right as positive and left as negative then V2 initial is equal to negative one half V1 initial. Let's use the equation for one dimensional elastic collisions. Specifically, the one involving V1 final, V1 initial, and V2 initial. Let's have mass 1 be the 1 kg mass and mass 2 be the 3 kg mass.

$$V_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)V_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)V_{2i}$$

$$V_{1f} = -6\ m/s \qquad V_{2i} = -\frac{1}{2}V_{1i}$$

$$-6 = \left(\frac{1-3}{1+3}\right)V_{1i} + \left(\frac{2(3)}{1+3}\right)\left(-\frac{1}{2}V_{1i}\right)$$

$$-6 = -\frac{1}{2}V_{1i} - \frac{3}{4}V_{1i}$$

$$-6 = \left(-\frac{1}{2} - \frac{3}{4}\right)V_{1i} \qquad V_{1i} = \frac{-6}{\left(-\frac{1}{2} - \frac{3}{4}\right)}$$

$$V_{1i} = 4.8\ \frac{m}{s}$$

If V1 initial is 4.8 m/s to the right then V2 initial is 2.4 m/s to the left (-2.4 m/s). Plugging in these values for the second elastic collision equation will give us V2 final.

$$V_{2f} = \left(\frac{2m_1}{m_1+m_2}\right)V_{1i} + \left(\frac{m_2-m_1}{m_1+m_2}\right)V_{2i}$$

$$V_{2f} = \left(\frac{2(1)}{1+3}\right)(4.8) + \left(\frac{3-1}{1+3}\right)(-2.4)$$

$$V_{2f} = \frac{1}{2}(4.8) + \frac{1}{2}(-2.4)$$

$$V_{2f} = 2.4 - 1.2$$

$$V_{2f} = 1.2 \frac{m}{s}$$

So $(V_1)_i$ =4.8 m/s, $(V_2)_i$ = -2.4 m/s, and $(V_2)_f$= 1.2 m/s.

6. The easiest way to calculate angular momentum in this situation is by multiplying moment of inertia by angular velocity. The formula for calculating moment of inertia for a rod rotated around its end is 1/3 m L^2 where L is the length of the rod (not the angular momentum). So we calculate I = 1/3 (100 kg)(4.2 m)^2 = 588 kg m^2. We now need ω which can be found using 2π/T where T is the period of revolution. Because it is the minute hand it make 1 revolution every 1 hour or 60 minutes or 3600 seconds. 2π/3600 seconds = 0.001745 rads/sec. Multiply these together and we have L = 1.026 kg m^2/s. Because the rotation is clockwise the angular velocity and the angular momentum is pointing into the clock face. A clock facing south will have it minute hand rotate in the opposite direction the clock face is facing. So north.