RIVER MEANDERS

The striking geometric regularity of a winding river is no accident. Meanders appear to be the form in which a river does the least work in turning; hence they are the most probable form a river can take

by Luna B. Leopold and W. B. Langbein

Is there such a thing as a straight river? Almost anyone can think of a river that is more or less straight for a certain distance, but it is unlikely that the straight portion is either very straight or very long. In fact, it is almost certain that the distance any river is straight does not exceed 10 times its width at that point.

The sinuosity of river channels is clearly apparent in maps and aerial photographs, where the successive curves of a river often appear to have a certain regularity. In many instances the repeating pattern of curves is so pronounced that it is the most distinctive characteristic of the river. Such curves are called meanders, after a winding stream in Turkey known in ancient Greek times as the Maiandros and today as the Menderes. The nearly geometric regularity of river meanders has attracted the interest of geologists for many years, and at the U.S. Geological Survey we have devoted considerable study to the problem of understanding the general mechanism that underlies the phenomenon. In brief, we have found that meanders are not mere accidents of nature but the form in which a river does the least work in turning, and hence are the most probable form a river can take.

Regular Forms from Random Processes

Nature of course provides many opportunities for a river to change direction. Local irregularities in the bounding medium as well as the chance emplacement of boulders, fallen trees, blocks of sod, plugs of clay and other obstacles can and do divert many rivers from a straight course. Although local irregularities are a sufficient reason for a river's not being straight, however, they are not a necessary reason. For one thing, such irregularities cannot account for the rather consistent geometry of meanders. Moreover, laboratory studies indicate that streams meander even in "ideal," or highly regular, mediums [*see illustration on page 64*].

That the irregularity of the medium has little to do with the formation of meanders is further demonstrated by the fact that meandering streams have been observed in several naturally homogeneous mediums. Two examples are ocean currents (notably the Gulf Stream) and water channels on the surface of a glacier. The meanders in both cases are as regular and irregular as river meanders.

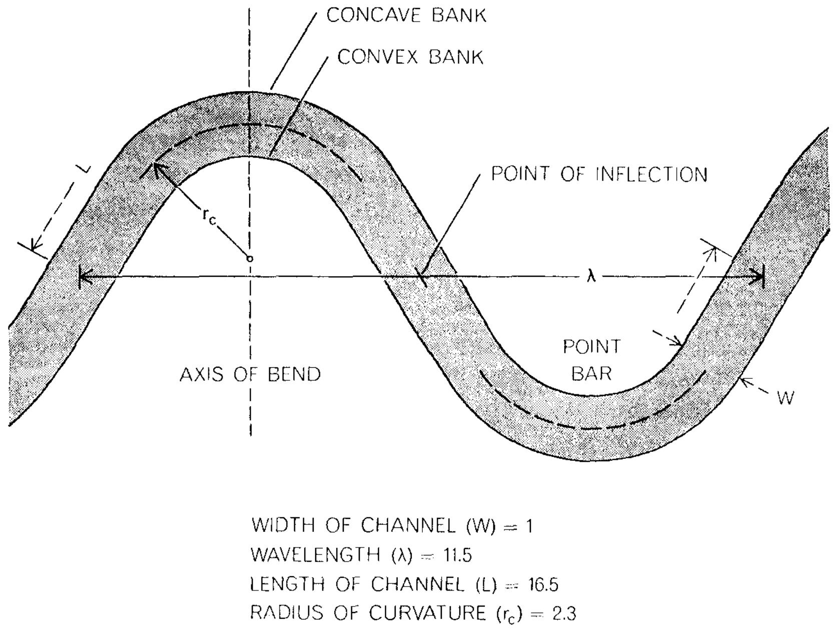
The fact that local irregularities cannot account for the existence of river meanders does not rule out other random processes as a possible explanation. Chance may be involved in subtler and more continuous ways, for example in turbulent flow, in the manner in which the riverbed and banks are formed, or in the interaction of the flow and the bed. As it turns out, chance operating at this level can explain the formation of regular meanders. It is a paradox of nature that such random processes can produce regular forms, and that regular processes often produce random forms.

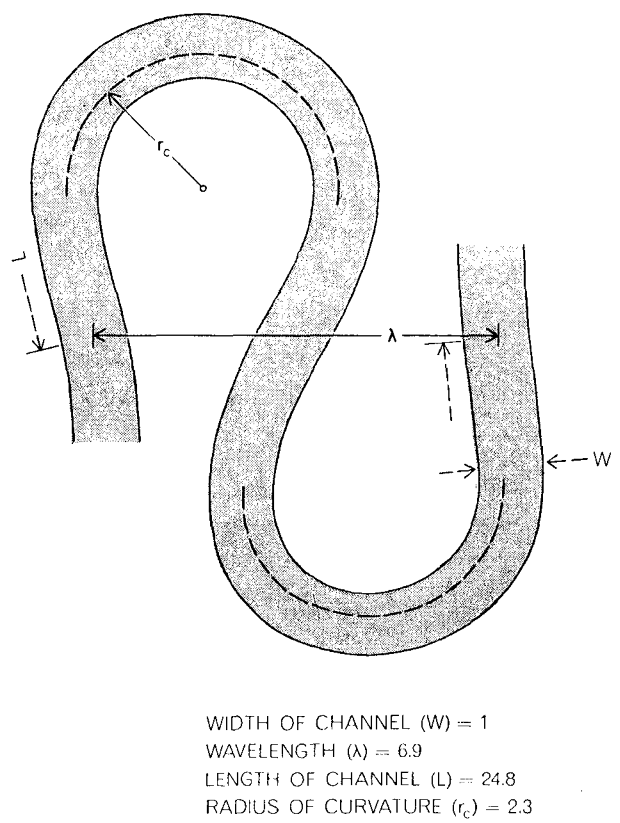
Meanders commonly form in alluvium (water-deposited material, usually unconsolidated), but even when they occur in other mediums they are invariably formed by a continuous process of erosion, transportation and deposition of the material that composes the medium. In every case material is eroded from the concave portion of a meander, transported downstream and deposited on the convex portion, or bar, of a meander. The material is often deposited on the same side of the stream from which it was eroded. The conditions in which meanders will be formed in rivers can be stated rather simply, albeit only in a general way: Meanders will usually appear wherever the river traverses a gentle slope in a medium consisting of fine-grained material that is easily eroded and transported but has sufficient cohesiveness to provide firm banks.

A given series of meanders tends to have a constant ratio between the wavelength of the curve and the radius of curvature. The appearance of regularity depends in part on how constant this ratio is. In the two drawings on page 62 the value of this ratio for the meander that looks rather like a sine wave (*top*) is five for the wavelength to one for the radius; the more tightly looped meander (*bottom*) has a corresponding value of three to one. A sample of 50 typical meanders on many different rivers and streams has yielded an average value for this ratio of about 4.7 to one. Another property that is used to describe meanders is sinuosity, or tightness of bend, which is expressed as the ratio of the length of the channel in a given curve to the wavelength of the curve. For the large majority of meandering rivers the value of this ratio ranges between 1.3 to one and four to one.

Close inspection of the photographs

ENTRENCHED MEANDERS of the Colorado River in southern Utah were photographed from a height of about 3,000 feet. The meanders were probably formed on the surface of a gently sloping floodplain at about the time the entire Colorado Plateau began to rise at least a million years ago. The meanders later became more developed as river cut deep into layers of sediment. Mean downstream direction is toward right.





PROPERTIES used to describe river meanders are indicated for two typical meander curves. A series of meanders has a regular appearance on a map whenever there tends to be a constant ratio between the wavelength (λ) of the curve and its radius of curvature (r_c). The value of this ratio for the meander that looks rather like a sine wave (*top*) is five to one; the more tightly looped meander (*bottom*) has a corresponding value of three to one. An average value for this ratio is about 4.7 to one. Sinuosity, or tightness of bend, is expressed as the ratio of the length of the channel (L) in a given curve to the wavelength of curve. The value of this ratio for the top curve is 1.4 to one and for the bottom curve 3.6 to one. On the average the value of this ratio ranges between 1.3 to one and four to one.

and maps that accompany this article will show that typical river meanders do not exactly follow any of the familiar curves of elementary geometry. The portion of the meander near the axis of bend (the center of the curve) does resemble the arc of a circle, but only approximately. Neither is the curve of a meander quite a sine wave. Generally the circular segment in the bend is too long to be well described by a sine wave. The straight segment at the point of inflection—the point where the curvature of the channel changes direction—prevents a meander from being simply a series of connected semicircles.

Sine-generated Curves

We first recognized the principal characteristics of the actual curve traced out by a typical river meander in the course of a mathematical analysis aimed at generating meander-like curves by means of "random walk" techniques. A random walk is a path described by successive moves on a surface (for example a sheet of graph paper); each move is generally a fixed unit of distance, but the direction of any move is determined by some random process (for example the turn of a card, the throw of a die or the sequence of a table of random numbers). Depending on the purpose of the experiment, there is usually at least one constraint placed on the direction of the move. In our random-walk study one of the constraints we adopted was that the path was to begin at some point A and end at some other point B in a given number of steps. In other words, the end points and the length of the path were fixed but the path itself was "free."

The mathematics involved in finding the average, or most probable, path taken by a random walk of fixed length had been worked out in 1951 by Hermann von Schelling of the General Electric Company. The exact solution is expressed by an elliptic integral, but in our case a sufficiently accurate approximation states that the most probable geometry for a river is one in which the angular direction of the channel at any point with respect to the mean down-valley direction is a sine function of the distance measured along the channel [*see illustration on opposite page*].

The curve that is traced out by this most probable random walk between two points in a river valley we named a "sine-generated" curve. As it happens, this curve closely approximates the shape of real river meanders [*see illustration on next page*]. At the axis of bend the channel is directed in the mean down-valley direction and the angle of deflection is zero, whereas at the point of inflection the angle of deflection reaches a maximum value.

A sine-generated curve differs from a sine curve, from a series of connected semicircles or from any other familiar geometric curve in that it has the smallest variation of the changes of direction. This means that when the changes in direction are tabulated for a given distance along several hypothetical meanders, the sums of the squares of these changes will be less for a sine-generated curve than for any other regular curve of the same length. This operation was performed for four different curves of the same length, wavelength and sinuosity—a parabolic curve, a sine curve, a circular curve and a sine-generated curve—in the illustration on page 65. When the squares of the changes in direction were measured in degrees over 10 equally spaced intervals for each curve, the resulting values were: parabolic curve, 5,210; sine curve, 5,200; circular curve, 4,840; sine-generated curve, 3,940.

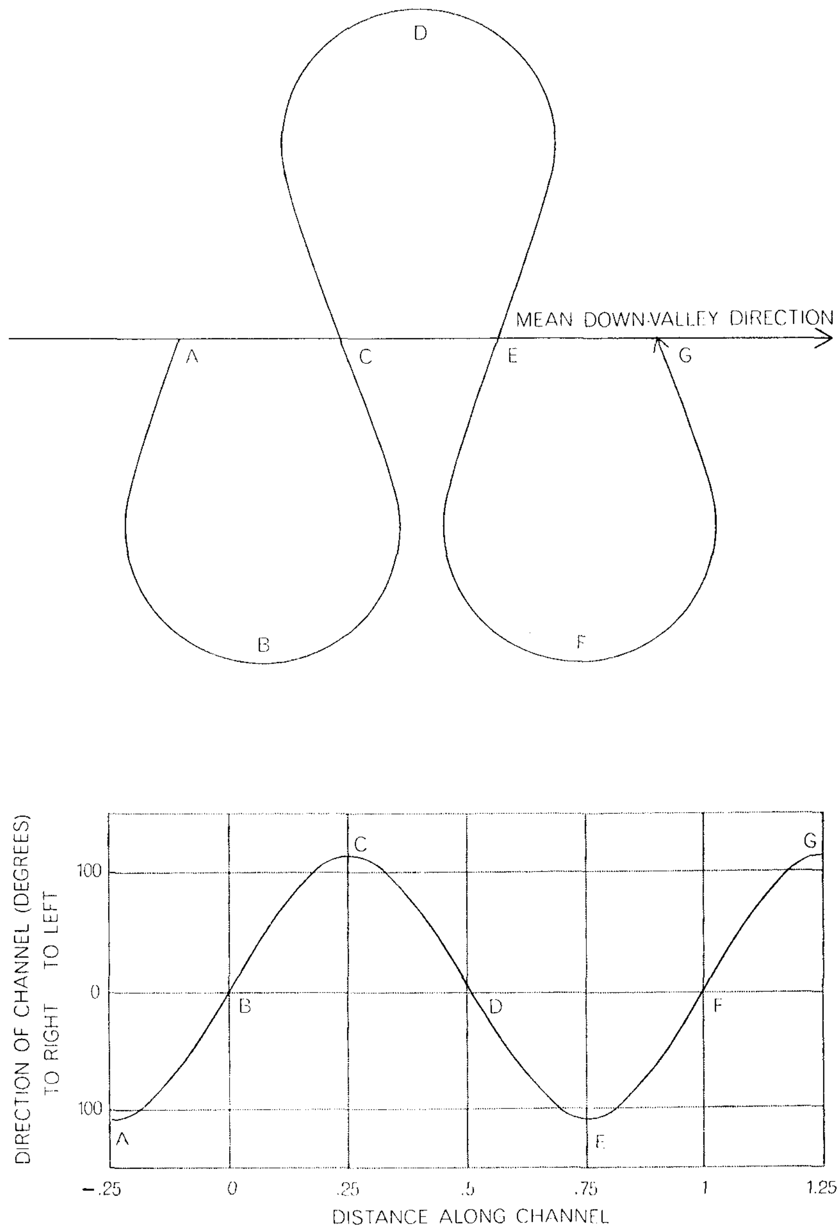
Curve of Minimum Total Work

Another property closely associated with the fact that a sine-generated curve minimizes the sum of the squares of the changes in direction is that it is also the curve of minimum total work in bending. This property can be demonstrated by bending a thin strip of spring steel into various configurations by holding the strip firmly at two points and allowing the length between the fixed points to assume an unconstrained shape [*see top illustration on pages 66 and 67*]. The strip will naturally avoid any concentration of bending and will assume a shape in which the bend is as uniform as possible. In effect the strip will assume a shape that minimizes total work, since the work done in each element of length is proportional to the square of its angular deflection. The shapes assumed by the strip are sine-generated curves and indeed are good models of river meanders.

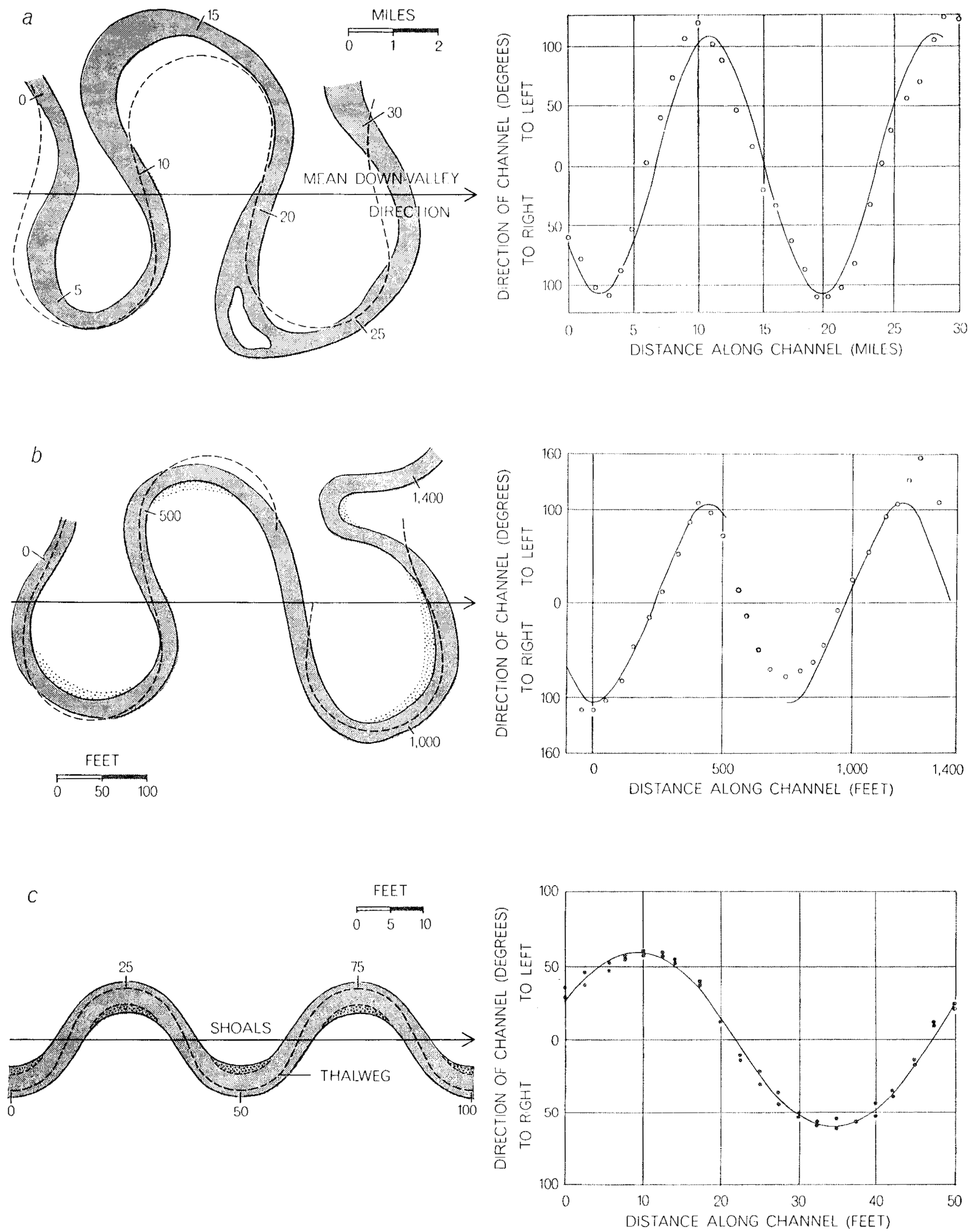
A catastrophic example of a sine-generated curve on a much larger scale was provided by the wreck of a Southern Railway freight train near Greenville, S.C., on May 31, 1965 [*see bottom illustration on page 67*]. Thirty adjacent flatcars carried as their load 700-foot sections of track rail chained in a bundle to the car beds. The train, pulled by five locomotives, collided with a bulldozer and was derailed. The violent compressive strain folded the trainload of rails into a drastically foreshortened snakelike configuration. The elastic properties of the steel rails tended to minimize total bending exactly as in the case of the spring-steel strip, and as a result the wrecked train assumed the shape of a sine-generated curve that distributed the bending as uniformly as possible. This example is particularly appropriate to our discussion of river meanders because, like river meanders, the bent rails deviate in a random way from the perfect symmetry of a sine-generated curve while preserving its essential form.

The Shaping Mechanism

The mechanism for changing the course of a river channel is contained



SINE-GENERATED CURVE (*top*) closely approximates the shape of real river meanders. This means that the angular direction of the channel at any point with respect to the mean down-valley direction (*toward the right*) is a sine function of the distance measured along the channel (*graph at bottom*). At the axis of each bend (*B, D and F*) the channel is directed in the mean down-valley direction and the angle of deflection is zero, whereas at each point of inflection (*A, C, E and G*) the angle of deflection reaches a maximum value.

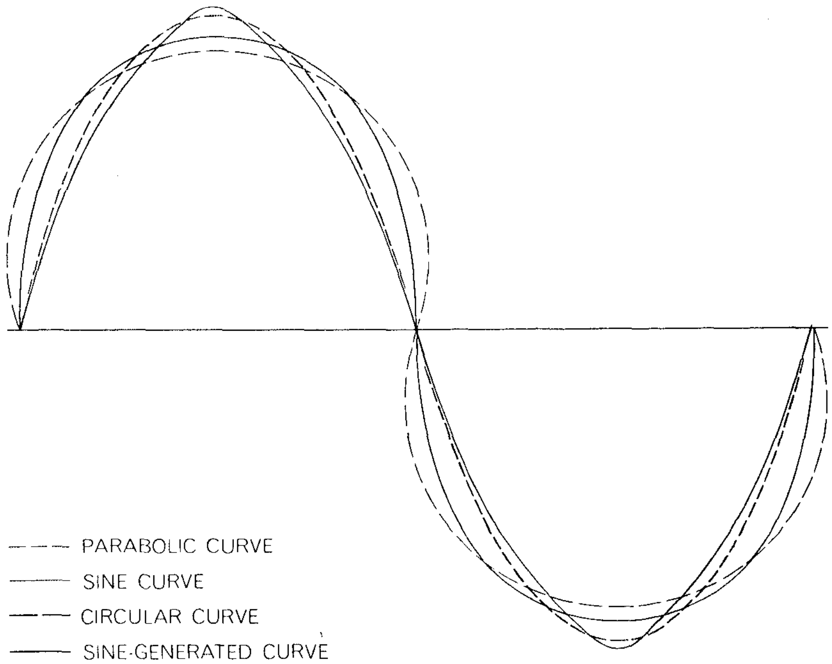


MAPS at left depict segments of two typical meandering streams, the Mississippi River near Greenville, Miss. (*a*), and Blackrock Creek in Wyoming (*b*), as well as a segment of an experimental meander formed in a homogeneous medium in the laboratory (*c*). Measurements of the angular direction of the channels with respect to the mean downstream direction were made at regular intervals along the center lines of the two natural meanders and along the thalweg, or deepest part of the channel, of the experimental meander. When these measurements were plotted against the distance of each channel, the resulting curves closely approximated sine waves (*right*). The corresponding sine-generated curves are superposed on their respective channel maps (*broken black curves*).

in the ability of water to erode, transport and deposit the material of the river's medium. Especially on a curve, the velocity gradient against the channel bank sets up local eddies that concentrate the expenditure of energy and localize erosion. An idealized flow pattern in a typical meander is shown in the top illustration on page 68. The left side of the illustration indicates the velocity vectors at various points for five cross sections along the curve. As the cross sections indicate, the depth of the channel changes systematically along the curve, the shallowest section being at the point of inflection and the deepest section at the axis of bend. At the same time the cross-sectional shape itself changes; it is symmetrical across the channel just downstream from the point of inflection and most asymmetrical at the axis of bend, the deeper section being always nearer the concave bank. The velocity vectors show a normal decrease in velocity with depth except at the axis of bend and near the concave bank, where the highest velocity at any point in the meander occurs somewhat below the surface of the water.

The right side of the same illustration shows the streamlines of flow at the surface of the meander. The maximum-velocity streamline is in the middle of the channel just downstream from the point of inflection; it crosses toward the concave bank at the axis of bend and continues to hug the concave bank past the next point of inflection. Riverboatmen navigating upstream on a large river face the problem that the deepest water, which they usually prefer, tends to coincide with the streamline of highest velocity. Their solution is to follow the thalweg (the deepest part of the river, from the German for "valley way") where it crosses over the center line of the channel as the channel changes its direction of curvature but to cut as close to the convex bank as possible in order to avoid the highest velocity near the concave bank. This practice led to the use of the term "crossover" as a synonym for the point of inflection.

The lack of identity between the maximum-velocity streamline and the center line of the channel arises from the centrifugal force exerted on the water as it flows around the curve. The centrifugal force is larger on the faster-moving water near the surface than on the slower-moving water near the bed. Thus in a meander the surface water is deflected toward the concave bank, requiring the bed water to move toward the convex bank. A circulatory system is set up in the cross-sectional plane, with surface water plunging toward the bed near the concave bank and bed water rising toward the surface near the convex bank. This circulation, together with the general downstream motion, gives each discrete element of water a roughly helical path that reverses its direction of rotation with each successive meander. As a result of this helical motion of water, material eroded from the concave bank tends to be swept toward the convex bank, where it is deposited, forming what is called a point bar.



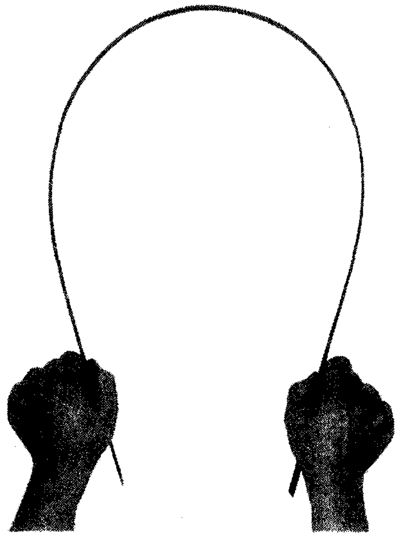
VARIATION IN CURVATURE of a sine-generated curve is less than for any other regular geometric curve. This means that when the changes in direction are tabulated for small distances along several hypothetical meanders, the sums of the squares of the changes in direction will be less for a sine-generated curve than for any other curve. The changes in direction were measured in degrees over 10 equally spaced intervals for each of the four curves depicted here. When the squares of these changes were summed, the following values were obtained: parabolic curve, 5,210; sine curve, 5,200; circular curve, 4,840; sine-generated curve, 3,940. The four curves are equal in length, wavelength and sinuosity.

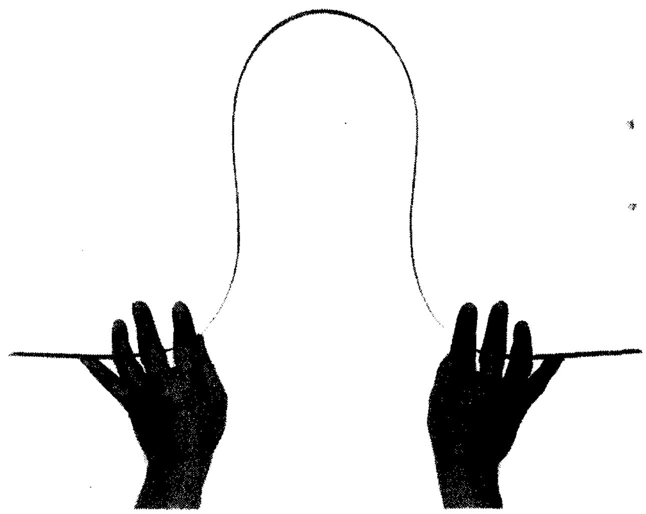
Erosion of the concave banks and deposition on the convex banks tends to make meander curves move laterally across the river valley. Because of the randomness of the entire process, the channel as a whole does not move steadily in any one direction, but the combined lateral migration of the meanders over a period of many years results in the river channel's occupying every possible position between the valley walls. The deposition on the point bars, combined with the successive occupation by the river of all possible positions, results in the formation of the familiar broad, flat floor of river valleys—the "floodplain" of the river. The construction of a floodplain by the lateral movement of a single meander can be observed even in the course of a few years; this is demonstrated in the bottom illustration on page 68, which is made up of four successive cross sections surveyed between 1953 and 1964 on Watts Branch, a small tributary of the Potomac River near Washington.

The overall geometry of a meandering river is an important factor in determining the rate at which its banks will be eroded. In general the banks are eroded at a rate that is proportional to the degree with which the river channel is bent. Any curve other than a sine-generated curve would tend to concentrate bank erosion locally or, by increasing the total angular bending, would add to the total erosion. Thus the sine-generated curve assumed by most meandering rivers tends to minimize total erosion.

Riffles and Pools

In the light of the preceding discussion it is possible to examine some of the hydraulic properties of meanders in greater detail. If a river channel is re-





STRIP OF SPRING STEEL is used to demonstrate that a sine-generated curve is the curve of minimum total work. The strip is bent into various configurations by holding it firmly at two points and allowing the length between the fixed points to assume an un-

garded as being in a steady state, the form it assumes should be such as to avoid concentrating variations in *any* property at the expense of another property.

For example, variations in depth and velocity are inherent in all river channels, whether they are straight or curved. Even a reach, or length of channel, that is quite straight has a more or less uneven bed that consists of alternating deeps and shallows. Although this is not so obvious in a period of high flow, it becomes quite apparent at low flow, when the shallow sections tend to ripple in the sunlight as water backs up behind each hump in the bed before pouring over its downstream slope. To a trout fisherman this fast reach is known as a riffle. Alternating with the riffles are deeps, which the fisherman would call pools, through which the water flows slower and more smoothly.

The alternation of riffles and pools in a trout stream at low flow is noteworthy for another reason. The humps in the stream bed that give rise to the riffles tend to be located alternately on each side of the stream [*see top illustration on page 69*]. As a consequence the stream at low flow seems to follow a course that wanders successively from one side of the channel to the other, in a manner having an obvious similarity to meandering.

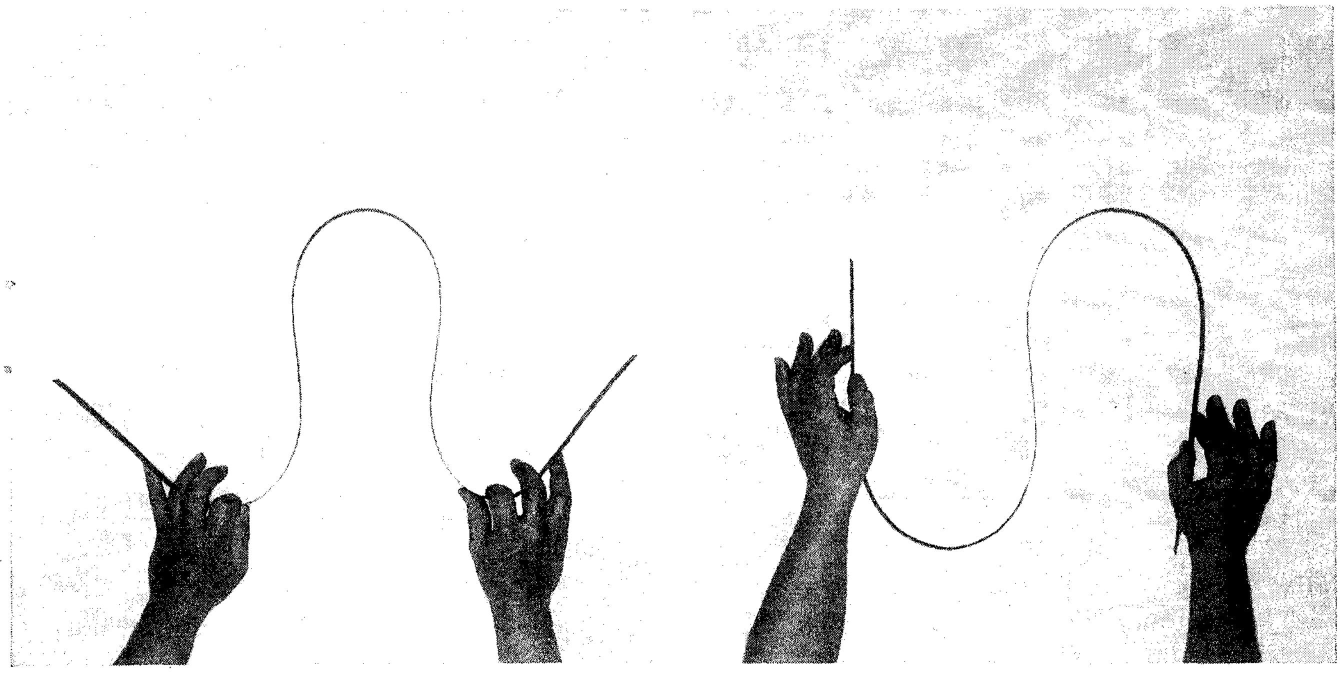
The analogy between this temporary sinuosity and full-scale meandering is strengthened by the fact that the riffles occur at roughly equal intervals along the channel. Moreover, the spacing of the riffles is correlated with the width of the channel. Successive riffles are located at intervals equal to about five to seven times the local channel width, or roughly twice the wavelength of a typical meander. This surprisingly consistent ratio seems even more remarkable when one realizes that each meander contains two riffles, one at each point of inflection. This observation led us to hypothesize that the same mechanism that causes meanders must also be at work in straight channels, and that a detailed study of the form and the hydraulic properties of two segments of channel that differ only in their degree of curvature might shed some light on the formation of meanders.

Obtaining Meander Profiles

In order to test this hypothesis it was necessary to obtain accurate data for all the pertinent hydraulic factors: depth, velocity, water-surface profile and bed profile. For several years we had attempted to measure such factors in small rivers near Washington just after every heavy rainstorm, when there was a rapid increase in streamflow. The water level changed so quickly in such storms, however, that there was never enough time to measure all the hydraulic factors in detail through a succession of two riffles and an intervening pool. Then in 1959 we tried another strategy: we decided to measure a small stream in Wyoming, named Baldwin Creek, in early June, a period of maximum runoff from melting snow. Measurements were made in two places, a meandering reach and a straight reach, that were comparable in all outward aspects except sinuosity. The stream was about 20 feet wide and was nearly overflowing its banks, so that we could just barely walk in it wearing chest-high rubber waders.

Robert M. Myrick, an engineer with the Geological Survey, and one of us (Leopold) began a series of measurements in the midafternoon of June 19, surveying water-surface and bed profiles with a level and a rod, and making velocity and depth measurements with a current meter and a rod. When darkness came, we lighted lanterns and continued our measurements. At about daybreak we slept for a few hours and then resumed the survey, grateful that the melting snow had kept the stream at a steady high flow for such a long time.

Several days later we were able to sit down under a tree and plot the profiles, velocities and depths on graph paper. What emerged was a quite unexpected contrast between meandering reach and straight reach [*see bottom illustration on page 69*]. The slope of the water surface in the meandering reach



constrained shape. The strip will naturally avoid any concentration of bending and will assume a shape in which the bend is as uniform as possible. In each of the four cases shown here this shape is a sine-generated curve and indeed a good model of a river meander.

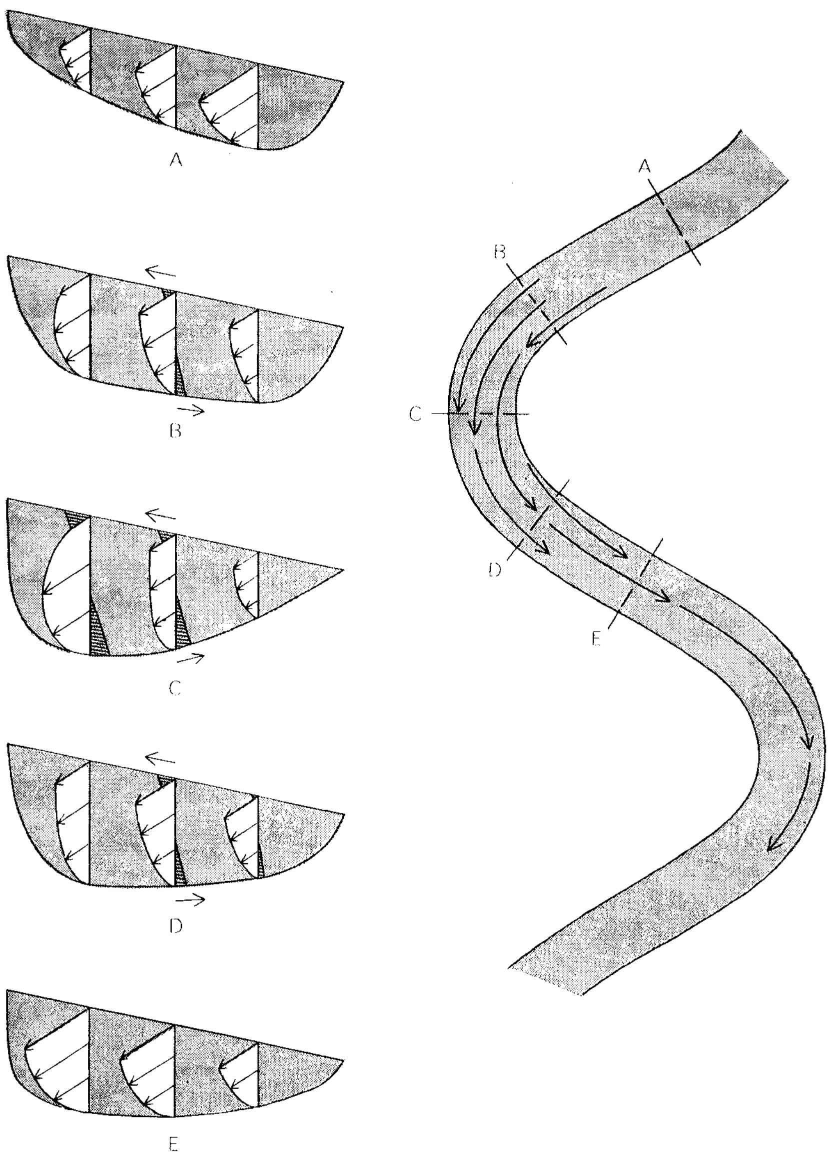
was clearly steeper than that in the straight reach; moreover, the water-surface profile of the meandering reach was nearly a straight sloping line, whereas the straight reach had a stepped profile, steep over the riffle bars and comparatively flat over the intervening pools.

What did this mean? It was as if the river had, to use somewhat anthropomorphic terms, chosen to cut a meander curve in order to achieve a more uniform water-surface profile. This suggested that the river had chosen the curved path in order to achieve the objective of uniform energy loss for each unit of distance along the channel, but had paid a price in terms of the larger total energy loss inherent in a curved path.

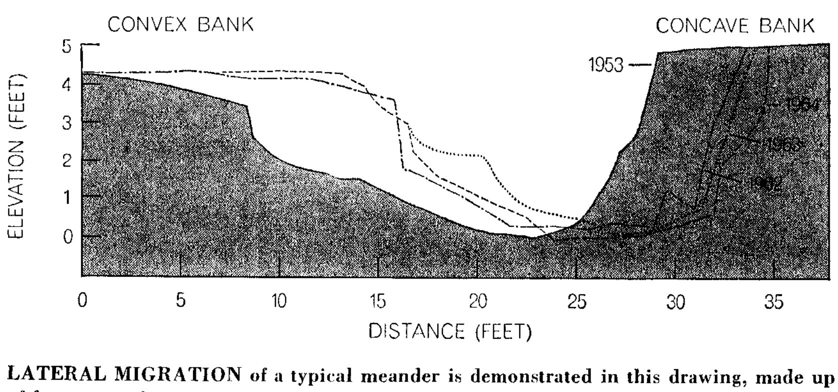
Conclusions

These data provided the key to further research, which ultimately resulted in several conclusions. First, it appears that a meandering channel more

CATASTROPHIC EXAMPLE of a sine-generated curve on a much larger scale was provided by the wreck of a Southern Railway freight train near Greenville, S.C., on May 31, 1965. Thirty adjacent flatcars carried as their load 700-foot sections of track rails chained in a bundle to the car beds. The train, pulled by five locomotives, collided with a bulldozer and was derailed. The violent compressive strain folded the trainload of rails into the drastically foreshortened configuration shown in this aerial photograph. The elastic properties of the steel rails tended to minimize total bending exactly as in the case of the spring-steel strip shown at top of these two pages, and the wrecked train assumed the shape of a sine-generated curve that distributed the bending as uniformly as possible.



IDEALIZED FLOW PATTERN of a typical meander is shown here. The left side of the illustration indicates the velocity vectors in a downstream direction for five cross sections across the curve; the lateral component of the velocity is indicated by the triangular hatched areas. The right side of the illustration shows the streamlines at the surface of the meander.



LATERAL MIGRATION of a typical meander is demonstrated in this drawing, made up of four successive cross sections surveyed between 1953 and 1964 on Watts Branch, a small tributary of the Potomac River near Washington. The lateral migration of meanders by the erosion of the concave banks and deposition on the convex banks over many years results in a river channel's occupying every possible position between the valley walls.

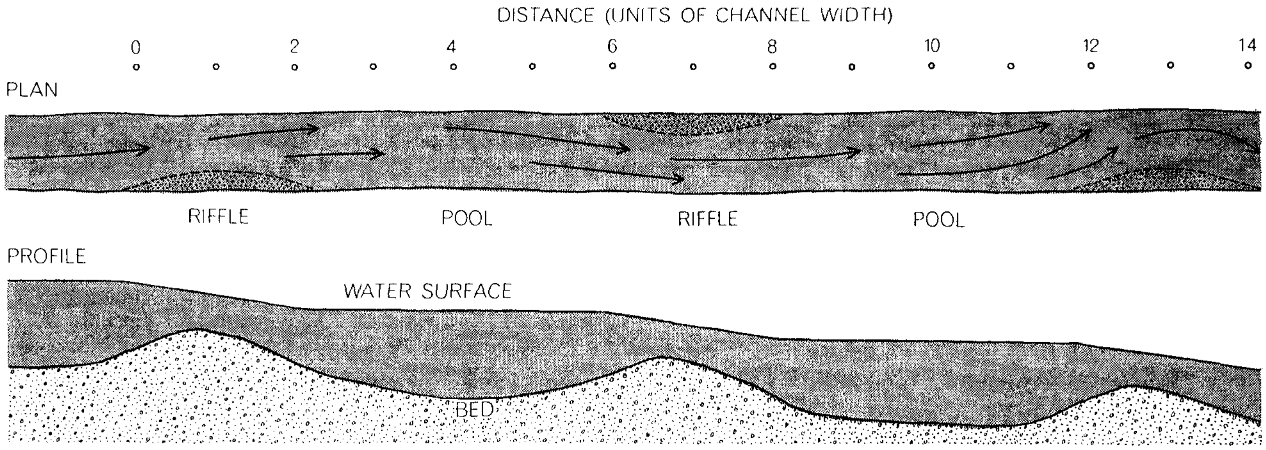
closely approaches uniformity in the rate of work over the various irregularities of the riverbed than a straight channel does. Of course the slope of the water surface is, with a slight correction for velocity, an accurate indicator of the rate at which energy is lost in the form of frictional heat along the length of the stream. Therefore a uniform longitudinal water-surface slope signifies a uniform expenditure of energy for each unit of distance along the channel.

A meander attains a more uniform rate of energy loss by the introduction of a form of energy loss not present in a straight reach, namely the curved path. It is evident that work is required to change the direction of a flowing liquid. Thus the slope of the water surface should increase wherever a curve is encountered by a river. In a meander it is at the deep pools, where the water-surface slope would be less steep than the average, that the introduction of a curve inserts enough energy loss to steepen the slope, thereby tending to make the slope for each unit of river length nearly the same. Accordingly the alternation of straight shallow reaches with curved deep reaches in a meander appears to be the closest possible approach to a configuration that results in uniform energy expenditure.

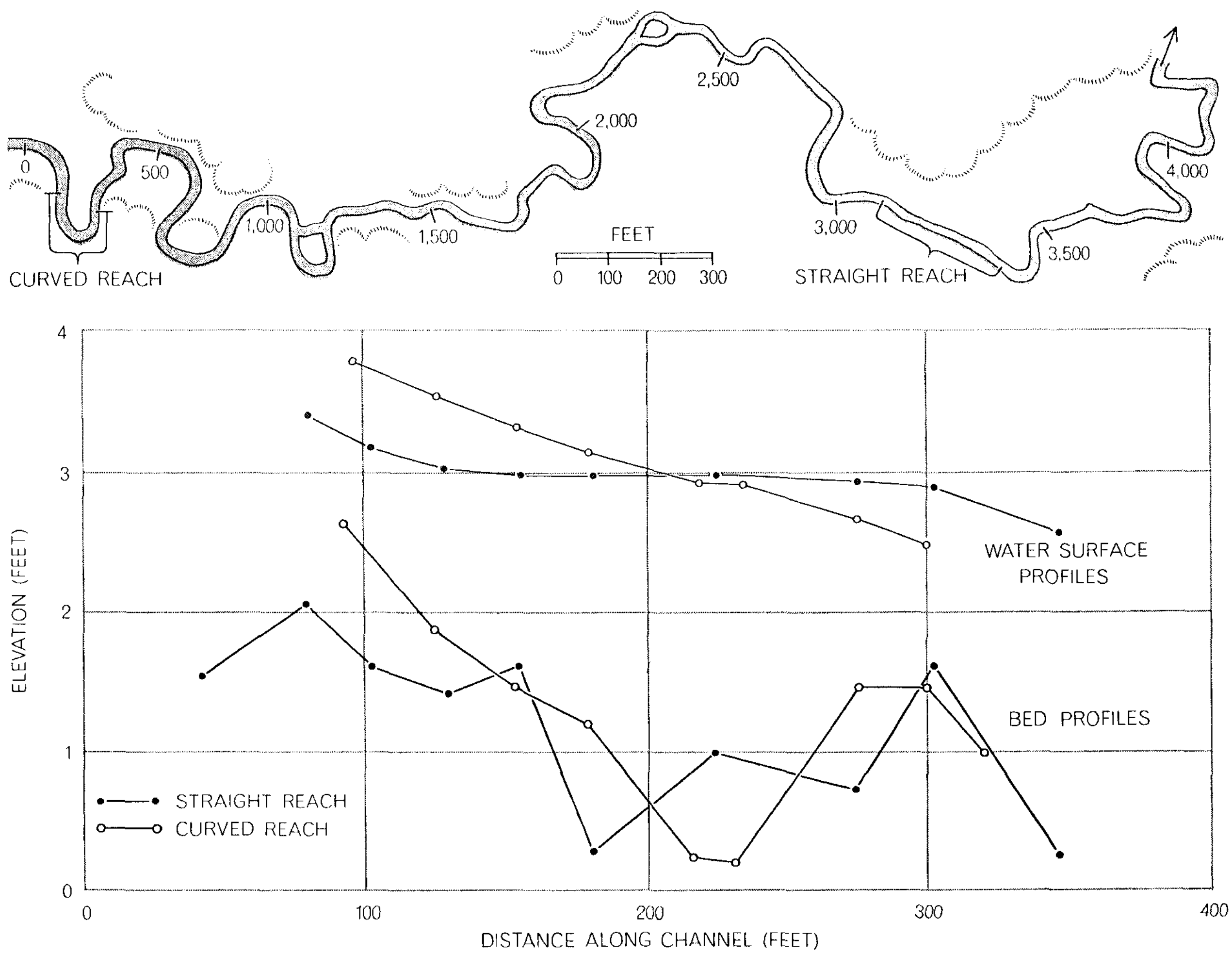
It is now possible to say something about the development of meandering in rivers. Although one can construct in a laboratory an initially straight channel that will in time develop a meandering pattern, a real meandering river should not be thought of as having an "origin." Instead we think of a river as having a heritage. When a continent first emerges from the ocean, small rills must form almost immediately; thereafter they change progressively in response to the interaction of uplift and other processes, including irregularities in the hardness of the rock.

Today the continuous changes that occur in rivers are primarily wrought by the erosion and deposition of sedimentary material. As we have seen, rivers tend to avoid concentrating these processes in any one place. Hence any irregularity in the slope of a river—for example a waterfall or a lake—is temporary on a geological time scale; the hydraulic forces at work in the river tend to eliminate such concentrations of change.

The formation of meander curves of a particular shape is an instance of this adjustment process. The meandering form is the most probable result of the processes that on the one hand tend to



STRAIGHT REACH of a river has a more or less uneven bed that consists of alternating deeps and shallows, known to trout fishermen as riffles and pools. The humps in the stream bed that give rise to the riffles tend to be located alternately on each side of the stream at intervals roughly equal to five to seven times the local stream width. As a consequence the stream at low flow seems to follow a course that wanders from one side of the channel to the other, in a manner having an obvious similarity to meandering.



PROFILES of the water surface and bed of a small stream in Wyoming named Baldwin Creek were obtained by one of the authors (Leopold) and a colleague in 1959 during a period of maximum runoff from melting snow. Measurements were made in two places, a meandering reach and a straight reach, that were comparable in all outward aspects except sinuosity (*map at top*). What emerged was a quite unexpected contrast between the two reaches (*bottom*). The slope of the water surface in the meandering reach was clearly steeper than that in the straight reach; moreover, the water-surface profile of the meandering reach was nearly a straight sloping line, whereas the straight reach had a stepped profile, steep over the riffle bars and comparatively flat over the intervening pools.

eliminate concentrations of energy loss and on the other tend to reduce the total energy loss to a minimum rate. The sine-generated curvature assumed by meanders achieves these ends more satisfactorily than any other shape.

The same tendencies operate through the erosion-deposition mechanism both in the river system as a whole and in a given segment of the river. The tendency toward uniform power expenditure in the entire river leads toward a longitudinal profile of the river that is highly concave, inasmuch as uniformity in the rate of work per unit of length of channel would be achieved by concentrating the steepest slopes near the headwaters, where the tributaries and hence discharges are small. The longitudinal concavity of the river's profile also minimizes work in the system as a whole.

Such a longitudinal concave profile, however, would lead to considerable variation in the rate of energy expenditure over each unit area of channel bed. Uniformity in this rate would be best achieved by a longitudinal profile that was nearly straight rather than by one that was highly concave. Actual river profiles lie between these two extremes, and meanders must be considered in both contexts: first, as they occur within the river system as a whole, and second, as they occur in a given segment of channel.

In the context of the entire river system a meander will occur where the material constituting the banks is comparatively uniform. This will be more likely to take place downstream in a floodplain area than upstream in a headwater area. To the extent that the meandering pattern tends to lengthen the downstream reaches more than those upstream, it promotes concavity in the longitudinal profile of the system, thereby promoting uniformity in the rate of energy expenditure per unit of channel length.

In the local context of a given segment of channel the average slope of the channel is fixed by the relation of that segment to the whole profile. Any local change in the channel must maintain that average slope. Between any two points on a valley floor, however, a variety of paths are possible, any one of which would maintain the same slope and hence the same length. The typical meander shape is assumed because, in the absence of any other constraints, the sine-generated curve is the most probable path of a fixed length between two fixed points.