CALCULUS

LIMITS AND DERIVATIVES

LIMIT PROPERTIES

Assume that the limits of $f(x)$ and $g(x)$ exist as x approaches a.

$$\lim_{x\to a}[f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$$

$$\lim_{x\to a}[cf(x)] = c\lim_{x\to a} f(x)$$

$$\lim_{x\to a}[f(x)g(x)] = \left(\lim_{x\to a} f(x)\right)\left(\lim_{x\to a} g(x)\right)$$

$$\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} \text{ if } \lim_{x\to a} g(x) \neq 0$$

$$\lim_{x\to a}[f(x)]^n = \left[\lim_{x\to a} f(x)\right]^n$$

$$\lim_{x\to a}\sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$$

FUNDAMENTAL LIMITS

$$\lim_{x\to a} c = c$$

$$\lim_{x\to\infty}\frac{1}{x} = 0 \text{ and } \lim_{x\to-\infty}\frac{1}{x} = 0$$

$$\lim_{x\to\infty}\frac{1}{x^p} = 0 \text{ for any } p > 0$$

$$\lim_{x\to\infty} x^p = \infty \text{ for any } p > 0$$

$$\lim_{x\to-\infty} x^p = \infty \text{ for even } p \text{ and } \lim_{x\to-\infty} x^p = -\infty \text{ for odd } p$$

$$\lim_{x\to\infty} e^x = \infty \text{ and } \lim_{x\to-\infty} e^x = 0$$

$$\lim_{x\to\infty} \ln x = \infty \text{ and } \lim_{x\to 0^+} \ln x = -\infty$$

$$\lim_{x\to\infty} \tan^{-1} x = \frac{\pi}{2} \text{ and } \lim_{x\to-\infty} \tan^{-1} x = -\frac{\pi}{2}$$

$$\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1 \text{ and } \lim_{\theta\to 0}\frac{\theta}{\sin\theta} = 1$$

$$\lim_{\theta\to 0}\frac{1-\cos\theta}{\theta} = 0$$

DERIVATIVE FORMULAS

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(x) = 1$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$$

$$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(b^x) = b^x \ln b$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \quad (x > 0)$$

$$\frac{d}{dx}(\log_b x) = \frac{1}{x\ln b} \quad (x > 0)$$

DERIVATIVE NOTATION

If $y = f(x)$, then the following are equivalent notations for the derivative.

$$\frac{dy}{dx} = y' = f'(x) = \frac{df}{dx} = \frac{d}{dx}(f(x))$$

DERIVATIVE DEFINITION

$$\frac{d}{dx}f(x) = f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

POWER RULE

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

PRODUCT RULE

$$\frac{d}{dx}(f(x)g(x)) = f(x)g'(x) + g(x)f'(x)$$

QUOTIENT RULE

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

CHAIN RULE

$$\frac{d}{dx}\big(f(g(x))\big) = f'(g(x))g'(x)$$

DERIVATIVE PROPERTIES

$$\frac{d}{dx}(f(x)+g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

$$\frac{d}{dx}(f(x)-g(x)) = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

$$\frac{d}{dx}(cf(x)) = c\cdot\frac{d}{dx}f(x)$$

L'HOPITAL'S RULE

If $\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$ then $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$

If $\lim_{x\to a} f(x)g(x) = 0\cdot(\pm\infty)$ then $\lim_{x\to a} f(x)g(x) = \lim_{x\to a}\frac{f(x)}{1/g(x)} = \lim_{x\to a}\frac{g(x)}{1/f(x)}$

If $\lim_{x\to a}[f(x)]^{g(x)} = 0^0$ or ∞^0 or 1^∞ then $\lim_{x\to a}[f(x)]^{g(x)} = \lim_{x\to a} e^{\ln[f(x)]^{g(x)}} = e^{\lim_{x\to a} g(x)\ln[f(x)]}$

THE SQUEEZE THEOREM

If $f(x) \leq g(x) \leq h(x)$ for all x near a (except possibly at a), and $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$, then

$$\lim_{x\to a} g(x) = L$$

CHAIN RULE FORMS

$$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1}g'(x)$$

$$\frac{d}{dx}\left(e^{g(x)}\right) = e^{g(x)}g'(x)$$

$$\frac{d}{dx}\left(b^{g(x)}\right) = b^{g(x)}g'(x)\ln b$$

$$\frac{d}{dx}\ln(g(x)) = \frac{1}{g(x)}g'(x) = \frac{g'(x)}{g(x)}$$

$$\frac{d}{dx}\log_b(g(x)) = \frac{g'(x)}{g(x)\ln b}$$

$$\frac{d}{dx}\sin(g(x)) = g'(x)\cos(g(x))$$

$$\frac{d}{dx}\cos(g(x)) = -g'(x)\sin(g(x))$$

$$\frac{d}{dx}\tan(g(x)) = g'(x)\sec^2(g(x))$$

$$\frac{d}{dx}\tan^{-1}(g(x)) = \frac{g'(x)}{1+[g(x)]^2}$$

$$\frac{d}{dx}f(ax+b) = af(ax+b)$$

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CALCULUS

INTEGRALS

COMMON CALCULUS 1 INTEGRALS

$$\int k\,dx = kx + C$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$$

$$\int x^{-1}\,dx = \int \frac{1}{x}dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C$$

$$\int b^x\,dx = \frac{b^x}{\ln b} + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \sec x \tan x\,dx = \sec x + C$$

$$\int \csc^2 x\,dx = -\cot x + C$$

$$\int \csc x \cot x\,dx = -\csc x + C$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$$

$$\int \frac{dx}{1+x^2} = \tan^{-1} x + C$$

$$\int \frac{dx}{|x|\sqrt{x^2-1}} = \sec^{-1} x + C$$

DEFINITE INTEGRAL DEFINITION

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x$$

where $\Delta x = \frac{b-a}{n}$ and $x_k = a + k\Delta x$

FUNDAMENTAL THEOREM OF CALCULUS, PART I

Assume $f(x)$ is continuous on $[a,b]$. If $F(x)$ is an antiderivative of $f(x)$ on $[a,b]$, then

$$\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$$

FUNDAMENTAL THEOREM OF CALCULUS, PART II

$$\frac{d}{dx}\int_a^x f(t)dt = f(x)$$

$$\frac{d}{dx}\int_a^{g(x)} f(t)dt = f\big(g(x)\big)g'(x) \qquad \text{(chain rule version)}$$

BASIC INTEGRATION PROPERTIES

$$\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

$$\int_a^b cf(x)dx = c\int_a^b f(x)dx$$

$$\int_a^a f(x)dx = 0$$

$$\int_b^a f(x)dx = -\int_a^b f(x)dx$$

$$\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx \quad (a \le b \le c)$$

$$\int_a^b k\,dx = k(b-a)$$

MORE INTEGRATION PROPERTIES

$$\left|\int_a^b f(x)dx\right| \le \int_a^b |f(x)|dx$$

If $f(x) \ge 0$ for $a \le x \le b$, then $\int_a^b f(x)dx \ge 0$

If $f(x) \ge g(x)$ for $a \le x \le b$, then

$$\int_a^b f(x)dx \ge \int_a^b g(x)dx$$

If $m \le f(x) \le M$ for $a \le x \le b$, then

$$m(b-a) \le \int_a^b f(x)dx \le M(b-a)$$

COMMON CALCULUS 2 INTEGRALS

$$\int \frac{dx}{ax+b} = \frac{1}{a}\ln|ax+b| + C$$

$$\int \tan x\,dx = \ln|\sec x| + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

$$\int \cot x\,dx = \ln|\sin x| + C$$

$$\int \csc x\,dx = -\ln|\csc x + \cot x| + C$$

$$\int \ln x\,dx = x\ln x - x + C$$

$$\int \tan^{-1} x\,dx = x\tan^{-1} x - \frac{1}{2}\ln(1+x^2) + C$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int f(kx)dx = \frac{1}{k}F(kx) + C$$

where $F(x)$ is any antiderivative of $f(x)$ and k is any nonzero constant. For example,

$$\int e^{kx}dx = \frac{1}{k}e^{kx} + C \quad \text{and} \quad \int \sin(kx)\,dx = -\frac{1}{k}\cos(kx) + C$$

INTEGRATION BY SUBSTITUTION

$$\int f\big(g(x)\big)g'(x)dx = \int f(u)\,du$$

or

$$\int_a^b f\big(g(x)\big)g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$$

where $u = g(x)$ and $du = g'(x)dx$

INTEGRATION BY PARTS

$$\int u\,dv = uv - \int v\,du$$

or

$$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$$

Remember the acronym **ILATE** when choosing u.
Inverse Trig, **L**ogarithmic, **A**lgebraic, **T**rigonometric, **E**xponential

ARC LENGTH FORMULA

The arc length differentiable function $y = f(x)$ over the interval $[a,b]$ is given by

$$\int_a^b \sqrt{1+[f'(x)]^2}dx = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

VOLUMES OF SOLIDS OF REVOLUTION

DISK METHOD: $\int_a^b \pi(\text{Radius})^2 dx = \int_a^b \pi\big(R(x)\big)^2 dx$

WASHER METHOD: $\int_a^b \pi\left(\binom{\text{Outer}}{\text{Radius}}^2 - \binom{\text{Inner}}{\text{Radius}}^2\right)dx = \int_a^b \pi\left(\big(R(x)\big)^2 - \big(r(x)\big)^2\right)dx$

SHELL METHOD: $\int_a^b 2\pi\binom{\text{Shell}}{\text{Radius}}\binom{\text{Shell}}{\text{Height}}dx$

TRIGONOMETRIC SUBSTITUTION

EXPRESSION	SUBSTITUTION	EVALUATION
$\sqrt{a^2-x^2}$	$x = a\sin\theta$ $dx = a\cos\theta\,d\theta$	$\sqrt{a^2-a^2\sin^2\theta}$ $= a\cos\theta$
$\sqrt{a^2+x^2}$	$x = a\tan\theta$ $dx = a\sec^2\theta\,d\theta$	$\sqrt{a^2+a^2\tan^2\theta}$ $= a\sec\theta$
$\sqrt{x^2-a^2}$	$x = a\sec\theta$ $dx = a\sec\theta\tan\theta\,d\theta$	$\sqrt{a^2\sec^2\theta-a^2}$ $= a\tan\theta$