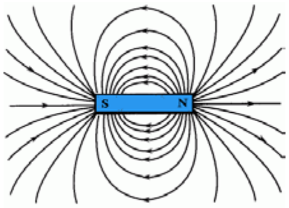
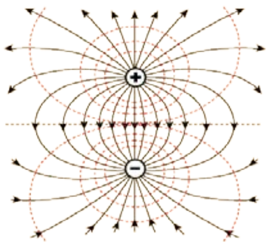
Practice Problem Set – Magnetic Fields - With Solutions

Question 1 (1 point)

Draw the magnetic field lines emanating from a magnetic dipole. How does the shape of the field compare to that from an electric dipole?



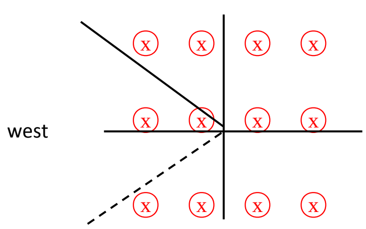
generated from magnetic loops; field lines loop, but don't end



generated from charges; field lines start and end

Question 2 (3 points)

(a) A proton is moving at 12% of the speed of light in the direction which is 20 degrees up from west. It passes through the earth's magnetic field which points due north with a strength of 0.5 x 10^{-4} T. What is the resultant force on the proton? What will the radius of curvature of its path be?



$$\vec{F} = q\vec{v} \; x \; \vec{B} \longrightarrow |\vec{F}| = q|v||B|\sin\theta$$

$$= (1.6 \; x \; 10^{-27} kg)\left(3 \; x \; 10^{8} \frac{m}{s}\right)(0.5 \; x \; 10^{-4} T)(0.12)$$

$$= \mathbf{2.88 \; x \; 10^{-16} N} \text{ 20° west of down or 70° down of west}$$

$$r = \frac{mv}{qB} = \frac{(1.6 \; x \; 10^{-27} kg)(.12)\left(3 \; x \; 10^{8} \frac{m}{s}\right)}{(1.6 \; x \; 10^{-19} C)(0} = \mathbf{7500 m}$$

(b) A cyclotron is used to accelerate protons to a velocity of 35,000 m/s. If the magnetic field for the cyclotron is 0.75 Tesla, how large does the cyclotron have to be? If the protons are directed from the cyclotron to a velocity selector with the same magnetic field, what electric field is needed for the protons to pass through the selector?

$$KE = {}^{1}\!/_{2}\, mv^2 = \frac{q^2B^2R^2}{2m} \longrightarrow r = \frac{mv}{qB}$$

$$r = \frac{(1.67\ x\ 10^{-27}kg)(35{,}000\frac{m}{s})}{(1.6\ x\ 10^{-19}C)(0.75T)} = \mathbf{0.49\ mm}$$

$$v = \frac{E}{B} \longrightarrow E = vB$$

$$= \left(35{,}000\frac{m}{s}\right)(0.75T) = \mathbf{26{,}000\frac{v}{m}}$$

Question 3 (3 points)

A wire loop is bent into the shape of a square with each side of length 4.5 cm. The loop is placed horizontally on a tabletop with two of the sides oriented north/south and two of the sides oriented east/west. A battery is connected so that a current of 24 mA is produced around the loop; the current flows in the clockwise direction looking from the top. What is the force produced by the earth's magnetic field on each section of current-carrying wire? What is the overall torque on the loop? What would the torque be if the same length of wire were bent into a circle instead of a square (assuming the same current)?

$$\vec{F} = I\vec{l}\ x\ \vec{B}$$

$$|\vec{F}| = (24mA)(4.5cm)(0.5\ x\ 10^{-4}T)$$

$$\overrightarrow{F_1} = 5.4\ x\ 10^{-8}N\ up \qquad \overrightarrow{F_2} = 5.4\ x\ 10^{-8}N\ down$$

$$Torque = I\vec{A}\ x\ \vec{B}$$

$$= (24mA)(0.045m)^2(0.5\ x\ 10^{-4}T)$$

$$= \mathbf{2.4\ x\ 10^{-9}N \cdot m\ East}$$

$$A_{circle} = \pi r^2$$

$$= IA\ x\ B \qquad 4l = circumference = 2\pi r$$

$$= I\left(\frac{4}{\pi}\right)l^2B = \mathbf{3\ x\ 10^{-9}N \cdot m\ East} \qquad r = \frac{2l}{\pi} \qquad A = \pi\left(\frac{2l}{\pi}\right)^2 = \frac{4l^2}{\pi}$$

Question 4 (3 points)

a) A wire of length 24 cm is bent into a square and placed flat on a table. A current of 45 mA is passed through the wire in a counter-clockwise direction (looking from above).

What is the magnitude and direction of the resulting magnetic field at the center of the square?

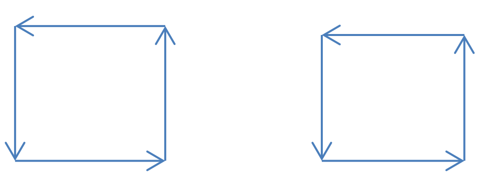
$\vec{B} = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{s}\times\hat{r}}{r^2} = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{s}\hat{r}sin\theta}{r^2}$ ➔ $sin\theta = \frac{l}{2r}$ ➔ $\frac{\mu_0 I}{4\pi}\int\frac{d\vec{s}(l)}{r^2(2r)} = \frac{\mu_0 Il}{8\pi}\int\frac{d\vec{s}}{r^3} = \frac{\mu_0 Il}{8\pi}\int_0^{\frac{l}{2}}\frac{d\vec{s}}{\sqrt{s^2+(\frac{l}{2})^2}^3}$ ➔

From an integral table you can find that: $\int\frac{dx}{\sqrt{x^2+a^2}^3} = \frac{x}{a^2\sqrt{x^2+a^2}}$ ➔ $|B| = \frac{\mu_0 Il}{8\pi}\frac{s}{\frac{l}{4}\sqrt{s^2+(\frac{l}{2})^2}} = \frac{\mu_0 I}{2\pi\sqrt{2}l}$

$$|B| = \frac{(4\pi\times10^{-7})(0.45mA)}{(2\pi\sqrt{2})(0.03m)} = 0.85\mu T$$

Part (b)

Two squares of wire like that in the previous question are placed side by side on a table with a distance of 8 cm between the closest sides of the two squares. A 45 mA current passes counter-clockwise through both squares. What is the resulting force between the two squares? Is it attractive or repulsive?



1 2 3 4

There are no forces in the y direction.

There are four forces in the x direction.

$$F = \frac{\mu_0 I^2 l}{2\pi}\frac{1}{r}$$

$$F_{13}(attractive) = \frac{(4\pi\times10^{-7})(0.45mA)^2(0.06m)}{2\pi}\frac{1}{(0.14m)}$$

$$F_{23}(repulsive) = \frac{(4\pi\times10^{-7})(0.45mA)^2(0.06m)}{2\pi}\frac{1}{(0.08m)}$$

$$F_{14}(repulsive) = \frac{(4\pi\times10^{-7})(0.45mA)^2(0.06m)}{2\pi}\frac{1}{(0.20m)}$$

$$F_{24}(attractive) = \frac{(4\pi \times 10^{-7})(0.45mA)^2(0.06m)}{2\pi} \frac{1}{(0.14m)}$$

$$\sum F = \frac{(4\pi\times10^{-7})(0.45mA)^2(0.06m)}{2\pi} \left(\frac{1}{(0.14m)} - \frac{1}{(0.20m)} - \frac{1}{(0.08m)} + \frac{1}{(0.14m)}\right) = 7.8 \times 10^{-11}\text{N}$$

This is an overall repulsive force between the two squares.