Exam 2 Review Questions: Math 2335 (Ritter)

Sections Covered: 4.1, 4.2, 4.3, 4.5-4.6, 5.1, 5.2, 5.3

This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.

(1) Use a linear interpolation of $f(x) = \sin x$ to approximate $\sin(0.35)$ using the points on the graph $(0.3, 0.2955202)$ and $(0.4, 0.3894183)$.

(2) Find the unique polynomial of degree ≤ 2 that passes through the points $(1, 1)$, $(2, 4)$ and $(3, -1)$.

(3) Given $f(x) = e^{-x}$ and the x-values $x_0 = 0, x_1 = 0.1, x_2 = 0.2$, compute the divided differences

$$f[x_0, x_1], \quad f[x_1, x_2], \quad \text{and} \quad f[x_0, x_1, x_2].$$

Use the result to write the quadratic interpolation of $f(x)$ through the points $(0, 1)$, $(0.1, e^{0.1})$, $(0.2, e^{0.2})$. Use a TI89 or similar calculator in float 7 mode.

(4) Consider using a 3^{rd} order interpolating polynomial $P_3(t)$ to approximate the function $f(t) = \tan^{-1} t$ on the interval $-1 \leq t \leq 1$. Find the x-values x_0, x_1, x_2, x_3 that will minimize the error.

(5) Use the fact that $|f^{(4)}(t)| \leq 24$ for $-1 \leq t \leq 1$ to bound the error $|f(t) - P_3(t)|$ from problem (4).

(6) Find the piece-wise linear interpolating function for the data set $\{(1, 3), (1.5, 2), (2, 3.5)\}$.

(7) Find the natural cubic spline that interpolates the data in problem (6).

(8) Use the trapazoid rule with two subintervals $T_2(f)$ and the trapazoid rule with 4 subintervals $T_4(f)$ to approximate

$$\int_0^2 \frac{1}{4+x^2}\,dx.$$

Find the error for each case (compute the exact value using the Fundamental Thm of Calculus).

(9) Repeat problem number (8) except using the Simpson's rules $S_2(f)$ and $S_4(f)$.

(10) Use the results from problems (8) and (9) to approximate

$$\int_0^2 \frac{1}{4+x^2}\,dx.$$

using the Richardson extrapolation methods $R_4(f)$ for both the trapazoid and the Simpson's rules.

(11) Use Gaussian numerical integration $I_2(f)$ to approximate

$$\int_{-1}^1 \sqrt[3]{x}\,e^{-x}\,dx.$$