Optimal Hour-Ahead Commitment and Storage Decisions of Wind Power Producers

Ece Cigdem Karakoyun
Econometric Institute, Erasmus University Rotterdam, 3062 PA, Rotterdam, The Netherlands
karakoyun@ese.eur.nl

Harun Avci
Department of Economics, Finance, and Quantitative Analysis, Kennesaw State University, Kennesaw, GA, USA
havci@kennesaw.edu

Woonghee Tim Huh
Sauder School of Business, University of British Columbia, Vancouver, British Columbia V6T 1Z2, Canada
tim.huh@sauder.ubc.ca

Ayse Selin Kocaman, Emre Nadar
Department of Industrial Engineering, Bilkent University, 06800 Ankara, Turkey
selin.kocaman@bilkent.edu.tr, emre.nadar@bilkent.edu.tr

Renewable energy generators often rely on their battery deployments to meet their commitments in electricity markets. We consider the joint energy commitment and storage problem for a wind farm paired with a battery. The power producer decides, in each hour of a finite planning horizon, how much energy to commit to dispatching or purchasing for the next hour, how much wind energy to generate, and how much energy to charge or discharge. The power producer pays a penalty cost if they do not fully meet their commitment. Using a Markov decision process model under uncertainties in electricity price (assumed to be positive) and wind speed, we first prove the optimality of a state-dependent threshold policy for the power producer's problem. This policy partitions the state space into several disjoint domains, each associated with a different action type, making it optimal to bring storage and commitment levels to different threshold pairs in each domain. We then employ our structural results to develop a heuristic solution procedure in a more general setting where the electricity price can also be negative. Numerical results show the high efficiency and scalability of this procedure. It provides solutions with an average deviation of only 0.3% from optimality and achieves a speedup of two to three orders of magnitude compared to the standard dynamic programming algorithm, reducing computation times from several hours to just a few minutes.

Key words: renewable energy; energy storage; electricity markets; Markov decision processes

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1. Introduction

1.1. Motivation and Overview of the Problem

Technological advancements, government support, and cost reductions have led to rapid growth in the use of renewable energy sources, such as wind and solar (Beyene and Tsao 2024). Renewable energy sources are projected to account for over one-third of global electricity generation soon, surpassing coal for the first time (EIA 2024a). While these sources contribute more to the overall energy supply, renewable power producers participate in electricity markets and make advance commitment decisions for energy delivery and purchase. Making these commitment decisions effectively is challenging due to the inherent intermittency of renewable energy sources, increasingly

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volatile electricity prices, and penalty costs arising from energy imbalances – situations where the actual energy generated differs from the committed amount.

One of the most promising solutions to address these challenges is the integration of renewable energy generation with energy storage, particularly through batteries (EIA 2024b). Battery storage helps stabilize renewable energy supply by storing excess energy during periods of overproduction and discharging it during shortages. Batteries also enable energy arbitrage, allowing producers to store electricity when prices are low and sell it when prices are high, further enhancing profitability. [1] Coupled with technological advancements and supportive policies, these factors have driven substantial growth in battery storage in recent years. For example, in the U.S., battery storage additions surged by 64% in 2024, making it the second-largest source of new capacity after renewables (Deloitte 2024). Policymakers are also refining regulatory frameworks to accelerate battery deployment (e.g., the U.S. Federal Energy Regulatory Commission's updated wholesale market rules, EPRI 2024, and the Department of Energy's grid energy storage strategy, DOE 2024). As these policy-driven efforts gain momentum, renewable power producers are at the forefront of a rapidly evolving market landscape, where advance commitment decisions for energy supply and purchase are becoming increasingly critical. In this context, the joint optimization of renewable energy generation and storage is essential for achieving both operational and financial objectives.

We study the energy commitment, generation, and storage problem of a wind power producer who owns a battery and participates as a price-taker (i.e., a producer who does not have the power to affect market prices) in a spot market entailing hourly commitments and settlements. In each time period, the producer decides how much energy to commit for sale or purchase in the next time period, how much wind energy to generate, and how much energy to charge or discharge. The battery installed helps mitigate energy imbalances caused by fluctuations in wind speed, while also enabling price arbitrage over time in the same market. The producer is subject to market regulatory constraints and settlement penalties for imbalances, defined as deviations between the real-time delivered energy (which includes both wind generation and battery operations) and the previously committed amount. A positive imbalance occurs when the delivered amount exceeds the commitment, while a negative imbalance reflects a shortfall. The transmission line and battery storage capacities limit the producer's ability to utilize all available wind energy. As a result, physical curtailment may arise endogenously when wind potential exceeds the combined system capacity,

[1] Although the high capital cost of procuring battery storage and its degradation over time are critical considerations for investment planning, these factors are exogenous to our study. We focus on short-term operational scheduling, assuming a battery of fixed size is already installed. In this context, battery storage is viewed primarily as an operational tool for managing price volatility and market exposure.

which changes dynamically with the battery storage level. We model this problem as a Markov decision process (MDP) by accounting for the electricity-price and wind-speed uncertainties.

1.2. Related Work

Our study contributes to two research streams in the energy literature: The first stream focuses on the energy commitment problem of renewable power generators in electricity markets (e.g., Kim and Powell 2011, Löhndorf et al. 2013, Jiang and Powell 2015, Gönsch and Hassler 2016, He et al. 2016, Hassler 2017, Khaloie et al. 2020, Kim et al. 2020, Zhang et al. 2020, Finnah and Gönsch 2021, Mansouri and Sioshansi 2022, Jeong et al. 2023, Karakoyun et al. 2023, and Chen et al. 2024). The second stream studies the joint optimization of renewable power generation and energy storage (e.g., Harsha and Dahleh 2014, Grillo et al. 2015, Tang et al. 2015, Zhou et al. 2019, Avci et al. 2021, Ma et al. 2022, Tsao and Vu 2023, Peng et al. 2024, and Tsao et al. 2025). Given the extensive body of work in these areas, we refer the reader to Parker et al. (2019) for a comprehensive survey of the related literature. We will review the most relevant papers that employ MDP formulations.

The problem setting we consider is similar to that explored in Karakoyun et al. (2023), which falls within the first research stream. While their work provides valuable insights into the complex interplay between battery deployments, energy imbalances, and penalty parameters, it presents neither optimal policy structure nor efficient solution algorithms. Our work complements theirs by offering a theoretical explanation of the complex system dynamics involved in optimizing energy commitments for intermittent generation assets. Specifically, we analytically derive the optimal policy structure for the joint optimization of energy commitment, generation, and storage decisions within a multidimensional state space. Furthermore, we employ this policy structure to develop computationally efficient solution algorithms. Karakoyun et al. (2023) solve their problem instances with a standard dynamic programming (DP) algorithm, which is difficult to implement in practice due to its long solution times (several hours). However, our solution methods accounting for the structural results in this paper yield solutions in only a few minutes.

Another closely related paper in the first research stream is Kim and Powell (2011), which derives a closed-form solution for the optimal hour-ahead commitment decision of a wind power producer, assuming that wind speed follows a uniform distribution. Their model focuses solely on commitment decisions, where the power producer determines how much energy to sell in advance. Our study extends this work in several key ways. First, we consider a more general model that incorporates energy generation and storage decisions. Second, our model takes into account power capacity constraints and transmission line limitations. Third, while the producer in their model can only commit to selling and cannot induce positive imbalances, our model allows the producer to buy energy and induce both positive and negative imbalances. Finally, our analysis does not rely on any specific probability distribution for wind speed.

The most closely related papers in the second research stream are Zhou et al. (2019) and Avci et al. (2021). Zhou et al. (2019) consider a wind farm co-located with an industrial battery, including the battery storage level as the only endogenous state variable in their MDP. Avci et al. (2021) consider a wind farm co-located with a pumped hydro energy storage facility consisting of two connected reservoirs, including the water levels in the upper and lower reservoirs as the two endogenous state variables in their MDP. Our study extends these two papers in several ways. First, both Zhou et al. (2019) and Avci et al. (2021) focus on electricity markets that are free of advance commitment decisions, whereas our model explicitly incorporates these decisions. This increases the complexity of the problem, as we must treat the commitment decision from the previous period as an additional endogenous state variable. Second, in terms of optimal policy characterization, the above two papers show concavity of the optimal value function in a single dimension (i.e., storage level), while we prove joint concavity in two dimensions (i.e., storage and commitment levels). Therefore, their optimal policy structures involve only one target level (for the battery storage level in Zhou et al. 2019 and for the upper-reservoir water level in Avci et al. 2021), whereas ours involves two target levels for storage and commitment decisions that must be jointly optimized.

1.3. Contributions of Our Study

We contribute to the energy literature in two different ways: (i) We characterize the optimal policy structure for the hourly energy commitment, generation, and storage problem. Our structural insights not only deepen market regulators' understanding of participant behaviors under uncertainty but also aid wind power producers in developing smarter and more practical heuristic solution methods. (ii) Building on our structural results, we design computationally efficient heuristic algorithms capable of delivering high-quality solutions.

In our structural analysis, assuming positive electricity prices, we first formulate the optimal amount of wind energy that should be generated in any given period as a function of the state variables in that period. We then prove the joint concavity of the optimal value function in storage and commitment levels. This result allows us to show that the optimal storage and commitment policy follows a state-dependent threshold policy. The optimal decisions in each period can be guided by thresholds that vary based on the state variables, which can be reduced to exogenous state variables through proper state-space partitioning. Specifically, the state space in each period can be partitioned into disjoint domains that correspond to the optimal decisions of 'positive imbalance' and 'negative imbalance' as well as to the optimal decisions of 'charge and purchase,' 'charge and sell,' and 'discharge and sell.' It is then optimal to bring the storage and commitment levels to a different exogenous-state-dependent threshold pair in each domain.

We construct a heuristic solution procedure that employs our structural results to reduce the computational burden of the standard DP algorithm, while also accounting for the possibility of negative electricity prices over long planning horizons. In this procedure, we develop a backward induction algorithm that calculates the state-dependent threshold pairs for the storage and commitment levels in each period. While the storage and commitment actions in positive-price states are determined by these threshold pairs, those actions in negative-price states are obtained from the myopically optimal solution. We call this solution method HC (Heuristic via Complete state space). We also consider a variant of HC that uses the output of HC executed in a simpler setting in which we ignore the spike component of the price to reduce the state space. We call this variant HR (Heuristic via Reduced state space).

In order to evaluate the use of our solution methods in the general problem, we have compiled realistic instances by incorporating the electricity-price and wind-speed time-series models of Karakoyun et al. (2023) into the random evolution of the exogenous state variables in our MDP. HC yields the optimal solution in all instances by up to 69 times faster than the standard DP algorithm. HR yields near-optimal solutions 8 times faster than HC. The solution time of HR is less than one minute in each instance, while the standard DP algorithm has an average solution time of 237.6 minutes. All these findings highlight the practical importance of our structural analysis.

1.4. Organization of the Paper

The remainder of this paper proceeds as follows. Section 2 formulates the problem. Section 3 establishes the optimal policy structure. Section 4 illustrates the optimal policy structure under perfect efficiency. Section 5 offers the heuristic solution methods based on our structural results. Section 6 presents the numerical results for our heuristic methods and alternative solution approaches. Section 7 concludes. Proofs of the analytical results are contained in an online appendix.

2. Problem Formulation

We consider an energy system that was studied in several recent papers (e.g., Zhou et al. 2019, Bhattacharjee et al. 2020, and Karakoyun et al. 2023). In this system, the energy generated in the wind power plant and/or purchased from the market can be utilized to charge the battery. The

Table 1 Notations used in the MDP model.

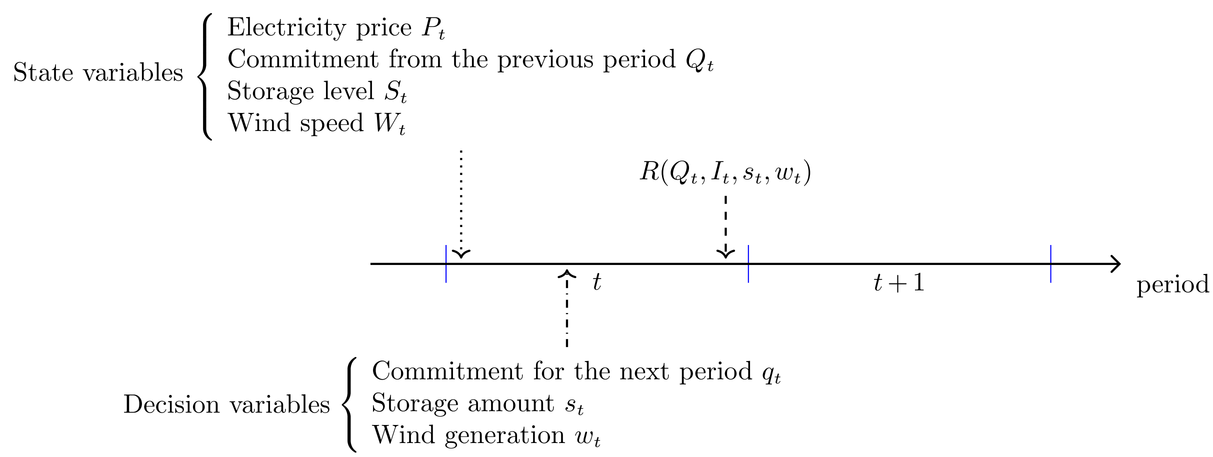
Parameters	**Descriptions**
C_S	Battery's energy capacity (MWh).
C_D	Maximum energy that can be discharged from the battery during a time period (MWh).
C_C	Maximum energy that can be charged into the battery during a time period (MWh).
C_T	Maximum energy that can be transmitted during a time period (MWh).
γ	Electricity conversion efficiency in the generation mode of the battery ($\gamma \in (0,1]$).
θ	Electricity conversion efficiency in the storage mode of the battery ($\theta \in (0,1]$).
τ	Electricity transmission efficiency ($\tau \in (0,1]$).
K_p	Positive imbalance price parameter in positive-price states ($0 \le K_p < 1$).
K_n	Negative imbalance price parameter in positive-price states ($1 < K_n$).
K_p^-	Positive imbalance price parameter in negative-price states ($1 < K_p^-$).
K_n^-	Negative imbalance price parameter in negative-price states ($0 \le K_n^- < 1$).
State variables	
S_t	Accumulated amount of energy in the battery at the beginning of period t, i.e., the storage level ($S_t \in [0, C_S]$).
Q_t	Amount of energy that the producer should sell in period t if $Q_t \ge 0$ and should buy in period t if $Q_t < 0$, as per the producer's commitment in period $t-1$, i.e., the commitment level ($Q_t \in \mathbb{R}$).
P_t	Electricity price in period t ($P_t \in \mathbb{R}$).
W_t	Wind speed in period t ($W_t \in \mathbb{R}_+$).
I_t	Exogenous state of the system ($I_t := (P_\kappa, W_\kappa)_{\kappa \le t}$ with $\mathbb{E}_{P_\kappa \mid I_t}[\lvert P_\kappa \rvert] < \infty$ for $\kappa \ge t$).
$f(W_t)$	Maximum amount of energy that can be generated in the wind power plant in period t.
Decision variables	
q_t	Amount of energy to commit to selling or purchasing in period $t+1$ ($q_t \in \mathbb{R}$).
w_t	Amount of energy to generate in the wind farm in period t ($w_t \in \mathbb{R}_+$).
s_t	Amount of energy to generate or store in the battery in period t ($s_t \in \mathbb{R}$).

power producer participates as a price-taker in a spot market that mandates hour-ahead commitments and hourly settlements.[2] The future electricity prices and wind availability are uncertain. The producer decides, in each hour, how much energy to generate in the wind power plant, how much energy to charge into or discharge from the battery, and how much energy to commit to selling to or buying from the market in the next hour. Since a similar setting was also studied in Karakoyun et al. (2023), our presentation and notation below closely follow the corresponding material in Karakoyun et al. (2023). The notations used in our model are listed in Table 1.

The power producer seeks to optimize its energy commitment, generation, and storage decisions over T periods (hours). $\mathcal{T} := \{1, 2, \ldots, T\}$ is the set of periods. We consider an MDP in which the storage and commitment levels are endogenous state variables while the electricity price and wind speed are exogenous state variables. In any period $t \in \mathcal{T}$, upon observing the state variables (P_t, Q_t, S_t, and W_t), the producer determines the amount of energy to commit to selling or purchasing in period $t+1$, $q_t \in \mathbb{R}$; the amount of energy to generate in the wind farm in period t, $w_t \in \mathbb{R}_+$; and the amount of energy to generate or store in the battery in period t, $s_t \in \mathbb{R}$. The battery is

[2] The price-taking assumption is benign if the system makes a very limited contribution to the overall energy supply in the market. This assumption was made in many related papers (e.g., Kim and Powell 2011, Ding et al. 2015, Gönsch and Hassler 2016, Hassler 2017, Díaz et al. 2019, and Karakoyun et al. 2023).

Figure 1 Sequence of events in each period.



charged if $s_t < 0$ and discharged if $s_t \geq 0$. See Figure 1 for an illustration of the sequence of events. The state variables S_t and Q_t evolve over time as follows: $S_{t+1} = S_t - s_t$ and $Q_t = q_{t-1}$.

The energy storage and generation decisions fall into one of the following three types: (i) Suppose that the battery is discharged ($s_t \geq 0$). The energy generated from the battery and wind farm is sold in this case. We call this decision type DS (the initials of 'discharge' and 'sell'). (ii) Suppose that the battery is charged ($s_t < 0$) and the generated wind energy is sufficient to charge the battery ($s_t/\theta \geq -w_t$). The excess wind energy is sold in this case. We call this decision type CS (the initials of 'charge' and 'sell'). (iii) Suppose that the battery is charged ($s_t < 0$) and the generated wind energy is not sufficient to charge the battery ($s_t/\theta < -w_t$). The required additional energy is purchased in this case. We call this decision type CP (the initials of 'charge' and 'purchase'). The actual amount of energy sold or purchased in period t is

$$E(s_t, w_t) := \begin{cases} (\gamma s_t + w_t)\tau & \text{if } s_t \geq 0, \\ (s_t/\theta + w_t)\tau & \text{if } -w_t \leq s_t/\theta < 0, \\ (s_t/\theta + w_t)/\tau & \text{if } s_t/\theta < -w_t \leq 0. \end{cases}$$

The energy is sold if $E(s_t, w_t) \geq 0$ and purchased otherwise. Note $E(s_t, w_t) = \min\{(\gamma s_t + w_t)\tau, (s_t/\theta + w_t)\tau, (s_t/\theta + w_t)/\tau\}$. Since the minimum of affine functions is concave, $E(\cdot, \cdot)$ is concave. Let $\mathbb{U}(Q_t, S_t, I_t)$ denote the set of action triplets (q_t, s_t, w_t) that are admissible in state (Q_t, S_t, I_t). Any action triplet in $\mathbb{U}(Q_t, S_t, I_t)$ satisfies the following conditions: $0 \leq w_t \leq f(W_t)$ (due to the wind-farm power capacity), $-\min\{C_S - S_t, C_C\} \leq s_t \leq \min\{S_t, C_D\}$ (due to the battery's energy and power capacities), and $-C_T \leq E(s_t, w_t) \leq \tau C_T$ (due to the transmission power capacity).

Deviations from contractual commitments can occur due to participants' forecast errors and/or profit-maximization behavior. Although such deviations are closely monitored by the market regulators, some occurrences still take place (Bergler et al. 2017 and Eicke et al. 2021). The market operators impose penalty payments to discourage deviations from scheduled commitments. We consider a general model that is flexible enough to reflect such market realities and participant behavior by incorporating the penalty costs. These penalties vary depending on the magnitude and direction of the imbalance as well as market regulations and conditions.

The energy storage and generation decisions can be grouped into two types based on the sign of deviations: (i) Suppose that the amount of energy supplied to (or withdrawn from) the market in real-time is greater (or lower) than the commitment level (i.e., $E(s_t, w_t) > Q_t$). This leads to a 'positive imbalance.' We call this decision type pi. (ii) Suppose that the amount of energy supplied to (or withdrawn from) the market in real-time is lower (or greater) than the commitment level (i.e., $E(s_t, w_t) < Q_t$). This leads to a 'negative imbalance.' We call this decision type ni. The producer is settled with a penalized price in both cases. Similar settings were studied in many papers (e.g., Kim and Powell 2011, Jiang and Powell 2015, Hassler 2017, Sunar and Birge 2019, and Finnah and Gönsch 2021). The payoff in period t can be expressed as a function of state variables Q_t and I_t as well as actions s_t and w_t:

$$R(Q_t, I_t, s_t, w_t) = \begin{cases} Q_tP_t + K_pP_t(E(s_t, w_t) - Q_t) & \text{if } P_t \geq 0 \text{ and } Q_t < E(s_t, w_t) \quad (\mathsf{pi}), \\ Q_tP_t - K_nP_t(Q_t - E(s_t, w_t)) & \text{if } P_t \geq 0 \text{ and } Q_t \geq E(s_t, w_t) \quad (\mathsf{ni}), \\ Q_tP_t + K_p^-P_t(E(s_t, w_t) - Q_t) & \text{if } P_t < 0 \text{ and } Q_t < E(s_t, w_t) \quad (\mathsf{pi}), \\ Q_tP_t - K_n^-P_t(Q_t - E(s_t, w_t)) & \text{if } P_t < 0 \text{ and } Q_t \geq E(s_t, w_t) \quad (\mathsf{ni}), \end{cases}$$

where K_pP_t and K_nP_t (or $K_p^-P_t$ and $K_n^-P_t$) are the imbalance prices in positive-price (or negative-price) states in the cases of pi and ni, respectively. The imbalance parameters are chosen so that $0 \leq K_p < 1 < K_n$ and $0 \leq K_n^- < 1 < K_p^-$. While the first term of the payoff function is the instant revenue Q_tP_t, the second term has the following implications: The producer in the case of pi receives $K_pP_t(E(s_t, w_t) - Q_t)$ if the price is positive or pays $-K_p^-P_t(E(s_t, w_t) - Q_t)$ otherwise, by selling the excess amount at an imbalance price lower than P_t. Likewise, the producer in the case of ni pays $K_nP_t(Q_t - E(s_t, w_t))$ if the price is positive or receives $-K_n^-P_t(Q_t - E(s_t, w_t))$ otherwise, by purchasing the excess amount at an imbalance price higher than P_t.[3]

A control policy π specifies the sequence of decision rules $(\eta_t^\pi(Q_t^\pi, S_t^\pi, I_t))_{t\in\mathcal{T}}$, where Q_t^π and S_t^π are the random state variables implied by policy π, $\forall t \in \mathcal{T}\backslash\{1\}$, and $\eta_t^\pi(Q_t^\pi, S_t^\pi, I_t) := (q_t^\pi(Q_t^\pi, S_t^\pi, I_t), s_t^\pi(Q_t^\pi, S_t^\pi, I_t), w_t^\pi(Q_t^\pi, S_t^\pi, I_t))$. We define Π as the set of all admissible control policies. The optimal expected total cash flow over the finite horizon, conditional on the initial state (Q_1, S_1, I_1), can be written as

$$\max_{\pi\in\Pi} \mathbb{E}\left[\sum_{t\in\mathcal{T}} R(Q_t^\pi, I_t, s_t^\pi(Q_t^\pi, S_t^\pi, I_t), w_t^\pi(Q_t^\pi, S_t^\pi, I_t)) \middle| Q_1, S_1, I_1\right].$$

[3] Similar pricing mechanisms appear in practice. In Spain, for instance, the imbalance prices in the cases of pi and ni are $(1-K)P_t$ and $(1+K)P_t$, respectively, where $0 < K \leq 1$ (e.g., Díaz et al. 2019 and Shinde et al. 2020).

For each period $t \in \mathcal{T}$ and each state (Q_t, S_t, I_t), the optimal value function $v_t^*(Q_t, S_t, I_t)$ can be calculated with the following DP recursion:

$$v_t^*(Q_t, S_t, I_t) = \max_{(q_t, s_t, w_t) \in \mathbb{U}(Q_t, S_t, I_t)} \left\{ R(Q_t, I_t, s_t, w_t) + \mathbb{E}_{I_{t+1}|I_t} \left[v_{t+1}^*(q_t, S_t - s_t, I_{t+1}) \right] \right\}$$

where $v_T(Q_T, S_T, I_T) = 0$. Note that $v_1^*(Q_1, S_1, I_1)$ is the optimal expected total cash flow for the initial state (Q_1, S_1, I_1) over the finite horizon.

3. Structural Analysis

In this section, we establish several structural properties of our optimal value function and use these properties to characterize the optimal policy structure. We first introduce several bounds on the optimal energy commitment decision.

Lemma 1. *Without loss of optimality, the commitment levels can be constrained as follows:* $-\min\{(C_S - (S_t - s_t))/(\theta\tau), \underline{E}\} \le q_t \le \min\{\tau(\gamma(S_t - s_t) + \bar{f}), \bar{E}\}$, $\forall t \ge 1$, *and* $-\min\{(C_S - S_t)/(\theta\tau), \underline{E}\} \le Q_t \le \min\{\tau(\gamma S_t + \bar{f}), \bar{E}\}$, $\forall t > 1$, *where* $\underline{E} := \min\{C_C/(\theta\tau), C_T\}$, $\bar{E} := \tau \min\{\gamma C_D + \bar{f}, C_T\}$, *and* $\bar{f} := \max_{t \in \mathcal{T}} \{f(W_t)\}$.

Lemma 1 states that the optimal amount of energy to be committed to selling/buying never exceeds the maximum amount of energy that can be sold/purchased in the next period.

We now assume that the electricity price is always nonnegative:

Assumption 1. $P_t \ge 0$, $\forall t \in \mathcal{T}$.

Assumption 1 implies that $R(Q_t, I_t, s_t, w_t) = \min\{Q_t P_t + K_p P_t (E(s_t, w_t) - Q_t), Q_t P_t + K_n P_t (E(s_t, w_t) - Q_t)\}$. Since the minimum of affine functions is concave, $R(Q_t, I_t, s_t, w_t)$ is jointly concave in Q_t and $E(s_t, w_t)$. Since $E(s_t, w_t)$ is jointly concave and increasing in s_t and w_t, $R(Q_t, I_t, s_t, w_t)$ is jointly concave and increasing in s_t and w_t as well. Under Assumption 1, we establish the following structural property of our optimal value function.

Lemma 2. *Under Assumption 1,* $v_t^*(Q_t, S_t, I_t) \le v_t^*(Q_t, S_t + \alpha, I_t)$ *where* $\alpha > 0$, $\forall t \in \mathcal{T}$.

Lemma 2 states that the system becomes more profitable as the amount of energy accumulated in the battery grows. This is because if the stored energy is higher, the producer can sell more energy from the battery by charging it less in the long run. Using Lemma 2, we formulate the optimal amount of wind energy that should be generated in any period.

Lemma 3. *Under Assumption 1,* $w_t^*(Q_t, S_t, I_t) = \min\{f(W_t), C_T + \min\{C_S - S_t, C_C\}/\theta\}$. *Moreover, if* $w_t^*(Q_t, S_t, I_t) = C_T + \min\{C_S - S_t, C_C\}/\theta$, *then* $s_t^*(Q_t, S_t, I_t) = -\min\{C_S - S_t, C_C\}$.

Lemma 3 states that it is optimal to generate as much wind energy as possible. If the wind energy potential is large enough, it is optimal to sell and store as much energy as possible. The curtailed amount of wind energy is given by $f(W_t) - (C_T + \min\{C_S - S_t, C_C\}/\theta)$ if $w_t^*(Q_t, S_t, I_t) < f(W_t)$. This lemma allows us to restrict our optimal policy characterization to the energy commitment and storage decisions.

Using Lemma 1 and Assumption 1, we also prove the concavity of our optimal value function.

PROPOSITION 1. *Under Assumption 1, $v_t^*(Q_t, S_t, I_t)$ is jointly concave in (Q_t, S_t), $\forall t \in \mathcal{T}$.*

The joint concavity of the value function implies that increasing the commitment level exhibits diminishing returns. A small commitment level raises the risk of missing the opportunity to sell more energy at the market price, while a large commitment level increases the risk of incurring penalty costs due to the difficulty of meeting the commitment. Moreover, a high commitment level may prompt the producer to consume more of the available energy in the current period, potentially limiting energy availability in the future. Similarly, the joint concavity implies that increasing the storage level also exhibits diminishing returns. As the battery approaches its capacity, each additional unit of stored energy provides a smaller benefit in terms of future operational flexibility.

An important implication of Proposition 1 is that the optimal energy commitment and storage policy can be characterized as following a threshold policy. Specifically, we introduce the optimal state-dependent target levels for commitment and storage decisions, which we respectively denote by $Y_t^*(Q_t, S_t, I_t)$ and $Z_t^*(Q_t, S_t, I_t)$, in an unconstrained problem free of certain capacity limits. We implement these target levels into the optimal policy structure in the original constrained problem:

$$(Y_t^*(Q_t, S_t, I_t), Z_t^*(Q_t, S_t, I_t)) := \underset{(q_t, z_t) \in [-\underline{E}, \bar{E}] \times [0, C_S]}{\arg\max} \left\{ R(Q_t, I_t, S_t - z_t, w_t) + \mathbb{E}\left[v_{t+1}^*(q_t, z_t, I_{t+1}) \right] \right\}, \tag{1}$$

where $z_t := S_t - s_t$ is the storage level at the end of period t if the action s_t is taken in period t. Since $Z_t^*(Q_t, S_t, I_t)$ may be inaccessible when the omitted capacity limits are reconsidered, the optimal storage level at the end of period t may differ from $Z_t^*(Q_t, S_t, I_t)$, implying that $Y_t^*(Q_t, S_t, I_t)$ may no longer be optimal at this storage level. Hence, we also introduce the optimal state-dependent target commitment level after the storage decision is made in the constrained problem:

$$Y_t(S_{t+1}, I_t) := \underset{q_t \in [-\underline{E}, \bar{E}]}{\arg\max} \left\{ \mathbb{E}\left[v_{t+1}^*(q_t, S_{t+1}, I_{t+1}) \right] \right\}. \tag{2}$$

Note that $Y_t(Z_t^*(Q_t, S_t, I_t), I_t) = Y_t^*(Q_t, S_t, I_t)$.

Using Lemma 3 and Proposition 1, we now characterize the optimal energy storage and commitment actions.

THEOREM 1. *Under Assumption 1, letting $w = w_t^*(Q_t, S_t, I_t)$ and $Z = Z_t^*(Q_t, S_t, I_t)$, the optimal energy storage action is*

$$s_t^*(Q_t, S_t, I_t) = \begin{cases} -\min\{Z - S_t, \theta(\tau C_T + w), C_C\} & \text{if } C_T \le w \text{ and } S_t + \theta(w - C_T) < Z, \\ -\theta(w - C_T) & \text{if } C_T \le w \text{ and } Z \le S_t + \theta(w - C_T), \\ -\min\{Z - S_t, \theta(\tau C_T + w), C_C\} & \text{if } C_T > w \text{ and } S_t < Z, \\ \min\{S_t - Z, (C_T - w)/\gamma, C_D\} & \text{if } C_T > w \text{ and } Z \le S_t. \end{cases}$$

The optimal energy commitment action is $q_t^*(Q_t, S_t, I_t) = Y_t(S_t - s_t^*(Q_t, S_t, I_t), I_t)$.

The maximization problem in (1) should be solved for each state to calculate the optimal control policy. This calculation method, however, provides no computational benefit compared to the standard DP algorithm. In order to overcome this drawback, we consider an alternative formulation that restricts the optimal target levels to depend only on exogenous state variable I_t and decision type ν. We denote these target levels for the commitment and storage actions by $Y_t^{(\nu)}(I_t)$ and $Z_t^{(\nu)}(I_t)$, respectively. We consider a total of six decision types: the actions of 'charge' and 'purchase' leading to positive imbalance (piCP) and negative imbalance (niCP), the actions of 'charge' and 'sell' leading to positive imbalance (piCS) and negative imbalance (niCS), and the actions of 'discharge' and 'sell' leading to positive imbalance (piDS) and negative imbalance (niDS). For $\nu \in \{\mathsf{niCP}, \mathsf{niCS}, \mathsf{niDS}, \mathsf{piCP}, \mathsf{piCS}, \mathsf{piDS}\}$,

$$\left(Y_t^{(\nu)}(I_t), Z_t^{(\nu)}(I_t)\right) := \underset{(q_t, z_t) \in [-\underline{E}, \bar{E}] \times [0, C_S]}{\arg\max} \left\{R^{(\nu)}(z_t, I_t) + \mathbb{E}\left[v_{t+1}^*(q_t, z_t, I_{t+1})\right]\right\}$$

where

$$R^{(\nu)}(z_t, I_t) = \begin{cases} -K_n P_t z_t/(\theta\tau) & \text{if } \nu = \mathsf{niCP}, \\ -K_n P_t \tau z_t/\theta & \text{if } \nu = \mathsf{niCS}, \\ -K_n P_t \gamma\tau z_t & \text{if } \nu = \mathsf{niDS}, \\ -K_p P_t z_t/(\theta\tau) & \text{if } \nu = \mathsf{piCP}, \\ -K_p P_t \tau z_t/\theta & \text{if } \nu = \mathsf{piCS}, \\ -K_p P_t \gamma\tau z_t & \text{if } \nu = \mathsf{piDS}. \end{cases}$$

Leveraging our structural results, we can easily calculate the optimal target storage level $Z_t^*(Q_t, S_t, I_t)$ in terms of $Z_t^{(\nu)}(I_t)$ (see Theorem 2). In the remainder of the paper, we occasionally suppress the dependencies of $Y_t^{(\nu)}(I_t)$ and $Z_t^{(\nu)}(I_t)$ on I_t for notational simplicity. This alternative calculation method enables significant computational savings (see Section 6).

We denote by Ω the domain of (Q_t, S_t, W_t), i.e., $\Omega := [-\underline{E}, \bar{E}] \times [0, C_S] \times [0, \infty)$. For storage policy characterization, we consider two different partitioning schemes for this domain. The first partitioning scheme is based on the relationship between $f(W_t)$ and C_T, leading to three disjoint sets Λ_1, Λ_2, and Λ_3. Set $\Lambda_1 := \{(Q_t, S_t, W_t) \in \Omega : f(W_t) \ge C_T + \min\{C_S - S_t, C_C\}/\theta\}$ represents the case where the wind power potential in period t exceeds the maximum total amount of energy that can be used for selling and storing in period t. Set $\Lambda_2 := \{(Q_t, S_t, W_t) \in \Omega : C_T + \min\{C_S - S_t, C_C\}/\theta > f(W_t) \geq C_T\}$ represents the case where the wind power potential in period t is greater than the transmission capacity but less than the maximum total amount of energy that can be used for selling and storing in period t. Set $\Lambda_3 := \{(Q_t, S_t, W_t) \in \Omega : C_T > f(W_t)\}$ represents the case where the wind power potential in period t is less than the transmission capacity.

The second partitioning scheme is based on the relationship between $f(W_t)$ and Q_t, leading to disjoint sets Ψ_1^+, Ψ_1^-, Ψ_2^+, Ψ_2^-, Ψ_3^+, and Ψ_3^-. The superscript gives the sign of commitment level. Sets $\Psi_1^+ := \{(Q_t, S_t, W_t) \in \Omega : f(W_t) \geq Q_t/\tau + \min\{C_S - S_t, C_C\}/\theta,\ Q_t \geq 0\}$ and $\Psi_1^- := \{(Q_t, S_t, W_t) \in \Omega : f(W_t) \geq \tau Q_t + \min\{C_S - S_t, C_C\}/\theta,\ Q_t < 0\}$ represent the cases where the wind power potential in period t is greater than the maximum total amount of energy that can be used for meeting the commitment and storing in period t. Sets $\Psi_2^+ := \{(Q_t, S_t, W_t) \in \Omega : Q_t/\tau + \min\{C_S - S_t, C_C\}/\theta > f(W_t) \geq Q_t/\tau,\ Q_t \geq 0\}$ and $\Psi_2^- := \{(Q_t, S_t, W_t) \in \Omega : \tau Q_t + \min\{C_S - S_t, C_C\}/\theta > f(W_t) \geq \tau Q_t,\ Q_t < 0\}$ represent the cases where the wind power potential in period t is greater than the amount of energy required to meet the commitment in period t but less than the maximum total amount of energy that can be used for meeting the commitment and storing in period t. Lastly, sets $\Psi_3^+ := \{(Q_t, S_t, W_t) \in \Omega : Q_t/\tau > f(W_t),\ Q_t \geq 0\}$ and $\Psi_3^- := \{(Q_t, S_t, W_t) \in \Omega : \tau Q_t > f(W_t),\ Q_t < 0\}$ represent the cases where the wind power potential in period t is less than the amount of energy required to meet the commitment in period t. Note $\Psi_3^- = \emptyset$.

Using the above sets, we construct ten subdomains of Ω: Λ_1, $\Lambda_2 \cap \Psi_1^+$, $\Lambda_2 \cap \Psi_1^-$, $\Lambda_2 \cap \Psi_2^+$, $\Lambda_2 \cap \Psi_2^-$, $\Lambda_3 \cap \Psi_1^+$, $\Lambda_3 \cap \Psi_1^-$, $\Lambda_3 \cap \Psi_2^+$, $\Lambda_3 \cap \Psi_2^-$, and $\Lambda_3 \cap \Psi_3^+$. Note $\Lambda_2 \cap \Psi_3^+ = \emptyset$ since the condition $Q_t/\tau > f(W_t) \geq C_T$ is not possible from Lemma 1. Incorporating these subdomains and suppressing the dependency of $Z_t^*(Q_t, S_t, I_t)$ on (Q_t, S_t, I_t) for notational simplicity, we state the main result of this section:

Theorem 2. *The optimal state-dependent target storage levels can be calculated as follows.*

(i) If $(Q_t, S_t, W_t) \in \Lambda_1$, $Z_t^*(Q_t, S_t, I_t) = C_S$.

(ii) If $(Q_t, S_t, W_t) \in \Lambda_2 \cap \Psi_1^+$, $Z_t^*(Q_t, S_t, I_t) = \max\left\{Z_t^{(\mathsf{piCS})}, S_t\right\}$.

(iii) If $(Q_t, S_t, W_t) \in \Lambda_2 \cap \Psi_1^-$,

$$Z_t^*(Q_t, S_t, I_t) = \begin{cases} \max\left\{Z_t^{(\mathsf{piCP})}, S_t + \theta f(W_t)\right\} & \text{if } S_t \leq Z_t^{(\mathsf{piCS})} - \theta f(W_t), \\ \max\left\{Z_t^{(\mathsf{piCS})}, S_t\right\} & \text{if } Z_t^{(\mathsf{piCS})} - \theta f(W_t) < S_t. \end{cases}$$

(iv) If $(Q_t, S_t, W_t) \in \Lambda_2 \cap \Psi_2^+$,

$$Z_t^*(Q_t,S_t,I_t)=\begin{cases}\max\left\{Z_t^{(\mathsf{niCP})},S_t+\theta f(W_t)\right\}\\ \quad \textit{if } S_t\le Z_t^{(\mathsf{niCS})}-\theta f(W_t),\\ \max\left\{Z_t^{(\mathsf{niCS})},S_t+\theta(f(W_t)-Q_t/\tau)\right\}\\ \quad \textit{if } Z_t^{(\mathsf{niCS})}-\theta f(W_t)<S_t\le Z_t^{(\mathsf{piCS})}-\theta(f(W_t)-Q_t/\tau),\\ \max\left\{Z_t^{(\mathsf{piCS})},S_t\right\}\\ \quad \textit{if } Z_t^{(\mathsf{piCS})}-\theta(f(W_t)-Q_t/\tau)<S_t.\end{cases}$$

(v) *If* $(Q_t,S_t,W_t)\in\Lambda_2\cap\Psi_2^-$,

$$Z_t^*(Q_t,S_t,I_t)=\begin{cases}\max\left\{Z_t^{(\mathsf{niCP})},S_t+\theta(f(W_t)-\tau Q_t)\right\}\\ \quad \textit{if } S_t\le Z_t^{(\mathsf{piCP})}-\theta(f(W_t)-\tau Q_t),\\ \max\left\{Z_t^{(\mathsf{piCP})},S_t+\theta f(W_t)\right\}\\ \quad \textit{if } Z_t^{(\mathsf{piCP})}-\theta(f(W_t)-\tau Q_t)<S_t\le Z_t^{(\mathsf{piCS})}-\theta f(W_t),\\ \max\left\{Z_t^{(\mathsf{piCS})},S_t\right\}\\ \quad \textit{if } Z_t^{(\mathsf{piCS})}-\theta f(W_t)<S_t.\end{cases}$$

(vi) *If* $(Q_t,S_t,W_t)\in\Lambda_3\cap\Psi_1^+$,

$$Z_t^*(Q_t,S_t,I_t)=\begin{cases}\max\left\{Z_t^{(\mathsf{piCS})},S_t\right\} & \textit{if } S_t\le Z_t^{(\mathsf{piDS})}\\ Z_t^{(\mathsf{piDS})} & \textit{if } Z_t^{(\mathsf{piDS})}<S_t.\end{cases}$$

(vii) *If* $(Q_t,S_t,W_t)\in\Lambda_3\cap\Psi_1^-$,

$$Z_t^*(Q_t,S_t,I_t)=\begin{cases}\max\left\{Z_t^{(\mathsf{piCP})},S_t+\theta f(W_t)\right\} & \textit{if } S_t\le Z_t^{(\mathsf{piCS})}-\theta f(W_t),\\ \max\left\{Z_t^{(\mathsf{piCS})},S_t\right\} & \textit{if } Z_t^{(\mathsf{piCS})}-\theta f(W_t)\le S_t\le Z_t^{(\mathsf{piDS})},\\ Z_t^{(\mathsf{piDS})} & \textit{if } Z_t^{(\mathsf{piDS})}<S_t.\end{cases}$$

(viii) *If* $(Q_t,S_t,W_t)\in\Lambda_3\cap\Psi_2^+$,

$$Z_t^*(Q_t,S_t,I_t)=\begin{cases}\max\left\{Z_t^{(\mathsf{niCP})},S_t+\theta f(W_t)\right\}\\ \quad \textit{if } S_t\le Z_t^{(\mathsf{niCS})}-\theta f(W_t),\\ \max\left\{Z_t^{(\mathsf{niCS})},S_t+\theta(f(W_t)-Q_t/\tau)\right\}\\ \quad \textit{if } Z_t^{(\mathsf{niCS})}-\theta f(W_t)<S_t\le Z_t^{(\mathsf{piCS})}-\theta(f(W_t)-Q_t/\tau),\\ \max\left\{Z_t^{(\mathsf{piCS})},S_t\right\}\\ \quad \textit{if } Z_t^{(\mathsf{piCS})}-\theta(f(W_t)-Q_t/\tau)<S_t\le Z_t^{(\mathsf{piDS})},\\ Z_t^{(\mathsf{piDS})}\\ \quad \textit{if } Z_t^{(\mathsf{piDS})}<S_t.\end{cases}$$

(ix) *If* $(Q_t,S_t,W_t)\in\Lambda_3\cap\Psi_2^-$,

$$Z_t^*(Q_t,S_t,I_t)=\begin{cases}\max\left\{Z_t^{(\mathsf{niCP})},S_t+\theta(f(W_t)-\tau Q_t)\right\}\\ \quad \textit{if } S_t\le Z_t^{(\mathsf{piCP})}-\theta(f(W_t)-\tau Q_t),\\ \max\left\{Z_t^{(\mathsf{piCP})},S_t+\theta f(W_t)\right\}\\ \quad \textit{if } Z_t^{(\mathsf{piCP})}-\theta(f(W_t)-\tau Q_t)<S_t\le Z_t^{(\mathsf{piCS})}-\theta f(W_t),\\ \max\left\{Z_t^{(\mathsf{piCS})},S_t\right\}\\ \quad \textit{if } Z_t^{(\mathsf{piCS})}-\theta f(W_t)<S_t\le Z_t^{(\mathsf{piDS})},\\ Z_t^{(\mathsf{piDS})}\\ \quad \textit{if } Z_t^{(\mathsf{piDS})}<S_t.\end{cases}$$

(x) If $(Q_t,S_t,W_t)\in\Lambda_3\cap\Psi_3^+$,

$$Z_t^*(Q_t,S_t,I_t)=\begin{cases}\max\left\{Z_t^{(\mathsf{niCP})},S_t+\theta f(W_t)\right\}\\ \quad \textit{if } S_t\le Z_t^{(\mathsf{niCS})}-\theta f(W_t),\\ \max\left\{Z_t^{(\mathsf{niCS})},S_t\right\}\\ \quad \textit{if } Z_t^{(\mathsf{niCS})}-\theta f(W_t)<S_t\le Z_t^{(\mathsf{niDS})},\\ \max\left\{Z_t^{(\mathsf{niDS})},S_t-(Q_t/\tau-f(W_t))/\gamma\right\}\\ \quad \textit{if } Z_t^{(\mathsf{niDS})}<S_t\le Z_t^{(\mathsf{piDS})}+(Q_t/\tau-f(W_t))/\gamma,\\ Z_t^{(\mathsf{piDS})}\\ \quad \textit{if } Z_t^{(\mathsf{piDS})}+(Q_t/\tau-f(W_t))/\gamma<S_t.\end{cases}$$

Theorem 2 formulates the optimal target storage level $Z_t^*(Q_t,S_t,I_t)$ in terms of $Z_t^{(\nu)}(I_t)$, conditional on the ten subdomains defined earlier. It implies that when the available energy is too low, it is optimal to charge the battery and purchase energy from the market. In this case, the producer chooses to bring the storage level up to $Z_t^{(\mathsf{niCP})}(I_t)$ or $Z_t^{(\mathsf{piCP})}(I_t)$. When the available energy is modestly high, the producer gains the flexibility to charge the battery and sell energy to the market. In this case, the producer chooses to bring the storage level up to $Z_t^{(\mathsf{niCS})}(I_t)$ or $Z_t^{(\mathsf{piCS})}(I_t)$. When the available energy is high enough, it is optimal to keep the storage level unchanged. However, when the available energy is too high, it is optimal to discharge the battery and sell energy to the market. In this case, the producer chooses to bring the storage level down to $Z_t^{(\mathsf{niDS})}(I_t)$ or $Z_t^{(\mathsf{piDS})}(I_t)$.

For a special case of our problem when the system is perfectly efficient (i.e., when $\gamma=\theta=\tau=1$), we provide in Section 4 an illustration of the optimal target storage level (restricted to the decision types ni and pi), conditional on four disjoint subdomains of (Q_t,S_t,W_t). We also discuss in Section 4 how the optimal type of imbalance is affected by the available energy. For another special case of our problem when the producer is free of commitment decisions (i.e., when $K_p=K_n=1$), we refer the reader to Zhou et al. (2019) for an illustration of the optimal target storage level (restricted to the decision types CP, CS, and DS), conditional on three disjoint subdomains of (S_t,W_t).

PROPOSITION 2. *The optimal target storage levels obey the following properties:*

(a) $Z_t^{(\mathsf{niCP})} \le Z_t^{(\mathsf{piCP})}$, $Z_t^{(\mathsf{niCS})} \le Z_t^{(\mathsf{piCS})}$, *and* $Z_t^{(\mathsf{niDS})} \le Z_t^{(\mathsf{piDS})}$.

(b) $Z_t^{(\mathsf{niCP})} \le Z_t^{(\mathsf{niCS})} \le Z_t^{(\mathsf{niDS})}$ *and* $Z_t^{(\mathsf{piCP})} \le Z_t^{(\mathsf{piCS})} \le Z_t^{(\mathsf{piDS})}$.

Point (a) of Proposition 2 states that the optimal target storage level is higher when the optimal storage action leads to a positive imbalance than when it leads to a negative imbalance. This is because the optimality of positive (negative) imbalance is a consequence of excess (limited) energy availability. The optimal target storage level may thus increase as the current storage level grows for a fixed wind power potential. Point (b) of Proposition 2 states that the optimal storage level is highest if the optimal decision type is DS and lowest if it is CP. This ranking is a consequence of the battery and transmission line inefficiencies. While $Z_t^{(\mathsf{niCS})} = Z_t^{(\mathsf{niDS})}$ and $Z_t^{(\mathsf{piCS})} = Z_t^{(\mathsf{piDS})}$ if $\gamma = \theta = 1$, $Z_t^{(\mathsf{niCP})} = Z_t^{(\mathsf{niCS})}$ and $Z_t^{(\mathsf{piCP})} = Z_t^{(\mathsf{piCS})}$ if $\tau = 1$.

4. Illustration of the Optimal Storage Policy under Perfect Efficiency

In this section, in order to simplify our policy illustration, we consider a special case of our problem in which the system is perfectly efficient (i.e., $\gamma = \theta = \tau = 1$). This special case involves only two decision types: negative imbalance and positive imbalance. For $\nu \in \{\mathsf{ni}, \mathsf{pi}\}$, the optimal target levels for the commitment and storage actions can be calculated as follows:

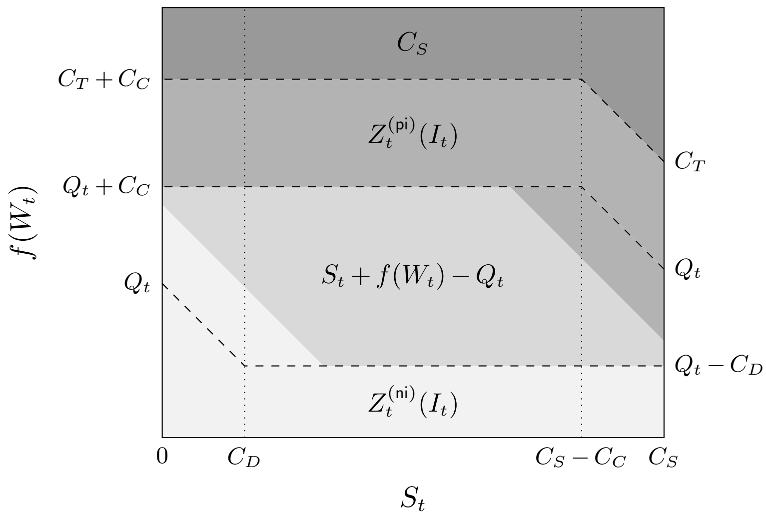
$$\left(Y_t^{(\nu)}(I_t), Z_t^{(\nu)}(I_t)\right) := \operatorname*{arg\,max}_{(q_t,z_t)\in[-\underline{E},\bar{E}]\times[0,C_S]} \left\{R^{(\nu)}(z_t, I_t) + \mathbb{E}\left[v_{t+1}^*(q_t, z_t, I_{t+1})\right]\right\}$$

where

$$R^{(\nu)}(z_t, I_t) = \begin{cases} -K_n P_t z_t & \text{if } \nu = \mathsf{ni}, \\ -K_p P_t z_t & \text{if } \nu = \mathsf{pi}. \end{cases}$$

For optimal storage policy characterization, unlike the original problem, we partition the domain Ω into four disjoint subdomains: We define the set $\Upsilon_1 := \{(Q_t, S_t, W_t) \in \Omega : f(W_t) \ge C_T + \min\{C_S - S_t, C_C\}\}$ as the subdomain where the wind power potential in period t is greater than the maximum total amount of energy that can be used for selling and storing in period t. We define the set $\Upsilon_2 := \{(Q_t, S_t, W_t) \in \Omega : C_T + \min\{C_S - S_t, C_C\} > f(W_t) \ge Q_t + \min\{C_S - S_t, C_C\}\}$ as the subdomain where the wind power potential in period t is less than the maximum total amount of energy that can be used for selling and storing in period t, but greater than the maximum total amount of energy that can be used for meeting the commitment and storing in period t. We define the set $\Upsilon_3 := \{(Q_t, S_t, W_t) \in \Omega : Q_t + \min\{C_S - S_t, C_C\} > f(W_t) \ge Q_t - \min\{S_t, C_D\}\}$ as the subdomain where the wind power potential in period t is less than the maximum total amount of energy that can be used for meeting the commitment and storing in period t, but greater than the amount of energy required to meet the commitment after the battery is discharged as much as possible in

Figure 2 Illustration of $Z_t^*(Q_t, S_t, I_t)$ for a fixed Q_t. Regions separated by dashed lines correspond to different subdomains of Ω (Υ_1 to Υ_4 from top to bottom). Different colors indicate different target levels.



period t. Finally, let $\Upsilon_4 := \{(Q_t, S_t, W_t) \in \Omega : Q_t - \min\{S_t, C_D\} > f(W_t)\}$. With these subdomains we identify the optimal storage policy structure:

COROLLARY 1. *Under Assumption 1, if $\gamma = \theta = \tau = 1$, the optimal state-dependent target storage levels can be calculated as follows.*

(i) If $(Q_t, S_t, W_t) \in \Upsilon_1$, $Z_t^*(Q_t, S_t, I_t) = C_S$.

(ii) If $(Q_t, S_t, W_t) \in \Upsilon_2$, $Z_t^*(Q_t, S_t, I_t) = Z_t^{(\mathsf{pi})}(I_t)$.

(iii) If $(Q_t, S_t, W_t) \in \Upsilon_3$,

$$Z_t^*(Q_t, S_t, I_t) = \begin{cases} Z_t^{(\mathsf{ni})}(I_t) & \text{if } S_t \le Z_t^{(\mathsf{ni})}(I_t) - f(W_t) + Q_t, \\ S_t + f(W_t) - Q_t & \text{if } Z_t^{(\mathsf{ni})}(I_t) - f(W_t) + Q_t < S_t \le Z_t^{(\mathsf{pi})}(I_t) - f(W_t) + Q_t, \\ Z_t^{(\mathsf{pi})}(I_t) & \text{if } Z_t^{(\mathsf{pi})}(I_t) - f(W_t) + Q_t < S_t. \end{cases}$$

(iv) If $(Q_t, S_t, W_t) \in \Upsilon_4$, $Z_t^*(Q_t, S_t, I_t) = Z_t^{(\mathsf{ni})}(I_t)$.

We provide an illustration of the optimal storage policy in Figure 2. (i) Suppose that the wind power potential in period t is extremely large (i.e., $(Q_t, S_t, W_t) \in \Upsilon_1$). See the top region in Figure 2. Then it is optimal to increase the battery storage level as much as possible. (ii) Suppose that the wind power potential in period t is large enough to only meet the commitment and charge the battery as much as possible (i.e., $(Q_t, S_t, W_t) \in \Upsilon_2$). See the second top region in Figure 2. Then it is optimal to bring the battery storage level as close to $Z_t^{(\mathsf{pi})}(I_t)$ as possible. (iii) Suppose that the wind power potential in period t is not sufficient to meet the commitment and charge the battery as much as possible (i.e., $(Q_t, S_t, W_t) \in \Upsilon_3$). See the third top region in Figure 2. If the battery storage level is low (i.e., $S_t \le Z_t^{(\mathsf{ni})}(I_t) - f(W_t) + Q_t$), it is optimal to bring it as close to $Z_t^{(\mathsf{ni})}(I_t)$ as possible. If it is modestly high (i.e., $Z_t^{(\mathsf{ni})}(I_t) - f(W_t) + Q_t < S_t \le Z_t^{(\mathsf{pi})}(I_t) - f(W_t) + Q_t$), it is optimal to meet the commitment as much as possible. If it is high (i.e., $Z_t^{(\mathsf{pi})}(I_t) - f(W_t) + Q_t < S_t$), it is optimal to bring it as close to $Z_t^{(\mathsf{pi})}(I_t)$ as possible. (iv) Suppose that the wind power potential in period t is extremely small (i.e., $(Q_t, S_t, W_t) \in \Upsilon_4$). See the bottom region in Figure 2. Then it is optimal to bring the battery storage level as close to $Z_t^{(\mathsf{ni})}(I_t)$ as possible.

These observations together indicate that the optimal target storage level $Z_t^*(Q_t, S_t, I_t)$ is likely to switch from $Z_t^{(\mathsf{pi})}(I_t)$ to $Z_t^{(\mathsf{ni})}(I_t)$ as the system moves from Υ_2 to Υ_4. In other words, the optimal imbalance type is likely to shift from positive to negative as the wind power potential decreases. The optimal imbalance type is also likely to shift from positive to negative as the storage level shrinks in Υ_3. This is because the marginal value of increasing storage is higher at lower levels, implying that a negative imbalance may be more desirable.

From a practical perspective, the above policy structure offers insights for both operational decision-making and policy design. Energy producers could benefit from dynamically adjusting their imbalance preferences in response to current market and wind conditions and battery storage levels. Once the imbalance type is determined, the target levels $Z_t(\mathsf{pi})$ and $Z_t(\mathsf{ni})$ play a central role in specifying the imbalance amount. In parallel, market regulators could implement penalty mechanisms that dynamically respond to these changing conditions to discourage imbalance behavior among market participants. When designed appropriately, such mechanisms induce target levels $Z_t(\mathsf{pi})$ and $Z_t(\mathsf{ni})$ that steer imbalances as close as possible to the desired levels.

While the threshold-type policy we derived may resemble the classical base-stock structure from single-item inventory control, our model differs in several important ways. First, the state space is more complex. Instead of a single inventory level, it involves two interdependent state variables: the storage level and the commitment level. This requires establishing joint concavity over a two-dimensional domain rather than relying on single-variable arguments. Second, the decision variables are inherently linked. Unlike classical models with largely independent decisions, our model requires simultaneous choices regarding wind generation, battery operation, and energy commitments, all under physical constraints such as storage and transmission limits and battery inefficiencies. Third, the payoff formulation departs from traditional settings. Rather than convex holding and shortage costs, the payoff reflects an asymmetric imbalance pricing mechanism, formulated as the minimum of multiple affine functions depending on the direction and magnitude of the energy imbalance. Finally, the complex interactions between energy-specific features such as conversion and transmission losses, storage and transmission limits, and wind availability segment the decision space into distinct operational regimes, each associated with a different configuration of actions. This complex policy structure has no direct counterpart in classical inventory control frameworks.

5. Heuristic Solution Approach

Structural insights from the problem domain aid in designing effective heuristic solution methods (e.g., Nadar et al. 2016, Zhou et al. 2019, Avci et al. 2021). Accordingly, Lemma 3 and Theorems 1–2 characterize the optimal policy structure under positive electricity prices, providing a foundation for our heuristic solution approach. However, unlike other commodities, electricity prices can become negative. This often results from supply-demand imbalances driven by high renewable energy generation, limited storage capacity, and rigid generation constraints.[4] To address this, we develop a heuristic solution procedure that explicitly accounts for negative electricity prices. Our heuristic procedure calculates the state-dependent target levels for the storage and commitment decisions in each period with a backward induction algorithm. The storage and commitment actions in positive-price states are determined by the state-dependent target levels, while those in negative-price states are determined by the myopically optimal storage decisions. We present below two variants of this heuristic procedure for the discrete-state and discrete-action version of our MDP. Let $\mathcal{Q}$, $\mathcal{S}$, and $\mathcal{I}$ denote the discrete spaces of the commitment level, storage level, and exogenous state variables (electricity price and wind speed), respectively.

5.1. Solution Approach via Complete State Space

The time-series models that we will adopt from the related literature for our numerical study involve autoregressive of order one, AR(1), processes. We refer the reader to Zhou et al. (2019), Avci et al. (2021), and Karakoyun et al. (2023) for detailed descriptions of this family of time-series models and their calibration with real-world data. It is thus sufficient to include the mean-reverting component of the price ρ_t, the spike component of the price J_t, and the deseasonalized wind speed ξ_t as the exogenous state variables in our MDP. The spike component here represents sudden and large moves in the price that are independent across periods. These jumps may arise for several reasons such as unexpected power plant or transmission line outages and extreme weather events (Seifert and Uhrig-Homburg 2007). With this new notation, we redefine $I_t = (\rho_t, J_t, \xi_t) \in \mathcal{I}$ as the exogenous state tuple in period t. We also define $\overline{I}_t = (\rho_t, \xi_t)$ as the exogenous state tuple when the spike component is omitted in period t, and $\overline{\mathcal{I}}$ as the discrete space of the exogenous state variables without the spike; this notation is needed for our heuristic construction.

[4] Despite its growing significance, negative pricing remains relatively underexplored in the operations research and energy literature. Several studies have examined the interaction between negative prices and battery storage operations (e.g., Zhou et al. 2016, Zhou et al. 2019, Kim et al. 2020, and Guerra et al. 2023), highlighting the need to incorporate it into the decision-making process.

Algorithm 1 Solution approach via complete state space.

$\bar{v}_T^{\mathsf{HC}}(Q_T, S_T, \bar{I}_T) \leftarrow 0$, $\forall (Q_T, S_T, \bar{I}_T) \in \mathcal{Q} \times \mathcal{S} \times \bar{\mathcal{I}}$.
for $t = T-1, \ldots, 1$ **do**
  for $I_t \in \mathcal{I}$ such that $P_t \geq 0$ **do**
    $\left(Y_t^{(\nu),\mathsf{HC}}(I_t), Z_t^{(\nu),\mathsf{HC}}(I_t)\right) \leftarrow \arg\max_{(q_t,z_t)\in\mathcal{Q}\times\mathcal{S}} \left\{R^{(\nu)}(z_t, I_t) + \mathbb{E}_{\bar{I}_{t+1}|\bar{I}_t}\left[\bar{v}_{t+1}^{\mathsf{HC}}(q_t, z_t, \bar{I}_{t+1})\right]\right\}$, $\forall \nu$.
  end for
  for $(Q_t, S_t, I_t) \in \mathcal{Q} \times \mathcal{S} \times \mathcal{I}$ **do**
    if $P_t \geq 0$ **then**
      Compute w_t^{HC} from Lemma 3.
      Compute $Z_t^{\mathsf{HC}}(Q_t, S_t, I_t)$ from Theorem 2 with $Z_t^{(\nu)}(I_t)$ replaced by $Z_t^{(\nu),\mathsf{HC}}(I_t)$, $\forall \nu$.
      Compute s_t^{HC} from Theorem 1 with $Z_t^*(Q_t, S_t, I_t)$ and $w_t^*(Q_t, S_t, I_t)$ replaced by $Z_t^{\mathsf{HC}}(Q_t, S_t, I_t)$ and w_t^{HC}.
    else
      $s_t^{\mathsf{HC}} \leftarrow -\min\{C_S - S_t, C_C, \theta\tau C_T\}$ and $w_t^{\mathsf{HC}} \leftarrow 0$.
    end if
    if $P_t \geq 0$ and $S_t - s_t^{\mathsf{HC}} = Z_t^{(\nu),\mathsf{HC}}(I_t)$ for some ν **then**
      $Y_t^{\mathsf{HC}}(S_t - s_t^{\mathsf{HC}}, I_t) \leftarrow Y_t^{(\nu),\mathsf{HC}}(I_t)$
    else
      $Y_t^{\mathsf{HC}}(S_t - s_t^{\mathsf{HC}}, I_t) \leftarrow \arg\max_{q_t\in\mathcal{Q}} \left\{\mathbb{E}_{\bar{I}_{t+1}|\bar{I}_t}\left[\bar{v}_{t+1}^{\mathsf{HC}}(q_t, S_t - s_t^{\mathsf{HC}}, \bar{I}_{t+1})\right]\right\}$.
    end if
    $q_t^{\mathsf{HC}} \leftarrow Y_t^{\mathsf{HC}}(S_t - s_t^{\mathsf{HC}}, I_t)$
    $v_t^{\mathsf{HC}}(Q_t, S_t, I_t) \leftarrow R(Q_t, I_t, s_t^{\mathsf{HC}}, w_t^{\mathsf{HC}}) + \mathbb{E}_{\bar{I}_{t+1}|\bar{I}_t}\left[\bar{v}_{t+1}^{\mathsf{HC}}(q_t^{\mathsf{HC}}, S_t - s_t^{\mathsf{HC}}, \bar{I}_{t+1})\right]$.
  end for
  $\bar{v}_t^{\mathsf{HC}}(Q_t, S_t, \bar{I}_t) \leftarrow \mathbb{E}_{J_t}\left[v_t^{\mathsf{HC}}(Q_t, S_t, I_t)\right]$, $\forall (Q_t, S_t, \bar{I}_t) \in \mathcal{Q} \times \mathcal{S} \times \bar{\mathcal{I}}$.
end for

For each period $t \in \mathcal{T}$ and each state (Q_t, S_t, I_t), we define $v_t^{\mathsf{HC}}(Q_t, S_t, I_t)$ as the value function of our heuristic method, and $Z_t^{(\nu),\mathsf{HC}}(I_t)$ and $Y_t^{(\nu),\mathsf{HC}}(I_t)$ as the state-dependent target levels of our heuristic method. Following Lemma 3 and Theorems 1-2, we compute these value functions and target levels, as well as the corresponding action triplets $(q_t^{\mathsf{HC}}, s_t^{\mathsf{HC}}, w_t^{\mathsf{HC}})$, by backward induction. See Algorithm 1. We call this method HC. We consider two main scenarios in this method:

- Suppose that the price is positive in the current state. We first calculate w_t^{HC} from Lemma 3. We next compute the state-dependent target level $Z_t^{\mathsf{HC}}(Q_t, S_t, I_t)$ from Theorem 2 with $Z_t^{(\nu)}(I_t)$ replaced by $Z_t^{(\nu),\mathsf{HC}}(I_t)$, $\forall \nu$. We then compute s_t^{HC} from Theorem 1 with $Z_t^*(Q_t, S_t, I_t)$ replaced by $Z_t^{\mathsf{HC}}(Q_t, S_t, I_t)$. Finally, if $Z_t^{(\nu),\mathsf{HC}}(I_t)$ is accessible, $q_t^{\mathsf{HC}} = Y_t^{(\nu),\mathsf{HC}}(I_t)$. Otherwise, we calculate q_t^{HC} from equation (2).
- Suppose that the price is negative in the current state. We obtain $w_t^{\mathsf{HC}} = 0$ (i.e., generate no wind energy) and $s_t^{\mathsf{HC}} = -\min\{C_S - S_t, C_C, \theta\tau C_T\}$ (i.e., purchase as much energy as possible) from the myopically optimal solution, but we calculate q_t^{HC} again from equation (2).

The number of states in which we need to compute the target levels in each period is of order $O(|\mathcal{I}|)$ and the number of feasible action triplets that we need to consider for target level computation in each state is of order $O(|\mathcal{Q}||\mathcal{S}|)$: the total number of operations required to exhaustively search the action triplets is of order $O(T|\mathcal{Q}||\mathcal{S}||\mathcal{I}|)$. To speed up computation and save memory, we calculate first the value function $\bar{v}^{\mathsf{HC}}_{t+1}(S_{t+1}, Q_{t+1}, \overline{I}_{t+1}) := \mathbb{E}_{J_{t+1}}\left[v^{\mathsf{HC}}_{t+1}(Q_{t+1}, S_{t+1}, I_{t+1})\right]$ in state $(S_{t+1}, Q_{t+1}, \overline{I}_{t+1})$ in period $t+1$ and then the action triplet in state (Q_t, S_t, I_t) in period t, using the target levels calculated with $\bar{v}^{\mathsf{HC}}_{t+1}(S_{t+1}, Q_{t+1}, \overline{I}_{t+1})$ and the expectation taken with respect to $\overline{I}_{t+1}|\overline{I}_t$. This heuristic method accelerates the standard DP algorithm of our problem, without loss of optimality for systems with positive prices.

5.2. Solution Approach via Reduced State Space

Our method HC calculates the target levels in period t for each exogenous state tuple I_t in the set $\mathcal{I}$; see step 3 of Algorithm 1. If the spike component of the price is assumed to be zero, the set $\mathcal{I}$ can be reduced to the set $\overline{\mathcal{I}}$ in Algorithm 1. Since $|\overline{\mathcal{I}}| = |\mathcal{I}|/|\mathcal{J}|$ where $\mathcal{J}$ is the discrete set of the spike component, the zero-spike assumption significantly reduces the computations of Algorithm 1. We thus consider a reduced state-space version of our method HC that calculates the target levels, which we denote by $Z^{\mathsf{HR}}_t(Q_t, S_t, \overline{I}_t)$ and $Y^{\mathsf{HR}}_t(S_{t+1}, \overline{I}_t)$, by executing Algorithm 1 with $\mathcal{J}$ replaced by $\mathcal{J}^{\mathsf{HR}} := \{0\}$. This variant of our method HC determines the storage action in each state with a zero spike value and a positive price via the target level $Z^{\mathsf{HR}}_t(Q_t, S_t, \overline{I}_t)$, but takes a myopic approach in other states. It also myopically determines the wind power generation action in each state. Finally, it determines the commitment action in each state via the target level $Y^{\mathsf{HR}}_t(S_{t+1}, \overline{I}_t)$. We call this method HR. We construct Algorithm 2 to calculate the expected total cash flow of the resulting heuristic policy with action triplets $(q^{\mathsf{HR}}_t, s^{\mathsf{HR}}_t, w^{\mathsf{HR}}_t)$.

In methods HC and HR, we choose to impose the myopic actions in negative-price states because the optimal policy structure is unavailable under negative prices, and the myopic actions are immediately obtained and are expected to be optimal in many negative-price states. The number of positive-price states is much greater than the number of negative-price states in each of our data-calibrated instances in Section 6. This has two implications: First, the myopic action in a negative-price state – charging the battery as much as possible – is also sensible from a forward-looking perspective since the price will very likely be positive in the next period. Second, the myopic actions in negative-price states, if suboptimal, can only slightly drain the total profit since the negative-price states have only a limited contribution to the total profit. In method HR, we choose to impose the myopic actions in nonzero-spike states because the existence of a spike in any

Algorithm 2 Solution approach via reduced state space.

$\bar{v}_T^{\mathsf{HR}}(Q_T, S_T, \bar{I}_T) \leftarrow 0$, $\forall (Q_T, S_T, \bar{I}_T) \in \mathcal{Q} \times \mathcal{S} \times \bar{\mathcal{I}}$.
for $t = T-1, \ldots, 1$ **do**
  for $(Q_t, S_t, I_t) \in \mathcal{Q} \times \mathcal{S} \times \mathcal{I}$ **do**
    if $P_t \geq 0$ **then**
      Compute w_t^{HR} from Lemma 3.
      Compute

$$Z_t \leftarrow \begin{cases} Z_t^{\mathsf{HR}}(Q_t, S_t, \bar{I}_t) & \text{if } J_t = 0, \\ 0 & \text{if } J_t > 0, \\ C_S & \text{if } J_t < 0. \end{cases}$$

      Compute s_t^{HR} from Theorem 1 with $Z_t^*(Q_t, S_t, I_t)$ and $w_t^*(Q_t, S_t, I_t)$ replaced by Z_t and w_t^{HR}.
    else
      $s_t^{\mathsf{HR}} \leftarrow -\min\{C_S - S_t, C_C, \theta\tau C_T\}$ and $w_t^{\mathsf{HR}} \leftarrow 0$.
    end if
    $q_t^{\mathsf{HR}} \leftarrow Y_t^{\mathsf{HR}}(S_t - s_t^{\mathsf{HR}}, \bar{I}_t)$.
    $v_t^{\mathsf{HR}}(Q_t, S_t, I_t) \leftarrow R(Q_t, I_t, s_t^{\mathsf{HR}}, w_t^{\mathsf{HR}}) + \mathbb{E}_{\bar{I}_{t+1}|\bar{I}_t}\left[\bar{v}_{t+1}^{\mathsf{HR}}(q_t^{\mathsf{HR}}, S_t - s_t^{\mathsf{HR}}, \bar{I}_{t+1})\right]$.
  end for
  $\bar{v}_t^{\mathsf{HR}}(Q_t, S_t, \bar{I}_t) \leftarrow \mathbb{E}_{J_t}\left[v_t^{\mathsf{HR}}(Q_t, S_t, I_t)\right]$, $\forall (Q_t, S_t, \bar{I}_t) \in \mathcal{Q} \times \mathcal{S} \times \bar{\mathcal{I}}$.
end for

period leads to an extremely high or low price, making the forward-looking perspective less critical and suggesting that the myopic action is likely to be optimal. Our numerical study in Section 6.2 verifies our intuition and justifies our use of myopic actions.

6. Numerical Results

We now conduct numerical experiments to evaluate the use of our heuristic methods HC and HR in the general problem, comparing them to the standard DP algorithm (yielding the optimal solution) with respect to objective value and computation time (Section 6.1). We also investigate the potential cost of enforcing myopic actions in non-zero spike states as in our methods HC and HR (Section 6.2). Finally, we examine the performance of several alternative solution methods with the same experimental setup (Section 6.3).

We consider instances in which the planning horizon spans the first week of August ($T = 168$ hours). The wind farm consists of 100 wind turbines, each with a capacity of 1.5 MW, for a total power capacity of 150 MW. This capacity reflects widely used models globally, such as the General Electric GE 1.5 series (Sheridan et al. 2024). The energy capacity of the battery (C_S) is 500 MWh and the charging/discharging capacities (C_C and C_D) are 40 or 60 MWh (see California Energy Commission 2023 and Power Engineering 2023 for similar settings). The battery can thus be fully charged or discharged in approximately ten hours, consistent with several examples in the related literature (Zhou et al. 2019 and U.S. Department of Energy 2020). The battery's round-trip efficiency ($r = \theta\gamma$) varies between 0.70 and 0.80 for a lead-acid battery, between 0.75 and 0.85 for a zinc-bromine battery, and between 0.90 and 0.95 for a lithium-ion battery (Zhou et al. 2016 and Arfeen et al. 2020). We thus restrict r to values from the set $\{0.7, 0.8, 0.9, 1\}$ (including the perfect efficiency). The charging and discharging efficiencies of the battery (θ and γ) are the same and equal to the square-root of the round-trip efficiency. This assumption is common in the literature (Ranaweera and Midtgård 2016, Zhou et al. 2016, Singh and Fernandez 2018, and Shabani et al. 2021). The energy capacity of the transmission line (C_T) is 200 MWh (Fernandez et al. 2016, Zhou et al. 2019, Marins et al. 2020, Karakoyun et al. 2023, and Glaum and Hofmann 2023). The transmission line efficiency (τ) is 0.95 or 1, reflecting typical efficiency losses observed in practice (EIA 2023 and NYISO 2024). We restrict the imbalance parameters (K_p, K_n, K_n^-, and K_p^-) to values from the set $\{0.6, 0.7, ..., 1.4\} \setminus \{1\}$. These values are consistent with those commonly observed in practice and reported in the literature (Löhndorf and Minner 2010, Chaves-Ávila et al. 2014, Hassler 2017, RTE 2022, and PWC 2023).

We implement the time-series models described by Karakoyun et al. (2023) into the exogenous stochastic process of our MDP. The negative price occurrence frequency (NPF) is 4.07% according to these models. We consider this original setting as well as a hypothetical one where the price is assumed to be always nonnegative. In all instances, the discretization level is 20 MWh, the initial storage level S_1 is the closest state to $C_S/2$, the initial commitment level Q_1 is the maximum amount of energy that can be committed to selling, and the initial exogenous state $I_1 = (\rho_1, J_1, \xi_1)$ is $(0, 0, 5)$. We construct a total of 48 instances with the above specifications. All computations were executed on a dual 3.7 GHz Intel Xeon W-2255 CPU server with 96 GB of RAM.

6.1. Performance of Our Solution Approaches

Tables 2 and 3 exhibit the optimality gaps and computation times of our solution approaches. Our method HC yields the optimal solution when the price is always positive (as shown in Lemma 3 and Theorems 1–2). We also observe that its optimal performance extends to instances where the price can also be negative; its optimality gap is no larger than 0.00022% in Tables 2 and 3. Our method HC reduces the computation time of the standard DP algorithm by up to 69 times.

Our method HR performs only slightly worse than our method HC with respect to objective value: it yields solutions with a maximum distance of 0.85% and an average distance of 0.30% from optimal. Our method HR, however, provides a further significant advantage in computations: the execution of our method HR takes only half a minute while that of the standard DP algorithm

Table 2 Optimality gaps and computation times when $K_p = K_n^- = 0.9$, $K_n = K_p^- = 1.1$, $C_S = 500$ MWh, $C_T = 200$ MWh.

$C_C = C_D$	NPF	τ	r	Optimality gaps		Computation times (minutes)		
				HC	HR	Optimal policy	HC	HR
40	0	0.95	0.7	0.00%	0.40%	205.4	4.0	0.5
			0.8	0.00%	0.32%	203.7	3.8	0.5
			0.9	0.00%	0.28%	204.3	3.8	0.5
			1	0.00%	0.24%	203.1	3.8	0.5
		1	0.7	0.00%	0.40%	202.2	3.8	0.5
			0.8	0.00%	0.32%	201.9	3.8	0.5
			0.9	0.00%	0.27%	202.7	3.8	0.5
			1	0.00%	0.24%	157.5	3.4	0.4
	4.07%	0.95	0.7	0.00%	0.21%	204.5	3.8	0.5
			0.8	0.00%	0.16%	204.5	3.8	0.5
			0.9	0.00%	0.15%	204.2	3.8	0.5
			1	0.00%	0.13%	203.1	3.8	0.5
		1	0.7	0.00%	0.21%	202.6	3.8	0.5
			0.8	0.00%	0.17%	202.9	3.8	0.5
			0.9	0.00%	0.14%	202.3	3.8	0.5
			1	0.00%	0.13%	157.1	3.4	0.4
60	0	0.95	0.7	0.00%	0.85%	318.8	4.7	0.5
			0.8	0.00%	0.65%	319.5	5.2	0.6
			0.9	0.00%	0.52%	319.3	5.4	0.6
			1	0.00%	0.44%	317.3	5.4	0.6
		1	0.7	0.00%	0.84%	319.2	4.7	0.5
			0.8	0.00%	0.65%	317.3	5.1	0.5
			0.9	0.00%	0.48%	321.4	5.3	0.6
			1	0.00%	0.40%	248.7	4.8	0.5
	4.07%	0.95	0.7	0.00%	0.49%	318.8	4.8	0.5
			0.8	0.00%	0.36%	319.4	5.1	0.6
			0.9	0.00%	0.29%	319.3	5.4	0.6
			1	0.00%	0.25%	318.0	5.4	0.6
		1	0.7	0.00%	0.48%	322.1	4.7	0.5
			0.8	0.00%	0.37%	317.5	5.1	0.5
			0.9	0.00%	0.27%	317.1	5.3	0.6
			1	0.00%	0.22%	249.1	4.8	0.5

takes several hours. All these results highlight the high efficiency and scalability of our solution methods constructed with structural knowledge.

We observe from Table 2 that our method HR induces lower optimality gaps when the battery charging/discharging capacities are small and the negative electricity prices are observed: The decisions of our method HR, if suboptimal, can only slightly deviate from the optimal decisions when the producer is better motivated for energy arbitrage but has limited flexibility in its charging/discharging decisions. We observe from Table 3 that the performance of our method HR is relatively robust to changes in the imbalance parameters.

We also tested the performance of our methods on instances with NPF values up to 25.8%,

Table 3 Optimality gaps and computation times when $C_S = 500$ MWh, $C_C = C_D = 40$ MWh, $C_T = 200$ MWh, NPF $= 4.07\%$, $\tau = 0.95$, $r = 0.8$.

$K_p = K_n^-$	$K_n = K_p^-$	Optimality gaps		Computation times (minutes)		
		HC	HR	Optimal policy	HC	HR
0.6	1.1	0.00%	0.30%	204.8	3.8	0.5
	1.2	0.00%	0.26%	204.4	3.8	0.5
	1.3	0.00%	0.19%	205.0	3.8	0.5
	1.4	0.00%	0.16%	205.1	3.8	0.5
0.7	1.1	0.00%	0.23%	205.6	3.8	0.5
	1.2	0.00%	0.21%	204.7	3.9	0.5
	1.3	0.00%	0.16%	204.0	3.8	0.5
	1.4	0.00%	0.16%	204.2	3.8	0.5
0.8	1.1	0.00%	0.21%	205.9	3.8	0.5
	1.2	0.00%	0.19%	204.4	3.8	0.5
	1.3	0.00%	0.18%	204.5	3.8	0.5
	1.4	0.00%	0.19%	205.3	3.8	0.5
0.9	1.1	0.00%	0.16%	206.5	3.8	0.5
	1.2	0.00%	0.16%	204.0	3.8	0.5
	1.3	0.00%	0.18%	205.2	3.8	0.5
	1.4	0.00%	0.18%	205.8	3.8	0.5

obtained by increasing the spike occurrence probabilities.[5] Our method HC continues to yield the optimal solution regardless of the NPF level. In contrast, the performance of our method HR degrades as the NPF level increases. This decline is primarily due to its target-level calculation method, which relies on the zero-spike assumption that becomes more restrictive at higher NPF values. Nevertheless, even in the extreme case of 25.8% NPF, HR performs only 3.62% worse than the optimal solution.

6.2. Numerical Investigation of the Impact of Myopic Actions

Our method HC takes a myopic approach when the price is negative. Our method HR takes a myopic approach when the price is positive and the spike is nonzero, or when the price is negative. On our experimental test bed, on average, 5.7% of the positive-price states have nonzero spikes, 91.9% of the positive-price nonzero-spike states have positive spikes, and all of the negative-price states have negative spikes. In our experiments, since the negative prices can only arise due to negative spikes, our method HR indeed imposes the myopic actions in all nonzero-spike states, while taking the forward-looking actions in all zero-spike states.

We intuitively expect the myopic actions of HR to be often optimal. We performed experiments to test our intuition: We define MD as the percentage of the nonzero-spike states in which the

[5] Negative prices were observed in the U.S. about 4% of the time in 2021. These were concentrated in specific regions such as parts of Kansas and Oklahoma within the Southwest Power Pool, where they accounted for over 25% of all hours (Berkeley Lab 2021).

Table 4 Numerical results when $K_p = K_n^- = 0.9$, $K_n = K_p^- = 1.1$, $C_S = 500$ MWh, $C_T = 200$ MWh.

$C_C = C_D$	NPF	τ	r	MD	MS-TCF	FS-TCF	NS-TCF
40	0	0.95	0.7	100.00%	26.99%	0.00%	73.01%
			0.8	100.00%	27.08%	0.00%	72.92%
			0.9	100.00%	27.09%	0.00%	72.91%
			1	100.00%	27.06%	0.00%	72.94%
		1	0.7	100.00%	26.99%	0.00%	73.01%
			0.8	100.00%	27.08%	0.00%	72.92%
			0.9	100.00%	27.08%	0.00%	72.92%
			1	100.00%	27.05%	0.00%	72.95%
	4.02%	0.95	0.7	99.30%	27.48%	0.04%	72.48%
			0.8	99.40%	27.45%	0.04%	72.51%
			0.9	99.49%	27.42%	0.03%	72.55%
			1	99.57%	27.36%	0.02%	72.61%
		1	0.7	99.33%	27.37%	0.04%	72.58%
			0.8	99.43%	27.35%	0.04%	72.62%
			0.9	99.51%	27.33%	0.03%	72.64%
			1	99.60%	27.27%	0.02%	72.70%
60	0	0.95	0.7	69.80%	16.55%	11.93%	71.52%
			0.8	71.53%	17.26%	11.19%	71.55%
			0.9	74.91%	18.67%	9.68%	71.65%
			1	78.14%	19.84%	8.40%	71.76%
		1	0.7	69.67%	16.88%	12.27%	70.86%
			0.8	71.40%	17.67%	11.53%	70.80%
			0.9	73.18%	18.99%	10.09%	70.92%
			1	79.83%	20.27%	8.65%	71.08%
	4.02%	0.95	0.7	81.42%	17.82%	11.93%	70.25%
			0.8	82.60%	18.41%	11.19%	70.40%
			0.9	84.48%	19.56%	9.88%	70.56%
			1	86.81%	20.86%	8.39%	70.75%
		1	0.7	81.43%	17.62%	11.99%	70.39%
			0.8	82.60%	18.18%	11.24%	70.58%
			0.9	83.77%	18.85%	10.45%	70.70%
			1	87.63%	21.30%	7.82%	70.87%

myopic decision is optimal according to the exact solution algorithm (i.e., the percentage of the myopic actions of HR that are indeed optimal). We also define FS-TCF as the percentage of the total cash flow that comes from the revenues collected in the nonzero-spike states in which the optimal decision is forward-looking (as opposed to myopic in HR). See Tables 4 and 5 for our results on the same test bed as in Section 6.1. We have found that MD is 92.55% and FS-TCF is only 3.49% on average. These results verify our intuition and show that the myopic decisions of HR, if not optimal, can only slightly drain the profits. We note that MD is substantially lower when the charging/discharging capacities are large and thus the level of flexibility in charging/discharging decisions is high.

We also examined the contribution of the nonzero-spike states to the total cash flow according to the exact solution algorithm: We define MS-TCF as the percentage of the total cash flow that

Table 5 Numerical results when $C_S = 500$ MWh, $C_C = C_D = 40$ MWh, $C_T = 200$ MWh, NPF $= 4.07\%$, $\tau = 0.95$, $r = 0.8$.

$K_p = K_n^-$	$K_n = K_p^-$	MD	MS-TCF	FS-TCF	NS-TCF
0.6	1.1	99.37%	25.95%	0.04%	74.01%
	1.2	99.23%	25.81%	0.05%	74.14%
	1.3	99.12%	25.68%	0.06%	74.26%
	1.4	99.05%	25.61%	0.07%	74.32%
0.7	1.1	99.38%	26.50%	0.04%	73.46%
	1.2	99.24%	26.41%	0.05%	73.54%
	1.3	99.13%	26.34%	0.06%	73.60%
	1.4	99.06%	26.33%	0.07%	73.60%
0.8	1.1	99.39%	27.01%	0.04%	72.95%
	1.2	99.26%	26.96%	0.05%	72.99%
	1.3	99.17%	26.94%	0.05%	73.01%
	1.4	99.09%	26.95%	0.06%	72.99%
0.9	1.1	99.40%	27.45%	0.04%	72.51%
	1.2	99.28%	27.47%	0.04%	72.49%
	1.3	99.20%	27.49%	0.05%	72.46%
	1.4	99.12%	27.49%	0.06%	72.45%

comes from the revenues collected in the nonzero-spike states in which the optimal decision is myopic. We define NS-TCF as the percentage of the total cash flow that comes from the revenues collected in the zero-spike states (in which HR takes the forward-looking perspective). Note that the sum of MS-TCF, FS-TCF, and NS-TCF equals one in each instance. See again Tables 4 and 5. We observe that the percentage of the total cash flow that comes from the revenues collected in the nonzero-spike states (MS-TCF plus FS-TCF) is 27.40% on average: the zero-spike states have a much greater impact on total cash flow than the nonzero-spike states, making the forward-looking perspective critical in our experiments. The myopic decisions of HR (in the nonzero-spike states) alone cannot guarantee a near-optimal performance. The higher level of sophistication that HR involves via forward-looking decisions (in the zero-spike states) is clearly useful.

6.3. Alternative Solution Methods

Our method HC takes a myopic approach in negative-price states and our method HR extends this approach to nonzero-spike states. We now evaluate the use of a myopically optimal policy that adopts the optimal solution of the two-period problem in each state (even when the price is positive and the spike component is zero) as an alternative heuristic approach for our problem. We call this method M. We consider here the two-period problem because the commitment decision in the current period can only affect the payoff in the next period. See Tables 6 and 7 for our results on the same test bed as in Section 6.1. We have found that the myopic policy yields solutions with an average distance of 8.88%, a maximum distance of 11.38%, and a minimum distance of 7.26% from

Table 6 Optimality gaps for alternative solution methods when $K_p = K_n^- = 0.9$, $K_n = K_p^- = 1.1$, $C_S = 500$ MWh, $C_T = 200$ MWh.

$C_C = C_D$	NPF	τ	r	Optimality gaps			
				M	F1	F2	DR
40	0	0.95	0.7	7.63%	11.30%	11.28%	3.13%
			0.8	7.96%	8.20%	10.42%	3.43%
			0.9	8.54%	6.20%	8.45%	2.27%
			1	9.03%	5.10%	7.41%	2.25%
		1	0.7	7.67%	7.50%	7.75%	3.18%
			0.8	7.96%	5.20%	6.86%	3.50%
			0.9	8.44%	4.20%	5.27%	2.30%
			1	9.01%	3.70%	4.92%	2.45%
	4.02%	0.95	0.7	7.26%	10.80%	10.78%	2.54%
			0.8	7.67%	7.70%	9.82%	2.77%
			0.9	8.46%	5.80%	7.92%	2.17%
			1	8.84%	4.80%	6.98%	2.10%
		1	0.7	7.29%	7.00%	7.19%	2.59%
			0.8	7.68%	4.80%	6.48%	2.83%
			0.9	8.47%	3.90%	4.96%	2.19%
			1	8.83%	3.50%	4.64%	2.34%
60	0	0.95	0.7	9.73%	14.90%	14.89%	4.05%
			0.8	10.19%	11.50%	13.75%	4.34%
			0.9	10.86%	7.60%	9.53%	3.12%
			1	11.38%	5.40%	8.23%	3.02%
		1	0.7	9.80%	10.00%	10.52%	4.08%
			0.8	10.19%	6.60%	8.94%	4.41%
			0.9	10.75%	4.50%	6.33%	3.20%
			1	11.35%	4.00%	5.42%	3.31%
	4.02%	0.95	0.7	8.99%	13.90%	13.92%	3.30%
			0.8	9.57%	10.80%	12.7%	3.52%
			0.9	10.55%	6.90%	8.65%	2.95%
			1	10.92%	4.90%	7.52%	2.81%
		1	0.7	9.09%	9.00%	9.51%	3.33%
			0.8	9.63%	6.00%	8.18%	3.60%
			0.9	10.60%	4.10%	5.71%	3.04%
			1	10.93%	3.80%	4.90%	3.15%

optimal. Comparing these results with our results for HC and HR, a forward-looking approach is arguably more suitable in positive-price states that are not dominantly large.

We also evaluate the use of fixed threshold policies as another heuristic approach for our problem. The target storage and commitment levels vary with the exogenous state variables in Theorems 1–2. For the fixed threshold policy, however, the target levels remain constant within each period but vary from one period to another. We consider two variants of the fixed threshold policy, which we call F1 and F2, respectively. F1 calculates the target levels in period t by restricting the exogenous state tuple I_t to its prediction $\hat{I}_t$ based on the initial state tuple I_1 in the backward algorithm of our method HC. The prediction for k periods later is found by first raising the transition probability matrices (obtained from the time-series models) to the kth power and then taking the expectations

Table 7 Optimality gaps for alternative solutions methods when $C_S = 500$ MWh, $C_C = C_D = 40$ MWh, $C_T = 200$ MWh, NPF $= 4.07\%$, $\tau = 0.95$, $r = 0.8$.

$K_p = K_n^-$	$K_n = K_p^-$	Optimality gaps			
		M	F1	F2	DR
0.6	1.1	7.99%	4.80%	4.92%	4.14%
	1.2	8.52%	4.20%	4.21%	3.28%
	1.3	8.44%	4.20%	4.16%	3.18%
	1.4	8.53%	4.40%	4.36%	3.30%
0.7	1.1	7.86%	4.40%	4.42%	3.28%
	1.2	8.40%	4.00%	3.95%	2.70%
	1.3	8.36%	4.00%	4.03%	2.72%
	1.4	8.33%	4.30%	4.33%	2.94%
0.8	1.1	7.77%	4.10%	4.76%	3.24%
	1.2	8.37%	4.00%	5.03%	3.05%
	1.3	8.24%	4.40%	5.04%	3.27%
	1.4	8.13%	4.70%	5.34%	3.64%
0.9	1.1	7.67%	7.70%	9.82%	2.77%
	1.2	8.21%	7.30%	7.92%	3.13%
	1.3	8.12%	7.80%	8.12%	3.56%
	1.4	7.93%	7.80%	7.41%	4.12%

via the resulting distributions. F2 calculates the target levels in period t by taking the expectation over the exogenous state tuple I_t conditional on the initial state tuple I_1. Specifically, we change step 4 in the backward algorithm of our method HC to

$$\left(Y_t^{(\nu),\mathsf{HC}}, Z_t^{(\nu),\mathsf{HC}}\right) \leftarrow \underset{\left(Y_t^{(\nu)}, Z_t^{(\nu)}\right) \in \mathcal{Q}\times\mathcal{S}}{\arg\max} \left\{ \mathbb{E}_{I_t|I_1}\left[R^{(\nu)}(Z_t^{(\nu)}, I_t) + \mathbb{E}_{\overline{I}_{t+1}|\overline{I}_t}\left[\overline{v}_{t+1}^{\mathsf{HC}}(Y_t^{(\nu)}, Z_t^{(\nu)}, \overline{I}_{t+1}) \right]\right]\right\}.$$

Like our method HR, both F1 and F2 use the target levels to determine the storage and commitment decisions in states with a zero spike value and a positive price, and take the myopic approach in all other states. See again Tables 6 and 7 for our results. We have found that F1 yields solutions with an average distance of 6.36%, a maximum distance of 14.90%, and a minimum distance of 3.50% from optimal. F2 yields solutions with an average distance of 7.45%, a maximum distance of 14.89%, and a minimum distance of 3.95% from optimal. These results imply that ignoring the state information in target level calculation causes a significant loss of optimality, demonstrating the usefulness of state-dependent policies in our problem.

Finally, we consider a deterministic reoptimization heuristic that solves a simpler version of our problem in each period obtained by replacing the random components with their expected values conditional on the current state. We call this method DR. The deterministic problem in state (Q_t, S_t, I_t) is given by

$$\max_{\left\{\left(q_\eta, s_\eta, w_\eta, S'_\eta\right)\right\}_{\eta\in\mathcal{T}:\eta\geq t}} \sum_{\eta\in\mathcal{T}:\eta\geq t} R(q_{\eta-1}, I_{t,\eta}, s_\eta, w_\eta)$$

where $S'_t = S_t,\ S'_{\eta+1} = S'_\eta - s_\eta,\ \forall \eta \geq t,\ q_{t-1} = Q_t,\ (q_\eta, s_\eta, w_\eta) \in \mathbb{U}(q_{\eta-1}, S'_\eta, I_{t,\eta}),\ \forall \eta \geq t$, and $I_{t,\eta} := (P_{t,\eta}, W_{t,\eta})$ is the expected exogenous state in period η conditional on the exogenous state I_t in period t. The objective is to maximize the total cash flow in periods from t through T. The objective function can be linearized when $P_{t,\eta} > 0,\ \forall \eta \geq t$, since the payoff function in each period can be shown to be the minimum of affine functions in this case. This heuristic restricts all price expectations to be nonnegative and solves a linear program when $P_t > 0$: the actions in period t are given by the optimal actions of period t obtained from the linear program. It takes a myopic approach when $P_t \leq 0$: the actions in period t are given by the optimal actions of the two-period problem with states $I_{t,t}$ and $I_{t,t+1}$. See again Tables 6 and 7 for our results. This heuristic yields solutions with an average distance of 3.12%, a maximum distance of 4.41%, and a minimum distance of 2.10% from optimal, performing worse than HC and HR. The precise modeling of uncertainties as in HC and HR thus seems useful in our problem setting.

7. Concluding Remarks

We have examined the energy commitment, generation and storage problem for a wind farm paired with a battery in the presence of uncertainties in the electricity price and wind speed. The power producer participates in an electricity market that operates with hourly commitments and settlements. Modeling this problem as an MDP, we establish the optimality of a state-dependent threshold policy for the storage and commitment problem under positive electricity prices. Leveraging this policy structure, we construct two heuristic solution methods (HC and HR) for the more general problem in which the price can also be negative. With data-calibrated time series models for the electricity price and wind speed, we numerically test the performance of these solution methods in the general problem.

Our experiments have revealed the optimal performance of our method HC in all instances. Furthermore, our method HC has an average solution time of 4.2 minutes, whereas the standard DP algorithm has an average solution time of almost 4 hours. Our method HR, on the other hand, provides near-optimal solutions with an average distance of 0.30% from optimal within half a minute. Our experiments have also revealed the relatively poor performance of simpler alternative solution methods (purely myopic solution approach, fixed threshold policies, and deterministic reoptimization heuristic) with respect to objective value.

Future research may consider other market settings in which the power producer makes commitments for a longer time window (e.g., day-ahead markets) or for multiple time windows of different lengths (e.g., sequential markets). Despite the added complexity of these settings, our results could inform the design of simplified policy structures that serve as effective heuristic solutions. One such setting is a two-settlement electricity market where the producer first makes commitments in the day-ahead market and then settles real-time deviations at imbalance prices in the balancing market. In this setting, our policy structure could be adapted to determine hourly energy generation and storage levels, conditional on the prespecified day-ahead commitments and prices. This approach may perform well by accounting for intraday price and supply uncertainties that have immediate effects, without requiring full consideration of day-ahead market dynamics that have more indirect effects. Future research may also extend our work by incorporating investment-related decisions and examining the impact of cycle-induced battery degradation on long-term profitability and operational strategy.

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Online Technical Appendix. Proofs of the Analytical Results

Proof of Lemma 1. Let $\underline{Q}_t$ and $\bar{Q}_t$ denote some bounds on $E(s_t, w_t)$ in state (Q_t, S_t, I_t) such that $\underline{Q}_t \leq E(s_t, w_t) \leq \bar{Q}_t$. Note that $R(\cdot, I_t, s_t, w_t)$ is a decreasing function for $Q_t \geq E(s_t, w_t)$ since $P_t(1-K_n) < 0$ if $P_t \geq 0$ and $P_t(1-K_n^-) < 0$ if $P_t < 0$. Thus $R(\bar{Q}_t + \alpha, I_t, s_t, w_t) < R(\bar{Q}_t, I_t, s_t, w_t)$ for $\alpha > 0$. This implies that

$$\begin{aligned} v_t^*(\bar{Q}_t + \alpha, S_t, I_t) &= \max_{(q_t, s_t, w_t) \in \mathbb{U}(\bar{Q}_t + \alpha, S_t, I_t)} \left\{ R(\bar{Q}_t + \alpha, I_t, s_t, w_t) + \mathbb{E}_{I_{t+1}|I_t} \left[v_{t+1}^*(q_t, S_{t+1}, I_{t+1}) \right] \right\} \\ &< \max_{(q_t, s_t, w_t) \in \mathbb{U}(\bar{Q}_t, S_t, I_t)} \left\{ R(\bar{Q}_t, I_t, s_t, w_t) + \mathbb{E}_{I_{t+1}|I_t} \left[v_{t+1}^*(q_t, S_{t+1}, I_{t+1}) \right] \right\} = v_t^*(\bar{Q}_t, S_t, I_t). \end{aligned} \tag{EC.1}$$

Also, note that $R(\cdot, I_t, s_t, w_t)$ is an increasing function for $Q_t \leq E(s_t, w_t)$ since $P_t(1-K_p) > 0$ when $P_t \geq 0$ and $P_t(1-K_p^-) > 0$ when $P_t < 0$. Thus $R(\underline{Q}_t - \alpha, I_t, s_t, w_t) < R(\underline{Q}_t, I_t, s_t, w_t)$ for $\alpha > 0$. This implies that

$$\begin{aligned} v_t^*(\underline{Q}_t - \alpha, S_t, I_t) &= \max_{(q_t, s_t, w_t) \in \mathbb{U}(\underline{Q}_t - \alpha, S_t, I_t)} \left\{ R(\underline{Q}_t - \alpha, I_t, s_t, w_t) + \mathbb{E}_{I_{t+1}|I_t} \left[v_{t+1}^*(q_t, S_{t+1}, I_{t+1}) \right] \right\} \\ &< \max_{(q_t, s_t, w_t) \in \mathbb{U}(\underline{Q}_t, S_t, I_t)} \left\{ R(\underline{Q}_t, I_t, s_t, w_t) + \mathbb{E}_{I_{t+1}|I_t} \left[v_{t+1}^*(q_t, S_{t+1}, I_{t+1}) \right] \right\} = v_t^*(\underline{Q}_t, S_t, I_t). \end{aligned} \tag{EC.2}$$

Let $\eta_t^*(Q_t, S_t, I_t) = (q, s, w)$. Assume to the contrary that $q > \bar{Q}_{t+1}$. Then $v_t^*(Q_t, S_t, I_t) = R(Q_t, I_t, s, w) + \mathbb{E}\left[v_{t+1}^*(q, S_t - s, I_{t+1}) \right] \geq R(Q_t, I_t, s, w) + \mathbb{E}\left[v_{t+1}^*(\bar{Q}_{t+1}, S_t - s, I_{t+1}) \right]$. But this leads to a contradiction since $v_{t+1}^*(q, S_t - s, I_{t+1}) < v_{t+1}^*(\bar{Q}_{t+1}, S_t - s, I_{t+1})$ from (EC.1). Hence $q \leq \bar{Q}_{t+1}$. Now, assume to the contrary that $q < \underline{Q}_{t+1}$. Then $v_t^*(Q_t, S_t, I_t) = R(Q_t, I_t, s, w) + \mathbb{E}\left[v_{t+1}^*(q, S_t - s, I_{t+1}) \right] \geq R(Q_t, I_t, s, w) + \mathbb{E}\left[v_{t+1}^*(\underline{Q}_{t+1}, S_t - s, I_{t+1}) \right]$. But this leads to a contradiction since $v_{t+1}^*(q, S_t - s, I_{t+1}) < v_{t+1}^*(\underline{Q}_{t+1}, S_t - s, I_{t+1})$ from (EC.2). Hence $q \geq \underline{Q}_{t+1}$. We showed that $\underline{Q}_{t+1} \leq q_t \leq \bar{Q}_{t+1}$. Since $-\min\{(C_S - S_{t+1})/(\theta\tau), \underline{E}\} \leq E(s_{t+1}, w_{t+1}) \leq \min\{\tau(\gamma S_{t+1} + \bar{f}), \bar{E}\}$, note that $-\min\{(C_S - (S_t - s_t))/(\theta\tau), \underline{E}\} \leq q_t \leq \min\{\tau(\gamma(S_t - s_t) + \bar{f}), \bar{E}\}$, $\forall t \in \mathcal{T}$. Since $Q_{t+1} = q_t$, we have $-\min\{(C_S - S_t)/(\theta\tau), \underline{E}\} \leq Q_t \leq \min\{\tau(\gamma S_t + \bar{f}), \bar{E}\}$, $\forall t \in \mathcal{T} \setminus \{1\}$. $\square$

Proof of Lemma 2. Note that $v_T^*(Q_T, S_T, I_T) = v_T^*(Q_T, S_T + \alpha, I_T) = 0$ for $\alpha > 0$. Assuming $v_{t+1}^*(Q_{t+1}, S_{t+1}, I_{t+1}) \leq v_{t+1}^*(Q_{t+1}, S_{t+1} + \alpha, I_{t+1})$, we show $v_t^*(Q_t, S_t, I_t) \leq v_t^*(Q_t, S_t + \alpha, I_t)$. Let $\eta_t^*(Q_t, S_t, I_t) = (q, s, w)$. Also, let $\hat{s} = \max\{s, S_t + \alpha - C_S\}$ and

$$\hat{w} = \begin{cases} w & \text{if } \hat{s} = s, \\ \max\{0, w - (\hat{s} - s)/\theta\} & \text{if } \hat{s} \neq s. \end{cases}$$

We show that $(q, \hat{s}, \hat{w}) \in \mathbb{U}(Q_t, S_t + \alpha, I_t)$: If $s < S_t + \alpha - C_S$, since $(q, s, w) \in \mathbb{U}(Q_t, S_t, I_t)$, note that $-C_C \leq s < S_t + \alpha - C_S = \hat{s} \leq \min\{S_t + \alpha, C_D\}$. If $s \geq S_t + \alpha - C_S$, since $(q, s, w) \in \mathbb{U}(Q_t, S_t, I_t)$, note that $-\min\{C_S - S_t - \alpha, C_C\} \leq \hat{s} = s \leq \min\{S_t + \alpha, C_D\}$. Thus $-\min\{C_S - S_t - \alpha, C_C\} \leq \hat{s} \leq \min\{S_t + \alpha, C_D\}$. Since $s \leq \hat{s}$, we have $0 \leq \hat{w} \leq w \leq f(W_t)$. If $\hat{s} = s$, then $-C_T \leq E(s, w) = E(\hat{s}, \hat{w}) \leq \tau C_T$. If $s < \hat{s} = S_t + \alpha - C_S \leq 0$, then $\hat{s}/\theta + \hat{w} = \hat{s}/\theta + \max\{0, w - (\hat{s} - s)/\theta\} = \max\{\hat{s}/\theta, s/\theta + w\} \leq C_T$ and $-\tau C_T \leq s/\theta + w \leq \hat{s}/\theta + \hat{w}$. Hence $(q, \hat{s}, \hat{w}) \in \mathbb{U}(Q_t, S_t + \alpha, I_t)$. We consider the following three cases to show that $E(s, w) \leq E(\hat{s}, \hat{w})$:

(1) If $s \geq S_t + \alpha - C_S$, then $\hat{s} = s$ and $\hat{w} = w$. Thus $E(s, w) = E(\hat{s}, \hat{w})$.

(2) If $s < S_t + \alpha - C_S \leq 0$ and $-w \leq s/\theta < 0$, then $\hat{s} \leq 0$ and $\hat{s}/\theta + \hat{w} = \max\{\hat{s}/\theta, s/\theta + w\} = s/\theta + w \geq 0$. Thus $E(s, w) = (s/\theta + w)\tau = (\hat{s}/\theta + \hat{w})\tau = E(\hat{s}, \hat{w})$.

(3) If $s < S_t + \alpha - C_S \le 0$ and $s/\theta < -w \le 0$, then $\hat{s} \le 0$ and $\hat{s}/\theta + \hat{w} = \max\{\hat{s}/\theta, s/\theta + w\} \le 0$. Thus $E(s,w) = (s/\theta + w)/\tau \le (\hat{s}/\theta + \hat{w})/\tau = E(\hat{s},\hat{w})$.

Hence $E(s,w) \le E(\hat{s},\hat{w})$. Thus $R(Q_t, I_t, s, w) \le R(Q_t, I_t, \hat{s}, \hat{w})$. Note $\hat{s} \le s + \alpha$ since $(q,s,w) \in \mathbb{U}(Q_t, S_t, I_t)$. Thus $v_t^*(Q_t, S_t, I_t) = R(Q_t, I_t, s, w) + \mathbb{E}\left[v_{t+1}^*(q, S_t - s, I_{t+1})\right] \le R(Q_t, I_t, \hat{s}, \hat{w}) + \mathbb{E}\left[v_{t+1}^*(q, S_t + \alpha - \hat{s}, I_{t+1})\right] \le v_t^*(Q_t, S_t + \alpha, I_t)$. The first inequality holds since $v_{t+1}^*(Q_{t+1}, S_{t+1}, I_{t+1}) \le v_{t+1}^*(Q_{t+1}, S_{t+1} + \alpha, I_{t+1})$. □

Proof of Lemma 3. Let $\eta_t^*(Q_t, S_t, I_t) = (q,s,w)$. Also, let $\bar{w} := \min\{f(W_t), C_T + \min\{C_S - S_t, C_C\}/\theta\}$ denote the maximum amount of wind energy that can be generated in state (Q_t, S_t, I_t) and $(q, \hat{s}, \hat{w}) \in \mathbb{U}(Q_t, S_t, I_t)$ denote any feasible action triplet with $\hat{w} < \bar{w}$. We will show that it is possible to construct a feasible action triplet that is more profitable than the triplet $(q, \hat{s}, \hat{w})$ in each of the following four cases:

(1) Suppose that $E(\hat{s},\hat{w}) < 0$: We define $\Delta_1 = \min\{-\tau E(\hat{s},\hat{w}), \bar{w} - \hat{w}\} > 0$. Note that $-C_T \le E(\hat{s},\hat{w}) < E(\hat{s},\hat{w}) + \Delta_1/\tau = E(\hat{s}, \hat{w} + \Delta_1) \le 0$ and $0 \le \hat{w} < \hat{w} + \Delta_1 \le \bar{w} \le f(W_t)$. Hence, $(q, \hat{s}, \hat{w} + \Delta_1) \in \mathbb{U}(Q_t, S_t, I_t)$ and $R(Q_t, I_t, \hat{s}, \hat{w} + \Delta_1) > R(Q_t, I_t, \hat{s}, \hat{w})$: The triplet $(q, \hat{s}, \hat{w} + \Delta_1)$ is better than the triplet $(q, \hat{s}, \hat{w})$.

(2) Suppose that $E(\hat{s},\hat{w}) \ge 0$ and $\hat{s} = -\min\{C_S - S_t, C_C\}$: We define $\Delta_2 = \bar{w} - \hat{w} > 0$. Note that $0 \le E(\hat{s},\hat{w}) < E(\hat{s}, \hat{w} + \Delta_2) = (\hat{s}/\theta + \hat{w} + \Delta_2)\tau = (\hat{s}/\theta + \bar{w})\tau \le (-\min\{C_S - S_t, C_C\}/\theta + C_T + \min\{C_S - S_t, C_C\}/\theta)\tau = \tau C_T$ and $0 \le \hat{w} < \hat{w} + \Delta_2 = \bar{w} \le f(W_t)$. Hence, $(q, \hat{s}, \hat{w} + \Delta_2) \in \mathbb{U}(Q_t, S_t, I_t)$ and $R(Q_t, I_t, \hat{s}, \hat{w} + \Delta_2) > R(Q_t, I_t, \hat{s}, \hat{w})$: The triplet $(q, \hat{s}, \hat{w} + \Delta_2)$ is better than the triplet $(q, \hat{s}, \hat{w})$.

(3) Suppose that $E(\hat{s},\hat{w}) \ge 0$ and $-\min\{C_S - S_t, C_C\} < \hat{s} \le 0$: We define $\Delta_3 = \min\{\min\{C_S - S_t, C_C\}/\theta + \hat{s}/\theta, \bar{w} - \hat{w}\} > 0$. Since $0 < \Delta_3 \le \min\{C_S - S_t, C_C\}/\theta + \hat{s}/\theta$, note that $-\min\{C_S - S_t, C_C\} \le \hat{s} - \theta\Delta_3 < \hat{s} \le 0$. Also, note that $E(\hat{s},\hat{w}) = (\hat{s}/\theta + \hat{w})\tau = ((\hat{s} - \theta\Delta_3)/\theta + \hat{w} + \Delta_3)\tau = E(\hat{s} - \theta\Delta_3, \hat{w} + \Delta_3)$ and $0 \le \hat{w} < \hat{w} + \Delta_3 \le \bar{w} \le f(W_t)$. Hence, $(q, \hat{s} - \theta\Delta_3, \hat{w} + \Delta_3) \in \mathbb{U}(Q_t, S_t, I_t)$ and $R(Q_t, I_t, \hat{s}, \hat{w}) = R(Q_t, I_t, \hat{s} - \theta\Delta_3, \hat{w} + \Delta_3)$. Then, by Lemma 2,

$$R(Q_t, I_t, \hat{s}, \hat{w}) + \mathbb{E}\left[v_{t+1}^*(q, S_t - \hat{s}, I_{t+1})\right] < R(Q_t, I_t, \hat{s} - \theta\Delta_3, \hat{w} + \Delta_3) + \mathbb{E}\left[v_{t+1}^*(q, S_t - \hat{s} + \theta\Delta_3, I_{t+1})\right].$$

Thus, the triplet $(q, \hat{s} - \theta\Delta_3, \hat{w} + \Delta_3)$ is better than the triplet $(q, \hat{s}, \hat{w})$.

(4) Suppose that $E(\hat{s},\hat{w}) \ge 0$ and $\hat{s} > 0$: We define $\Delta_4 = \min\{\gamma\hat{s}, \bar{w} - \hat{w}\} > 0$. Since $0 < \Delta_4 \le \gamma\hat{s}$, note that $0 \le \hat{s} - \Delta_4/\gamma < \hat{s} \le \min\{S_t, C_D\}$. Also, note that $E(\hat{s},\hat{w}) = (\gamma\hat{s} + \hat{w})\tau = (\gamma(\hat{s} - \Delta_4/\gamma) + \hat{w} + \Delta_4)\tau = E(\hat{s} - \Delta_4/\gamma, \hat{w} + \Delta_4)$ and $0 \le \hat{w} < \hat{w} + \Delta_4 \le \bar{w} \le f(W_t)$. Hence, $(q, \hat{s} - \Delta_4/\gamma, \hat{w} + \Delta_4) \in \mathbb{U}(Q_t, S_t, I_t)$ and $R(Q_t, I_t, \hat{s}, \hat{w}) = R(Q_t, I_t, \hat{s} - \Delta_4/\gamma, \hat{w} + \Delta_4)$. Then, by Lemma 2,

$$R(Q_t, I_t, \hat{s}, \hat{w}) + \mathbb{E}\left[v_{t+1}^*(q, S_t - \hat{s}, I_{t+1})\right] < R(Q_t, I_t, \hat{s} - \Delta_4/\gamma, \hat{w} + \Delta_4) + \mathbb{E}\left[v_{t+1}^*(q, S_t - \hat{s} + \Delta_4/\gamma, I_{t+1})\right].$$

Thus, the triplet $(q, \hat{s} - \Delta_4/\gamma, \hat{w} + \Delta_4)$ is better than the triplet $(q, \hat{s}, \hat{w})$.

Hence we showed that $w = \bar{w}$. Next, suppose that $w = C_T + \min\{C_S - S_t, C_C\}/\theta$. We will show that $s = -\min\{C_S - S_t, C_C\}$. First, assume to the contrary that $s > 0$. In this case, $w\tau < (\gamma s + w)\tau = E(s,w) \le \tau C_T$. But this leads to a contradiction since $w \ge C_T$. Thus $s \le 0$. Note that $-\min\{C_S - S_t, C_C\} \le s$ since $(q,s,w) \in \mathbb{U}(Q_t, S_t, I_t)$. Also, note that $(s/\theta + C_T + \min\{C_S - S_t, C_C\}/\theta)\tau = E(s,w) \le \tau C_T$, implying that $-\min\{C_S - S_t, C_C\} \ge s$. Hence, the only feasible action is $s = -\min\{C_S - S_t, C_C\}$. □

Proof of Proposition 1. Note that $v_T^*(\cdot,\cdot,I_T)$ is jointly concave. Assuming $v_{t+1}^*(\cdot,\cdot,I_{t+1})$ is jointly concave, we will prove $v_t^*(\cdot,\cdot,I_t)$ is jointly concave. Taking a similar path to that in Zhou et al. (2019), we convert our problem to an equivalent one with linear constraints. We define the following decision variables:

- s_t^{CG}: The amount of energy charged into the battery from the wind energy generated;
- s_t^{CP}: The amount of energy charged into the battery from the energy purchased;
- s_t^{D}: The amount of energy discharged from the battery;
- w_t^{C}: The amount of wind energy generated and charged into the battery; and
- w_t^{S}: The amount of wind energy generated and sold in the market.

We define $\Gamma(Q_t,S_t,I_t)$ as the set of decision variable tuples $(q_t,s_t^{CG},s_t^{CP},s_t^D,w_t^C,w_t^S)\in\mathbb{R}\times\mathbb{R}_+^5$ that satisfy

$$-\underline{E}\le q_t\le\bar{E}, \tag{EC.3}$$

$$w_t^S+\gamma s_t^D\le C_T, \tag{EC.4}$$

$$s_t^{CP}/(\theta\tau)\le C_T, \tag{EC.5}$$

$$s_t^{CG}=\theta w_t^C, \tag{EC.6}$$

$$w_t^C+w_t^S\le f(W_t), \tag{EC.7}$$

$$s_t^D\le\min\{S_t,C_D\}, \tag{EC.8}$$

$$s_t^{CG}+s_t^{CP}\le\min\{C_S-S_t,C_C\}, \tag{EC.9}$$

$$s_t^{CG},s_t^{CP},s_t^D,w_t^C,w_t^S\ge 0. \tag{EC.10}$$

Constraint (EC.3) builds upon Lemma 1. Constraints (EC.4) and (EC.5) are the transmission capacity constraints for selling and buying energy, respectively. Constraint (EC.6) relates the decision w_t^C to the decision s_t^{CG}. Constraint (EC.7) says the wind energy generated is bounded by the available wind potential. Constraints (EC.8) and (EC.9) are the battery capacity constraints for discharging and charging energy, respectively. We now consider the following problem:

$$\max_{(q_t,s_t^{CG},s_t^{CP},s_t^D,w_t^C,w_t^S)\in\Gamma(Q_t,S_t,I_t)}\left\{R(Q_t,I_t,(w_t^S+\gamma s_t^D)\tau-s_t^{CP}/(\theta\tau))+\mathbb{E}\left[v_{t+1}^*(q_t,S_t+s_t^{CG}+s_t^{CP}-s_t^D,I_{t+1})\right]\right\} \tag{EC.11}$$

where

$$R(Q_t,I_t,e)=\begin{cases}Q_tP_t+K_pP_t(e-Q_t) & \text{if } Q_t<e,\\ Q_tP_t-K_nP_t(Q_t-e) & \text{if } Q_t\ge e.\end{cases}$$

We show that the above problem is equivalent to ours by constructing an optimal solution to (EC.11) that satisfies $s_t^D=0$ or $s_t^{CG}+s_t^{CP}=0$. Let $(\hat{q},\hat{s}^{CG},\hat{s}^{CP},\hat{s}^D,\hat{w}^C,\hat{w}^S)\in\Gamma(Q_t,S_t,I_t)$ denote a feasible solution to (EC.11). We consider the following two cases:

(1) Suppose that $\hat{s}^D>0$ and $\hat{s}^{CP}>0$: We define $\Delta_1=\min\{\hat{s}^{CP},\hat{s}^D\}>0$. Note that $(\hat{q},\hat{s}^{CG},\hat{s}^{CP}-\Delta_1,\hat{s}^D-\Delta_1,\hat{w}^C,\hat{w}^S)\in\Gamma(Q_t,S_t,I_t)$ and $(\hat{w}^S+\gamma(\hat{s}^D-\Delta_1))\tau-(\hat{s}^{CP}-\Delta_1)/(\theta\tau)>(\hat{w}^S+\gamma\hat{s}^D)\tau-\hat{s}^{CP}/(\theta\tau)$. Since

$R(Q_t, I_t, \cdot)$ is an increasing function, $(\hat{q}, \hat{s}^{CG}, \hat{s}^{CP} - \Delta_1, \hat{s}^D - \Delta_1, \hat{w}^C, \hat{w}^S)$ yields a larger objective value to (EC.11) than $(\hat{q}, \hat{s}^{CG}, \hat{s}^{CP}, \hat{s}^D, \hat{w}^C, \hat{w}^S)$. Thus, $(\hat{q}, \hat{s}^{CG}, \hat{s}^{CP}, \hat{s}^D, \hat{w}^C, \hat{w}^S)$ cannot be an optimal solution to (EC.11) if $\hat{s}^D > 0$ and $\hat{s}^{CP} > 0$.

(2) Suppose that $\hat{s}^D > 0$, $\hat{s}^{CP} = 0$, and $\hat{s}^{CG} > 0$: We define $\Delta_2 = \min\{\hat{s}^{CG}, \hat{s}^D\} > 0$. Note that $(\hat{q}, \hat{s}^{CG} - \Delta_2, \hat{s}^{CP}, \hat{s}^D - \Delta_2, \hat{w}^C - \Delta_2/\theta, \hat{w}^S + \gamma\Delta_2) \in \Gamma(Q_t, S_t, I_t)$ and $(\hat{w}^S + \gamma\Delta_2 + \gamma(\hat{s}^D - \Delta_2))\tau - \hat{s}^{CP}/(\theta\tau) = (\hat{w}^S + \gamma\hat{s}^D)\tau - \hat{s}^{CP}/(\theta\tau)$. Thus, if $(\hat{q}, \hat{s}^{CG}, \hat{s}^{CP}, \hat{s}^D, \hat{w}^C, \hat{w}^S)$ is an optimal solution to (EC.11), then $(\hat{q}, \hat{s}^{CG} - \Delta_2, \hat{s}^{CP}, \hat{s}^D - \Delta_2, \hat{w}^C - \Delta_2/\theta, \hat{w}^S + \gamma\Delta_2)$ is also optimal with $\hat{s}^{CP} + \hat{s}^{CG} - \Delta_2 = 0$ or $\hat{s}^D - \Delta_2 = 0$ depending on the value of Δ_2.

Thus $v_t^*(Q_t, S_t, I_t)$ equals the optimal objective value of (EC.11). Next, we show that $|v_t^*(Q_t, S_t, I_t)| < \infty$. Since $-C_T \le Q_t \le \tau C_T$ from Lemma 1 and $-C_T \le E(s_t, w_t) \le \tau C_T$, $|R(Q_t, I_t, s_t, w_t)| \le |P_t| C_T$. Hence, $|v_t^*(Q_t, S_t, I_t)| \le \sum_{\kappa=t}^{T-1} |\mathbb{E}_{P_\kappa|I_t}[P_\kappa]| C_T \le \sum_{\kappa=t}^{T-1} \mathbb{E}_{P_\kappa|I_t}[|P_\kappa|] C_T < \infty$ since $\mathbb{E}_{P_\kappa|I_t}[|P_\kappa|] < \infty$, $\forall \kappa \in \mathcal{T}$ with $\kappa \ge t$.

Finally, we define $\mathcal{C} := \{(Q_t, S_t, q_t, s_t^{CG}, s_t^{CP}, s_t^D, w_t^C, w_t^S) \mid (Q_t, S_t) \in \Theta,\ (q_t, s_t^{CG}, s_t^{CP}, s_t^D, w_t^C, w_t^S) \in \Gamma(Q_t, S_t, I_t)\}$ where $\Theta := \{(Q_t, S_t) \mid -\underline{E} \le Q_t \le \bar{E},\ 0 \le S_t \le C_S,\ (S_t - C_S)/(\theta\tau) \le Q_t\}$. Note that $\mathcal{C}$ is a convex set since Θ and $\Gamma(Q_t, S_t, I_t)$ are polyhedral and thus convex sets. The objective function of problem (EC.11) is a concave function on $\mathcal{C}$ since $v_{t+1}^*(\cdot, \cdot, I_{t+1})$ is jointly concave and $R(Q_t, I_t, \cdot)$ is concave. Since $v_t^*(Q_t, S_t, I_t) < \infty$, Theorem A.4 in Porteus (2002) implies that $v_t^*(Q_t, S_t, I_t)$ is a concave function on Θ. □

Proof of Theorem 1. Let $\eta_t^*(Q_t, S_t, I_t) = (q, s, w)$ denote the optimal action triplet in state (Q_t, S_t, I_t). We first characterize the optimal energy storage action. For notational convenience, we suppress the dependency of Z_t^* on (Q_t, S_t, I_t). We consider the following three scenarios:

(i) Suppose that $C_T \le w$. If $s > 0$, then $E(s, w) = (\gamma s + w)\tau > \tau C_T$. But this leads to a contradiction since $E(s, w) \le \tau C_T$. Thus $s \le 0$. Since $s/\theta + w \le C_T$, $S_t + \theta(w - C_T) \le S_t - s$. We then consider the following problem:

$$\max_{(q_t, z_t) \in [-\underline{E}, \bar{E}] \times [S_t + \theta(w - C_T), C_S]} \left\{ R(Q_t, I_t, S_t - z_t, w) + \mathbb{E}\left[v_{t+1}^*(q_t, z_t, I_{t+1}) \right] \right\}.$$

Since $R(Q_t, I_t, S_t - z_t, w) + \mathbb{E}\left[v_{t+1}^*(q_t, z_t, I_{t+1}) \right]$ is jointly concave in (q_t, z_t), note that (q_t^*, z_t^*) is a maximizer of this problem where $z_t^* = \max\{Z_t^*, S_t + \theta(w - C_T)\}$. We consider the following two cases:

- Suppose that $S_t + \theta(w - C_T) < Z_t^*$. Then $z_t^* = Z_t^*$. Since $-\tau C_T \le s/\theta + w$, $s \ge -\theta(\tau C_T + w)$. Hence, taking into account the capacity constraints, we obtain $s = -\min\{Z_t^* - S_t, \theta(\tau C_T + w), C_C\}$.
- Suppose that $Z_t^* \le S_t + \theta(w - C_T)$. Then $z_t^* = S_t + \theta(w - C_T)$. Recall from Lemma 3 that $w = \min\{f(W_t), C_T + \min\{C_S - S_t, C_C\}/\theta\}$. Thus, $w \le C_T + C_C/\theta$, that is, $\theta(w - C_T) \le C_C$. Since $s < 0$, taking into account the capacity constraints, we obtain $s = -\min\{\theta(w - C_T), C_C\} = -\theta(w - C_T)$.

(ii) Suppose that $C_T > w$ and $s < 0$. We consider the following problem:

$$\max_{(q_t, z_t) \in [-\underline{E}, \bar{E}] \times [S_t, C_S]} \left\{ R(Q_t, I_t, S_t - z_t, w) + \mathbb{E}\left[v_{t+1}^*(q_t, z_t, I_{t+1}) \right] \right\}.$$

Since $R(Q_t, I_t, S_t - z_t, w) + \mathbb{E}\left[v_{t+1}^*(q_t, z_t, I_{t+1}) \right]$ is jointly concave in (q_t, z_t), note that (q_t^*, z_t^*) is a maximizer of this problem where $z_t^* = \max\{S_t, Z_t^*\}$. Since $-\tau C_T \le s/\theta + w$, $s \ge -\theta(\tau C_T + w)$. Hence, taking into account the capacity constraints, we obtain $s = -\min\{Z_t^* - S_t, \theta(\tau C_T + w), C_C\}$ if $Z_t^* > S_t$.

(iii) Suppose that $C_T > w$ and $s \geq 0$. We consider the following problem:

$$\max_{(q_t,z_t)\in[-\underline{E},\bar{E}]\times[0,S_t]} \left\{ R(Q_t,I_t,S_t-z_t,w) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]\right\}.$$

Since $R(Q_t,I_t,S_t-z_t,w) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]$ is jointly concave in (q_t,z_t), note that (q^*_t,z^*_t) is a maximizer of this problem where $z^*_t = \min\{Z^*_t, S_t\}$. Since $\gamma s + w \leq C_T$, $s \leq (C_T - w)/\gamma$. Hence, taking into account the capacity constraints, we obtain $s = \min\{S_t - Z^*_t, (C_T-w)/\gamma, C_D\}$ if $Z^*_t \leq S_t$.

We next obtain the optimal energy commitment action from the problem $\max_{q_t\in[-\underline{E},\bar{E}]} \mathbb{E}\left[v^*_{t+1}(q_t, S_t - s, I_{t+1})\right]$. Since $Y_t(S_t-s,I_t)$ is a maximizer of this problem, $q = Y_t(S_t - s, I_t)$. □

Proof of Theorem 2. Let $\eta^*_t(Q_t,S_t,I_t) = (q,s,w)$ denote the optimal action triplet in state (Q_t,S_t,I_t). We make the following observations in four different scenarios:

(a) Suppose that $(Q_t,S_t,W_t)\in\Lambda_1$. Thus $f(W_t)\geq C_T + \min\{C_S - S_t, C_C\}/\theta$. Lemma 3 implies that $w = C_T + \min\{C_S - S_t, C_C\}/\theta$ and $s = -\min\{C_S - S_t, C_C\}$.

(b) Suppose that $(Q_t,S_t,W_t)\in\Lambda_2$. Thus $C_T + \min\{C_S - S_t, C_C\}/\theta > f(W_t) \geq C_T$. Lemma 3 implies that $w = f(W_t)$. Note that $f(W_t)\geq C_T \geq Q_t/\tau$ if $Q_t \geq 0$ from Lemma 1, and $f(W_t) \geq C_T \geq 0 > \tau Q_t$ if $Q_t < 0$. If $s > 0$, then $E(s,w) = (\gamma s + f(W_t))\tau > \tau C_T$. But this leads to a contradiction since $E(s,w)\leq \tau C_T$. Thus $s \leq 0$, leading to decision type CS (charge and sell) or CP (charge and purchase).

(c) Suppose that $(Q_t,S_t,W_t)\in\Lambda_3$. Thus $C_T > f(W_t)$. Lemma 3 implies that $w = f(W_t)$.

(d) Suppose that $(Q_t,S_t,W_t)\in\Psi^+_1\cup\Psi^-_1$. Thus, letting $\underline{s} = -\min\{C_S - S_t, C_C\}$, we have $f(W_t)\geq Q_t/\tau - \underline{s}/\theta$ if $Q_t \geq 0$, and $f(W_t) \geq \tau Q_t - \underline{s}/\theta$ if $Q_t < 0$. Note that $f(W_t) \geq \max\{Q_t/\tau, \tau Q_t\} - \underline{s}/\theta$. If $\underline{s} \leq s \leq 0$, then $s/\theta + f(W_t) \geq s/\theta + \max\{Q_t/\tau,\tau Q_t\} - \underline{s}/\theta \geq \max\{Q_t/\tau,\tau Q_t\}$. If $s > 0 \geq \underline{s}$, then $\gamma s + f(W_t) > \max\{Q_t/\tau,\tau Q_t\} - \underline{s}/\theta \geq \max\{Q_t/\tau,\tau Q_t\}$. Thus $E(s,f(W_t)) = \min\{(\gamma s + f(W_t))\tau, (s/\theta + f(W_t))\tau, (s/\theta + f(W_t))/\tau\} \geq \min\{\max\{Q_t,\tau^2 Q_t\}, \max\{Q_t/\tau^2, Q_t\}\} = Q_t$, leading to positive imbalance.

We now formulate the optimal state-dependent target storage levels:

(i) Suppose that $(Q_t,S_t,W_t)\in\Lambda_1$. Thus $s = -\min\{C_S - S_t, C_C\}$ from scenario (a). Hence $Z^*_t = C_S$.

(ii) Suppose that $(Q_t,S_t,W_t)\in\Lambda_2\cap\Psi^+_1$. Thus, $s\leq 0$ and $w = f(W_t)\geq Q_t/\tau$ from scenario (b), and $E(s,w)\geq Q_t \geq 0$ from scenario (d). We then consider the problem $\max_{(q_t,z_t)\in[-\underline{E},\bar{E}]\times[S_t,C_S]} \left\{R^{(\mathsf{piCS})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]\right\}$. Since $R^{(\mathsf{piCS})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]$ is jointly concave in (q_t,z_t), $Z^*_t = Z^{(\mathsf{piCS})}_t$ if $S_t \leq Z^{(\mathsf{piCS})}_t$ and $Z^*_t = S_t$ otherwise.

(iii) Suppose that $(Q_t,S_t,W_t)\in\Lambda_2\cap\Psi^-_1$. Thus, $s\leq 0$ and $w = f(W_t)$ from scenario (b), and $E(s,w)\geq Q_t$ from scenario (d). We now consider the following two cases:

- Suppose that $s/\theta + w > 0$. Thus $0 \geq s > -\theta f(W_t)$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\underline{E},\bar{E}]\times[S_t,S_t+\theta f(W_t)]} \left\{R^{(\mathsf{piCS})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]\right\}.$$

  Since $R^{(\mathsf{piCS})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]$ is jointly concave in (q_t,z_t), $Z^*_t = S_t + \theta f(W_t)$ if $S_t + \theta f(W_t) \leq Z^{(\mathsf{piCS})}_t$, $Z^*_t = Z^{(\mathsf{piCS})}_t$ if $S_t \leq Z^{(\mathsf{piCS})}_t < S_t + \theta f(W_t)$, and $Z^*_t = S_t$ otherwise.

- Suppose that $0 \geq s/\theta + w$. Thus $-\theta f(W_t) \geq s$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\underline{E},\bar{E}]\times[S_t+\theta f(W_t),C_S]} \left\{ R^{(\mathsf{piCP})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right] \right\}.$$

Since $R^{(\mathsf{piCP})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]$ is jointly concave in (q_t,z_t), $Z^*_t = Z^{(\mathsf{piCP})}_t$ if $S_t + \theta f(W_t) \leq Z^{(\mathsf{piCP})}_t$ and $Z^*_t = S_t + \theta f(W_t)$ otherwise.

We obtain the result by combining all of the above observations.

(iv) Suppose that $(Q_t,S_t,W_t) \in \Lambda_2 \cap \Psi_2^+$. Thus, $f(W_t) \geq Q_t/\tau \geq 0$, and $s \leq 0$ and $w = f(W_t)$ from scenario (b). We now consider the following three cases:

- Suppose that $s/\theta + w \geq Q_t/\tau > 0$. Thus $Q_t \leq E(s,w)$ and $0 \geq s > -\theta(f(W_t) - Q_t/\tau)$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\underline{E},\bar{E}]\times[S_t,S_t+\theta(f(W_t)-Q_t/\tau)]} \left\{ R^{(\mathsf{piCS})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right] \right\}.$$

Since $R^{(\mathsf{piCS})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]$ is jointly concave in (q_t,z_t), $Z^*_t = S_t + \theta(f(W_t) - Q_t/\tau)$ if $S_t + \theta(f(W_t) - Q_t/\tau) \leq Z^{(\mathsf{piCS})}_t$, $Z^*_t = Z^{(\mathsf{piCS})}_t$ if $S_t \leq Z^{(\mathsf{piCS})}_t < S_t + \theta(f(W_t) - Q_t/\tau)$, and $Z^*_t = S_t$ otherwise.

- Suppose that $Q_t/\tau > s/\theta + w > 0$. Thus $Q_t > E(s,w)$ and $-\theta(f(W_t) - Q_t/\tau) \geq s > -\theta f(W_t)$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\underline{E},\bar{E}]\times[S_t+\theta(f(W_t)-Q_t/\tau),S_t+\theta f(W_t)]} \left\{ R^{(\mathsf{niCS})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right] \right\}.$$

Since $R^{(\mathsf{niCS})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]$ is jointly concave in (q_t,z_t), $Z^*_t = S_t + \theta f(W_t)$ if $S_t + \theta f(W_t) \leq Z^{(\mathsf{niCS})}_t$, $Z^*_t = Z^{(\mathsf{niCS})}_t$ if $S_t + \theta(f(W_t) - Q_t/\tau) \leq Z^{(\mathsf{niCS})}_t < S_t + \theta f(W_t)$, and $Z^*_t = S_t + \theta(f(W_t) - Q_t/\tau)$ otherwise.

- Suppose that $0 \geq s/\theta + w$. Thus $Q_t \geq 0 \geq E(s,w)$ and $-\theta f(W_t) \geq s$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\underline{E},\bar{E}]\times[S_t+\theta f(W_t),C_S]} \left\{ R^{(\mathsf{niCP})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right] \right\}.$$

Since $R^{(\mathsf{niCP})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]$ is jointly concave in (q_t,z_t), $Z^*_t = Z^{(\mathsf{niCP})}_t$ if $S_t + \theta f(W_t) \leq Z^{(\mathsf{niCP})}_t$ and $Z^*_t = S_t + \theta f(W_t)$ otherwise.

We obtain the result by combining all of the above observations.

(v) Suppose that $(Q_t,S_t,W_t) \in \Lambda_2 \cap \Psi_2^-$. Thus, $Q_t < 0$, and $s \leq 0$ and $w = f(W_t) \geq 0 > \tau Q_t$ from scenario (b). We now consider the following three cases:

- Suppose that $s/\theta + w > 0$. Thus $E(s,w) > 0 \geq Q_t$ and $0 \geq s > -\theta f(W_t)$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\underline{E},\bar{E}]\times[S_t,S_t+\theta f(W_t)]} \left\{ R^{(\mathsf{piCS})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right] \right\}.$$

Since $R^{(\mathsf{piCS})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]$ is jointly concave in (q_t,z_t), $Z^*_t = S_t + \theta f(W_t)$ if $S_t + \theta f(W_t) \leq Z^{(\mathsf{piCS})}_t$, $Z^*_t = Z^{(\mathsf{piCS})}_t$ if $S_t \leq Z^{(\mathsf{piCS})}_t < S_t + \theta f(W_t)$, and $Z^*_t = S_t$ otherwise.

- Suppose that $0 \geq s/\theta + w \geq \tau Q_t$. Thus $Q_t \leq E(s,w)$ and $-\theta f(W_t) \geq s \geq -\theta(f(W_t) - \tau Q_t)$. We then consider the following problem:
$$\max_{(q_t,z_t)\in[-\underline{E},\bar{E}]\times[S_t+\theta f(W_t),S_t+\theta(f(W_t)-\tau Q_t)]} \left\{R^{(\mathsf{piCP})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]\right\}.$$
Since $R^{(\mathsf{piCP})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]$ is jointly concave in (q_t,z_t), $Z^*_t = S_t + \theta(f(W_t) - \tau Q_t)$ if $S_t + \theta(f(W_t) - \tau Q_t) \leq Z^{(\mathsf{piCP})}_t$, $Z^*_t = Z^{(\mathsf{piCP})}_t$ if $S_t + \theta f(W_t) \leq Z^{(\mathsf{piCP})}_t < S_t + \theta(f(W_t) - \tau Q_t)$, and $Z^*_t = S_t + \theta f(W_t)$ otherwise.
- Suppose that $0 \geq \tau Q_t > s/\theta + w$. Thus $Q_t > E(s,w)$ and $-\theta(f(W_t) - \tau Q_t) > s$. We then consider the following problem:
$$\max_{(q_t,z_t)\in[-\underline{E},\bar{E}]\times[S_t+\theta(f(W_t)-\tau Q_t),C_S]} \left\{R^{(\mathsf{niCP})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]\right\}.$$
Since $R^{(\mathsf{niCP})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]$ is jointly concave in (q_t,z_t), $Z^*_t = Z^{(\mathsf{niCP})}_t$ if $S_t + \theta(f(W_t) - \tau Q_t) \leq Z^{(\mathsf{niCP})}_t$ and $Z^*_t = S_t + \theta(f(W_t) - \tau Q_t)$ otherwise.

We obtain the result by combining all of the above observations.

(vi) Suppose that $(Q_t, S_t, W_t) \in \Lambda_3 \cap \Psi^+_1$. Thus, $w = f(W_t)$ and $E(s,w) \geq Q_t \geq 0$ from scenarios (c)–(d).

- Suppose that $s \geq 0$. We then consider the following problem:
$$\max_{(q_t,z_t)\in[-\underline{E},\bar{E}]\times[0,S_t]} \left\{R^{(\mathsf{piDS})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]\right\}.$$
Since $R^{(\mathsf{piDS})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]$ is jointly concave in (q_t,z_t), $Z^*_t = S_t$ if $S_t \leq Z^{(\mathsf{piDS})}_t$ and $Z^*_t = Z^{(\mathsf{piDS})}_t$ otherwise.
- Suppose that $s < 0$. We then consider the following problem:
$$\max_{(q_t,z_t)\in[-\underline{E},\bar{E}]\times[S_t,C_S]} \left\{R^{(\mathsf{piCS})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]\right\}.$$
Since $R^{(\mathsf{piCS})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]$ is jointly concave in (q_t,z_t), $Z^*_t = Z^{(\mathsf{piCS})}_t$ if $S_t \leq Z^{(\mathsf{piCS})}_t$ and $Z^*_t = S_t$ otherwise.

We obtain the result by combining all of the above observations.

(vii) Suppose that $(Q_t, S_t, W_t) \in \Lambda_3 \cap \Psi^-_1$. Thus, $Q_t < 0$, $w = f(W_t)$, and $E(s,w) \geq Q_t$ from scenarios (c)–(d).

- Suppose that $s \geq 0$. We then consider the following problem:
$$\max_{(q_t,z_t)\in[-\underline{E},\bar{E}]\times[0,S_t]} \left\{R^{(\mathsf{piDS})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]\right\}.$$
Since $R^{(\mathsf{piDS})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]$ is jointly concave in (q_t,z_t), $Z^*_t = S_t$ if $S_t \leq Z^{(\mathsf{piDS})}_t$ and $Z^*_t = Z^{(\mathsf{piDS})}_t$ otherwise.
- Suppose that $s < 0$ and $s/\theta + w > 0$. Thus $0 > s \geq -\theta f(W_t)$. We then consider the following problem:
$$\max_{(q_t,z_t)\in[-\underline{E},\bar{E}]\times[S_t,S_t+\theta f(W_t)]} \left\{R^{(\mathsf{piCS})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]\right\}.$$
Since $R^{(\mathsf{piCS})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]$ is jointly concave in (q_t,z_t), $Z^*_t = S_t + \theta f(W_t)$ if $S_t + \theta f(W_t) \leq Z^{(\mathsf{piCS})}_t$, $Z^*_t = Z^{(\mathsf{piCS})}_t$ if $S_t \leq Z^{(\mathsf{piCS})}_t < S_t + \theta f(W_t)$, and $Z^*_t = S_t$ otherwise.

- Suppose that $s<0$ and $0\geq s/\theta+w$. Thus $-\theta f(W_t)\geq s$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\underline{E},\bar{E}]\times[S_t+\theta f(W_t),C_S]}\left\{R^{(\mathsf{piCP})}(z_t,I_t)+\mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]\right\}.$$

  Since $R^{(\mathsf{piCP})}(z_t,I_t)+\mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]$ is jointly concave in (q_t,z_t), $Z^*_t=Z^{(\mathsf{piCP})}_t$ if $S_t+\theta f(W_t)\leq Z^{(\mathsf{piCP})}_t$ and $Z^*_t=S_t+\theta f(W_t)$ otherwise.

We obtain the result by combining all of the above observations.

(viii) Suppose that $(Q_t,S_t,W_t)\in\Lambda_3\cap\Psi_2^+$. Thus, $f(W_t)\geq Q_t/\tau\geq 0$, and $w=f(W_t)$ from scenario (c).

- Suppose that $s\geq 0$. Thus $Q_t\leq E(s,w)$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\underline{E},\bar{E}]\times[0,S_t]}\left\{R^{(\mathsf{piDS})}(z_t,I_t)+\mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]\right\}.$$

  Since $R^{(\mathsf{piDS})}(z_t,I_t)+\mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]$ is jointly concave in (q_t,z_t), $Z^*_t=S_t$ if $S_t\leq Z^{(\mathsf{piDS})}_t$ and $Z^*_t=Z^{(\mathsf{piDS})}_t$ otherwise.

- Suppose that $s<0$ and $s/\theta+w\geq Q_t/\tau\geq 0$. Thus $Q_t\leq E(s,w)$ and $0>s\geq-\theta(f(W_t)-Q_t/\tau)$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\underline{E},\bar{E}]\times[S_t,S_t+\theta(f(W_t)-Q_t/\tau)]}\left\{R^{(\mathsf{piCS})}(z_t,I_t)+\mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]\right\}.$$

  Since $R^{(\mathsf{piCS})}(z_t,I_t)+\mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]$ is jointly concave in (q_t,z_t), $Z^*_t=S_t+\theta(f(W_t)-Q_t/\tau)$ if $S_t+\theta(f(W_t)-Q_t/\tau)\leq Z^{(\mathsf{piCS})}_t$, $Z^*_t=Z^{(\mathsf{piCS})}_t$ if $S_t\leq Z^{(\mathsf{piCS})}_t<S_t+\theta(f(W_t)-Q_t/\tau)$, and $Z^*_t=S_t$ otherwise.

- Suppose that $s<0$ and $Q_t/\tau>s/\theta+w\geq 0$. Thus $Q_t>E(s,w)$ and $-\theta(f(W_t)-Q_t/\tau)>s>-\theta f(W_t)$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\underline{E},\bar{E}]\times[S_t+\theta(f(W_t)-Q_t/\tau),S_t+\theta f(W_t)]}\left\{R^{(\mathsf{niCS})}(z_t,I_t)+\mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]\right\}.$$

  Since $R^{(\mathsf{niCS})}(z_t,I_t)+\mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]$ is jointly concave in (q_t,z_t), $Z^*_t=S_t+\theta f(W_t)$ if $S_t+\theta f(W_t)\leq Z^{(\mathsf{niCS})}_t$, $Z^*_t=Z^{(\mathsf{niCS})}_t$ if $S_t+\theta(f(W_t)-Q_t/\tau)\leq Z^{(\mathsf{niCS})}_t<S_t+\theta f(W_t)$, and $Z^*_t=S_t+\theta(f(W_t)-Q_t/\tau)$ otherwise.

- Suppose that $s<0$ and $0>s/\theta+w$. Thus $Q_t\geq 0>E(s,w)$ and $-\theta f(W_t)\geq s$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\underline{E},\bar{E}]\times[S_t+\theta f(W_t),C_S]}\left\{R^{(\mathsf{niCP})}(z_t,I_t)+\mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]\right\}.$$

  Since $R^{(\mathsf{niCP})}(z_t,I_t)+\mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]$ is jointly concave in (q_t,z_t), $Z^*_t=Z^{(\mathsf{niCP})}_t$ if $S_t+\theta f(W_t)\leq Z^{(\mathsf{niCP})}_t$ and $Z^*_t=S_t+\theta f(W_t)$ otherwise.

We obtain the result by combining all of the above observations.

(ix) Suppose that $(Q_t,S_t,W_t)\in\Lambda_3\cap\Psi_2^-$. Thus, $f(W_t)\geq 0>\tau Q_t$, and $w=f(W_t)$ from scenario (c).

- Suppose that $s \geq 0$. Thus $Q_t < 0 \leq E(s,w)$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\underline{E},\bar{E}]\times[0,S_t]} \left\{ R^{(\mathsf{piDS})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]\right\}.$$

Since $R^{(\mathsf{piDS})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]$ is jointly concave in (q_t,z_t), $Z^*_t = S_t$ if $S_t \leq Z^{(\mathsf{piDS})}_t$ and $Z^*_t = Z^{(\mathsf{piDS})}_t$ otherwise.

- Suppose that $s < 0$ and $s/\theta + w \geq 0$. Thus $Q_t < 0 \leq E(s,w)$ and $0 \geq s > -\theta f(W_t)$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\underline{E},\bar{E}]\times[S_t,S_t+\theta f(W_t)]} \left\{ R^{(\mathsf{piCS})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]\right\}.$$

Since $R^{(\mathsf{piCS})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]$ is jointly concave in (q_t,z_t), $Z^*_t = S_t + \theta f(W_t)$ if $S_t + \theta f(W_t) \leq Z^{(\mathsf{piCS})}_t$, $Z^*_t = Z^{(\mathsf{piCS})}_t$ if $S_t \leq Z^{(\mathsf{piCS})}_t < S_t + \theta f(W_t)$, and $Z^*_t = S_t$ otherwise.

- Suppose that $s < 0$ and $0 > s/\theta + w \geq \tau Q_t$. Thus $Q_t \leq E(s,w)$ and $-\theta f(W_t) \geq s \geq -\theta(f(W_t) - \tau Q_t)$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\underline{E},\bar{E}]\times[S_t+\theta f(W_t),S_t+\theta(f(W_t)-\tau Q_t)]} \left\{ R^{(\mathsf{piCP})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]\right\}.$$

Since $R^{(\mathsf{piCP})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]$ is jointly concave in (q_t,z_t), $Z^*_t = S_t + \theta(f(W_t) - \tau Q_t)$ if $S_t + \theta(f(W_t) - \tau Q_t) \leq Z^{(\mathsf{piCP})}_t$, $Z^*_t = Z^{(\mathsf{piCP})}_t$ if $S_t + \theta f(W_t) \leq Z^{(\mathsf{piCP})}_t < S_t + \theta(f(W_t) - \tau Q_t)$, and $Z^*_t = S_t + \theta f(W_t)$ otherwise.

- Suppose that $s < 0$ and $0 > \tau Q_t > s/\theta + w$. Thus $Q_t > E(s,w)$ and $-\theta(f(W_t) - \tau Q_t) > s$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\underline{E},\bar{E}]\times[S_t+\theta(f(W_t)-\tau Q_t),C_S]} \left\{ R^{(\mathsf{niCP})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]\right\}.$$

Since $R^{(\mathsf{niCP})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]$ is jointly concave in (q_t,z_t), $Z^*_t = Z^{(\mathsf{niCP})}_t$ if $S_t + \theta(f(W_t) - \tau Q_t) \leq Z^{(\mathsf{niCP})}_t$ and $Z^*_t = S_t + \theta(f(W_t) - \tau Q_t)$ otherwise.

We obtain the result by combining all of the above observations.

(x) Suppose that $(Q_t, S_t, W_t) \in \Lambda_3 \cap \Psi^+_3$. Thus, $Q_t > \tau f(W_t) \geq 0$, and $w = f(W_t)$ from scenario (c).

- Suppose that $s \geq 0$ and $\gamma s + w \geq Q_t/\tau$. Thus $Q_t \leq E(s,w)$ and $s \geq (Q_t/\tau - f(W_t))/\gamma$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\underline{E},\bar{E}]\times[0,S_t-(Q_t/\tau-f(W_t))/\gamma]} \left\{ R^{(\mathsf{piDS})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]\right\}.$$

Since $R^{(\mathsf{piDS})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]$ is jointly concave in (q_t,z_t), $Z^*_t = S_t - (Q_t/\tau - f(W_t))/\gamma$ if $S_t - (Q_t/\tau - f(W_t))/\gamma \leq Z^{(\mathsf{piDS})}_t$ and $Z^*_t = Z^{(\mathsf{piDS})}_t$ otherwise.

- Suppose that $s \geq 0$ and $\gamma s + w < Q_t/\tau$. Thus $Q_t > E(s,w)$ and $(Q_t/\tau - f(W_t))/\gamma > s \geq 0$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\underline{E},\bar{E}]\times[S_t-(Q_t/\tau-f(W_t))/\gamma,S_t]} \left\{ R^{(\mathsf{niDS})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]\right\}.$$

Since $R^{(\mathsf{niDS})}(z_t,I_t) + \mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]$ is jointly concave in (q_t,z_t), $Z^*_t = S_t$ if $S_t \leq Z^{(\mathsf{niDS})}_t$, $Z^*_t = Z^{(\mathsf{niDS})}_t$ if $S_t - (Q_t/\tau - f(W_t))/\gamma \leq Z^{(\mathsf{niDS})}_t < S_t$, and $Z^*_t = S_t - (Q_t/\tau - f(W_t))/\gamma$ otherwise.

- Suppose that $s<0$ and $s/\theta+w\geq 0$. Thus, $Q_t>(s/\theta+f(W_t))\tau=E(s,w)$ since $f(W_t)<Q_t/\tau$, and $0>s\geq -\theta f(W_t)$. We then consider the following problem:
$$\max_{(q_t,z_t)\in[-\underline{E},\bar{E}]\times[S_t,S_t+\theta f(W_t)]}\left\{R^{(\mathsf{niCS})}(z_t,I_t)+\mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]\right\}.$$
Since $R^{(\mathsf{niCS})}(z_t,I_t)+\mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]$ is jointly concave in (q_t,z_t), $Z^*_t=S_t+\theta f(W_t)$ if $S_t+\theta f(W_t)\leq Z_t^{(\mathsf{niCS})}$, $Z^*_t=Z_t^{(\mathsf{niCS})}$ if $S_t\leq Z_t^{(\mathsf{niCS})}<S_t+\theta f(W_t)$, and $Z^*_t=S_t$ otherwise.
- Suppose that $s<0$ and $0>s/\theta+w$. Thus $Q_t\geq 0>E(s,w)$ and $-\theta f(W_t)>s$. We then consider the following problem:
$$\max_{(q_t,z_t)\in[-\underline{E},\bar{E}]\times[S_t+\theta f(W_t),C_S]}\left\{R^{(\mathsf{niCP})}(z_t,I_t)+\mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]\right\}.$$
Since $R^{(\mathsf{niCP})}(z_t,I_t)+\mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]$ is jointly concave in (q_t,z_t), $Z^*_t=Z_t^{(\mathsf{niCP})}$ if $S_t+\theta f(W_t)\leq Z_t^{(\mathsf{niCP})}$ and $Z^*_t=S_t+\theta f(W_t)$ otherwise.

We obtain the result by combining all of the above observations. □

Proof of Proposition 2. Let $u^*_t(q_t,z_t)=\mathbb{E}\left[v^*_{t+1}(q_t,z_t,I_{t+1})\right]$.

(a) We show that $Z_t^{(\mathsf{niCP})}\leq Z_t^{(\mathsf{piCP})}$: By definition of $Z_t^{(\nu)}$ and $Y_t^{(\nu)}$, the following inequalities hold.
$$u^*_t\left(Y_t^{(\mathsf{niCP})},Z_t^{(\mathsf{niCP})}\right)-K_nP_tZ_t^{(\mathsf{niCP})}/(\theta\tau)\geq u^*_t\left(Y_t^{(\mathsf{piCP})},Z_t^{(\mathsf{piCP})}\right)-K_nP_tZ_t^{(\mathsf{piCP})}/(\theta\tau),$$
$$u^*_t\left(Y_t^{(\mathsf{piCP})},Z_t^{(\mathsf{piCP})}\right)-K_pP_tZ_t^{(\mathsf{piCP})}/(\theta\tau)\geq u^*_t\left(Y_t^{(\mathsf{niCP})},Z_t^{(\mathsf{niCP})}\right)-K_pP_tZ_t^{(\mathsf{niCP})}/(\theta\tau).$$
The summation of the above inequalities implies $(K_p-K_n)P_tZ_t^{(\mathsf{niCP})}/(\theta\tau)\geq(K_p-K_n)P_tZ_t^{(\mathsf{piCP})}/(\theta\tau)$. Since $\theta>0$, $\tau>0$, and $K_n>K_p$, $Z_t^{(\mathsf{niCP})}\leq Z_t^{(\mathsf{piCP})}$. Similarly, we can show that $Z_t^{(\mathsf{niCS})}\leq Z_t^{(\mathsf{piCS})}$ and $Z_t^{(\mathsf{niDS})}\leq Z_t^{(\mathsf{piDS})}$.

(b) We show that $Z_t^{(\mathsf{niCP})}\leq Z_t^{(\mathsf{niCS})}\leq Z_t^{(\mathsf{niDS})}$: By definition of $Z_t^{(\nu)}$ and $Y_t^{(\nu)}$, the following inequalities hold.
$$u^*_t\left(Y_t^{(\mathsf{niCP})},Z_t^{(\mathsf{niCP})}\right)-K_nP_tZ_t^{(\mathsf{niCP})}/(\theta\tau)\geq u^*_t\left(Y_t^{(\mathsf{niCS})},Z_t^{(\mathsf{niCS})}\right)-K_nP_tZ_t^{(\mathsf{niCS})}/(\theta\tau),$$
$$u^*_t\left(Y_t^{(\mathsf{niCS})},Z_t^{(\mathsf{niCS})}\right)-\tau K_nP_tZ_t^{(\mathsf{niCS})}/\theta\geq u^*_t\left(Y_t^{(\mathsf{niCP})},Z_t^{(\mathsf{niCP})}\right)-\tau K_nP_tZ_t^{(\mathsf{niCP})}/\theta.$$
The summation of the above inequalities implies $K_nP_t(\tau-1/\tau)Z_t^{(\mathsf{niCP})}/\theta\geq K_nP_t(\tau-1/\tau)Z_t^{(\mathsf{niCS})}/\theta$. Since $\theta>0$, $1\geq\tau>0$, and $K_n>0$, $Z_t^{(\mathsf{niCP})}\leq Z_t^{(\mathsf{niCS})}$. By definition of $Z_t^{(\nu)}$ and $Y_t^{(\nu)}$, the following inequalities hold.
$$u^*_t\left(Y_t^{(\mathsf{niCS})},Z_t^{(\mathsf{niCS})}\right)-\tau K_nP_tZ_t^{(\mathsf{niCS})}/\theta\geq u^*_t\left(Y_t^{(\mathsf{niDS})},Z_t^{(\mathsf{niDS})}\right)-\tau K_nP_tZ_t^{(\mathsf{niDS})}/\theta,$$
$$u^*_t\left(Y_t^{(\mathsf{niDS})},Z_t^{(\mathsf{niDS})}\right)-\gamma\tau K_nP_tZ_t^{(\mathsf{niDS})}\geq u^*_t\left(Y_t^{(\mathsf{niCS})},Z_t^{(\mathsf{niCS})}\right)-\gamma\tau K_nP_tZ_t^{(\mathsf{niCS})}.$$
The summation of the above inequalities implies $\tau K_nP_t(\gamma-1/\theta)Z_t^{(\mathsf{niCS})}\geq\tau K_nP_t(\gamma-1/\theta)Z_t^{(\mathsf{niDS})}$. Since $\tau>0$, $\gamma\leq 1$, $1\geq\theta>0$, and $K_n>0$, $Z_t^{(\mathsf{niCS})}\leq Z_t^{(\mathsf{niDS})}$. Similarly, we can show that $Z_t^{(\mathsf{piCP})}\leq Z_t^{(\mathsf{piCS})}\leq Z_t^{(\mathsf{piDS})}$. □

The proof of Corollary 1 is available upon request from the authors.

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