1   Poker

Poker is an ideal setting to study probabilities. Computing the probabilities of different hands will require a variety of approaches. Our focus, however, is not on playing or betting strategies. Our perspective will be solely a combinatorial one. A standard deck of playing cards with no jokers will be used. Each standard deck contains four suits: clubs , diamonds , hearts and spades . Each deck also contains thirteen different ordered ranks: 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King, Ace. The Ace can be considered a 1 if so desired in the ordering of the ranks. There is exactly one card of each rank and suit for a total of $13 * 4 = 52$ cards. There exist a variety of different hands in poker. In this section, only five card hands will be considered. Variations on the game of poker will be left for the homework problems. The margin note describes the types of poker hands and ranks them in order, i.e., a hand higher up in the table beats all the hands below it in the table.

Royal Flush
Straight Flush
Four of a Kind
Full House
Flush
Straight
Three of a Kind
Two Pair
One Pair
Other

Using a single deck of cards and discarding the jokers, there are $\binom{52}{5} = 2598\,960$ [1] different five card hands possible. The next step is to count the number of different ways each hand can possibly occur. The chart below indicates the winning order of the various hands from best to worst.

How many different poker hands of four of a kind exist? A four of a kind consists of one rank represented by all four suits and one other card. The first task is to select the rank in which to get all four suits. The second task is to select all four suits of that rank. The third and final task is to pick the fifth card. This can be done in exactly $13\binom{4}{4}48 = 624$ ways and the probability of drawing this hand is (see Table 1.5.2)

Next, consider the three different flush hands. The royal flush is a 10, Jack, Queen, King, Ace of the same suit. It is plain to see that there are only four such hands. A straight flush is a straight of the same suit that is not a royal flush. First, count the number of straights. There exist ten different straights.

A, 2, 3, 4, 5

After selecting one of the ten straights, a suit must then be selected. Hence, there exist $4 * 10 = 40$ straights of the same suit. However, four of those would be considered a royal flush. No sane poker player would hold a royal flush and call it a straight flush. The four hands that are a royal flush must now be subtracted from the total and there are 36 different straight flush hands in poker. This concept will be needed again. It is crucial that when counting a particular hand that a better hand is not included while enumerating the lesser hand.

2, 3, 4, 5, 6
3, 4, 5, 6, 7
4, 5, 6, 7, 8
5, 6, 7, 8, 9
6, 7, 8, 9, 10
7, 8, 9, 10, J
8, 9, 10, J, Q
9, 10, J, Q, K

A flush consists of five cards of the same suit. There are thirteen cards of each suit and $\binom{13}{5} = 1287$ ways to pick five cards of any one suit. However, previous work has shown that nine of those hands are actually a straight flush and one of them is a royal flush. Hence, there are 1277 hands that are a flush for any given suit. Next, multiply by four to select the suit for the flush and there are 5,108 different flush hands. Three more hands have their information entered in Table

10, J, Q, K, A

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[1] "More hands than there are gopher holes" according to Bret Maverick.

Hand	Ways	Probability
Royal Flush	4	$\frac{4}{\binom{52}{5}} = 1.5391 \times 10^{-6}$
Straight Flush	36	$\frac{36}{\binom{52}{5}} = 1.3852 \times 10^{-5}$
Four of a Kind	624	$\frac{624}{\binom{52}{5}} = 2.4010 \times 10^{-4}$
Full House		
Flush	5,108	$\frac{5108}{\binom{52}{5}} = 1.9654 \times 10^{-3}$
Straight		
Three of a Kind		
Two Pair		
One Pair		
Other		

The remaining information will be computed in the homework section. When counting the number of different five card hands of a particular type, it is very important that you not improve on the type of hand that you are enumerating.

Problem 1 *Complete the table of probabilities for the remaining poker hands.*

Problem 2 *Does the ordering of the hands in poker make sense according to the probabilities associated with each hand? Explain.*

Problem 3 *Shot in the back while playing poker, Wild Bill Hickok's final hand was a pair of aces and a pair of eights, now known as the dead man's hand. How many different dead man's hands exist?*

Problem 4 *What is the probability that a five card hand will contain no face cards?*

Problem 5 *What is the probability that a five card hand will contain at least one of each suit?*

Problem 6 *How many five card hands contain at least one of each of the three face cards (King, Queen, Jack)?*

Problem 7 *If a deck of cards contains the two jokers (one red, one black) that can be any desired card, what is the probability of four of a kind?*

Problem 8 *If a deck of cards contains the two jokers that can be any desired card, what is the probability of a full house?*

Problem 9 *If a deck of cards contains the two jokers that can be any desired card, which hand should win: a five of a kind or a royal flush? Explain your reasoning.*

Problem 10 *A player holds the five cards 2, 3, 6, 10 and Ace in his hand. If he discards the 10, (without returning it to the deck) and draws one more card, what is the probability that the result will be a flush?*

Problem 11 *A player holds the five cards 5, 5, King, King and Ace in her hand. If she discards the Ace (without replacing it into the deck) and draws an additional card, what is the probability that the result will be a full house?*

Problem 12 *A player holds the five cards 2, 5, King, King and Ace in her hand. If she discards the 2, 5 and Ace (without replacing them into the deck) and draws three more cards, what is the probability that the result will be   i. a three of a kind;   ii. a three of a kind or better?*

Problem 13 *In seven card stud, a player is dealt seven cards and then picks the best five card hand from those seven. Compute the probabilities for each of the following hands in seven card stud.*
i. royal flush;
ii. straight flush;
iii. four of a kind.