Directed Graphs (Digraphs) Definitions and Examples

MATH 8020

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1 Definitions

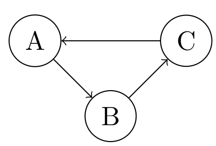
1.1 Directed Graph (Digraph)

A **directed graph** (or **digraph**) G is defined as a pair $G = (V, E)$, where:

- V is a set of vertices (or nodes).
- E is a set of directed edges (or arcs), which are ordered pairs of vertices.

1.2 Example of a Digraph

Consider the following digraph:



1.3 Vertices

The elements of the set V are called **vertices**.

1.4 Arcs

The elements of the set E are called **arcs**. An arc connects two vertices and has a direction.

1.5 Neighborhood

The **neighborhood** of a vertex v, denoted $N(v)$, is the set of all vertices that can be reached by a directed edge from v:

$$N(v) = \{u \in V \mid (v, u) \in E\}$$

1.6 Example of Neighborhood

For the digraph above: - $N(A) = \{B\}$ - $N(B) = \{C\}$ - $N(C) = \{A\}$

1.7 In-Degree

The **in-degree** of a vertex v, denoted $\deg^-(v)$, is the number of arcs directed into v.

1.8 Example of In-Degree

In the digraph: - $\deg^-(A) = 1$ - $\deg^-(B) = 1$ - $\deg^-(C) = 1$

1.9 Out-Degree

The **out-degree** of a vertex v, denoted $\deg^+(v)$, is the number of arcs directed out of v.

1.10 Example of Out-Degree

In the digraph: - $\deg^+(A) = 1$ - $\deg^+(B) = 1$ - $\deg^+(C) = 1$

2 Handshaking Lemma for Digraphs

The **handshaking lemma** for digraphs states that the sum of the in-degrees of all vertices is equal to the sum of the out-degrees:

$$\sum_{v \in V} \deg^-(v) = \sum_{v \in V} \deg^+(v) = |E|$$

2.1 Example of Handshaking Lemma

For the digraph with edges $E = \{(A, B), (B, C), (C, A)\}$: - The in-degrees are $\deg^-(A) = 1$, $\deg^-(B) = 1$, $\deg^-(C) = 1$. - The out-degrees are $\deg^+(A) = 1$, $\deg^+(B) = 1$, $\deg^+(C) = 1$. - Thus, $1 + 1 + 1 = 3$.

3 Conclusion

Understanding directed graphs and their properties is essential for various applications in computer science, including network analysis, scheduling, and more.