# Exam 2 Math 2306 sec. 54

Fall 2021

**Name:** (4 pts) ______________________________

Your signature (required) confirms that you agree to practice academic honesty.

**Signature:** ______________________________

| Problem | Points |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| Total (+4) | |

INSTRUCTIONS: There are 6 problems worth 16 points each. You may use one sheet (8.5" $\times$ 11") of your own prepared notes/formulas.
**No use of a calculator, text book, smart device, or other resource is permitted. Illicit use of any additional resource will result in a grade of zero on this exam as well as a formal allegation of academic misconduct.**

Show all of your work on the paper provided to receive full credit.

**1.** Show that if $a$ and $b$ are different real numbers then the functions $f_1(x) = e^{ax}$ and $f_2(x) = e^{bx}$ are linearly independent on the interval $(-\infty, \infty)$.

Using the Wronskian

$$W(f_1, f_2)(x) = \begin{vmatrix} e^{ax} & e^{bx} \\ ae^{ax} & be^{bx} \end{vmatrix}$$

$$= e^{ax}(be^{bx}) - ae^{ax}(e^{bx}) = (b-a)e^{(a+b)x}$$

The exponential part, $e^{(a+b)x} > 0$.

Since $b \neq a$, $b - a \neq 0$. Hence

$W \neq 0$ and they are linearly

independent.

**2.** A dying battery imparts a force of $E(t) = 30e^{-5t}$ volts to an LR circuit with an inductance of 2 henries and a resistance of 4 ohms. If the initial current $i(0) = 0$, determine the current $i(t)$ in the circuit.

$$Li' + Ri = E$$

Here, $L = 2$, $R = 4$, and $E = 30e^{-5t}$

$$2i' + 4i = 30e^{-5t}, \quad i(0) = 0.$$

Standard form $\quad i' + 2i = 15e^{-5t}$

$$P(t) = 2, \quad \mu = e^{\int 2dt} = e^{2t}$$

$$\frac{d}{dt}\left(e^{2t} i\right) = 15e^{-5t} \cdot e^{2t} = 15e^{-3t}$$

$$e^{2t} i = \int 15e^{-3t}\, dt = \frac{-15}{3} e^{-3t} + k$$

$$i = -5e^{-5t} + ke^{-2t}$$

$$i(0) = -5 + k = 0 \implies k = 5$$

The current

$$\boxed{i(t) = 5e^{-2t} - 5e^{-5t}}$$

**3.** Find the general solution of the nonhomogeneous ODE.

$y'' - 4y' + 3y = 2e^x$

Find $y_c$: $m^2 - 4m + 3 = 0$

$(m-1)(m-3) = 0 \quad m = 1 \text{ or } m = 3$

$y_1 = e^x, \quad y_2 = e^{3x}, \quad y_c = C_1 e^x + C_2 e^{3x}$

Find $y_p$: $g(x) = 2e^x$

$y_p = (Ae^x)x = Axe^x$

$y_p' = Ae^x + Axe^x$

$y_p'' = Ae^x + Ae^x + Axe^x$

$y_p'' - 4y_p' + 3y_p = 2e^x$

$2Ae^x + Axe^x - 4(Ae^x + Axe^x) + 3Axe^x = 2e^x$

$xe^x(A - 4A + 3A) + e^x(2A - 4A) = 2e^x$

$-2A = 2$

$A = -1$

So $y_p = -xe^x$

$$\boxed{\begin{array}{l}\text{The general solution} \\ y = C_1 e^x + C_2 e^{3x} - xe^x\end{array}}$$

**4.** Find a particular solution of the nonhomogeneous ODE. A fundamental solution set $\{y_1, y_2\}$ for the associated homogeneous equation is given.

$x^2y'' - 2xy' = 9, \quad x > 0 \qquad y_1 = 1, \quad y_2 = x^3$

Will use variation of parameters.

Standard form : $y'' - \frac{2}{x}y' = \frac{9}{x^2}$

$g(x) = \frac{9}{x^2}, \quad y_1 = 1, \quad y_2 = x^3$

$W = \begin{vmatrix} 1 & x^3 \\ 0 & 3x^2 \end{vmatrix} = 3x^2$

$y_p = u_1y_1 + u_2y_2$

$u_1 = \int -\frac{g y_2}{W}\,dx = \int \frac{-\frac{9}{x^2} \cdot x^3}{3x^2}\,dx = -3\int \frac{1}{x}\,dx$

$= -3\ln x$

$u_2 = \int \frac{g y_1}{W}\,dx = \int \frac{\frac{9}{x^2} \cdot 1}{3x^2}\,dx = 3\int x^{-4}\,dx = -x^{-3}$

$y_p = (-3\ln x) \cdot 1 - x^{-3}(x^3)$

$= -3\ln x - 1$

$$\boxed{y_p = -3\ln x - 1}$$

* Since $-1$ is part of $y_c$, we could say

$y_p = -3\ln x$

**5.** Find the general solution of the homogeneous ODE.

$y'' + 8y' + 17y = 0$ Characteristic equation

$m^2 + 8m + 17 = 0$

$m^2 + 8m + 16 = -1$

$(m+4)^2 = -1 \Rightarrow m+4 = \pm i$

$m = -4 \pm i$

$y_1 = e^{-4x} \cos x \ , \ y_2 = e^{-4x} \sin x$

$$\boxed{y = c_1 e^{-4x} \cos x + c_2 e^{-4x} \sin x}$$

**6.** Find the **form** of the general solution of the nonhomogeneous ODE when using the method of undetermined coefficients. Don't bother finding the coefficients, $A$, $B$, etc.

$y''+8y'+17y = 4e^{-4x}\cos(x)+3x$ $y_c$ is in problem 5 above

Let $g_1(x) = 4e^{-4x} \cos x$

$y_{p_1} = (A e^{-4x} \cos x + B e^{-4x} \sin x) . x$

$$\boxed{y_{p_1} = Axe^{-4x} \cos x + Bx e^{-4x} \sin x}$$

Let $g_2(x) = 3x$

$$\boxed{y_{p_2} = Cx + D}$$

$$\boxed{y_p = y_{p_1} + y_{p_2}}$$