December 5 Math 2306 sec. 51 Fall 2022

Section 17: Fourier Series: Trigonometric Series

Suppose f is piecewise continuous on the interval $(-p, p)$. Then we can write f as a Fourier series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi x}{p}\right) + b_n \sin\left(\frac{n\pi x}{p}\right) \right)$$

where

$$\begin{aligned} a_0 &= \frac{1}{p}\int_{-p}^{p} f(x)\,dx, \\ a_n &= \frac{1}{p}\int_{-p}^{p} f(x)\cos\left(\frac{n\pi x}{p}\right)\,dx, \quad \text{and} \\ b_n &= \frac{1}{p}\int_{-p}^{p} f(x)\sin\left(\frac{n\pi x}{p}\right)\,dx \end{aligned}$$

Example

$a_0 = 3, \quad a_n = 0, \quad b_n = \frac{-3}{n\pi}\left((-1)^n - 1\right)$

$= \frac{1}{n\pi}\left(-3(-1)^n + 3\right)$

$= \frac{3}{n\pi}\left(1 - (-1)^n\right)$

Let's find the series for

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ 3, & 0 < x < \pi \end{cases}$$

$$f(x) = \frac{3}{2} + \sum_{n=1}^{\infty} \frac{3}{n\pi}\left(1 - (-1)^n\right)\sin(nx)$$