1 Pigeonhole Principle

The basic form of the pigeonhole principle is that if $k+1$ pigeons are placed into k pigeonholes then at least one pigeonhole will contain at least two pigeons.

Clearly, if three pigeons are placed into two pigeonholes then one hole will contain at least two pigeons. It could be the case that all three pigeons are placed into one hole so it is not appropriate to say that one pigeonhole will contain exactly two pigeons. If five pigeons were placed into four pigeonholes then at least one hole will contain at least two pigeons. It certainly could be the case that three pigeonholes contain exactly one pigeon each and the fourth contains two pigeons. However, it could also be the case that two pigeonholes contain two pigeons each, a third contains one pigeon and the remaining two pigeonholes are empty. Or once again, all five pigeons could be placed into one pigeonhole, leaving the other four pigeonholes empty.

An extended form of the pigeonhole principle is that if n pigeons are placed into k pigeonholes, then at least one pigeonhole will contain at least $\left\lceil \frac{n}{k} \right\rceil$ pigeons.

The ceiling function, $\left\lceil \frac{n}{k} \right\rceil = j$, where j is the smallest integer greater than or equal to $\frac{n}{k}$. Similarly, the floor function, $\left\lfloor \frac{n}{k} \right\rfloor = j$, where j is the largest integer less than or equal to $\frac{n}{k}$.

If 10 pigeons are placed into 3 pigeonholes then at least one pigeonhole will contain at least $\left\lceil \frac{10}{3} \right\rceil = 4$ pigeons. The same result will be true if 11 or 12 pigeons are placed into 3 pigeonholes. However, once 13 pigeons are placed into 3 pigeonholes then at least one pigeonhole will contain at least $\left\lceil \frac{13}{3} \right\rceil = 5$ pigeons.

For example, if Michael Jordan scored 57 points in four regulation quarters of a basketball game then there was at least one quarter where he scored at least $\left\lceil \frac{57}{4} \right\rceil = 15$ points.

In a set of 6 people, show that there exists a subset of at least 3 people who either all know every other person in the subset or there exists a subset of at least 3 people all of whom are strangers to one another. You may assume that if person A knows person B, then person B also knows person A. Show that this result is not true for a set of 5 people.

Exercise 1 *Does a class of 12 students have at least 2 students who share the same birth month?*

Exercise 2 *Does a class of 13 students have at least 2 students who share the same birth month?*

Exercise 3 *Does a class of 25 students have at least 2 students who share the same birth month?*

Exercise 4 *Does a class of 25 students have at least 3 students who share the same birth month?*

Exercise 5 *Does a class of 25 students have at least 4 students who share the same birth month?*

Exercise 6 *At a university of 13,000 students, at least how many must share the same birthday (not including the year)? Don't forget leap years. How many must have the birthday September 19th?*

Exercise 7 *At a university of 20,347 students, at least how many must share the same 4-digit pin number for their ATM card (assuming that each student has an ATM card)? Do we know that at least that many people have the pin 1134?*

Exercise 8 *John has 13 ordinary coins (pennies, nickels, dimes and quarters) in his pocket. At least how many of the same coin must John have? At least how many pennies must John have?*

Exercise 9 *Ariel wishes to encode every book in her personal library with a code consisting of an uppercase letter followed by two digits. If Ariel has 1000 books in her library can each book receive a unique code?*

Exercise 10 *Every day Don lunches at a restaurant that offers a lunch combo with a choice of beverage, entrée and side. If six drinks, twelve entrées and eight sides are available, on what day will Don first be forced to repeat a combo order?*

Exercise 11 *(Ducks text) In 2016, there were 3, 945, 875 live births in the US. (Source: http://www.w3.org/1999/xlink.) Did there have to be two of these births within the same second?*

Exercise 12 *(Ducks text) A cold-footed centipede has a drawer filled with many, many socks. And yes, that centipede does have 100 feet. If the centipede only owns green and brown socks, how many must it pull from the drawer in the dark of the morning to be assured that it has a matching set for all of its feet (100 socks of the same color)? What if the centipede also owns polka-dotted socks? What if the centipede's drawer has many, many socks of k different colors?*

Exercise 13 *(with apologies to Ducks) In Georgia there are a lot of Waffle Houses. It is not particularly unusual to find five Waffle Houses within one square mile. Must there be two of them within* $\frac{3}{4}$ *of a mile of each other? What about within* $\frac{1}{2}$ *mile of each other?*