Section 2.1: Sample Spaces and Events

Definition 1 *A* ***random phenomenon*** *is a situation in which we know what outcomes could happen, but we don't know which particular outcome did or will happen.*

Definition 2 *(crude & initial) A* ***sample space*** *is the collection of all possible outcomes of an experiment.*

For any random phenomenon, each attempt, or trial, generates an outcome. Something happens on each trial, and we call whatever happens the outcome. These outcomes are individual possibilities, like the number we see on top when we roll a die. Sometimes we are interested in a combination of outcomes (e.g., a die is rolled and comes up even). A combination of outcomes is called an **event**. In formal mathematical terms, events are **sets** and outcomes are **elements**.

Example 1 *An experiment consists of randomly picking a card from a standard deck of playing cards (no jokers). Some possible outcomes of this experiment are listed below.*

A - The 8 of clubs is selected.
B - A red card is selected.
C - The Jack of hearts is not selected.

Remark 1 *On many occasions we will want to perform mathematical operations on events.*

Definition 3 *The event* **A complement**, *denoted $\acute{A}$ (or $\bar{A}$ or A^c), is the event that A does not occur.*

Definition 4 *The event* **A union B**, *denoted $A \cup B$, is the event that A or B (or both) occur.*

Definition 5 *The event* **A intersect B**, *denoted $A \cap B$, is the event that A and B both occur.*

Problem 1 *Using a Venn diagram, shade each of the above operations.*

Example 2 *For the sample space* $S = \{1, 2, 3, 4, 5, 6\}$, $A = \{1, 2\}$, $B = \{2, 3\}$ *and* $C = \{2, 4, 6\}$ *determine each of the following:*

1. $\bar{A}$
2. $\bar{B}$
3. $\bar{C}$
4. $A \cup B$
5. $A \cap B$
6. $A \cap C$
7. $B \cup C$
8. $B \cap C$

Example 3 *For the experiment that consists of randomly picking a card from a standard deck of playing cards (no jokers) consider the following outcomes.*

A - The 8 of clubs is selected.
B - A red card is selected.
C - The Jack of hearts is not selected.

Problem 2 *Describe each of the following:*

1. $\bar{A}$
2. $\bar{B}$
3. $\bar{C}$
4. $A \cup B$
5. $A \cap B$
6. $A \cap C$
7. $B \cup C$
8. $B \cap C$

We also want to describe events as **disjoint** (or **mutually exclusive**). Two events are disjoint if they cannot occur at the same time. Mathematically speaking, A and B are disjoint if and only if $A \cap B = \emptyset$.

Example 4 *An experiment consists of randomly picking a card from a standard deck of playing cards (no jokers). Which pairs of the following events are disjoint?*

A - The 8 of clubs is selected.
B - A red card is selected.
C - The Jack of hearts is not selected.

Problem 3 *An experiment consist of flipping a fair coin twice. Compute the probabilities of the following events. Construct the sample space.*

HH	TH
HT	TT

Example 5 *A pair of fair dice is rolled. Construct the sample space.*

$(1,1)$	$(2,1)$	$(3,1)$	$(4,1)$	$(5,1)$	$(6,1)$
$(1,2)$	$(2,2)$	$(3,2)$	$(4,2)$	$(5,2)$	$(6,2)$
$(1,3)$	$(2,3)$	$(3,3)$	$(4,3)$	$(5,3)$	$(6,3)$
$(1,4)$	$(2,4)$	$(3,4)$	$(4,4)$	$(5,4)$	$(6,4)$
$(1,5)$	$(2,5)$	$(3,5)$	$(4,5)$	$(5,5)$	$(6,5)$
$(1,6)$	$(2,6)$	$(3,6)$	$(4,6)$	$(5,6)$	$(6,6)$