Graph Families: Definitions and Examples

MATH 8020

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1 Complete Graph

A **complete graph** K_n is defined as a graph in which every pair of distinct vertices is connected by a unique edge.

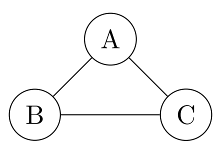
1.1 Set Definition

Let $V = \{v_1, v_2, \ldots, v_n\}$ be the set of vertices. The edge set is defined as:

$$E(K_n) = \{\{v_i, v_j\} \mid 1 \leq i < j \leq n\}$$

1.2 Example of a Complete Graph

For K_3 (a complete graph with 3 vertices):



2 Null Graph

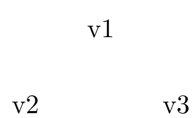
A **null graph** on n vertices is defined as a graph that has n vertices and no edges.

2.1 Set Definition

Let $V = \{v_1, v_2, \ldots, v_n\}$ and $E = \emptyset$. The null graph is denoted as N_n.

2.2 Example of a Null Graph

For N_3 (a null graph with 3 vertices):



3 Path Graph

A **path graph** P_n is defined as a graph that consists of n vertices arranged in a linear sequence.

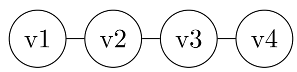
3.1 Set Definition

Let $V = \{v_1, v_2, \ldots, v_n\}$. The edge set is defined as:

$$E(P_n) = \{\{v_i, v_{i+1}\} \mid 1 \leq i < n\}$$

3.2 Example of a Path Graph

For P_4 (a path graph with 4 vertices):



4 Cycle Graph

A **cycle graph** C_n is defined as a graph that consists of a single cycle, meaning it is a closed loop with n vertices. You may build the cycle graph from the path graph by creating an edge between the two vertices of degree 1.

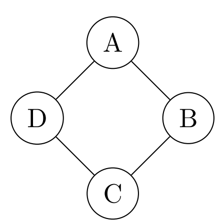
4.1 Set Definition

Let $V = \{v_1, v_2, \ldots, v_n\}$. The edge set is defined as:

$$E(C_n) = \{\{v_i, v_{i+1}\} \mid 1 \leq i < n\} \cup \{\{v_n, v_1\}\}$$

4.2 Example of a Cycle Graph

For C_4 (a cycle graph with 4 vertices):



5 Wheel Graph

A **wheel graph** W_n is defined as a graph formed by connecting a single central vertex to all vertices of a cycle graph C_n.

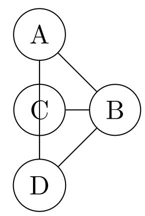
5.1 Set Definition

Let $V = \{v_0, v_1, v_2, \ldots, v_n\}$ where v_0 is the center. The edge set is defined as:

$$E(W_n) = \{\{v_0, v_i\} \mid 1 \leq i < n\} \cup E(C_n)$$

5.2 Example of a Wheel Graph

For W_3 (a wheel graph with 4 vertices):



6 Conclusion

Understanding different types of graphs is fundamental in graph theory and has numerous applications in computer science, network analysis, and more.