Complements and the Addition Rule

1 Finite Cardinality

Let A be an event or set. The number of outcomes in or size of A is denoted $|A|$.

Example 1 *Let* $A = \{q, w, e, r, t, y\}$. $|A| = 6$.

2 Complements

Let A be any event. Recall that the complement of A, $\bar{A}$ (also A^c or $A^{'}$) is the event that A does not occur.

Exercise 1 *For each of the following events, describe its complement.*

1. *When flipping a coin once, a head is observed.*

2. *When flipping a coin twice, exactly one head is observed.*

3. *When flipping a coin twice, at least one head is observed.*

4. *When flipping a coin ten times, at least one head is observed.*

Exercise 2 *For each of the following determine the number of ways the event can occur and repeat for its complement.*

1. *When flipping a coin once, a head is observed.*

2. *When flipping a coin twice, exactly one head is observed.*

3. *When flipping a coin twice, at least one head is observed.*

4. *When flipping a coin ten times, at least one head is observed.*

Exercise 3 *A pair of fair dice is rolled. How many ways can the sum of the dice be less than or equal to 10?*

3 The Addition Rule

Theorem 1 *If A and B are disjoint (mutually exclusive) sets or events then $|A \text{ or } B| = |A| + |B|$.*

Example 2 *Pick a single card from a deck. How many ways can you select an Ace or an 8?*
These are disjoint events. There are four of each rank in a deck of cards. Thus, the number of ways to select an Ace or an 8 is $4 + 4 = 8$

Exercise 4 *Pick a single card from a deck. How many ways can you select a club or a diamond?*

Exercise 5 *A study at a local bar found people of various ages playing games. Each patron participates in exactly one game.*

	21-29	30-39	40-49	50 and older	Total
Darts	4	12	15	6	37
Pool	8	17	16	11	52
Karaoke	17	5	0	1	23
Total	29	34	31	18	112

How many ways can you select a randomly selected person...

1. is playing pool;

2. is 30-39;

3. is playing pool or singing karaoke;

4. is 21-29 or 40-49.

Exercise 6 *A pair of fair dice is rolled. How many ways can the sum of the dice be 4 or 5?*

Exercise 7 *A pair of fair dice is rolled. How many ways can the sum of the dice be at least 10?*

Of course there are times when we want to determine$|A$ or $B|$when A and B are not disjoint. In such a case we must employ the general addition rule.

4 The General Addition Rule

For any events A and B, $|A$ or $B| = |A| + |B| - |A$ and $B|$.

Example 3 *A card is selected at random. How many ways can the card be a club or a 3?*

Exercise 8 *A card is selected at random. How many ways can it be a face card or a heart?*

Exercise 9 *Roll a pair of dice. How many ways can*

1. the sum be 7 or 8;

2. the sum be 5 or a 2 appears on at least one die.

Exercise 10 *A study at a local bar found people of various ages playing games.*

	21-29	30-39	40-49	50 and older	Total
Darts	4	12	15	6	37
Pool	8	17	16	11	52
Karaoke	17	5	0	1	23
Total	29	34	31	18	112

How many ways is a randomly selected person...

1. playing pool or darts;

2. 30-39 or throwing darts;

3. playing pool or 50 and older;

4. 21-29 or singing Karaoke.

5 Exercises

1. Grimaldi page 11: 1, 3-13, 15, 19, 29, 30